Georgi-Machacek Model
Beyond Tree Level

Cheng-Wei Chiang
National Taiwan University

CWC, AL Kuo, K Yagyu, PLB774 (2017) 119 [1707.04176]
CWC, AL Kuo, K Yagyu, 1804.02633
An Extended Higgs Sector

• Compared to fermion and gauge sectors, the scalar sector is less explored experimentally.

• Other than usual symmetries, no guiding principles in constructing the scalar sector:
  ➤ representations of scalar bosons
  ➤ numbers of scalar bosons
  ➤ extra symmetries (continuous/discrete)
  ➤ required by new physics
    (neutrino mass, DM, EWBG, SUSY, etc)

Study of predictions and constraints of models with an extended Higgs sector

cf. 3 generations of fermions and 3 gauge interactions
Probing Higgs Sector

• Two ways to probe an extended Higgs sector:
  • Direct search: to discover additional Higgs bosons that contribute to EWSB; and
  • Indirect search: to find deviations from SM in observables/quantities related to the 125-GeV Higgs boson (h).

• So far, no additional Higgs boson has been discovered at the LHC.
  ➤ making the indirect search more appealing
  ➤ Higgs couplings at loop level
Motivations for GM Model

• With the introduction of a complex Higgs triplet field $\Delta$, one can give Majorana mass to LH neutrinos.

• Relevant Yukawa interactions:

$- h_{ij} \psi_{iL}^T C i\sigma_2 \Delta \psi_{jL} + h.c.$

$$M_\nu = \sqrt{2} h v_\Delta = h \frac{\mu_1 v_0^2}{m_2^2}$$

Type-II seesaw
Motivations for GM Model

- The model realizes the minimal Higgs sector containing isospin triplet fields while maintaining custodial symmetry.
- The most general Higgs potential allowed by gauge and Lorentz symmetries:

\[
V(\Phi, \Delta) = \frac{1}{2} m_1^2 \text{tr}[\Phi^\dagger \Phi] + \frac{1}{2} m_2^2 \text{tr}[\Delta^\dagger \Delta] + \lambda_1 (\text{tr}[\Phi^\dagger \Phi])^2 + \lambda_2 (\text{tr}[\Delta^\dagger \Delta])^2 + \lambda_3 (\text{tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \text{tr}[\Phi^\dagger \Phi] \text{tr}[\Delta^\dagger \Delta] + \lambda_5 \text{tr} \left[ \frac{\Phi^\dagger \sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] \text{tr} \left[ \Delta^\dagger T^a \Delta T^b \right]
\]

\[
\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\Phi^+ & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} (\chi^0)^* & \xi^+ & \chi^{++} \\ -(\chi^+) & \xi^0 & \chi^+ \\ (\chi^{++}) & -(\xi^+) & \chi^0 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}
\]

- Decoupling limit: \( m_2 \to \infty \)
- \( v_\Delta \) induced by \( v_\Phi \) through \( \mu_1 \)
Features of GM Model

• A large triplet VEV $v_\Delta$ is allowed.
  ➤ focus on $v_\Delta \sim O(\text{GeV})$ regime
**Vacuum Expectation Values**

- The VEV’s are subject to the constraint
  \[ v^2 = v^2_\phi + 8v^2_\Delta = \frac{1}{\sqrt{2}G_F} = (246 \text{ GeV})^2 \]
  with two mixing-angle definitions seen in the literature:
  \[ \tan \theta_H = \frac{2\sqrt{2}v_\Delta}{v_\phi} \text{ or } \tan \beta = \frac{v_\phi}{2\sqrt{2}v_\Delta} \]

- One could attribute EWSB entirely to \( v_\Delta \) (\( \simeq 87 \text{ GeV} \)) while keeping \( v_\phi = 0 \).
- Perturbativity of top Yukawa coupling demands \( v_\Delta \lesssim 80 \text{ GeV} \).

