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Georgi-Machacek Model Beyond Tree Level

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CWC, AL Kuo, K Yagyu, PLB774 (2017) 119 [1707.04176]
CWC, AL Kuo, K Yagyu, 1804.02633

An Extended Higgs Sector

- Compared to fermion and gauge sectors, the scalar sector is less explored experimentally.

- Other than usual symmetries, no guiding principles in constructing the scalar sector:
  - ➠ representations of scalar bosons
  - ➠ numbers of scalar bosons
  - ➠ extra symmetries (continuous/discrete)
  - ➠ required by new physics
    (neutrino mass, DM, EWBG, SUSY, etc)

cf. 3 generations of fermions and 3 gauge interactions

Study of predictions and constraints of models with an extended Higgs sector

Probing Higgs Sector

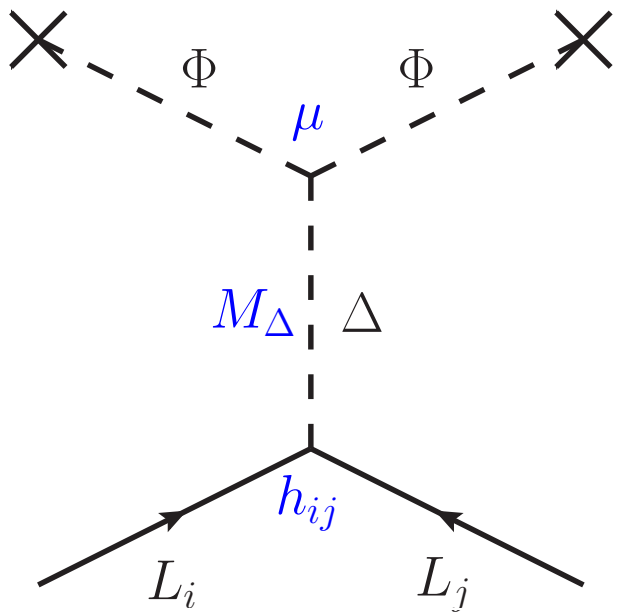
- Two ways to probe an extended Higgs sector:
  - Direct search: to discover additional Higgs bosons that contribute to EWSB; and
  - Indirect search: to find deviations from SM in observables/quantities related to the 125-GeV Higgs boson (h).

- So far, no additional Higgs boson has been discovered at the LHC.
  ➠ making the indirect search more appealing
  ➠ Higgs couplings at loop level

Motivations for GM Model

- With the introduction of a complex Higgs triplet field Δ, one can give Majorana mass to LH neutrinos.
- Relevant Yukawa interactions:

$$-h_{ij}\psi_{iL}^T C i\sigma_2 \Delta \psi_{jL} + \text{h.c.}$$



Type-II seesaw

$$M_\nu = \sqrt{2} h v_\Delta = h \frac{\mu_1 v_0^2}{m_2^2}$$

Motivations for GM Model

- The model realizes the minimal Higgs sector containing isospin triplet fields while maintaining custodial symmetry.
- The most general Higgs potential allowed by gauge and Lorentz symmetries:

assume $m_1^2 < 0$, but $m_2^2 > 0$

$$V(\Phi,\Delta) = \frac{1}{2}m_1^2 \mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2 \mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1(\mathrm{tr}[\Phi^\dagger\Phi])^2 + \lambda_2(\mathrm{tr}[\Delta^\dagger\Delta])^2$$

$$+ \lambda_3 \mathrm{tr}[(\Delta^\dagger\Delta)^2] + \lambda_4 \mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] + \lambda_5 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right] \mathrm{tr}\left[\Delta^\dagger T^a \Delta T^b\right]$$

$$+ \mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right] (P^\dagger \Delta P)_{ab} + \mu_2 \mathrm{tr}\left[\Delta^\dagger T^a \Delta T^b\right] (P^\dagger \Delta P)_{ab}$$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -(\phi^+)^* & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} (\chi^0)^* & \xi^+ & \chi^{++} \\ -(\chi^+)^* & \xi^0 & \chi^+ \\ (\chi^{++})^* & -(\xi^+)^* & \chi^0 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}$$

- Decoupling limit: $m_2 \to \infty$
- v_Δ induced by v_Φ through μ_1

Features of GM Model

- A large triplet VEV v_Δ is allowed.
  - ➠ focus on $v_\Delta \sim$ O(GeV) regime

Vacuum Expectation Values

- The VEV's are subject to the constraint

$$v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} = (246\ \mathrm{GeV})^2$$

with two mixing-angle definitions seen in the literature:

$$\tan\theta_H = \frac{2\sqrt{2}v_\Delta}{v_\phi} \ \text{ or } \ \tan\beta = \frac{v_\phi}{2\sqrt{2}v_\Delta}$$

- One could attribute EWSB entirely to v_Δ ($\simeq$ 87 GeV) while keeping $v_\phi = 0$.

Georgi, Machacek 1985
Chanowitz, Golden 1985

- Perturbativity of top Yukawa coupling demands $v_\Delta \lesssim 80$ GeV.

➠ other constraints later

Features of GM Model

- A large triplet VEV v_Δ is allowed.
  ➠ focus on $v_\Delta \sim O(\text{GeV})$ regime
- There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  ➠ providing a link between neutrino and LHC physics

Higgs Spectrum

1 complex triplet + 1 real triplet

1 complex doublet

$SU(2)_L \otimes SU(2)_R$

Δ: **3** $\otimes$ **3**

Φ: **2** $\otimes$ **2**

5 physical parameters to scan

$SU(2)_V$

5 $\oplus$ **3** $\oplus$ **1**

3 $\oplus$ **1**

mixing angle α

mixing angle β

$$H_5 \equiv \begin{bmatrix} H_5^{++} \\ H_5^{+} \\ H_5^{0} \\ H_5^{-} \\ H_5^{--} \end{bmatrix}$$

CP-even

m_{H5}

fermiophobic

$$H_3 \equiv \begin{bmatrix} H_3^{+} \\ H_3^{0} \\ H_3^{-} \end{bmatrix}$$

CP-odd

m_{H3}

gaugephobic

$$H_1 \equiv [H_1^0]$$

m_{H1}

h (125 GeV)

m_h

CP-even

$$\Phi_3 \equiv \begin{bmatrix} w^{+} \\ z^{0} \\ w^{-} \end{bmatrix}$$

$$\tan\beta = \frac{v_\phi}{2\sqrt{2}v_\Delta}$$

$$v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} = (246\ \text{GeV})^2$$

Features of GM Model

- A large triplet VEV v_Δ is allowed.
  ➠ focus on $v_\Delta \sim$ O(GeV) regime
- There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  ➠ providing a link between neutrino and LHC physics
- The SM-like Higgs can possibly have stronger/weaker couplings with weak bosons (simplest model for this).

