

# Interpretation of Angular Distributions of Z-boson Production at Colliders

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Based on the paper of JCP, Wen-Chen Chang, Evan McClellan, Oleg Teryaev, Phys. Lett. B758 (2016) 384, PRD 96 (2017) 054020, and preprints

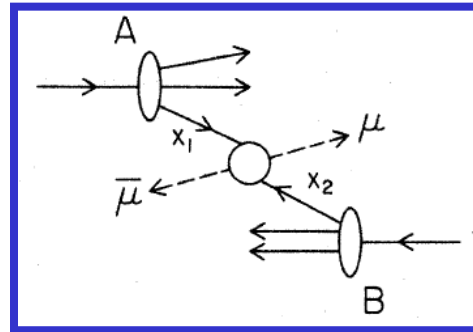
# Angular Distribution in the “Naïve” Drell-Yan

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES\*

Sidney D. Drell and Tung-Mow Yan

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 May 1970)

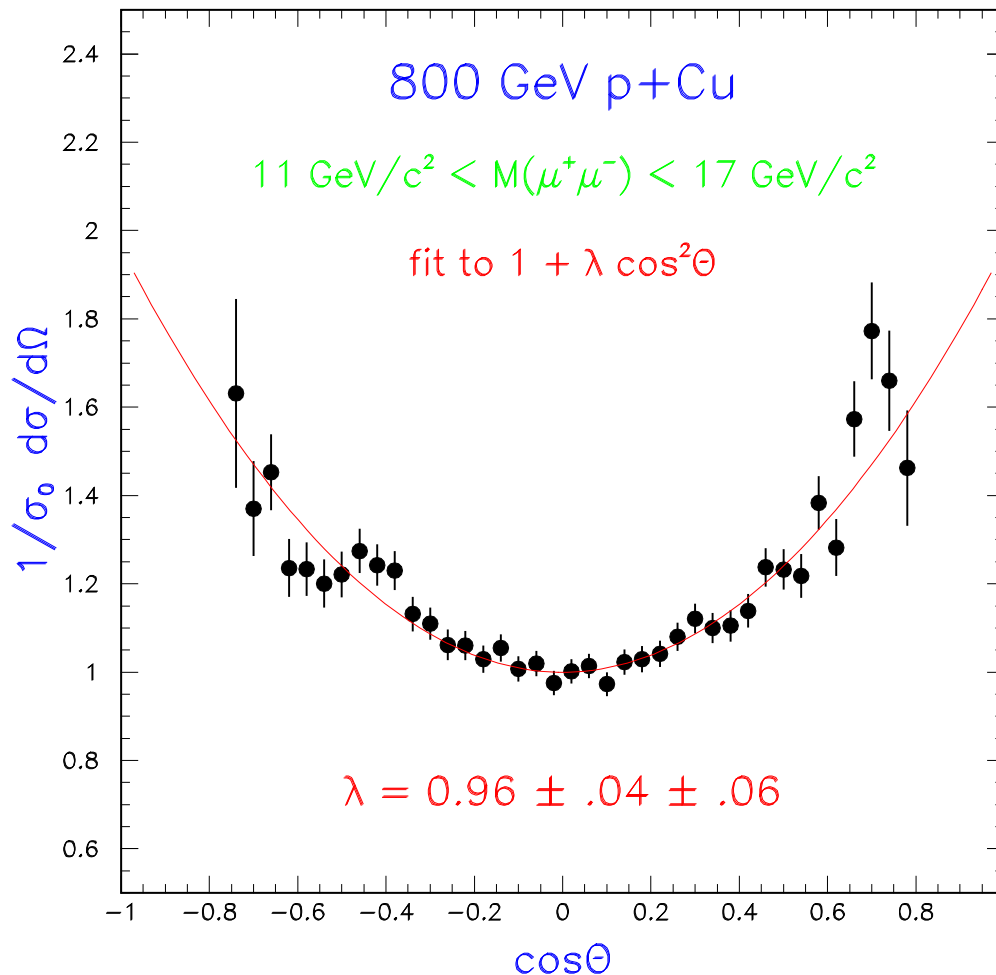


(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin- $\frac{1}{2}$  parton-antiparton pairs. This means a distribution in the di-muon rest system varying as  $(1 + \cos^2\theta)$  rather than  $\sin^2\theta$  as found in Sakurai's<sup>10</sup> vector-dominance model, where  $\theta$  is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

# Drell-Yan angular distribution

Lepton Angular Distribution of “naive” Drell-Yan:

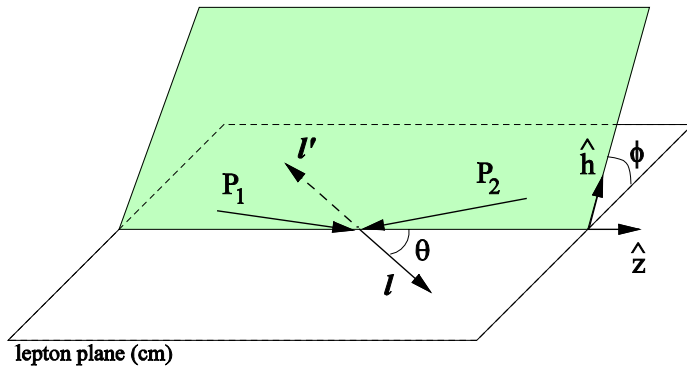
$$\frac{d\sigma}{d\Omega} = \sigma_0(1 + \lambda \cos^2 \theta); \quad \lambda = 1$$



Data from Fermilab  
E772

(Ann. Rev. Nucl. Part.  
Sci. 49 (1999) 217-253)

# Drell-Yan lepton angular distributions



$\Theta$  and  $\Phi$  are the decay polar and azimuthal angles of the  $\mu^-$  in the dilepton rest-frame

## Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]$$

**Lam-Tung relation:  $1 - \lambda = 2\nu$**

- Reflect the spin-1/2 nature of quarks  
(analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