\( \Rightarrow \) other constraints later

---

Georgi, Machacek 1985  
Chanowitz, Golden 1985
Features of GM Model

• A large triplet VEV $v_{\Delta}$ is allowed.
  ➤ focus on $v_{\Delta} \sim O(\text{GeV})$ regime

• There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  ➤ providing a link between neutrino and LHC physics
Higgs Spectrum

\begin{align*}
\Delta: & \quad 3 \otimes 3 \\
\Phi: & \quad 2 \otimes 2
\end{align*}

\begin{align*}
H_5 \equiv & \begin{bmatrix} H_{5}^{++} \\ H_{5}^{+} \\ H_{5}^{0} \\ H_{5}^{-} \\ H_{5}^{--} \end{bmatrix} \\
H_3 \equiv & \begin{bmatrix} H_3^+ \\ H_3^0 \\ H_3^- \end{bmatrix} \\
\Phi_3 \equiv & \begin{bmatrix} w^+ \\ z^0 \\ w^- \end{bmatrix}
\end{align*}

\begin{align*}
SU(2)_L \otimes SU(2)_R \\
SU(2)_V
\end{align*}

\begin{align*}
5 \oplus 3 \oplus 1 \quad & 3 \oplus 1
\end{align*}

5 physical parameters to scan

\begin{align*}
\text{CP-even} \\
\text{CP-odd}
\end{align*}

\begin{align*}
\text{fermiophobic} \\
\text{gaugephobic}
\end{align*}

\begin{align*}
m_{H5} \\
m_{H3}
\end{align*}

\begin{align*}
\tan \beta = \frac{v_\phi}{2\sqrt{2}v_\Delta} \\
v^2 = v_\phi^2 + 8v_\Delta^2 + \frac{1}{\sqrt{2}G_F} = (246 \text{ GeV})^2
\end{align*}

Cheng-Wei Chiang, National Taiwan University
Features of GM Model

• A large triplet VEV $v_\Delta$ is allowed.
  ➤ focus on $v_\Delta \sim O(\text{GeV})$ regime

• There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  ➤ providing a link between neutrino and LHC physics

• The SM-like Higgs can possibly have stronger/weaker couplings with weak bosons (simplest model for this).
**Neutral Higgs Couplings**

- Coupling scale factors \((V = W,Z; F = \text{quarks})\) at tree level:

\[
\kappa_F = \frac{g_{\phi FF}}{g_{hFF}^{SM}}, \quad \kappa_V = \frac{g_{\phi VV}}{g_{hVV}^{SM}},
\]

\[
\tan \theta_H = \frac{2\sqrt{2}v_{\Delta}}{v_{\phi}} \quad \text{or} \quad \tan \beta = \frac{v_{\phi}}{2\sqrt{2}v_{\Delta}}
\]

depending only on \(\alpha\) and \(\beta\) (or \(v_{\Delta}\))

<table>
<thead>
<tr>
<th>Higgs</th>
<th>(\kappa_F)</th>
<th>(\kappa_V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h)</td>
<td>(\cos \alpha)</td>
<td>(\sin \beta \cos \alpha - \sqrt{\frac{8}{3}} \cos \beta \sin \alpha)</td>
</tr>
<tr>
<td>(\sin \beta)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sin \alpha)</td>
<td>(\cos \beta \sin \alpha)</td>
<td>(\sqrt{\frac{8}{3}} \cos \beta \cos \alpha)</td>
</tr>
<tr>
<td>(\sin \beta)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

group factor that makes it possible for the entire factor to be > 1 (mixing required)

suppressed by \(\alpha\)

blue: \(\kappa_V\)
red: \(\kappa_F\)
Features of GM Model

- A large triplet VEV $v_\Delta$ is allowed.
  - focus on $v_\Delta \sim O(\text{GeV})$ regime
- There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  - providing a link between neutrino and LHC physics
- The SM-like Higgs can possibly have stronger/weaker couplings with weak bosons (simplest model for this).
- There exists a $H_5^\pm W^\mp Z$ vertex at tree level through mixing and proportional to $v_\Delta$.
  - cf. loop-induced in models such as 2HDM
- The Higgs decay pattern is mainly controlled by $v_\Delta$ and mass hierarchy.
HIGGS DECAY PATTERN

\[ \Delta m < 0 \ (m_{H_5} > m_{H_3}) \]

\[ H_{5}^{++} \rightarrow H_{3}^{+}W^{+} , \ H_{5}^{+} \rightarrow H_{3}^{+}Z/H_{3}^{0}W^{+} , \ H_{5}^{0} \rightarrow H_{3}^{\pm}W^{\mp}/H_{3}^{0}Z \]
\[ H_{3}^{+} \rightarrow H_{1}^{0}W^{+} , \ H_{3}^{0} \rightarrow H_{1}^{0}Z \]