Neutral Higgs Couplings

- Coupling scale factors (V = W,Z; F = quarks) at tree level:

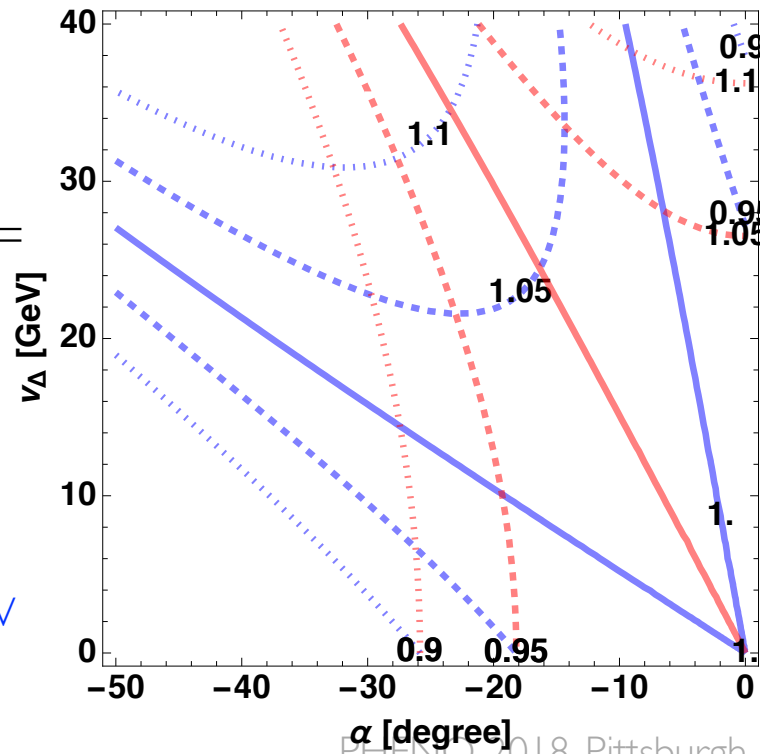
$$\kappa_F = \frac{g_{\varphi FF}}{g_{hFF}^{\rm SM}} \ , \ \kappa_V = \frac{g_{\varphi VV}}{g_{hVV}^{\rm SM}} \qquad \tan\theta_H = \frac{2\sqrt{2}v_\Delta}{v_\phi} \ \text{or} \ \tan\beta = \frac{v_\phi}{2\sqrt{2}v_\Delta}$$

depending only on α and β (or v_Δ)

group factor that makes it possible for the entire factor to be > 1 (mixing required)

Higgs	κ_F	κ_V
h	$\frac{\cos\alpha}{\sin\beta}$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$
H_1^0	$\frac{\sin\alpha}{\sin\beta}$	$\sin\beta\sin\alpha + \sqrt{\frac{8}{3}}\cos\beta\cos\alpha$

suppressed by α



blue: κ_V

red: κ_F

Features of GM Model

- A large triplet VEV v_Δ is allowed.
  ➠ focus on $v_\Delta \sim$ O(GeV) regime
- There exists a doubly-charged Higgs boson that can lead to like-sign LNV and possibly even LFV processes at tree level.
  ➠ providing a link between neutrino and LHC physics
- The SM-like Higgs can possibly have stronger/weaker couplings with weak bosons (simplest model for this).
- There exists a $H_5^\pm W^\mp Z$ vertex at tree level through mixing and proportional to v_Δ.
  ➠ cf. loop-induced in models such as 2HDM
- The Higgs decay pattern is mainly controlled by v_Δ and mass hierarchy.

Higgs Decay Pattern

CWC, Yagyu JHEP 2012

$\Delta m < 0 \;\; (m_{H_5} > m_{H_3})$

$H_5^{++} \to H_3^+ W^+ \;,\; H_5^+ \to H_3^+ Z / H_3^0 W^+ \;,\; H_5^0 \to H_3^\pm W^\mp / H_3^0 Z$