# Decay angular distributions in pion-induced Drell-Yan

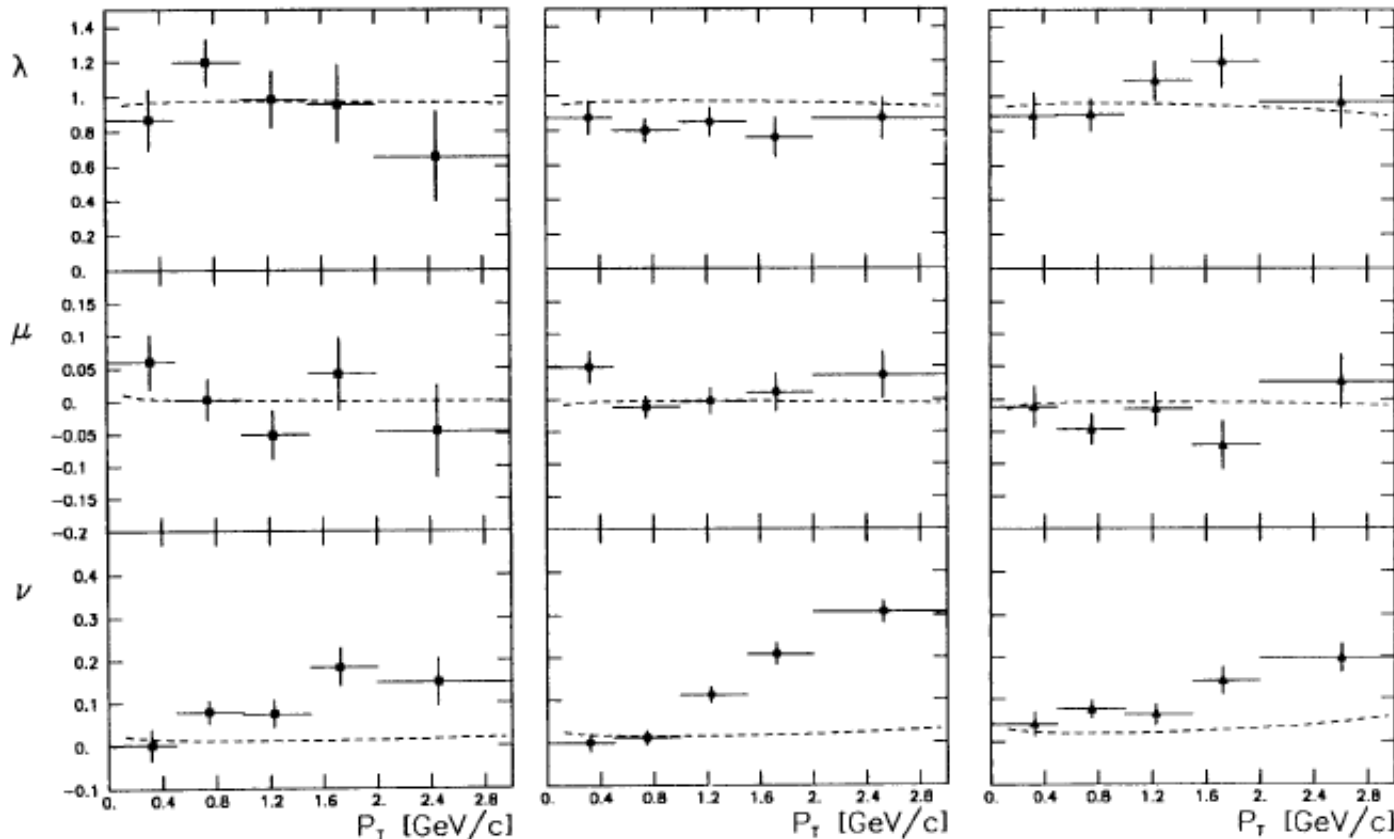
$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right] \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi\right]$$

140 GeV/c

194 GeV/c

286 GeV/c

NA10  $\pi^- + W$



Z. Phys.

37 (1988) 545

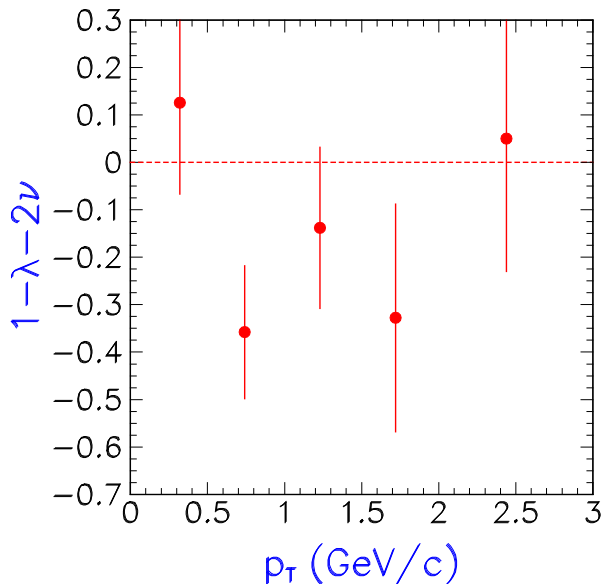
Dashed curves  
are from pQCD  
calculations

$\nu \neq 0$  and  $\nu$  increases with  $p_T$

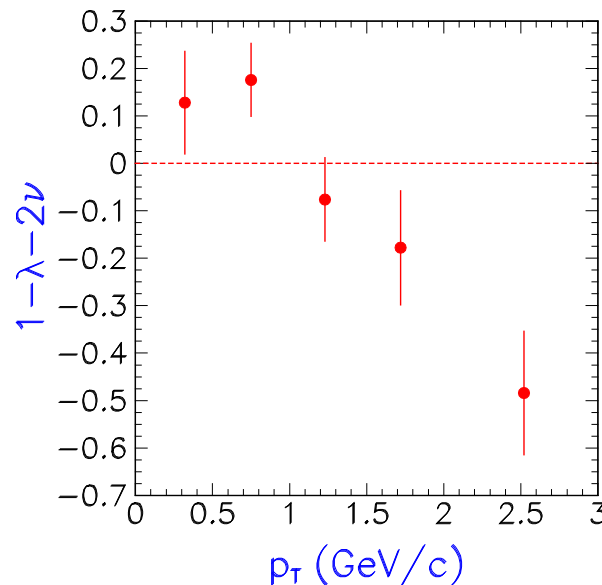
# Decay angular distributions in pion-induced Drell-Yan

Is the Lam-Tung relation ( $1-\lambda-2\nu=0$ ) violated?

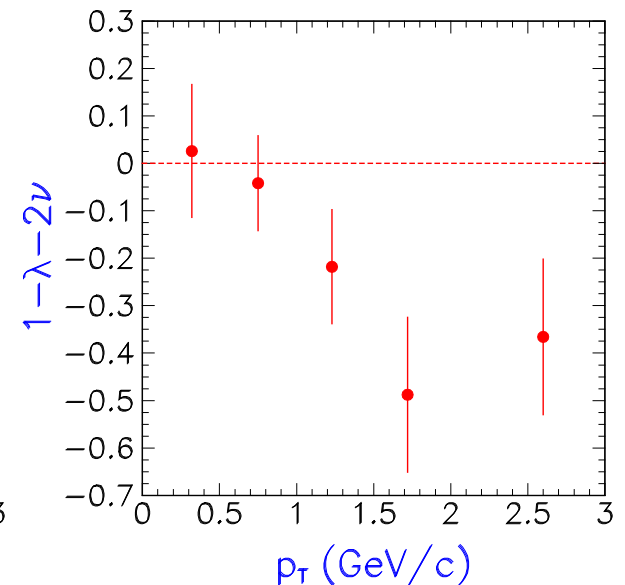
140 GeV/c



194 GeV/c



286 GeV/c



Data from NA10 (Z. Phys. 37 (1988) 545)