\[ \Delta m > 0 : \ (m_{H_3} > m_{H_5}) \]

\[ H_{5}^{++} \rightarrow W^{+}W^{+} \]
\[ H_{5}^{+} \rightarrow W^{+}Z \]
\[ H_{5}^{0} \rightarrow W^{+}W^{-}/ZZ \]
\[ H_{3}^{+} \rightarrow H_{5}^{+}W^{-}/H_{5}^{+}Z/H_{5}^{0}W^{+} \]
\[ H_{3}^{0} \rightarrow H_{5}^{\pm}W^{\mp}/H_{5}^{0}Z \]

\[ m_{H_5} > m_{H_3} \]
\[ m_{H_3} > m_{H_5} \]
\[ m_{H_3} > m_{H_5} \]

CWC, Yagyu JHEP 2012

\[ \Delta m > 0 : \ (m_{H_3} > m_{H_5}) \]

\[ H_{5}^{++} \rightarrow W^{+}W^{+} \]
\[ H_{5}^{+} \rightarrow W^{+}Z \]
\[ H_{5}^{0} \rightarrow W^{+}W^{-}/ZZ \]
\[ H_{3}^{+} \rightarrow H_{5}^{+}W^{-}/H_{5}^{+}Z/H_{5}^{0}W^{+} \]
\[ H_{3}^{0} \rightarrow H_{5}^{\pm}W^{\mp}/H_{5}^{0}Z \]

\[ H_{5}^{++} \rightarrow \ell^{+}\ell^{+} , \ H_{5}^{+} \rightarrow \ell^{+}\nu , \ H_{5}^{0} \rightarrow \nu\nu \]
\[ H_{3}^{+} \rightarrow \ell^{+}\nu , \ H_{3}^{0} \rightarrow \nu\nu .\]
Expected Coupling Precision

- All Higgs couplings will be determined by HL-LHC + ILC to $O(1)$ or sub percent level (particularly $hVV$ couplings).
  ➤ need to know radiative corrections
\textbf{Custodial Higgs Models}

- Couplings of $h$ are modified by exotic Higgs fields due to their EW charges and mixing with $\Phi_{SM}$.
- At tree level, their $hVV$ couplings satisfy
  \[ \kappa_W = \kappa_Z \]
- Three simplest custodial Higgs models:

<table>
<thead>
<tr>
<th>( \Phi_{\text{new}} )</th>
<th>rHSM</th>
<th>2HDM</th>
<th>GM model</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((m_h, v) = (125, 246)) GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha, m_S, \mu_S, \lambda_{\Phi S}, \lambda_S )</td>
<td>( m_H, m_A, m_{H^\pm}, \mu, \alpha, \tan \beta = v_2/v_1 )</td>
<td>( m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha, \tan \beta = v_\Phi/(2\sqrt{2}v_\Delta) )</td>
<td></td>
</tr>
<tr>
<td>( \kappa_{W,Z} = g_{hVV} / g_{hVV}^{\text{SM}} )</td>
<td>( \cos \alpha )</td>
<td>( \sin(\beta - \alpha) )</td>
<td>( \sin \beta \cos \alpha - \sqrt{\frac{8}{3}} \cos \beta \sin \alpha )</td>
</tr>
</tbody>
</table>

-\( \kappa_{W,Z} \) always \( \leq 1 \) through mixing
- Group factor that makes it possible for the entire factor to be \( > 1 \) (mixing required)
Radiative Corrections

• Radiative corrections can lead to at least two effects:
  • changes in the magnitudes of various couplings
  • deviations from tree-level relations among couplings
due to various custodial symmetry breaking parameters (couplings, masses).