$H_3^+ \to H_1^0 W^+ \;,\; H_3^0 \to H_1^0 Z$

$\Delta m > 0 : \;\; (m_{H_3} > m_{H_5})$

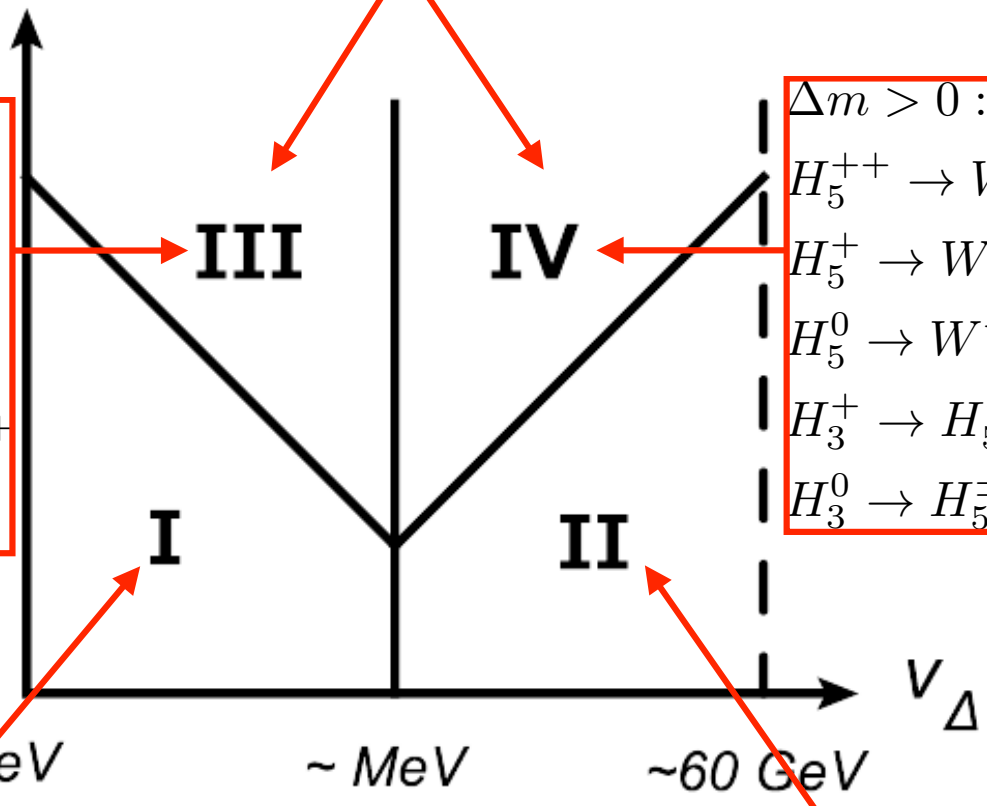
$H_5^{++} \to \ell^+ \ell^+$

$H_5^+ \to \ell^+ \nu$

$H_5^0 \to \nu\nu$

$H_3^+ \to H_5^{++} W^- / H_5^+ Z / H_5^0 W^+$

$H_3^0 \to H_5^\pm W^\mp / H_5^0 Z$



$\Delta m > 0 : \;\; (m_{H_3} > m_{H_5})$

$H_5^{++} \to W^+ W^+$

$H_5^+ \to W^+ Z$

$H_5^0 \to W^+ W^- / ZZ$

$H_3^+ \to H_5^{++} W^- / H_5^+ Z / H_5^0 W^+$

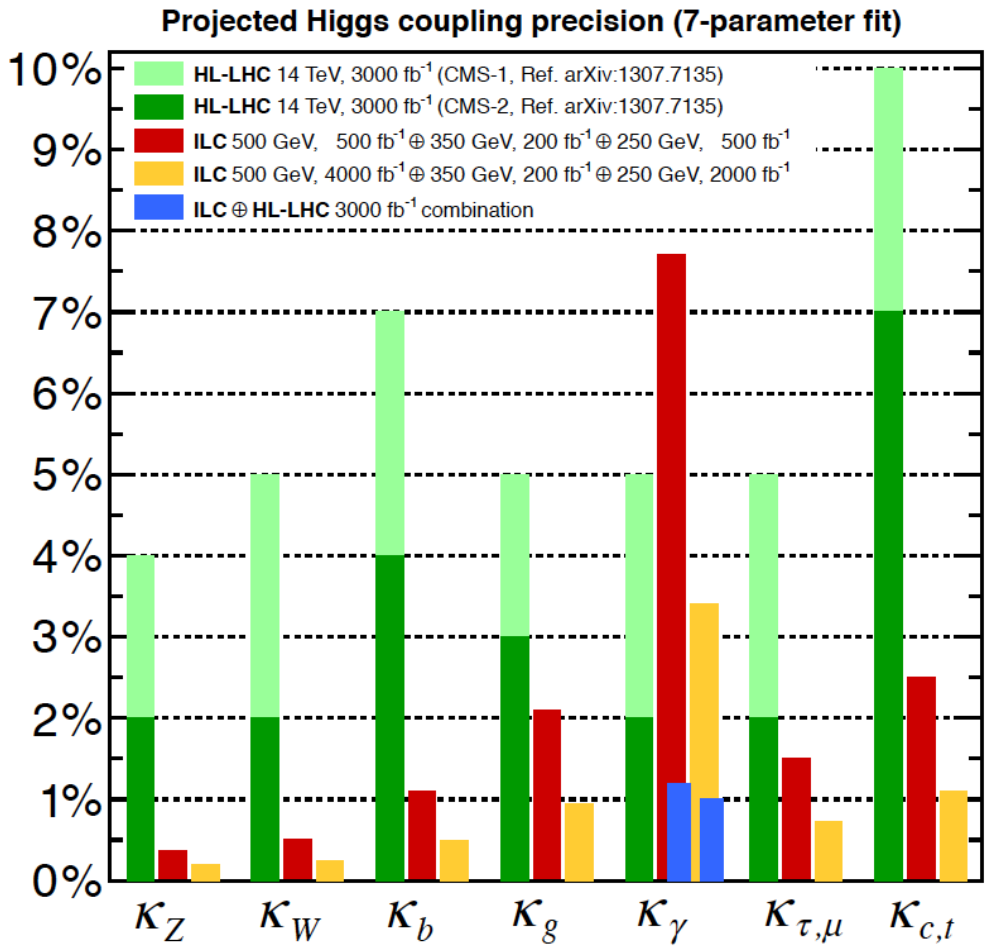
$H_3^0 \to H_5^\pm W^\mp / H_5^0 Z$

$H_5^{++} \to \ell^+ \ell^+ \;,\; H_5^+ \to \ell^+ \nu \;,\; H_5^0 \to \nu\nu \;,$
$H_3^+ \to \ell^+ \nu \;,\; H_3^0 \to \nu\nu.$

$H_5^{++} \to W^+ W^+ \;,\; H_5^+ \to W^+ Z \;,\; H_5^0 \to W^+ W^- / ZZ \;,$
$H_3^+ \to \tau^+ \nu / c\bar{s} / t\bar{b} \;,\; H_3^0 \to b\bar{b}.$

Expected Coupling Precision

- All Higgs couplings will be determined by HL-LHC + ILC to O(1) or sub percent level (particularly hVV couplings).
  ➠ need to know radiative corrections



Custodial Higgs Models

- Couplings of h are modified by exotic Higgs fields due to their EW charges and mixing with Φ_{SM}.
- At tree level, their hVV couplings satisfy

$$\kappa_W = \kappa_Z$$

- Three simplest custodial Higgs models:

	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^\pm$	$H_1, (H_3^0, H_3^\pm), (H_5^0, H_5^\pm, H_5^{\pm\pm})$
parameters $(m_h, v) = (125, 246)$ GeV	$\alpha, m_S,$ $\mu_S, \lambda_{\Phi S}, \lambda_S$	$m_H, m_A, m_{H^\pm},$ $\mu, \alpha, \tan\beta = v_2/v_1$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$ $\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$

always ≤ 1 through mixing

group factor that makes it possible for the entire factor to be > 1 (mixing required)

Radiative Corrections

- Radiative corrections can lead to at least two effects:
  - changes in the magnitudes of various couplings
  - deviations from tree-level relations among couplings

  due to various custodial symmetry breaking parameters (couplings, masses).

	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^\pm$	$H_1, (H_3^0, H_3^\pm), (H_5^0, H_5^\pm, H_5^{\pm\pm})$
parameters $(m_h, v) = (125, 246)$ GeV	$\alpha, m_S,$ $\mu_S, \lambda_{\Phi S}, \lambda_S$	$m_H, m_A, m_{H^\pm},$ $\mu, \alpha, \tan\beta = v_2/v_1$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$ $\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$
$\delta\kappa_V$	$-\sin\alpha\delta\alpha$	$\cos(\beta-\alpha)(\delta\beta-\delta\alpha)$	$\frac{\partial\kappa_V}{\partial\alpha}\delta\alpha + \frac{\partial\kappa_V}{\partial\beta}\delta\beta + \frac{\partial\kappa_V}{\partial\rho}\delta\rho$

- Want to know how big quantum corrections are.

Renormalization

- Independent counter terms in the model:

gauge sector

$$m_W^2 \to m_W^2 + \delta m_W^2 \ ,$$
$$m_Z^2 \to m_Z^2 + \delta m_Z^2 \ ,$$
$$\alpha_{\rm em} \to \alpha_{\rm em} + \delta\alpha_{\rm em} \ ,$$
$$B_\mu \to \left(1 + \frac{1}{2}\delta Z_B\right) B_\mu \ ,$$
$$W_\mu^a \to \left(1 + \frac{1}{2}\delta Z_W\right) W_\mu^a \ .$$

scalar sector

$$m_X^2 \to m_X^2 + \delta m_X^2 \ ,$$
$$(X = H_5, H_3, H_1, h)$$
$$\mu_i \to \mu_i + \delta\mu_i \ , (i = 1, 2)$$
$$v_\Delta \to v_\Delta + \delta v_\Delta \ ,$$
$$\nu \to 0 + \delta\nu \ , \ (\nu = v_\xi - v_\chi)$$
$$\alpha \to \alpha + \delta\alpha \ .$$

Renormalization

- In addition to the renormalization conditions in the SM to get physical G_F, m_Z, and α_{EM}, the GM model allows one additional condition, which we take to make

$$\alpha_{\rm em} T = \frac{\Pi^{\rm 1PI}_{ZZ}(0) - \Pi^{\rm 1PI}_{ZZ}(0)\big|_{\rm SM}}{m_Z^2} - \frac{\Pi^{\rm 1PI}_{WW}(0) - \Pi^{\rm 1PI}_{WW}(0)\big|_{\rm SM}}{m_W^2}$$

$$+ \delta\rho \qquad \delta\rho = \frac{8 v_\Delta \delta\nu}{v^2}$$

equal to zero or its experimental value.

