WHAT NEUTRINO WAVE PACKETS?
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We want to study how neutrino oscillations come about and when they cease to be. I’ll discuss only one aspect of this today.

What’s been said in the last 40+ years is fairly consistent, typically called the “neutrino wave packet picture”:

1. The moment a source particle decays, neutrino mass eigenstates are emitted.
2. Propagating neutrinos are described by wave packets.
3. These wave packets separate as they travel to the detector.
4. If the detector is sufficiently far away, oscillations can be damped because the wave packets stop overlapping.

First, I will illustrate this picture in more detail.

However, if we take one more step, this picture changes dramatically.
Say there is a pion at rest in the lab. The size of its wave function is $\Delta x$.

The wave function changes very little for times $t \ll M_\pi \Delta x^2$. 
The pion decays like $\pi \rightarrow \mu \nu_i$, where $i = 1, 2, 3$.

Only for simplicity, consider that $\pi \rightarrow e\nu$ is negligible, and there are only $\nu_1$ and $\nu_2$. The neutrinos have different velocities, because they have different masses.
It’s common to think of the $\mu$ and $\nu$ as also having wave functions, called wave packets.

The straight lines denote the center of the wave packet, associated with its semi-classical trajectory.
If the pion decays at time $t$, there are two contributions to the pion’s decay amplitude:

Here, they’re drawn on top of each other.
A neutrino detector, on for a given time window, is located in some region in spacetime, at a spatial distance $L \gg \Delta x$ from the pion.

There is no detector for the charged lepton.
As $L$ becomes large, some neutrinos might “miss” the detector.

The neutrino wave packets may no longer sufficiently overlap at the detector, which dampens the interference necessary for oscillations, e.g., see Giunti (hep-ph/0105319).
Until this point, this has been a rough review of the salient features of the neutrino wave packet picture.

However, there is an extra ingredient: neutrino oscillation experiments do not measure at what time the source particle decays.

“Unperformed experiments have no results.”
- Asher Peres

We cannot assign a value to the decay time $t$ of the pion, in our example.
If the decay time $t$ is not measured, then one must sum over all $t$ to calculate the amplitude.

\[ A = \sum_t \left( \ldots \right) \]

The probability is $P = |A|^2$. Doing this calculation gives \ldots
...two dominant contributions to the amplitude for the detector to measure something.

\[ \mathcal{A} \sim \nu_1 + \nu_2 \]

We calculated this by treating the neutrinos as quantum fields and doing time-dependent perturbation theory.
In the calculation, one can see that the contributions to the amplitude from neutrinos which do not “hit” the detector are suppressed by powers of $1/L$. 

$$\mathcal{A} \sim \nu_1 + \nu_2$$
Drawing them on top of each other:

If the pion’s decay time is unmeasured, and its wave function does not change with time, there is no damping of neutrino oscillations, for any baseline $L$.

*This differs from the neutrino wave packet picture, where the pion decays in a region in time, even though it’s unmeasured.*
• This does not mean real-world neutrino oscillations never get damped. There are many mechanisms that can give rise to oscillation damping, e.g., anything that causes the sources’s quantum state to change with time. These effects are typically negligible for terrestrial neutrino experiments.

• Our final picture is the result of a calculation for a two-body decay using quantum fields for the neutrinos and performing time-dependent perturbation theory. This automatically takes care of the fact that the decay time of the parent particle is not measured, since only measured quantities are referred to.

• Our results would change if the charged lepton is measured. But that’s a different calculation. See, e.g., Cohen, Glashow, Ligeti (2009).
“The problem is that people think of particles as quantum mechanical systems but think of [detectors] as classical mechanical systems. If you’re willing to realize that both the particle and the [detector] are two interacting parts of one quantum mechanical system, then there’s no problem.”

- Sidney Coleman