<table>
<thead>
<tr>
<th>$\Phi^{\text{new}}$</th>
<th>rHSM</th>
<th>2HDM</th>
<th>GM model</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameters</td>
<td>$S$</td>
<td>$H, A, H^\pm$</td>
<td>$H_1, (H^0_3, H^\pm_3), (H^0_5, H^\pm_5, H^{\pm\pm}_5)$</td>
</tr>
<tr>
<td>$(m_h, v)$ = (125, 246) GeV</td>
<td>$\alpha, m_S, \mu_S, \lambda_{FS}, \lambda_S$</td>
<td>$m_H, m_A, m_{H^\pm}$, $\mu, \alpha, \tan \beta = v_2/v_1$</td>
<td>$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha$, $\tan \beta = v_\Phi/(2\sqrt{2}v_\Delta)$</td>
</tr>
<tr>
<td>$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\text{SM}}$</td>
<td>$\cos \alpha$</td>
<td>$\sin(\beta - \alpha)$</td>
<td>$\sin \beta \cos \alpha - \sqrt{\frac{8}{3}} \cos \beta \sin \alpha$</td>
</tr>
<tr>
<td>$\delta \kappa_V$</td>
<td>$- \sin \alpha \delta \alpha$</td>
<td>$\cos(\beta - \alpha)(\delta \beta - \delta \alpha)$</td>
<td>$\frac{\partial \kappa_V}{\partial \alpha} \delta \alpha + \frac{\partial \kappa_V}{\partial \beta} \delta \beta + \frac{\partial \kappa_V}{\partial \rho} \delta \rho$</td>
</tr>
</tbody>
</table>

• Want to know how big quantum corrections are.
Renormalization

- Independent **counter terms** in the model:

  **Gauge sector**

  \[
  m_W^2 \rightarrow m_W^2 + \delta m_W^2 ,
  \]

  \[
  m_Z^2 \rightarrow m_Z^2 + \delta m_Z^2 ,
  \]

  \[
  \alpha_{em} \rightarrow \alpha_{em} + \delta \alpha_{em} ,
  \]

  \[
  B_\mu \rightarrow \left(1 + \frac{1}{2} \delta Z_B \right) B_\mu ,
  \]

  \[
  W^a_\mu \rightarrow \left(1 + \frac{1}{2} \delta Z_W \right) W^a_\mu .
  \]

  **Scalar sector**

  \[
  m_X^2 \rightarrow m_X^2 + \delta m_X^2 ,
  \]

  \[
  (X = H_5, H_3, H_1, h)
  \]

  \[
  \mu_i \rightarrow \mu_i + \delta \mu_i , (i = 1, 2)
  \]

  \[
  v_\Delta \rightarrow v_\Delta + \delta v_\Delta ,
  \]

  \[
  \nu \rightarrow 0 + \delta \nu , \ (\nu = v_\xi - v_\chi)
  \]

  \[
  \alpha \rightarrow \alpha + \delta \alpha .
  \]
RENORMALIZATION

• In addition to the renormalization conditions in the SM to get physical $G_F$, $m_Z$, and $\alpha_{EM}$, the GM model allows one additional condition, which we take to make equal to zero or its experimental value.

\[
\alpha_{em} T = \frac{\Pi^{1PI}_{ZZ}(0) - \Pi^{1PI}_{ZZ}(0)_{SM}}{m_Z^2} - \frac{\Pi^{1PI}_{WW}(0) - \Pi^{1PI}_{WW}(0)_{SM}}{m_W^2}
\]

\[+ \delta \rho \quad \delta \rho = \frac{8 \nu \Delta \delta \nu}{v^2}\]

equal to zero or its experimental value.

• Use the on-shell scheme to fix other counter terms.
**hVV Couplings**

- In general, the renormalized $hV_\mu V_\nu$ vertices can be decomposed as

$$\hat{\Gamma}^{\mu\nu} = \hat{\Gamma}_1 g^{\mu\nu} + \hat{\Gamma}_2 p_1^\mu p_2^\nu + \hat{\Gamma}_3 \epsilon^{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma$$

where the last two form factors start to appear at 1-loop level from 1-particle irreducible (1PI) diagram contributions, while the **first form factor** of interest to us

$$\hat{\Gamma}_1 = \frac{2m_V^2}{\nu} \kappa_V + \Gamma^{1\text{PI}} + \delta \Gamma$$

has contributions from the **tree-level coupling**, **1PI diagrams**, and **counter terms**.
$\kappa_Z$ AND $\kappa_W$

- $hVV$ scale factors at 1-loop with momentum dependence are defined as:

$$\hat{\kappa}_V(p^2) \equiv \frac{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{NP}}{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{SM}}$$