- Use the on-shell scheme to fix other counter terms.

hVV Couplings

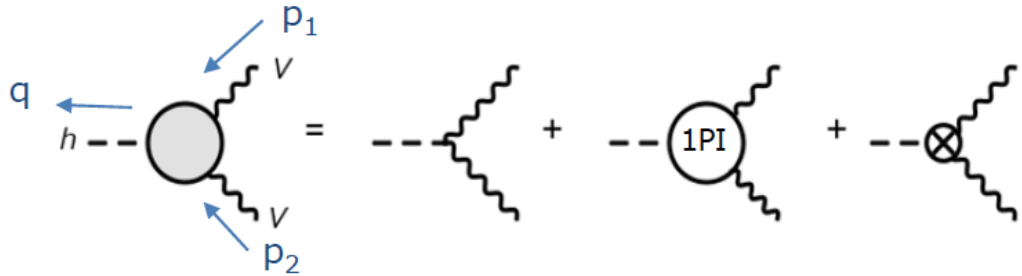
- In general, the renormalized $hV_\mu V_\nu$ vertices can be decomposed as

$$\hat{\Gamma}^{\mu\nu} = \hat{\Gamma}_1 g^{\mu\nu} + \hat{\Gamma}_2 p_1^\mu p_2^\nu + \hat{\Gamma}_3 \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}$$

where the last two form factors start to appear at 1-loop level from 1-particle irreducible (1PI) diagram contributions, while the first form factor of interest to us

$$\hat{\Gamma}_1 = \frac{2m_V^2}{v}\kappa_V + \Gamma^{1\mathrm{PI}} + \delta\Gamma$$

has contributions from the tree-level coupling, 1PI diagrams, and counter terms.



κ_Z AND κ_W

- hVV scale factors at 1-loop with momentum dependence are defined as:

momentum of off-shell V*

$$\hat{\kappa}_V(p^2) \equiv \frac{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm NP}}{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm SM}}$$

- At 1σ, $\kappa_{W,Z}$ are (will be) determined to be

LHC Run-I	$\kappa_Z = [0.94, 1.13]$	$\kappa_W = [0.78, 1.00]$	
HL-LHC	$\Delta\kappa_Z = 2 - 4\%$	$\Delta\kappa_W = 2 - 5\%$	14 TeV, 3000/fb
ILC	$\Delta\kappa_Z = 0.58\%$	$\Delta\kappa_W = 0.81\%$	500 GeV, 500/fb ⊕ 350 GeV, 200/fb ⊕ 250 GeV, 500/fb

1606.02266 [hep-ex]
1310.8361 [hep-ex]
1506.05992 [hep-ex]

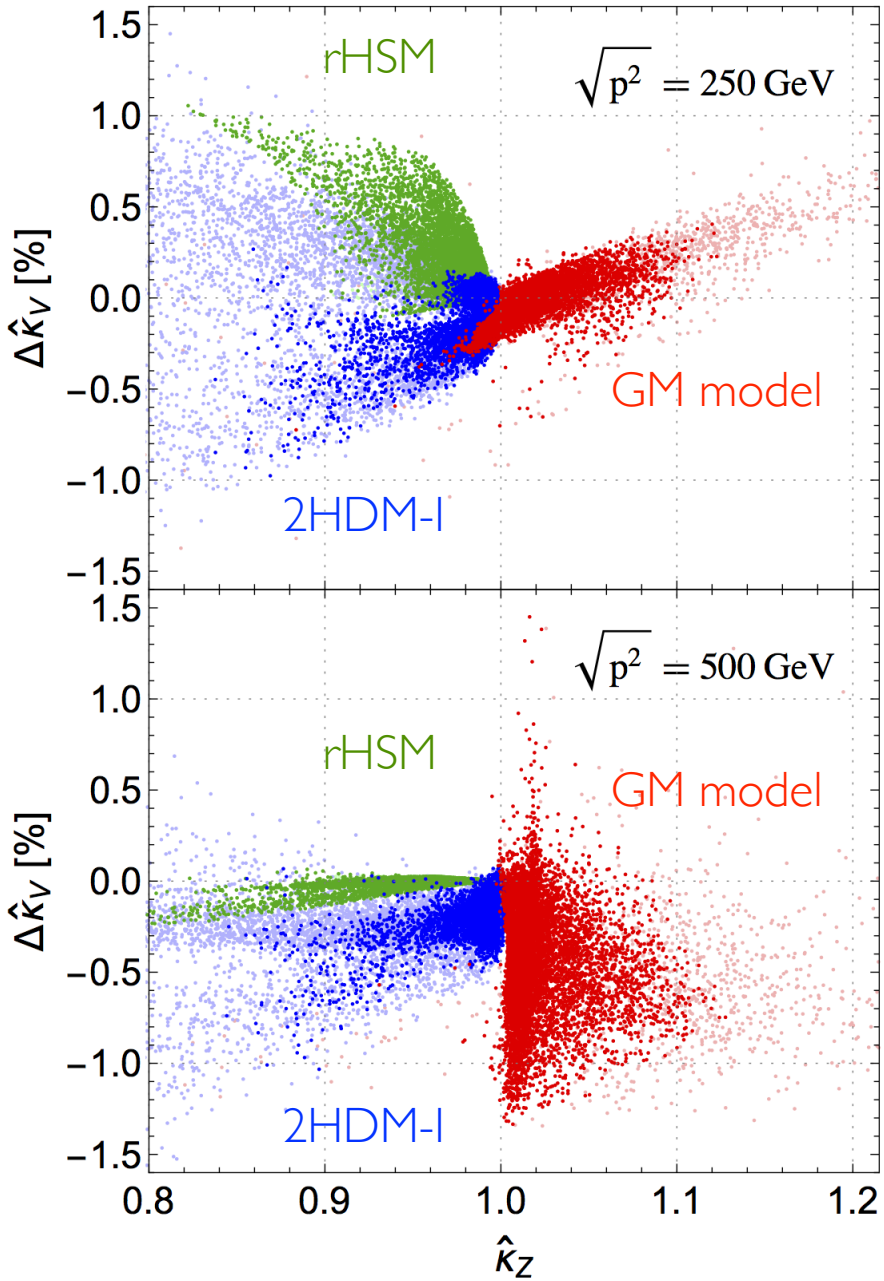
- Radiative corrections in SM:

$$\frac{g_{hVV}^{1-\rm loop}}{g_{hVV}^{\rm tree}} \simeq \begin{cases} -1.2 \;\; (+1.0)\ \% \;\; (hZZ)\ , \\ +0.4 \;\; (+1.3)\ \% \;\; (hWW)\ , \end{cases} \quad \text{for } \sqrt{p^2} = 250\ (500)\ \text{GeV}$$

