Clockworking FIMPs

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With

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Dark Matter and the Hierarchy Problem

Cosmic Microwave Background (CMB): Precision Cosmological Measure of Dark Matter

$\Omega_{DM} = 0.220 \pm 0.011$

CMB alone
Dark Matter and the Hierarchy Problem

Natural

Natural

Highly Unnatural

M. Strassler 2013

PhD comics
processes that contribute to the annihilation processes. The s channel diagrams connecting freedom of the DM particle. For the velocity averaged cross sections we have a variety of density, using the formula,

\[ \frac{d n_x}{dt} + 3 H n_x = \langle \sigma v \rangle [n_x^2 - n_{eq}^2] \]

Eventually as all bath particles decouples from the thermal bath at their respective thresholds. The number density keeps dropping but still tracks the equilibrium value Eq.

\[ \Omega_x h^2 = 0.1 \frac{x_f \sqrt{g_{eff}}}{28} \frac{2.10^{-26} \text{cm}^3 \text{s}^{-1}}{10} < \sigma_{xx} v_{\text{ann}} > \]

DM in thermal equilibrium with SM in early universe

Interaction Strength: Weak Scale -> A BSM theory that also solves Hierarchy problem
Weakly Interacting Massive Particle

**WIMPS**  
DM in thermal equilibrium with SM in early universe

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma v \rangle [n_\chi^2 - n_{eq}^2]
\]

Interaction Strength: Weak Scale \(\rightarrow\) A BSM theory that also solves Hierarchy problem

\[
\Omega_\chi h^2 = 0.1 \frac{x_f}{28} \frac{\sqrt{g_{\text{eff}}}}{10} \left[ 2.10^{-26} \text{cm}^3 \text{s}^{-1} \right] < \sigma_{\chi\chi}v_{\text{ann}} >
\]
Weakly Interacting Massive Particle

**WIMPS**

DM in thermal equilibrium with SM in early universe

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma v \rangle [n_\chi^2 - n_{eq}^2]
\]

Interaction Strength : Weak Scale -> A BSM theory that also solves Hierarchy problem

\[
\Omega_\chi h^2 = 0.1 \times \frac{x_f \sqrt{g_{eff}}}{28 \times 10} \times 2.10^{-26} \text{cm}^3 \text{s}^{-1} \langle \sigma_\chi v_{\text{ann}} \rangle
\]
The freedom of the DM particle. For the velocity averaged cross sections we have a variety of processes, one can then solve Eq. where

\[
\frac{dn_x}{dt} + 3H n_x = \langle \sigma v \rangle [n_x^2 - n_{eq}^2]
\]

and thus without any other particles to annihilate to, the DM freezes out and yields a relic density that we observe today. A brief summary of the calculations we perform can be found in Appendix. Given the above formula for the velocity averaged annihilation cross section for 2 WIMPS, the relic density is obtained by the solution of the Boltzmann equation, describing the evolution of the DM number density.

Eventually as all bath particles (SM and BSM) freeze out, the interaction rate

\[
\Omega_X h^2 = 0.1 \frac{x_f \sqrt{g_{eff}}}{28 \cdot 10} \frac{2.10^{-26} \text{cm}^3 \text{s}^{-1}}{< \sigma_{\chi\chi} v_{\text{ann}} >}
\]

The Møller velocity is given by,

\[
\frac{\bar{v}}{v} = \frac{1}{\sqrt{1 + x^2}}
\]

where,

\[
x = \frac{m}{T} = \frac{\text{interaction strength}}{\text{weak scale}}
\]

Interaction Strength: Weak Scale -> A BSM theory that also solves Hierarchy problem.
Interaction Strength : Weak Scale -> A BSM theory that also solves Hierarchy problem