- At 1σ, $\kappa_{W,Z}$ are (will be) determined to be

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\kappa_Z$</th>
<th>$\kappa_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC Run-I</td>
<td>[0.94, 1.13]</td>
<td>[0.78, 1.00]</td>
</tr>
<tr>
<td>HL-LHC</td>
<td>$\Delta \kappa_Z = 2 - 4%$</td>
<td>$\Delta \kappa_W = 2 - 5%$</td>
</tr>
<tr>
<td>ILC</td>
<td>$\Delta \kappa_Z = 0.58%$</td>
<td>$\Delta \kappa_W = 0.81%$</td>
</tr>
</tbody>
</table>

- Radiative corrections in SM:

$$\frac{g_{hVV}^{1-\text{loop}}}{g_{hVV}^{\text{tree}}} \simeq \begin{cases} -1.2 \ (\ +1.0 \) \% \ (hZZ) , \\ +0.4 \ (\ +1.3 \) \% \ (hWW) , \end{cases} \quad \text{for } \sqrt{p^2} = 250 \ (500) \ \text{GeV}$$

Cheng-Wei Chiang, National Taiwan University
1-LOOP RESULTS

- Lighter dots satisfy theoretical constraints (unitarity, stability, perturbativity, and oblique parameters [S and T]).
- Darker dots further satisfy Higgs data from LHC Run-I (20 channels).
- Other types of 2HDM are expected to have a similar result as 2HDM-I.
- It is possible to discriminate among the rHSM, 2HDMs and GM model.
1-LOOP RESULTS

• Same parameter sets in the two plots, only different in p.
• Green dots change little after imposing the Higgs data, while 2HDM and GM dots shrink significantly.
• GM prefers $\kappa_Z \in [0.88,1.12]$, while the others have $\kappa_Z \in [0.8,1.0]$.
• $\Delta \kappa_V$ may be observable.
• 250-GeV ILC is better than 500-GeV in distinguishing rHSM and 2HDM-I.
Distribution on $(\alpha, v_\Delta)$ Plane

\[ \hat{\kappa}_V (p_2^2) = \frac{\hat{\Gamma}^1_{hVV}(m^2_V, p_2^2, m^2_h)_{GM}}{\hat{\Gamma}^1_{hVV}(m^2_V, p_2^2, m^2_h)_{SM}} \]

|\Delta \hat{\kappa}_V| \equiv |\hat{\kappa}_W - \hat{\kappa}_Z|

- Larger values of $\kappa_Z$ are obtained in the upper-left region.
- The result does not change much when $p_2 = 500$ GeV.
- $|\Delta \kappa_V|$ does not depend much on $v_\Delta$ and $\alpha$. 
DISTRIBUTION ON \((\alpha, \nu_\Delta)\) PLANE

\[
\hat{\kappa}_{b, \tau} \equiv \frac{\hat{\Gamma}_{h\bar{b}b, h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\text{GM}}}{\hat{\Gamma}_{h\bar{b}b, h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\text{SM}}}
\]

\[
\hat{\kappa}_t \equiv \frac{\hat{\Gamma}_{h\bar{t}t}(m_t^2, (500 \text{ GeV})^2, m_h^2)_{\text{GM}}}{\hat{\Gamma}_{h\bar{t}t}(m_t^2, (500 \text{ GeV})^2, m_h^2)_{\text{SM}}}
\]

- Behaviors of \(\kappa_b\) (\(\kappa_T\)) and \(\kappa_t\) are virtually the same.
- In contrast to the \(\kappa_Z\) case, \(\kappa_b\) becomes smaller when \(|\alpha|\) becomes larger.
DISTRIBUTION ON (α, vΔ) PLANE

• While the variations from the SM predictions are typically ≲ 10% for κZ, κb and κt, the magnitude of the deviation in the hhh coupling, i.e., κh − 1, can be at a few 100% level.

\[ \hat{\kappa}_h \equiv \frac{\hat{\Gamma}_{hhh}(m_h^2, (500 \text{ GeV})^2, m_h^2)}{\hat{\Gamma}_{hhh}(m_h^2, (500 \text{ GeV})^2, m_h^2)}_{\text{GM}} \]

\[ \frac{\hat{\Gamma}_{hhh}(m_h^2, (500 \text{ GeV})^2, m_h^2)}{\hat{\Gamma}_{hhh}(m_h^2, (500 \text{ GeV})^2, m_h^2)}_{\text{SM}} \]

► large quantum corrections for hhh coupling

• In addition, κh does not depend much on α and vΔ as compared to others.
Some shifting between two plots.
Range of $\kappa_T$ gets more restricted for larger $\kappa_Z$.
At $\kappa_Z \approx 1.13$, $\kappa_T \approx 0.95$.