1-Loop Results

$\Delta\hat{\kappa}_V \equiv \hat{\kappa}_W - \hat{\kappa}_Z$

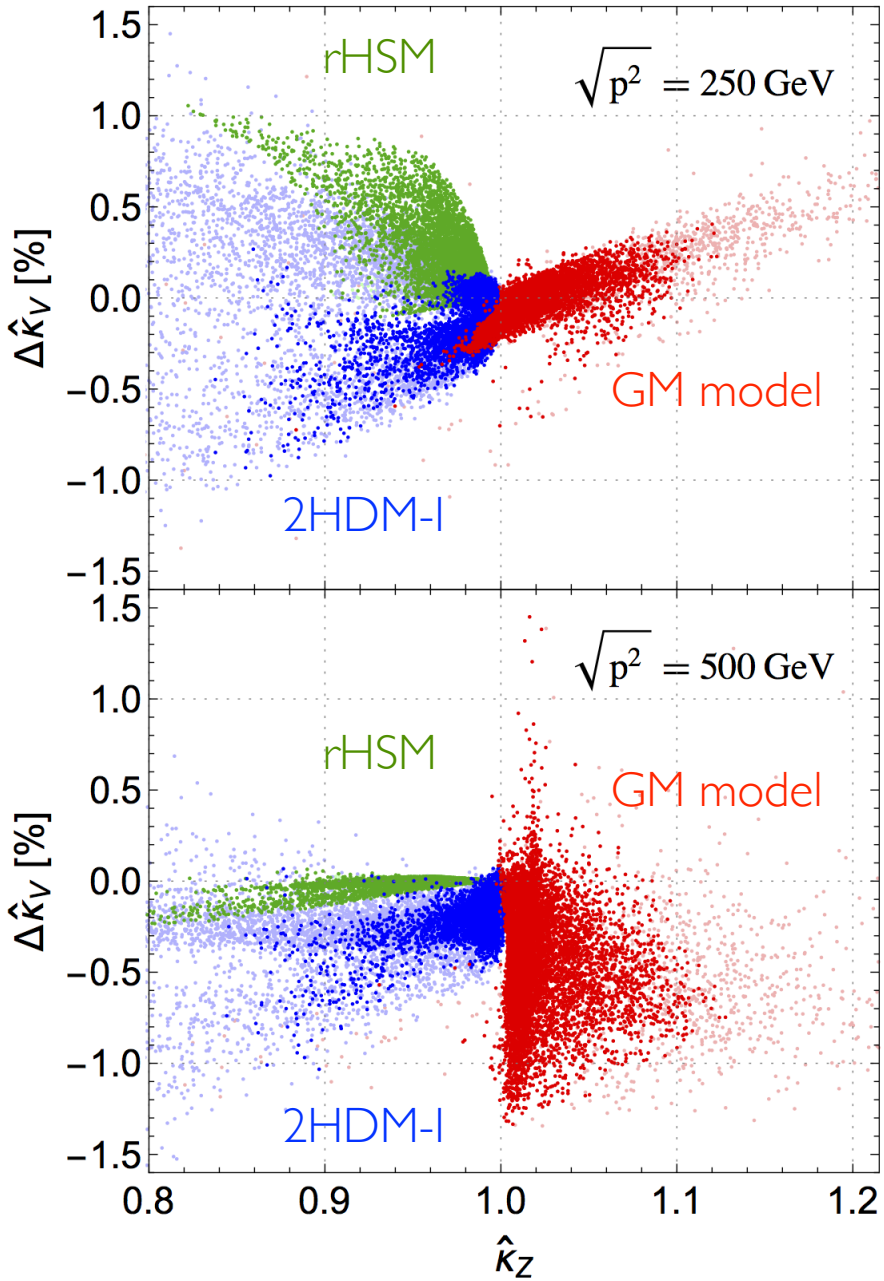
- Lighter dots satisfy theoretical constraints (unitarity, stability, perturbativity, and oblique parameters [S and T]).
- Darker dots further satisfy Higgs data from LHC Run-I (20 channels). ATLAS + CMS 2016
- Other types of 2HDM are expected to have a similar result as 2HDM-I.
- It is possible to discriminate among the rHSM, 2HDMs and GM model.



1-Loop Results

$\Delta\hat{\kappa}_V \equiv \hat{\kappa}_W - \hat{\kappa}_Z$

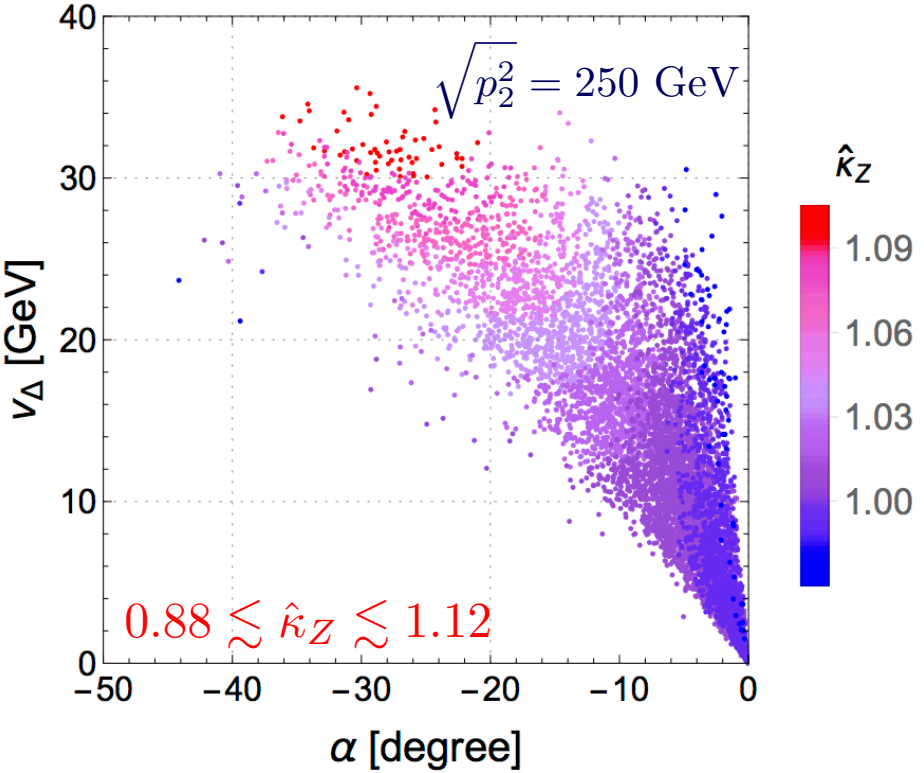
- Same parameter sets in the two plots, only different in p.
- Green dots change little after imposing the Higgs data, while 2HDM and GM dots shrink significantly.
- GM prefers $\kappa_Z \in [0.88, 1.12]$, while the others have $\kappa_Z \in [0.8, 1.0]$.
- $\Delta\kappa_V$ may be observable.
- 250-GeV ILC is better than 500-GeV in distinguishing rHSM and 2HDM-I.

