A new theorem for lepton number conservation in seesaw models

arXiv:1712.07611

Cédric Weiland
with Kristian Moffat and Silvia Pascoli

Institute for Particle Physics Phenomenology, Durham University

Pheno 2018
University of Pittsburgh
08 May 2018
Massive neutrinos and New Physics

- Observation of $\nu$ oscillations
  - at least 2 $\nu$ are massive

- BSM necessary for $\nu$ mass
  - Radiative models
  - Extra-dimensions
  - R-parity violation in supersymmetry
  - Seesaw mechanisms

- 3 minimal tree-level seesaw models $\Rightarrow$ 3 types of heavy fields
  - type I: right-handed neutrinos, SM gauge singlets
  - type II: scalar triplets
  - type III: fermionic triplets

$$m_\nu = -\frac{1}{2} Y_\nu \frac{v^2}{M_R} Y^T_\nu$$

$$m_\nu = -2 Y_\Delta v^2 \frac{\mu_\Delta}{M^2_\Delta}$$

$$m_\nu = -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y^T_\Sigma$$
Towards testable Type I variants

- Taking $M_R \gg m_D$ gives the "vanilla" type 1 seesaw
  \[ m_{\nu} = -m_D M_R^{-1} m_D^T \]
  \[ m_{\nu} \sim 0.1 \text{ eV} \Rightarrow \begin{cases} Y_{\nu} \sim 1 & \text{and} & M_R \sim 10^{14} \text{ GeV} \\ Y_{\nu} \sim 10^{-6} & \text{and} & M_R \sim 10^2 \text{ GeV} \end{cases} \]

- $m_{\nu}$ suppressed by small active-sterile mixing $m_D/M_R$

- Cancellation in matrix product to get large $m_D/M_R$

- Lepton number, e.g. low-scale type I [Ilakovac and Pilaftsis, 1995] and others
  inverse seesaw [Mohapatra and Valle, 1986, Bernabéu et al., 1987]
  linear seesaw [Akhmedov et al., 1996, Barr, 2004, Malinsky et al., 2005]

- Flavour symmetry, e.g. $A_4 \times \mathbb{Z}_2$ [Chao et al., 2010]
  $A_4$ or $\Sigma(81)$ [Chattopadhyay and Patel, 2017]
  $\mathbb{Z}(3)$ [Gu et al., 2009]

- Gauge symmetry, e.g. $U(1)_{B-L}$ [Pati and Salam, 1974] and others

\[ m_{\nu} = 0 \text{ equivalent to conserved } L \text{ for models with 3 } \nu_R \]

or less of equal mass [Kersten and Smirnov, 2007]
Extending the Kersten-Smirnov theorem

- Can the result of Kersten and Smirnov be generalized?
- Are lepton number violating processes suppressed in all low-scale seesaw models?

**Theorem**

If:
- no cancellation between different orders of the seesaw expansion\(^a\)
- no cancellations between different radiative orders\(^b\)

Then \( m_\nu = 0 \) equivalent to having the neutrino mass matrix, in the basis 
\( \nu^C_L, \{\nu_R^{(1)} \ldots \nu_R^{(1)}\}, \{\nu_R^{(2)} \ldots \nu_R^{(2)}\}, \{\nu_R^{(3)} \ldots \nu_R^{(3)}\} \)

\[
\tilde{M} = 
\begin{pmatrix}
0 & \alpha & \pm i\alpha & 0 \\
\alpha^T & M_1 & 0 & 0 \\
\pm i\alpha^T & 0 & M_1 & 0 \\
0 & 0 & 0 & M_2 \\
\end{pmatrix},
\tag{1}
\]

for an arbitrary number of \( \nu_R \) and to all radiative orders, with \( M_1 \) and \( M_2 \) diagonal matrices with positive entries and \( \alpha \) a generic complex matrix.

\(^a\)This is a necessary requirement to satisfy phenomenological constraints
\(^b\)These are highly fine-tuned solution that cannot be achieved solely by specific textures of the neutrino mass matrix
Corollary on lepton number violation

Using a unitary matrix $D$, let us construct

$$Q = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \pm \frac{i}{\sqrt{2}}D & \frac{1}{\sqrt{2}}D & 0 \\
0 & \frac{1}{\sqrt{2}}D & \pm \frac{i}{\sqrt{2}}D & 0 \\
0 & 0 & 0 & 1\
\end{pmatrix}$$

then through a change of basis

$$Q^T \tilde{M} Q = \begin{pmatrix}
0 & \pm i \sqrt{2} (D^T \alpha^T)^T & 0 & 0 \\
\pm i \sqrt{2} D^T \alpha^T & 0 & \pm i D^T M_1 D & 0 \\
0 & \pm i D^T M_1 D & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \sim \begin{pmatrix}
0 & M_D^T & 0 & 0 \\
M_D & 0 & M_R & 0 \\
0 & M_R^T & 0 & 0 \\
0 & 0 & 0 & M_2 \\
\end{pmatrix}$$

- Similar to the L conserving limit of inverse and/or linear seesaw
- Explicitly L conserving taking the L assignment $(+1, -1, +1, 0)$