Most of $\kappa_h$ values are predicted to be 1 to 5, though sometimes of $O(10)$.
Possible range of $\kappa_h$ is restricted for larger $\kappa_Z$.
Some $\kappa_h$ are predicted less than 1 or even negative.

$\kappa_Z$ vs $\kappa_T$ / $\kappa_Z$ vs $\kappa_h$
Summary

- We have done 1-loop radiative corrections to the Higgs couplings in the GM model, a model featured in giving Majorana mass and possibly large $hVV$ couplings.
- Theoretical (unitarity, stability, perturbativity, and oblique parameters) and experimental (Higgs signal strengths) constraints have been imposed to find viable parameter spaces.
- We have presented numerical results for the $hVV$ couplings in the rHSM, 2HDM-I and GM models at 250-GeV and 500-GeV ILC, showing power to discriminate among the models.
- We have also presented the results of $hff$ and $hhh$ couplings in the GM model, showing their correlations among themselves and with $hVV$ and the momentum dependence of these couplings.
Thank You!
Backup Slides
Some Benchmark Points

<table>
<thead>
<tr>
<th>α [degree]</th>
<th>$v_\Delta$ [GeV]</th>
<th>$\mu_1$ [GeV]</th>
<th>$\mu_2$ [GeV]</th>
<th>$\hat{\kappa}_W$</th>
<th>$\hat{\kappa}_Z$</th>
<th>$\hat{\kappa}_t$</th>
<th>$\hat{\kappa}_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP1</td>
<td>$-7.0$</td>
<td>10</td>
<td>$-100.3$</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
<td>1.08</td>
</tr>
<tr>
<td>BP2</td>
<td>$-8.0$</td>
<td>10</td>
<td>7.1</td>
<td>2789.5</td>
<td>0.99</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>BP3</td>
<td>$-15.1$</td>
<td>20</td>
<td>$-180.0$</td>
<td>1.04</td>
<td>1.04</td>
<td>0.99</td>
<td>1.34</td>
</tr>
<tr>
<td>BP4</td>
<td>$-16.1$</td>
<td>20</td>
<td>18.7</td>
<td>1338.3</td>
<td>1.02</td>
<td>1.01</td>
<td>0.95</td>
</tr>
<tr>
<td>BP5</td>
<td>$-22.4$</td>
<td>30</td>
<td>$-325.2$</td>
<td>$-53.3$</td>
<td>1.09</td>
<td>1.09</td>
<td>0.98</td>
</tr>
<tr>
<td>BP6</td>
<td>$-24.9$</td>
<td>30</td>
<td>10.0</td>
<td>755.0</td>
<td>1.07</td>
<td>1.06</td>
<td>0.93</td>
</tr>
</tbody>
</table>

TABLE I: Six benchmark points allowed by the perturbative unitarity and the vacuum stability. The masses of the extra Higgs bosons are taken to be $m_{H_5} = m_{H_3} = m_{H_1} = 400$ GeV. All the other input parameters are shown in the first four columns. The numbers given in the latter four columns show the output of the renormalized scale factors at $\sqrt{p^2} = 250$ (500) GeV for $\hat{\kappa}_{W,Z}$ ($\hat{\kappa}_t,\hat{\kappa}_h$).

- BP1,3,5 (BP2,4,6) are chosen such that predictions of one-loop corrected scale factors are close to (far from) the tree-level predictions for $v_\Delta = 10, 20, 30$ GeV, respectively.
Momentum Dependence

FIG. 3: Renormalized scale factors $\hat{k}_{W,Z}$ (left), $\hat{k}_t$ (middle) and $\hat{k}_h$ (right) as functions of $\sqrt{p^2}$ for $m_{H_5} = m_{H_3} = m_{H_1} = 400$ GeV. The upper panels show the cases of BP1 (black), BP3 (blue) and BP5 (red), while the lower panels show the cases of BP2 (black), BP4 (blue) and BP6 (red).