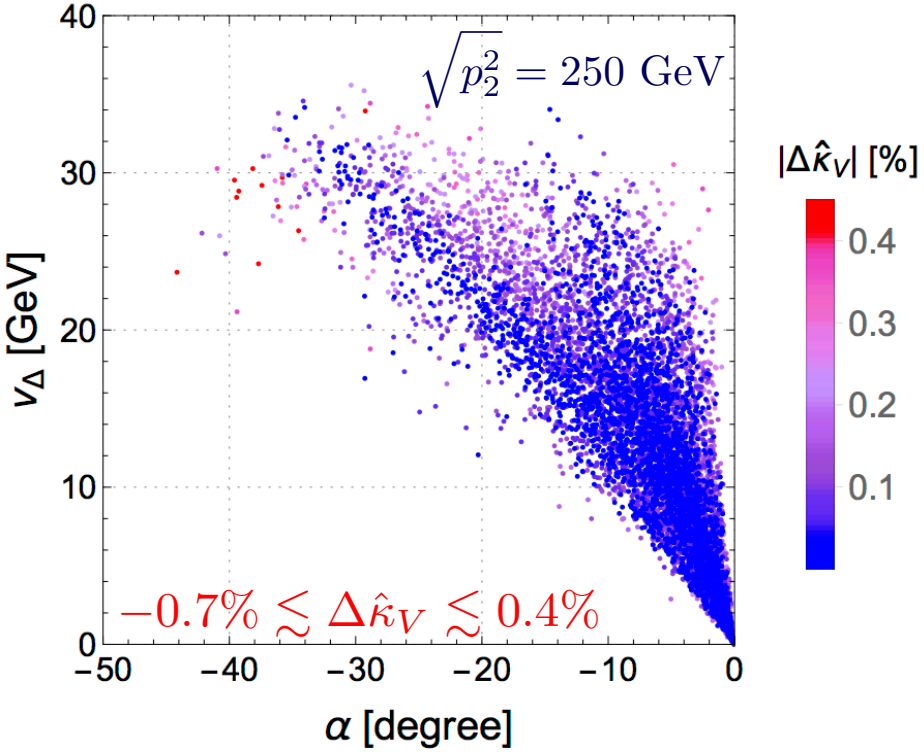


Distribution on (α, v_Δ) Plane

$$\hat{\kappa}_V(p_2^2) \equiv \frac{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm GM}}{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm SM}}$$

$$|\Delta\hat{\kappa}_V| \equiv |\hat{\kappa}_W - \hat{\kappa}_Z|$$

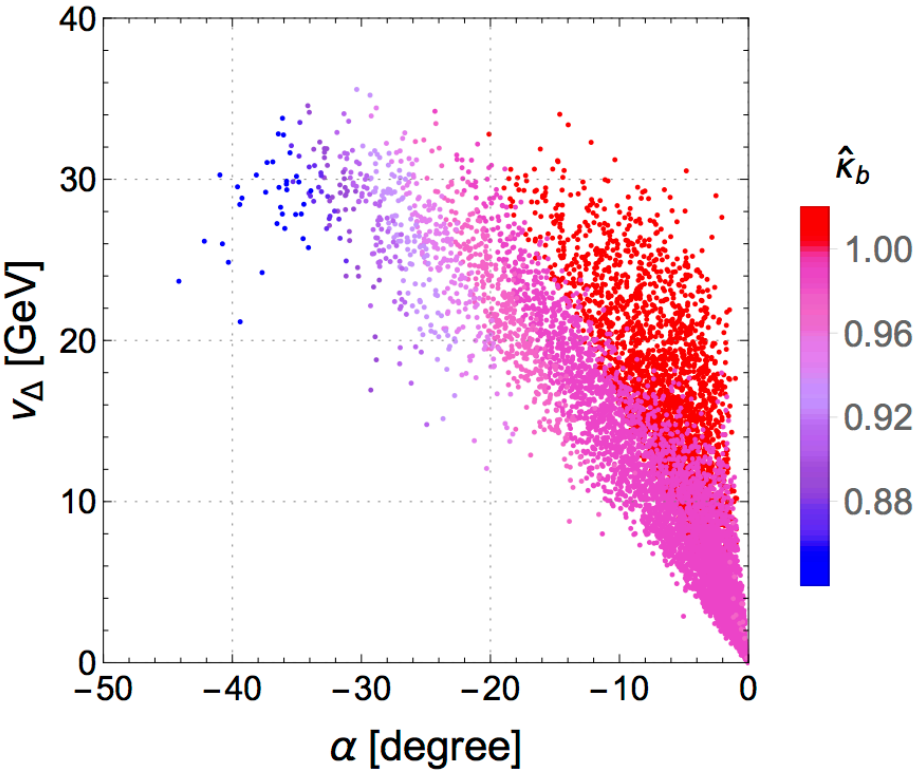


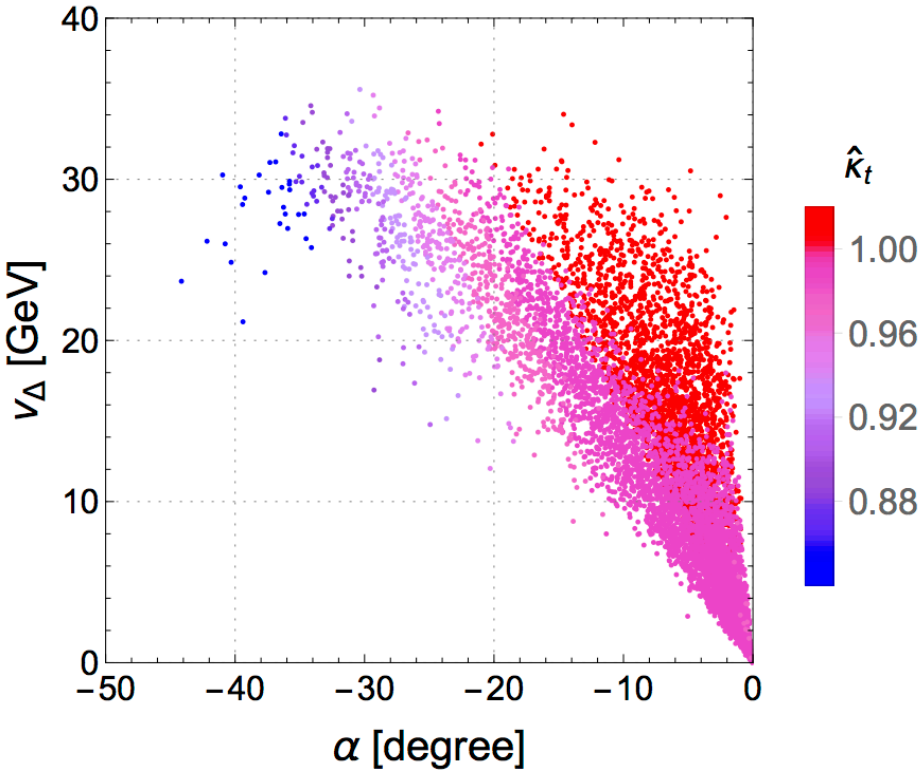


- Larger values of κ_Z are obtained in the upper-left region.
- The result does not change much when p_2 = 500 GeV
- $|\Delta\kappa_V|$ does not depend much on v_Δ and α.