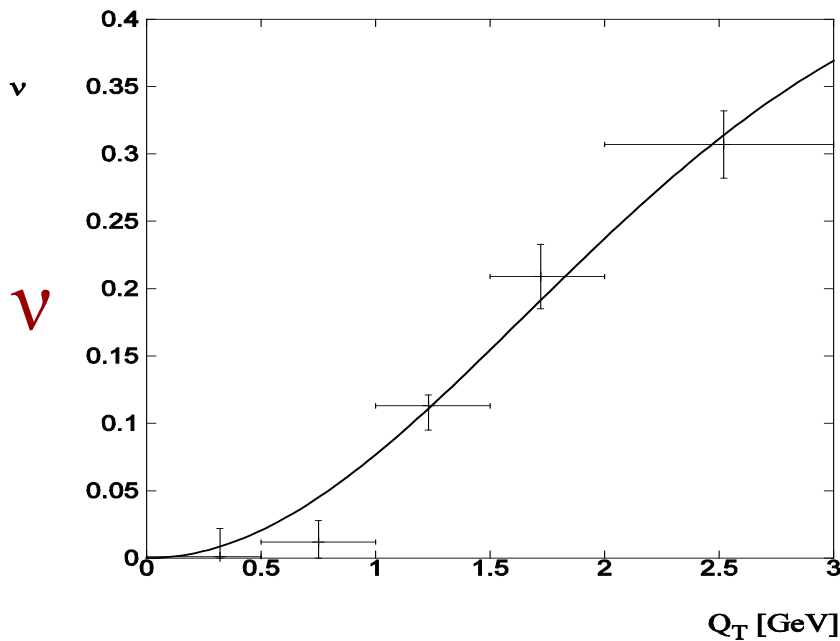
Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer, Vântinnen, Vogt, etc.)

# Boer-Mulders function $h_1^\perp$



- Boer pointed out that the  $\cos 2\phi$  dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^\perp$  can lead to an azimuthal dependence with  $v \propto \left(\frac{h_1^\perp}{f_1}\right) \left(\frac{\bar{h}_1^\perp}{\bar{f}_1}\right)$



$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$v = 16\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}$$

Boer, PRD 60 (1999) 014012

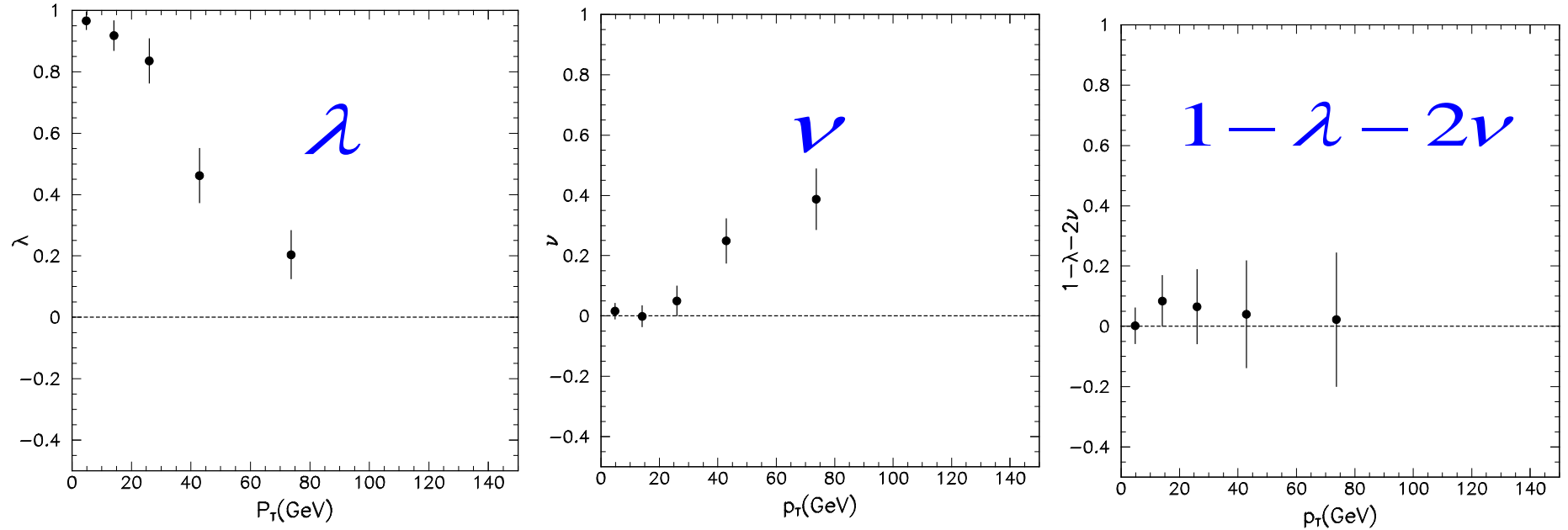
$$\kappa_1 = 0.47, M_C = 2.3 \text{ GeV}$$

$v > 0$  implies valence BM functions for pion and nucleon have same signs

# Lam-Tung relation from CDF Z-production

$$p + \bar{p} \rightarrow e^+ + e^- + X \text{ at } \sqrt{s} = 1.96 \text{ TeV}$$