**Corollary**

The most general gauge-singlet neutrino extensions of the SM with no cancellation between different orders of the seesaw expansion, no fine-tuned cancellations between different radiative orders and which lead to three massless neutrinos are L conserving.
Eq. (1) as a sufficient condition

1. Directly obtained from the corollary

---

1In the seesaw limit, light neutrinos are Majorana fermions whose mass violate L conservation. Eq. (1) being equivalent to L conservation implies that the light neutrinos are massless.
Necessary condition: tree level

- At tree-level and for the first order of the seesaw expansion

\[ m_\nu \approx -m_DM_R^{-1}m_D^T \]

- If \( m_DM_R^{-1}m_D^T = 0 \) and using \( Z = M_R^{-1}m_D^T \), then the exact block-diagonalisation of the full neutrino mass matrix gives

\[
\begin{align*}
m_\nu &= - (1 + Z^*Z^T)^{-\frac{1}{2}} Z^Tm_D^T (1 + Z^TZ)^{-\frac{1}{2}} \\
&\quad \quad - (1 + Z^TZ^*)^{-\frac{1}{2}} m_DZ (1 + ZZ^*)^{-\frac{1}{2}} \\
&\quad \quad + (1 + Z^*Z^T)^{-\frac{1}{2}} Z^TM_RZ (1 + ZZ^*)^{-\frac{1}{2}}
\end{align*}
\]

- All terms contain \( m_DM_R^{-1}m_D^T \) thus

\[ m_\nu = 0 \Rightarrow m_DM_R^{-1}m_D^T = 0 \]

to all orders of the seesaw expansion
Necessary condition: one-loop level

- When \( m_\nu = 0 \) at tree-level, the one-loop induced masses are

\[
\delta m_{ij} = \Re \left[ \frac{\alpha_W}{16\pi^2 m_W^2} C_{ik} C_{jk} f (m_k) \right]
\]

with \( C \) the mixing matrix in the neutral current and Higgs couplings and \( f \) the loop function.

- In the basis where \( M_R \) is diagonal, the full neutrino mass matrix \( M \) is

\[
M = \begin{pmatrix}
0 & m_{D1} & \cdots & m_{Dn} \\
m_{D1}^T & \mu_1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
m_{Dn}^T & 0 & \cdots & \mu_n
\end{pmatrix}
\]

and at the first order in the seesaw expansion

\[
\delta m = 0 \Rightarrow \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f (\mu_i) = 0
\]
Necessary condition: one-loop level

- Cancellation could still come from summation of non-zero terms 😊
- But a rescaling \( M \rightarrow \Lambda M \) does not affect the condition \( m_\nu = \delta m = 0 \)

\( f(x) \) being monotonically increasing and strictly convex,

\[
\sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\mu_i) = 0 \rightarrow \Lambda^{-2} \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\Lambda \mu_i) = 0
\]

generate linearly independent equations from which

\[
m_\nu = 0 \Rightarrow m_{Di} m_{Di}^T = 0
\]

since \( \mu_i > 0, f(\mu_i) > 0 \)
Necessary condition: one-loop level

- From a bit of algebra and by excluding trivial solutions,

\[ m_{Di}m_{Di}^T = 0 \Rightarrow \]

\[ m_{Di} = \begin{pmatrix} 
  u''_1 & \pm iu''_1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
  v''_1 & \pm iv''_1 & v'''_3 & \pm iv'''_3 & 0 & 0 & 0 & \cdots & 0 \\
  w''_1 & \pm iw''_1 & w'''_3 & \pm iw'''_3 & w'''_5 & \pm iw'''_5 & 0 & \cdots & 0 
\end{pmatrix} \]

- By rearranging the columns and rows, flavour-basis mass matrix becomes

\[ M = \begin{pmatrix} 
  0 & \alpha & \pm i\alpha & 0 \\
  \alpha^T & M_1 & 0 & 0 \\
  \pm i\alpha^T & 0 & M_1 & 0 \\
  0 & 0 & 0 & M_2 
\end{pmatrix} = \tilde{M} \]
Conclusions

- Any symmetry that leads to massless light neutrinos contains L as a subgroup or an accidental symmetry.

- Spectrum in the L conserving limit: 3 massless light neutrinos + heavy Dirac neutrinos + decoupled neutrinos.

- Nearly conserved L is a cornerstone of low-scale type I seesaw variants.

- Smallness of the light neutrino mass related to the smallness of the L breaking parameter, or equivalently to the degeneracy of the heavy neutrinos in pseudo-Dirac pairs in low-scale type I seesaw variants.

- Expect L violating signatures to be suppressed.
  → Needs to be quantitatively assessed.