Distribution on (α, v_Δ) Plane

$$\hat{\kappa}_{b,\tau} \equiv \frac{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m^2_{b,\tau}, m^2_{b,\tau}, m^2_h)_{\rm GM}}{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m^2_{b,\tau}, m^2_{b,\tau}, m^2_h)_{\rm SM}} \qquad \hat{\kappa}_t \equiv \frac{\hat{\Gamma}^1_{ht\bar{t}}(m^2_t, (500\ {\rm GeV})^2, m^2_h)_{\rm GM}}{\hat{\Gamma}^1_{ht\bar{t}}(m^2_t, (500\ {\rm GeV})^2, m^2_h)_{\rm SM}}$$



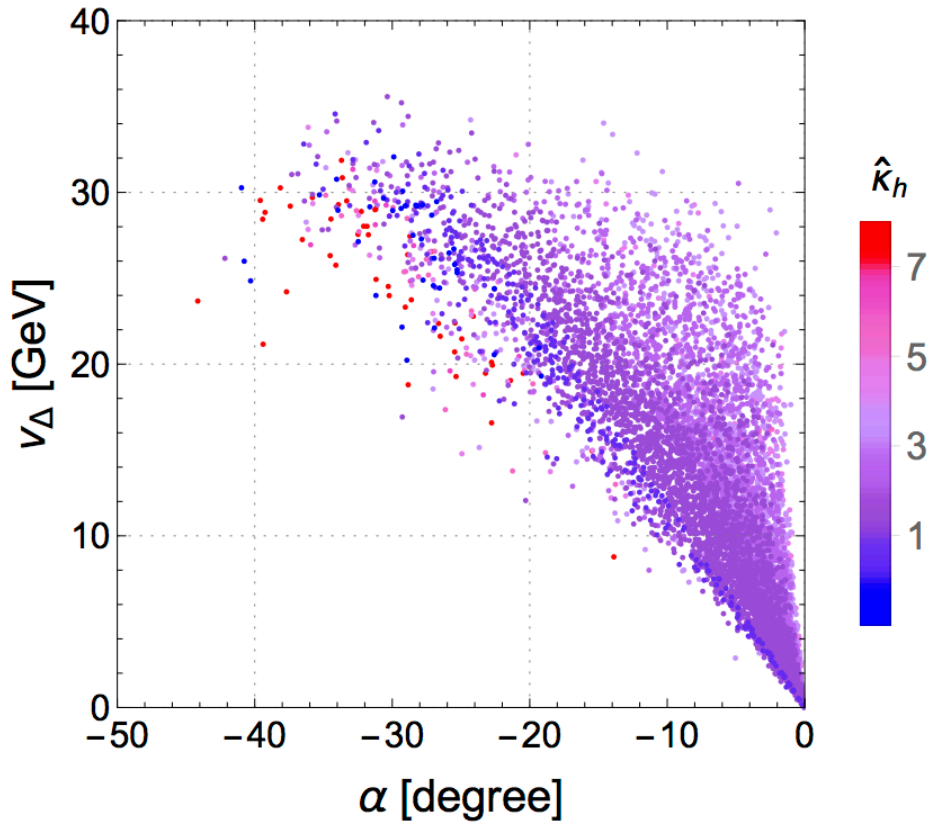


- Behaviors of κ_b (κ_τ) and κ_t are virtually the same.
- In contrast to the κ_Z case, κ_b becomes smaller when $|\alpha|$ becomes larger.

Distribution on (α, v_Δ) Plane

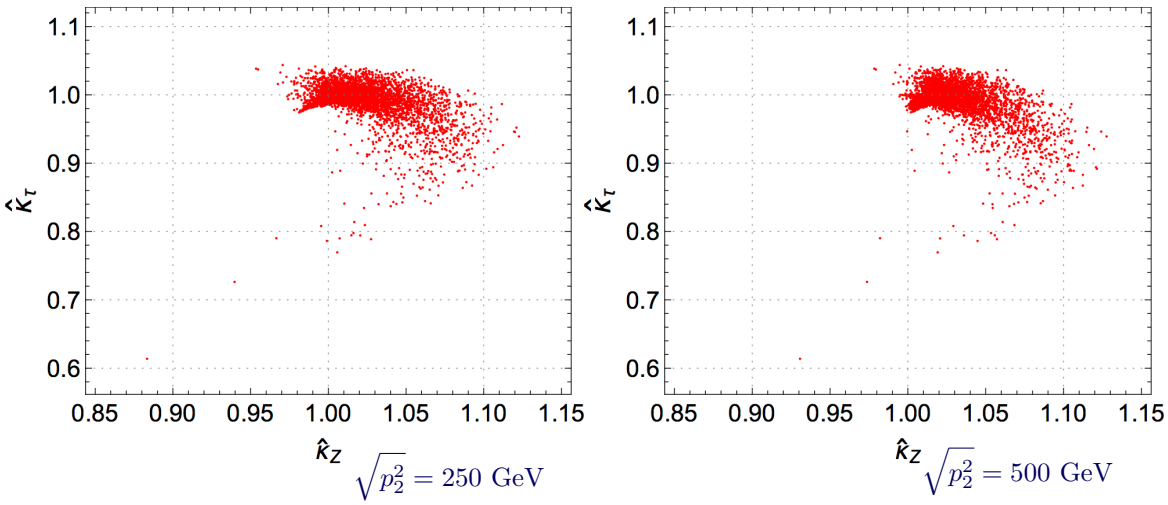
- While the variations from the SM predictions are typically $\lesssim$ 10% for κ_Z, κ_b and κ_t, the magnitude of the deviation in the hhh coupling, i.e., $\kappa_h - 1$, can be at a few 100% level.
  ➠ large quantum corrections for hhh coupling
- In addition, κ_h does not depend much on α and v_Δ as compared to others.

$$\hat{\kappa}_h \equiv \frac{\hat{\Gamma}^1_{hhh}(m_h^2, (500\ \mathrm{GeV})^2, m_h^2)_{\mathrm{GM}}}{\hat{\Gamma}^1_{hhh}(m_h^2, (500\ \mathrm{GeV})^2, m_h^2)_{\mathrm{SM}}}$$

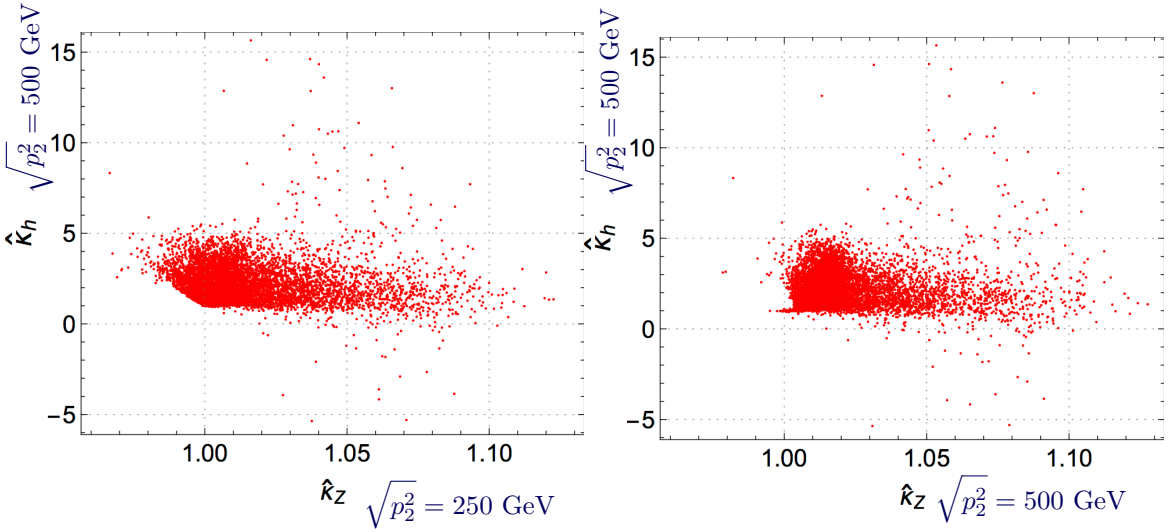


κ_Z vs κ_τ / κ_Z vs κ_h

- Some shifting between two plots.
- Range of κ_τ gets more restricted for larger κ_Z.
- At $\kappa_Z \simeq 1.13$, $\kappa_\tau \simeq 0.95$.