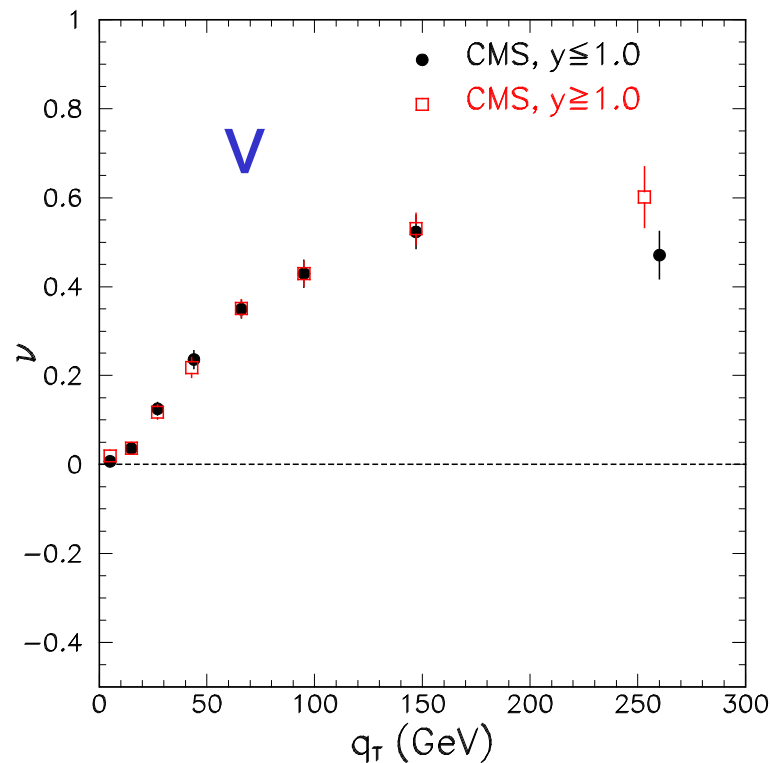
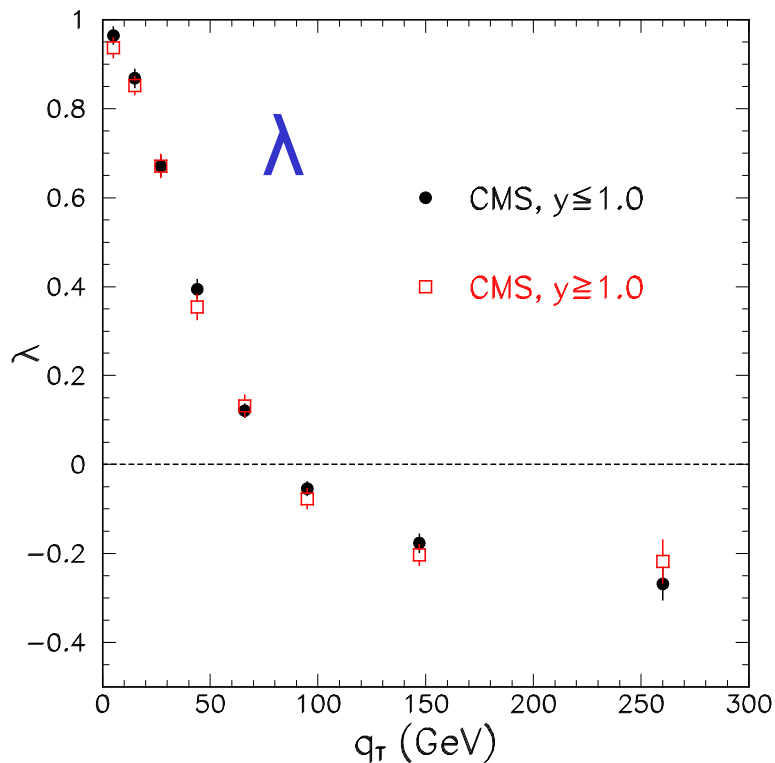
arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong  $p_T$  ( $q_T$ ) dependence of  $\lambda$  and  $\nu$
- Lam-Tung relation ( $1 - \lambda = 2\nu$ ) is satisfied within experimental uncertainties (TMD is not expected to be important at large  $p_T$ )



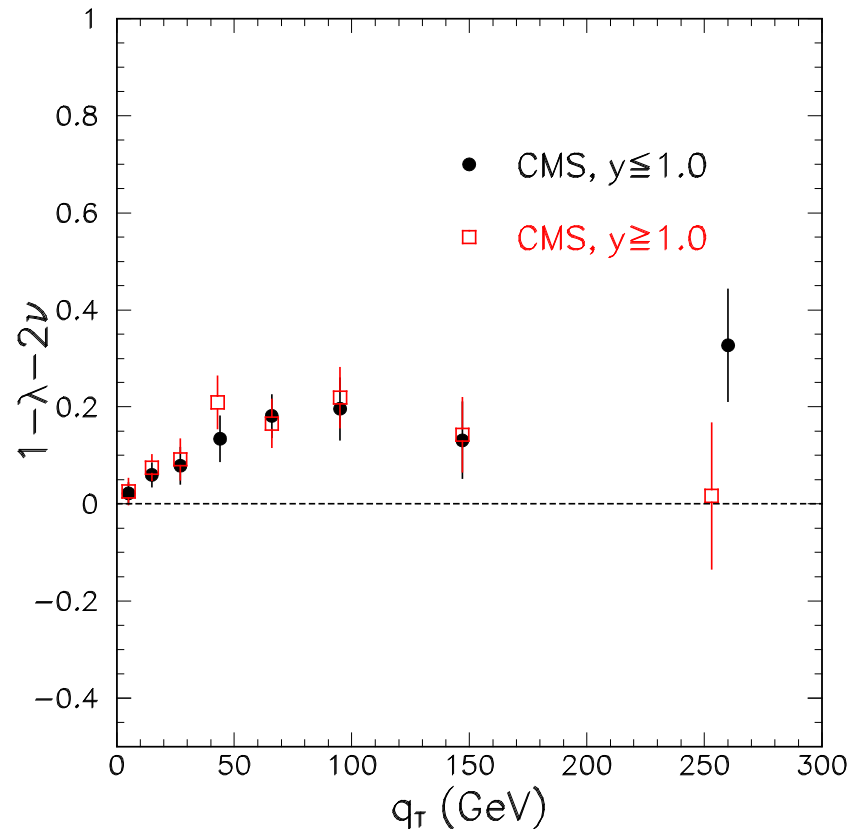
# Recent CMS (ATLAS) data for Z-boson production in $p+p$ collision at 8 TeV



(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking  $q_T$  dependencies for  $\lambda$  and  $\nu$  were observed at two rapidity regions
- Is Lam-Tung relation violated?

# Recent data from CMS for Z-boson production in $p+p$ collision at 8 TeV



- Yes, the Lam-Tung relation is violated ( $1 - \lambda > 2\nu$ )!
- Can one understand the origin of the violation of the Lam-Tung relation?

# Interpretation of the CMS Z-production results

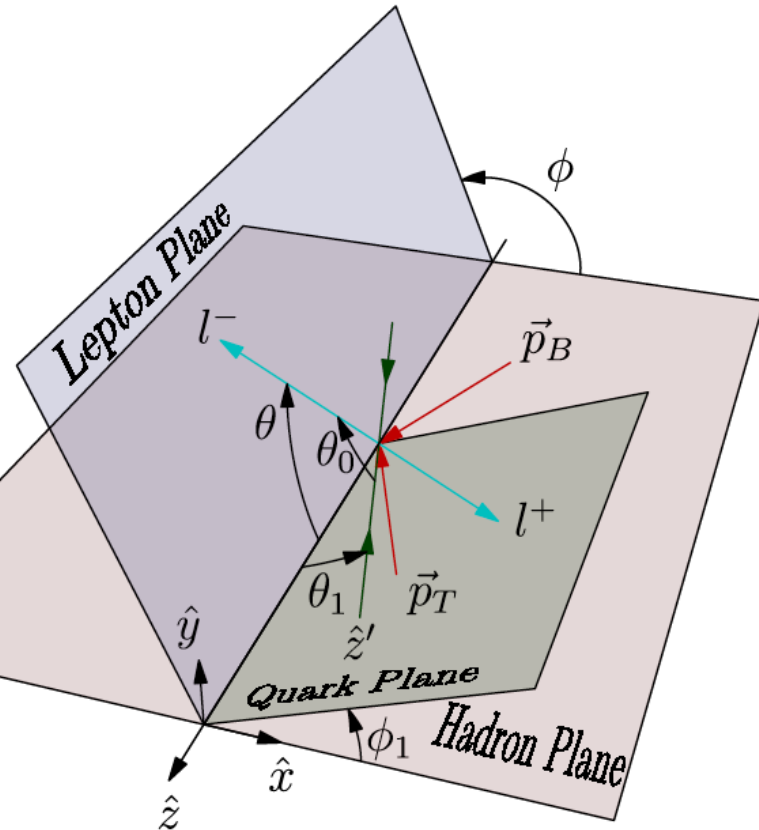
$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi\end{aligned}$$

## Questions:

- How is the above expression derived?
- Can one express  $A_0 - A_7$  in terms of some quantities?
- Can one understand the  $q_T$  dependence of  $A_0, A_1, A_2$ , etc?
- Can one understand the origin of the violation of Lam-Tung relation?