- Seems to be applicable to type III seesaw variants as well.
  → Currently investigating it.
Backup slides
Cancellation between different seesaw orders

- To second order in the expansion

\[ m_{\nu}^{(2)} = -m_{\nu}^{(1)} + \frac{1}{2} \left( m_{\nu}^{(1)} u \theta + \theta^T m_{\nu}^{(1)} \right) \]

with \( m_{\nu}^{(1)} \) the first order expression and \( \theta \) is \( Z^\dagger Z \) up to a unitary transformation

- Then

\[ (m_{\nu}^{(2)})_{ii} = 0 \leftrightarrow -\hat{m}_{lii}^{(1)} + \hat{m}_{lii}^{(1)} \theta_{ii} = 0 \]

and \( \theta_{ii} = 1 \)

- This contradicts [Fernandez-Martinez et al., 2016] which gives \( ||\theta|| \leq 0.0075 \)
An aside on the Kersten-Smirnov theorem

- Using tree-level contributions (\( m_\nu = 0 \Leftrightarrow m_D M_R^{-1} m_D^T = 0 \)), they get the general result if \( \#\nu_R \leq 3 \)

\[
m_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ ay_1 & ay_2 & ay_3 \\ by_1 & by_2 & by_3 \end{pmatrix}, \quad \text{and} \quad \frac{y_1^2}{M_{R,1}} = \frac{y_2^2}{M_{R,2}} = \frac{y_3^2}{M_{R,3}}
\]

- For \( \#\nu_R > 3 \), the system of linear equations in their proof is under-constrained.

- In general, no symmetry is present. Necessary to assume degenerate heavy neutrinos to make a statement.

- Justify this by requiring radiative stability but approach based on running of the Weinberg operator
  → Works only if Higgs boson lighter than all heavy neutrinos.
Details of one-loop proof I

The loop function is

\[ f(m_k) = m_k \left( 3m_Z^2 g_{kZ} + m_H^2 g_{kH} \right) \]

where

\[ g_{ab} = \frac{m_a^2}{m_a^2 - m_b^2} \log \frac{m_a^2}{m_b^2} \]

which gives

\[ U_i^T (1 + Z^T Z^*)^{-1} Z^T U_h^* f_h U_h^\dagger Z (1 + Z^\dagger Z)^{-1} U_i = 0 \]

\[ Z^T U_h^* f_h U_h^\dagger Z = 0 \]

to the first order in the seesaw expansion

\[ U_h \approx 1 \]

\[ Z^T F_h Z = 0 \]
Details of one-loop proof II

- We write $m^T_{Di} = (u^i, v^i, w^i)$, then
  
  $$m_{Di} m^T_{Di} = \begin{pmatrix}
  u^i u^i & u^i v^i & u^i w^i \\
  v^i u^i & v^i v^i & v^i w^i \\
  w^i u^i & w^i v^i & w^i w^i
  \end{pmatrix} = 0$$

- We construct $Y^i = u^i u^{iT} + u^i u^{i\dagger}$. Imposing $u^{iT} u^i = 0$ and excluding the trivial solution $u^i = 0$, rank$(Y^i) = 2$

- $Y^i$ symmetric and real: we can build a basis of real orthogonal eigenvectors $b^i_{1...ni}$.
  For the zero $n_i - 2$ eigenvalues,
  
  $$Y^i b^i_k = 0 \Rightarrow \|u^i\|^2 (u^{iT} b^i_k) = 0 \Rightarrow u^{iT} b^i_k = 0$$

- Then

  $$u^{i'} = R^{i'}_u u^i = \begin{pmatrix}
  b_1^{iT} u^i \\
  b_2^{iT} u^i \\
  b_3^{iT} u^i \\
  \vdots \\
  b_{n_i}^{iT} u^i
  \end{pmatrix} = \begin{pmatrix}
  u_1^{i'} \\
  u_2^{i'} \\
  0 \\
  \vdots \\
  0
  \end{pmatrix}$$
Details of one-loop proof III

- Once we have

\[ u^{i'} = \begin{pmatrix} u_1^{i'}, \pm iu_1^{i'}, 0, \ldots, 0 \end{pmatrix}^T \]

Under this transformation, we have

\[ u^{i'T}v^i = 0 \rightarrow u'^{iT}v'^i = 0 \]

leading us to conclude that

\[ v'^i = \begin{pmatrix} v_1'^i, \pm iv_1'^i, v_3'^i, v_4'^i, \ldots, v_n'^i \end{pmatrix}^T \]

- Similarly, we construct a second matrix \( R_v \) acting on \( \begin{pmatrix} v_3'^i, v_4'^i, \ldots, v_n'^i \end{pmatrix}^T \)

such that \( v'^i \) is reduced to

\[ v^{i''} = \begin{pmatrix} v_1^{i''}, \pm iv_1^{i''}, v_3^{i''}, \pm iv_3^{i''}, 0, \ldots, 0 \end{pmatrix}^T \]

- Rinse and repeat for \( w \)
Fine-tuning

We adopt here the idea of [Lopez-Pavon et al., 2015], where the tree-level and one-loop contributions cancel.

Evolution of $m_3$ as a function of the rescaling parameter $\Lambda$. Input masses and couplings where chosen to give $m_\nu = m_{\text{tree}} + m_{1\text{-loop}} = 0.046$ eV at $\Lambda = 1$. A deviation of less then $10^{-7}$ here, is enough to spoil the cancellation and contradict experimental limits.