- Most of κ_h values are predicted to be 1 to 5, though sometimes of O(10).
- Possible range of κ_h is restricted for larger κ_Z.



- Some κ_h are predicted less than 1 or even negative.

Summary

- We have done 1-loop radiative corrections to the Higgs couplings in the GM model, a model featured in giving Majorana mass and possibly large hVV couplings.
- Theoretical (unitarity, stability, perturbativity, and oblique parameters) and experimental (Higgs signal strengths) constraints have been imposed to find viable parameter spaces.
- We have presented numerical results for the hVV couplings in the rHSM, 2HDM-I and GM models at 250-GeV and 500-GeV ILC, showing power to discriminate among the models.
- We have also presented the results of hff and hhh couplings in the GM model, showing their correlations among themselves and with hVV and the momentum dependence of these couplings.

Thank You!

Backup Slides

Some Benchmark Points

	α [degree]	v_Δ [GeV]	μ_1 [GeV]	μ_2 [GeV]	$\hat{\kappa}_W$	$\hat{\kappa}_Z$	$\hat{\kappa}_t$	$\hat{\kappa}_h$
BP1	−7.0	10	−100.3	112.3	1.01	1.01	1.00	1.08
BP2	−8.0	10	7.1	2789.5	0.99	0.98	0.96	4.71
BP3	−15.1	20	−180.0	171.0	1.04	1.04	0.99	1.34
BP4	−16.1	20	18.7	1338.3	1.02	1.01	0.95	4.00
BP5	−22.4	30	−325.2	−53.3	1.09	1.09	0.98	1.39
BP6	−24.9	30	10.0	755.0	1.07	1.06	0.93	0.87

TABLE I: Six benchmark points allowed by the perturbative unitarity and the vacuum stability. The masses of the extra Higgs bosons are taken to be $m_{H_5} = m_{H_3} = m_{H_1} = 400$ GeV. All the other input parameters are shown in the first four columns. The numbers given in the latter four columns show the output of the renormalized scale factors at $\sqrt{p^2} = 250$ (500) GeV for $\hat{\kappa}_{W,Z}$ ($\hat{\kappa}_{t,h}$).

- BP1,3,5 (BP2,4,6) are chosen such that predictions of one-loop corrected scale factors are close to (far from) the tree-level predictions for v_Δ = 10, 20, 30 GeV, respectively.

Momentum Dependence

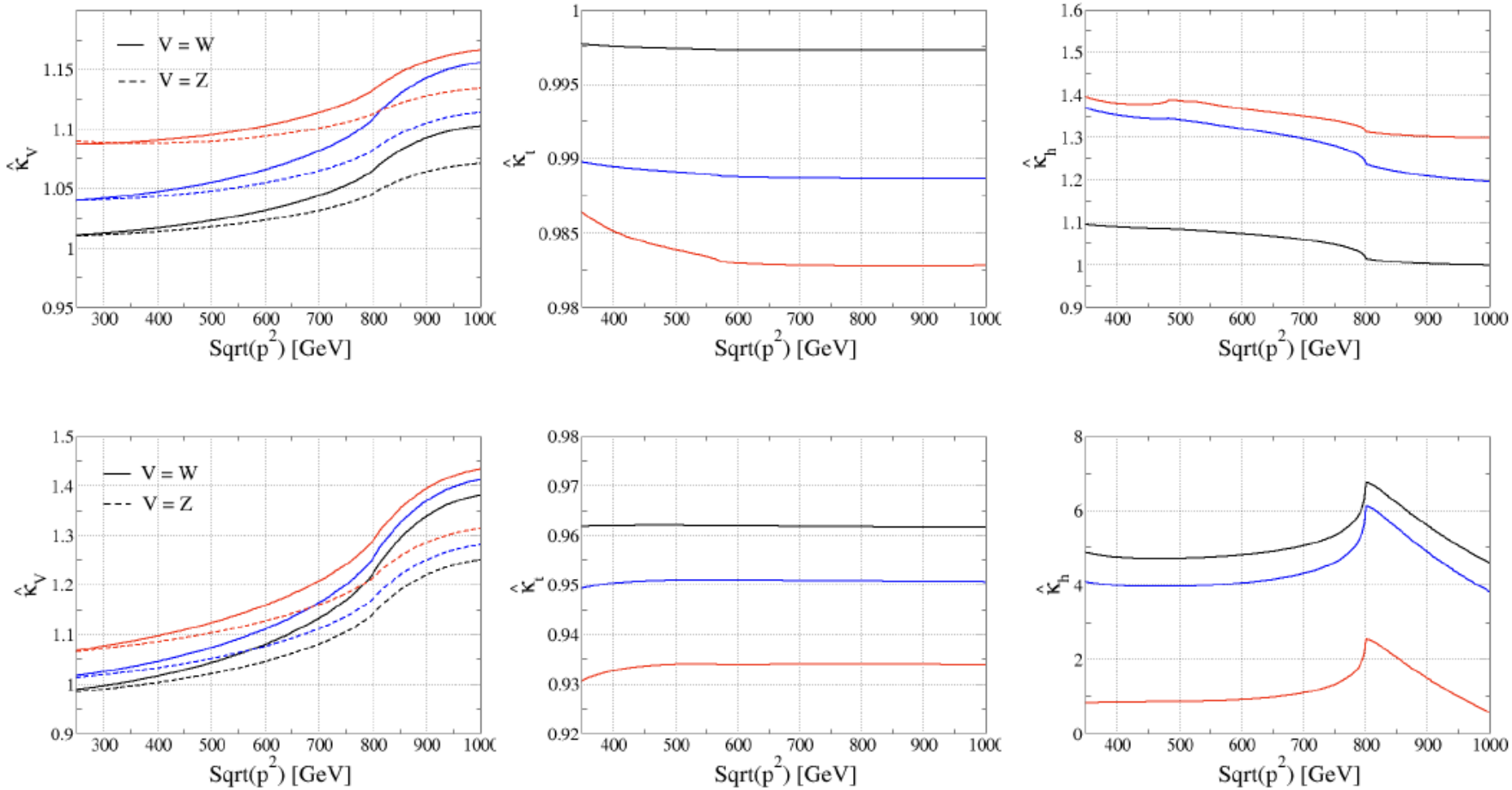


FIG. 3: Renormalized scale factors $\hat{\kappa}_{W,Z}$ (left), $\hat{\kappa}_t$ (middle) and $\hat{\kappa}_h$ (right) as functions of $\sqrt{p^2}$ for $m_{H_5} = m_{H_3} = m_{H_1} = 400$ GeV. The upper panels show the cases of BP1 (black), BP3 (blue) and BP5 (red), while the lower panels show the cases of BP2 (black), BP4 (blue) and BP6 (red).