# How is the angular distribution expression derived?

## Define three planes in the Collins-Soper frame



### 1) Hadron Plane

- Contains the beam  $\vec{P}_B$  and target  $\vec{P}_T$  momenta
- Angle  $\beta$  satisfies the relation  $\tan \beta = q_T / Q$

### 2) Quark Plane

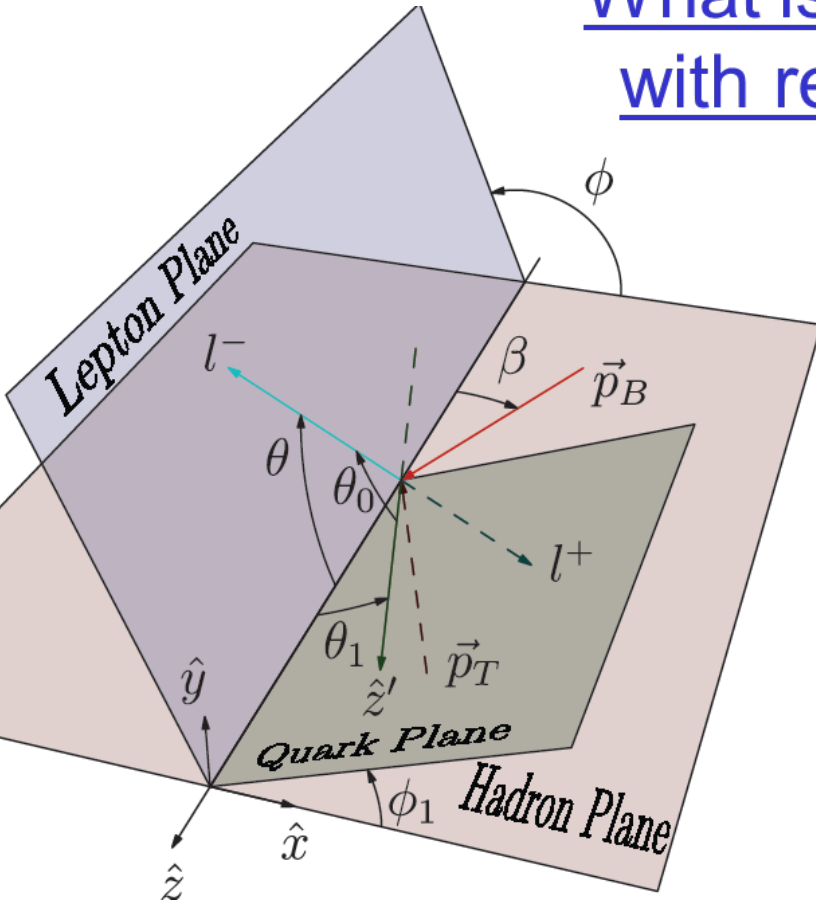
- $q$  and  $\bar{q}$  have head-on collision along the  $\hat{z}'$  axis
- $\hat{z}'$  axis has angles  $\theta_1$  and  $\phi_1$  in the C-S frame

### 3) Lepton Plane

- $l^-$  and  $l^+$  are emitted back-to-back with equal  $|\vec{P}|$
- $l^-$  and  $\hat{z}$  form the lepton plane
- $l^-$  is emitted at angle  $\theta$  and  $\phi$  in the C-S frame

# How is the angular distribution expression derived?

What is the lepton angular distribution with respect to the  $\hat{z}'$  (natural) axis?



$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0$$

**Azimuthally symmetric !**

How to express the angular distribution in terms of  $\theta$  and  $\phi$ ?

**Use the following relation:**

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos(\phi - \phi_1)$$



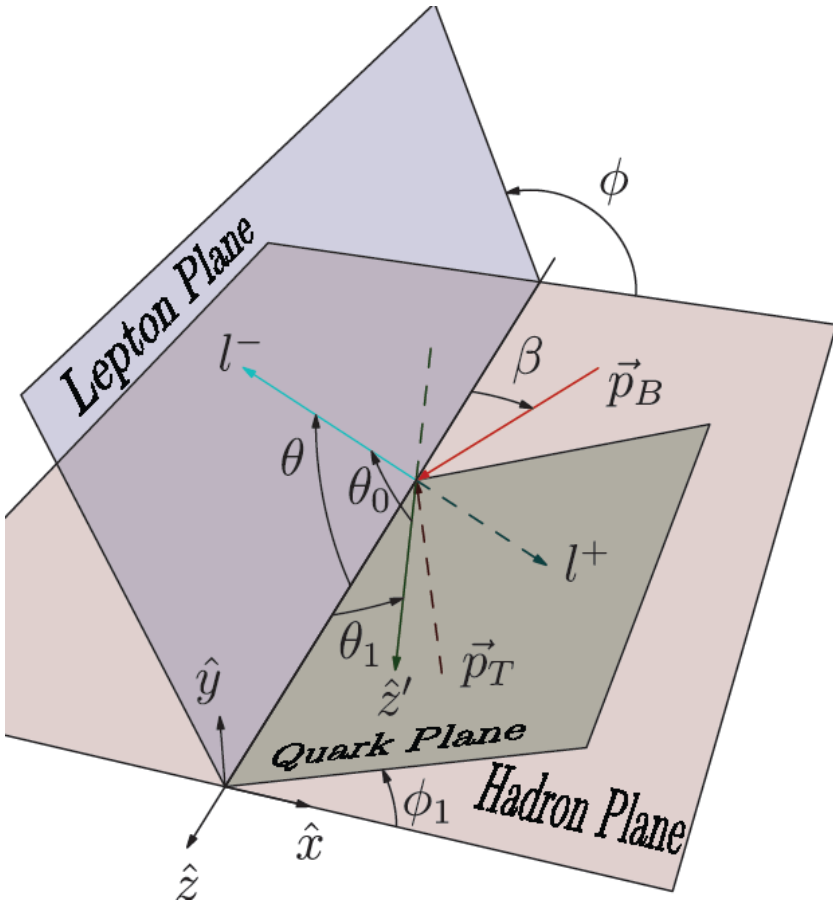
# All eight angular distribution terms are obtained!