\( \frac{dn_\chi}{dt} + 3Hn_\chi = <\sigma v>[n_\chi^2 - n_{eq}^2] \)

\[ \Omega_\chi h^2 = 0.1 \frac{x_f}{28} \sqrt{g_{_{eff}}} \frac{2.10^{-26} \text{cm}^3 \text{s}^{-1}}{10} <\sigma_{\chi\chi}v_{\text{ann}}> \]

Equilibrium

Freeze-out
Key Ingredient: DM not in thermal equilibrium with SM

Negligible initial DM density

\[ \lambda X B_1 B_2 \quad B_1 \rightarrow B_2 X. \]

\[ \dot{n}_X + 3Hn_X \approx 2g_{B_1} \int d\Pi_{B_1} \Gamma_{B_1} m_{B_1} f_{B_1} = g_{B_1} \int \frac{d^3 p_{B_1}}{(2\pi)^3} \frac{f_{B_1} \Gamma_{B_1}}{\gamma_{B_1}} = \frac{g_{B_1} m_{B_1}^2 \Gamma_{B_1}}{2\pi^2} T K_1(m_{B_1}/T) \]

\[ Y_{1 \rightarrow 2} \approx \frac{135 g_{B_1}}{8\pi^3 (1.66) g^S \sqrt{g^I}} \left( \frac{M_{Pl} \Gamma_{B_1}}{m_{B_1}^2} \right) \]

\[ \Omega_X h^2|_{tot} \approx \frac{1.09 \times 10^{27}}{g^S \sqrt{g^I}} m_X \sum_i \frac{g_{B_i} \Gamma_{B_i}}{m_{B_i}^2} \]

\[ \lambda \approx 1.5 \times 10^{-13} \left( \frac{m_B}{m_X} \right)^{1/2} \left( \frac{g^*_B |m_B|}{10^2} \right)^{3/4} \left( \frac{g_{bath}}{10^2} \right)^{-1/2}. \]

Exponentially small coupling required

How to dynamically and “naturally” generate such small couplings with order 1 numbers?
If there is a lightest observable sector particle (LOSP)

Typical models:
1. Moduli with weak scale SUSY
2. FIMPs from kinetic mixing with hidden U(1)
3. Right handed sneutrino in weak scale SUSY
Theory of N+1 copies of U(1) global symmetry broken down to a single U(1) at a scale f

Below f, N+1 massless Goldstones

Introduce N “soft” mass parameters $m^2_i$ that break this symmetry explicitly

Background of N spurion fields

Nearest neighbor interactions $Q_i = \delta_{ij} - q \delta_{i,j+1}, q > 1$ Non-diagonal charges of U(1)

The unbroken generator $Q = \sum_{j=0}^{N} \frac{Q_j}{q^j}$

Low energy effective lagrangian

$$\mathcal{L} = -\frac{f^2}{2} \sum_{j=0}^{N} \partial_{\mu} U_j^\dagger \partial^\mu U_j + \frac{m^2 f^2}{2} \sum_{j=0}^{N-1} (U_j^\dagger U_j^q + \text{h.c.})$$
A clockwork scalar

\[ \mathcal{L}_{SCW} = -\frac{1}{2} \sum_{j=0}^{N} \partial_k \phi_j^\dagger \partial^k \phi_j + \sum_{j=0}^{N-1} \frac{m^2}{2} (\phi_j - q \phi_{j+1})^2 + \sum_{j=0}^{N-1} \frac{m^2}{24 f^2} (\phi_j - q \phi_{j+1})^4 + \mathcal{O}(\phi_6) \]

\[ V(\phi) = \sum_{j=0}^{N-1} \frac{m^2}{2} (\phi_j - q \phi_{j+1})^2 + \sum_{j=0}^{N-1} \frac{m^2}{24 f^2} (\phi_j - q \phi_{j+1})^4 + \mathcal{O}(\phi_6) \]

\[ = \frac{1}{2} \sum_{i,j=0}^{N} \phi_i M_{ij}^2 \phi^j + \frac{m^2}{24 f^2} \sum_{i,j=0}^{N} (\phi_i M_{ij}^2 \phi^j)^2 + \mathcal{O}(\phi_6) \]