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{\sin^2 \theta_1}{2} (1 - 3 \cos^2 \theta) \\ & + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1\right) \sin 2\theta \cos \phi \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1\right) \sin^2 \theta \cos 2\phi \\ & + (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\ & + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1\right) \sin^2 \theta \sin 2\phi \\ & + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1\right) \sin 2\theta \sin \phi \\ & + (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega} \propto & (1 + \cos^2 \theta) + \frac{A_0}{2} (1 - 3 \cos^2 \theta) \\ & + A_1 \sin 2\theta \cos \phi \\ & + \frac{A_2}{2} \sin^2 \theta \cos 2\phi \\ & + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\ & + A_5 \sin^2 \theta \sin 2\phi \\ & + A_6 \sin 2\theta \sin \phi \\ & + A_7 \sin \theta \sin \phi\end{aligned}$$

$A_0 - A_7$  are entirely described by  $\theta_1$ ,  $\phi_1$  and  $a$

# Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$



# Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

- $A_0 \geq A_2$  (or  $1 - \lambda - 2\nu \geq 0$ )
- Lam-Tung relation ( $A_0 = A_2$ ) is satisfied when  $\phi_1 = 0$
- Forward-backward asymmetry,  $a$ , is reduced by a factor of  $\langle \cos \theta_1 \rangle$  for  $A_4$
- $A_5, A_6, A_7$  are odd function of  $\phi_1$  and must vanish from symmetry consideration
- Some equality and inequality relations among  $A_0 - A_7$  can be obtained

# Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

$$-1 < A_2 < 1$$

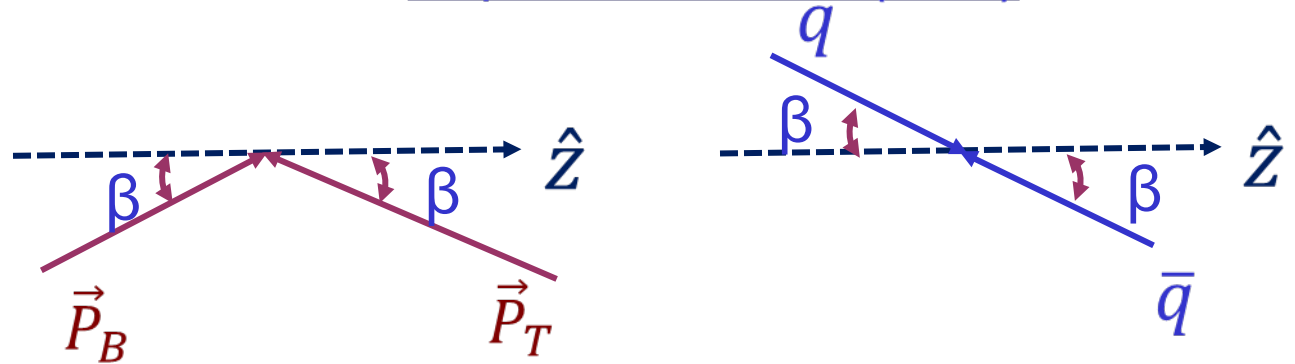
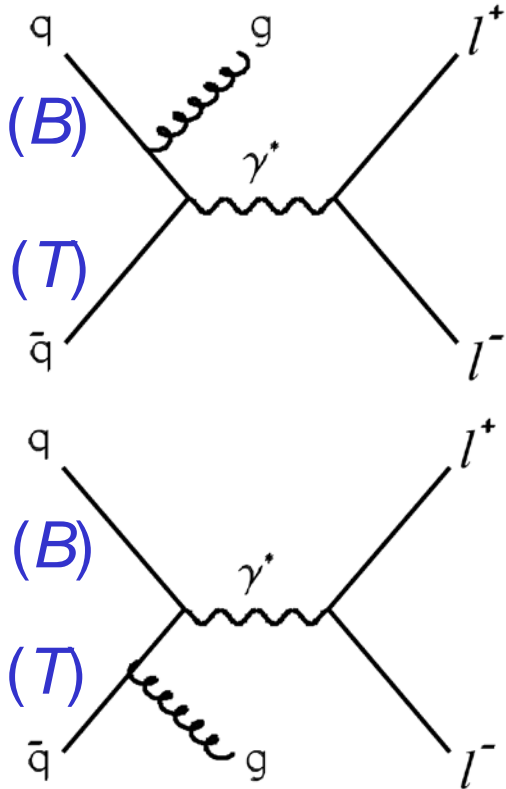
$$-a < A_3 < a$$

$$-a < A_4 < a$$

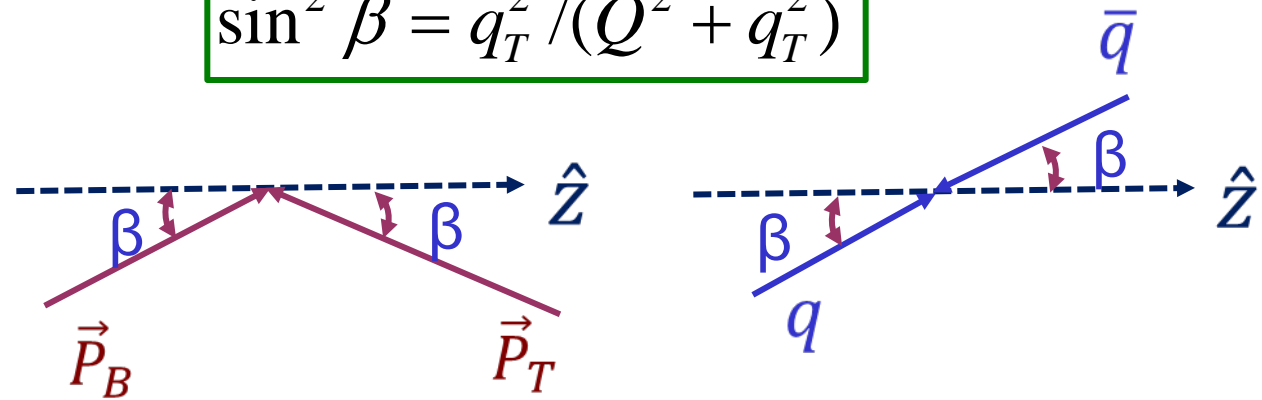
# What are the values of $\theta_1$ and $\phi_1$ at order $\alpha_s$ ?

1)  $q\bar{q} \rightarrow \gamma^*(Z^0)g$

In  $\gamma^*$  rest frame (C-S)



$$\sin^2 \beta = q_T^2 / (Q^2 + q_T^2)$$

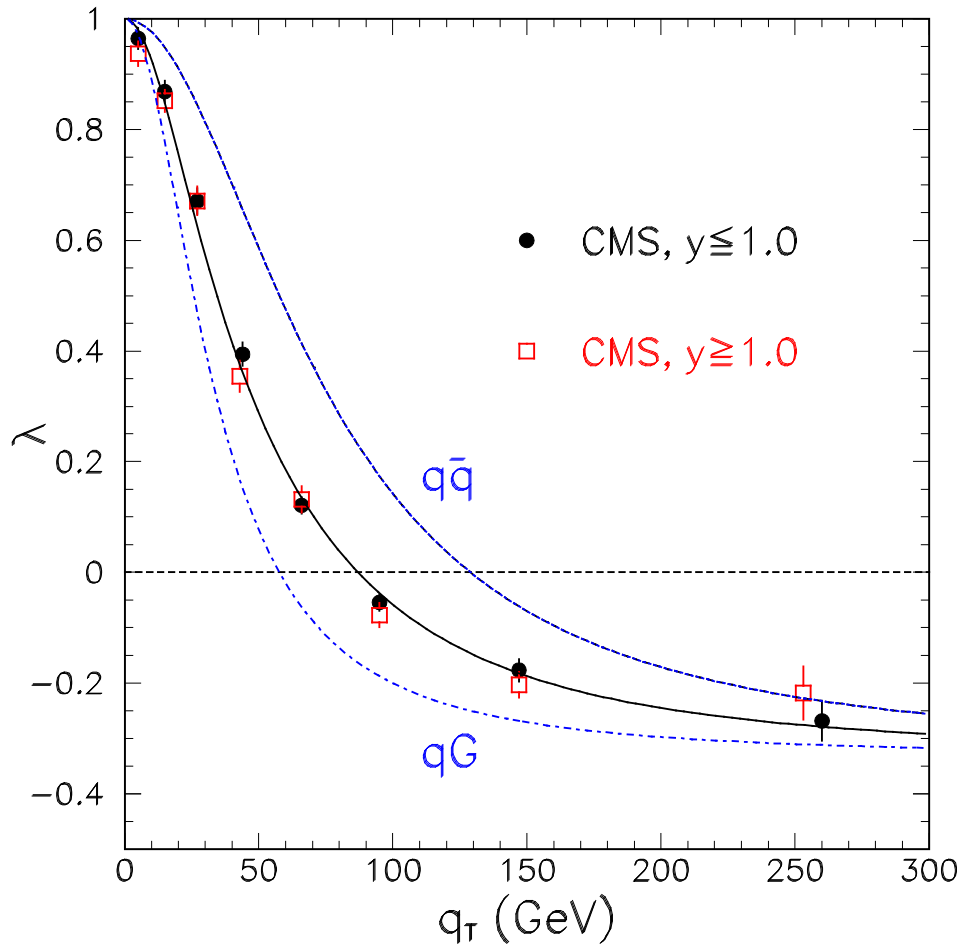


$$\theta_1 = \beta \text{ and } \phi_1 = 0; \quad A_0 = A_2 = \sin^2 \beta$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \quad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

# Compare with CMS data on $\lambda$

(Z production in  $p+p$  collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for } q\bar{q} \rightarrow Zg$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for } qG \rightarrow Zq$$

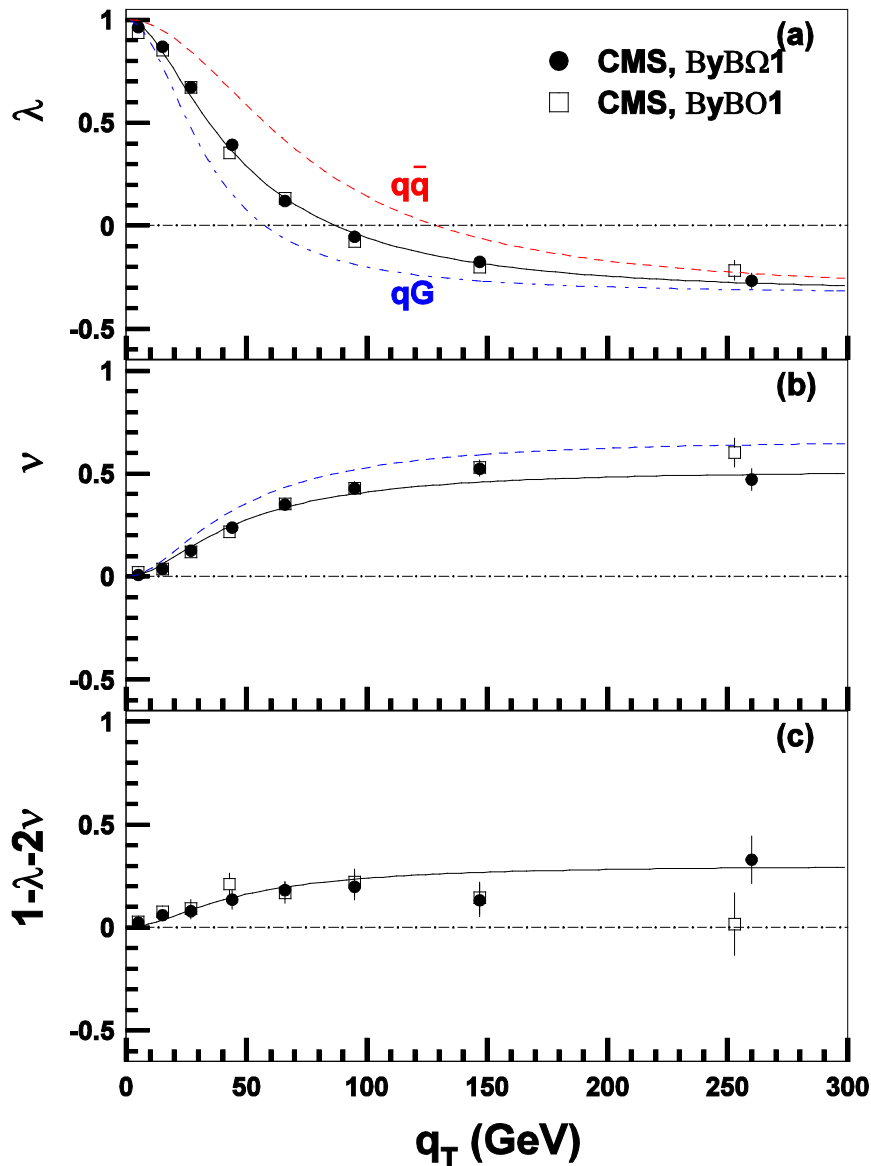
For both processes

$$\lambda = 1 \text{ at } q_T = 0 \quad (\theta_1 = 0^\circ)$$

$$\lambda = -1/3 \text{ at } q_T = \infty \quad (\theta_1 = 90^\circ)$$

Data can be well described  
 with a mixture of 58.5%  $qG$   
 and 41.5%  $q\bar{q}$  processes

# Compare with CMS data on Lam-Tung relation



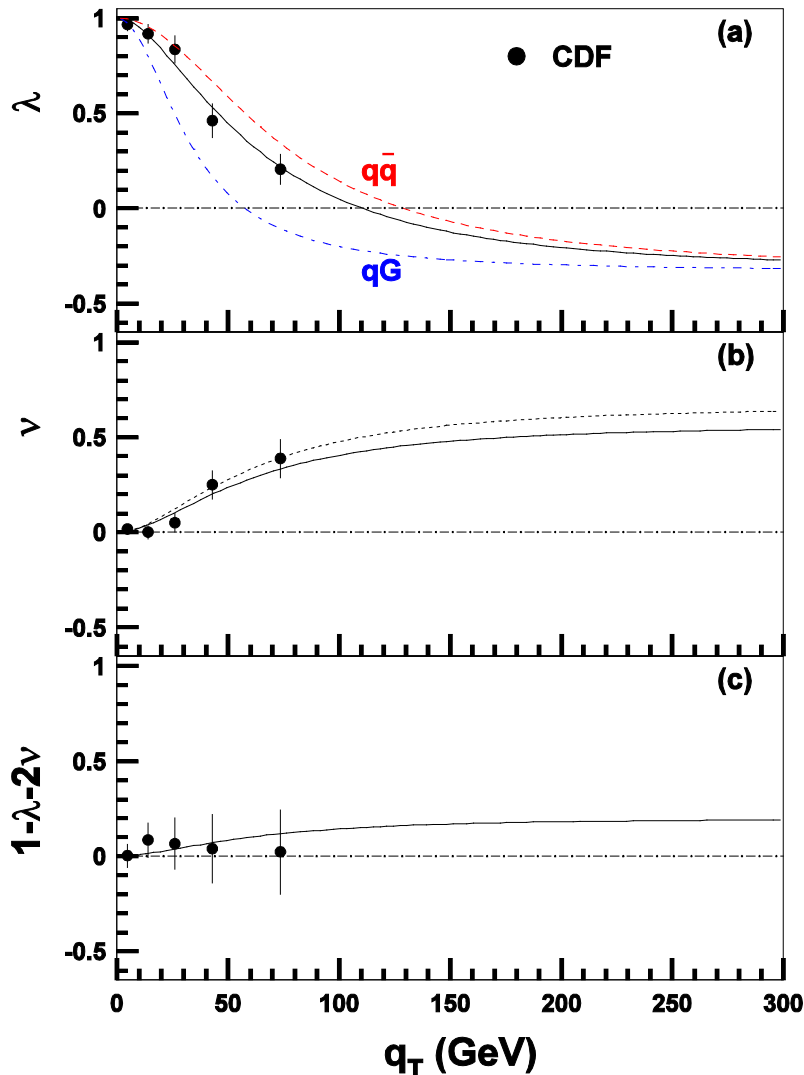
Solid curves correspond to a mixture of 58.5%  $qG$  and 41.5%  $q\bar{q}$  processes, and

$$\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.77$$

Violation of Lam-Tung relation is well described

# Compare with CDF data

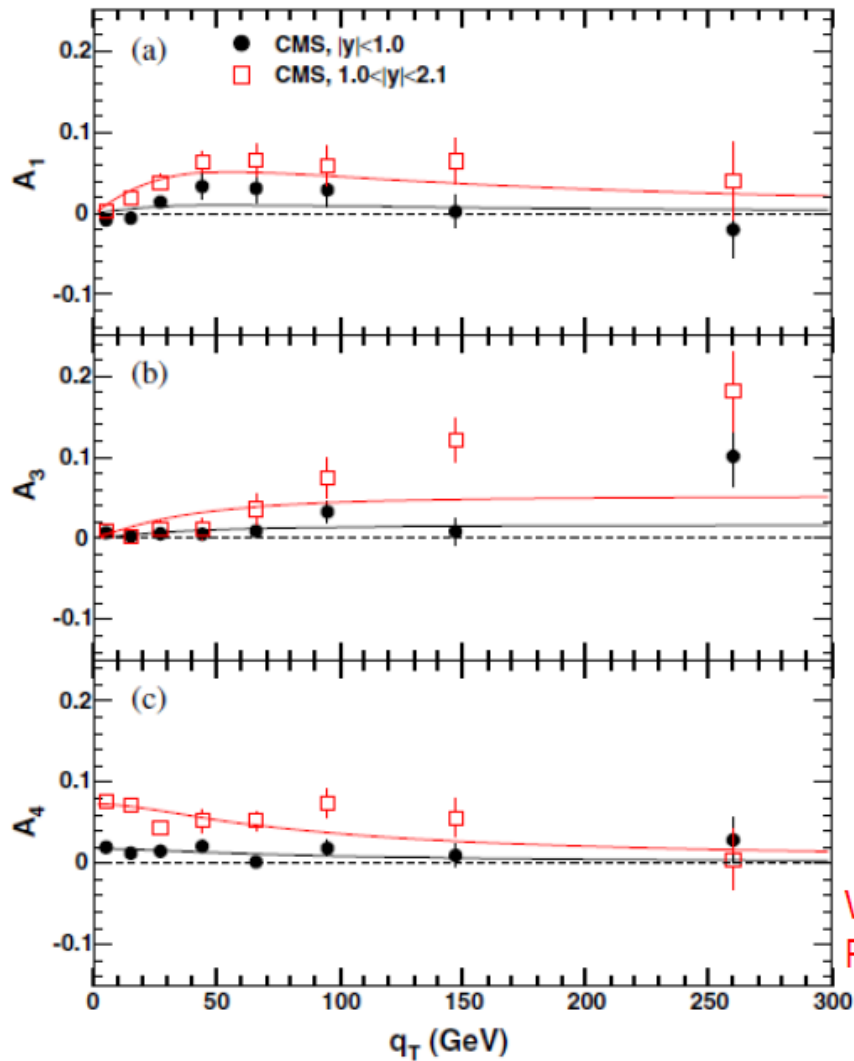
(Z production in  $p + \bar{p}$  collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5%  $qG$  and 72.5%  $q\bar{q}$  processes, and  $\langle \sin^2 \theta_1 \cos 2\phi_1 \rangle / \langle \sin^2 \theta_1 \rangle = 0.85$

Violation of Lam-Tung relation is not ruled out

# Compare with CMS data on $A_1$ , $A_3$ and $A_4$



$$A_1 = r_1 \left[ f \frac{q_T Q}{Q^2 + q_T^2} + (1-f) \frac{\sqrt{5} q_T Q}{Q^2 + 5q_T^2} \right]$$

$$A_3 = r_3 \left[ f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5q_T^2}} \right]$$

$$A_4 = r_4 \left[ f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1-f) \frac{Q}{\sqrt{Q^2 + 5q_T^2}} \right]$$

Rapidity of  $A_1$ ,  $A_3$  and  $A_4$   
are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev  
Phys. Rev. D 96, 054020 (2017)

# Future prospects

- Extend this study to W-boson production
  - Preliminary results show that the data can be well described
- Extend this study to fixed-target Drell-Yan data
  - Extraction of Boer-Mulders functions must take into account the QCD effects
- Extend this study to dihadron production in  $e^- e^+$  collision (inverse of the Drell-Yan)
  - Analogous angular distribution coefficients and analogous Lam-Tung relation