\[
\begin{pmatrix}
1 & -q & 0 & \cdots & 0 \\
-q & 1 + q^2 & -q & \cdots & 0 \\
0 & -q & 1 + q^2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 + q^2 & -q & q^2
\end{pmatrix}
\]

\[ m_{a_0}^2 = 0, \quad m_{a_k}^2 = \lambda_k m^2; \quad \lambda_k = q^2 + 1 - 2q \cos \frac{k \pi}{N + 1}, \quad k = 1, \ldots, N \]

\[ \Delta m \sim 2m \]

\[ m_1 \sim (q - 1)m \]

**Mass spectrum**

**Band Gap**

**Discrete Clockwork**

Very weakly coupled state.
A clockwork scalar

\[ O^T M^2 O = \text{diag}(m_{a_0}^2, \ldots, m_{a_n}^2) \]

\[ O_{j0} = \frac{N_0}{q^j}, \quad O_{jk} = N_k \left[ q \sin \frac{j k \pi}{N + 1} - \sin \frac{(j + 1) k \pi}{N + 1} \right]; \quad j = 0, \ldots, N; \quad k = 1, \ldots, N \]

Elements of rotation matrix

The exponential suppression.

Theory(SM) coupled to the nth site of the clockwork

\[ \mathcal{L} = \frac{\pi N}{16\pi^2 f} G_{\mu\nu} \tilde{G}^{\mu\nu} \]

\[ \mathcal{L} = \frac{1}{16\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \left( \frac{a_0}{f_0} - \sum_{k=1}^{N} (-1)^k \frac{a_k}{f_k} \right) \]

The clockwork axion

\[ f_0 = \frac{f q^N}{N_0}, \quad f_k = \frac{N_k}{q} \sin \frac{k \pi}{N + 1}. \]
A model of scalar FIMP

\[ \mathcal{L}_{sFIMP} = -\frac{1}{2} \sum_{j=0}^{N} \partial_\mu \phi_j^\dagger \partial^\mu \phi_j - \frac{1}{2} \sum_{i,j=0}^{N} \phi_i M_{ij}^2 \phi_j^* - \frac{m^2}{24 f^2} \sum_{i,j=0}^{N} (\phi_i \tilde{M}_{ij}^2 \phi_j^*)^2 \]

Diagonal mass term for every site

\[ \mathcal{L}_{int} = \kappa |H^\dagger H| \phi_n^2 \]

SM coupled to the nth site

\[ \mathcal{L}_{int} = \kappa \sum_{j=0,k=0}^{n} O_{kl} O_{jm} a_k a_l (v^2 + 2vh + h^2)/2 \]

FIMP mass suppressed by clockwork

\[ \phi_i \tilde{M}_{ij} \phi_j = \phi_i M_{ij} \phi_j + \kappa v^2 \delta_{in} \delta_{jn} \phi_n \phi_n \]

Modified mass matrix

\[ M_t = m \cdot \begin{bmatrix} 1 + t^2/m^2 & -q & 0 & \cdots & 0 & 0 \\ -q & 1 + q^2 + t^2/m^2 & -q & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 + q^2 + t^2/m^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 + t^2/m^2 \end{bmatrix} \]

Diagonal mass term unsuppressed by clockwork
Clockworking a scalar FIMP

FIMP production

\[ \frac{\kappa}{q^n} O_{jn} \]

\[ \frac{\kappa}{q^n} O_{jn} \]

\begin{align*}
\text{m}=1. \text{TeV} & \\
N & \begin{cases}
25 & \text{t}=0 \text{GeV} \\
20 & \text{t}=1 \text{GeV} \\
15 & \text{t}=100 \text{GeV}
\end{cases}
\end{align*}

\begin{align*}
\text{t}=1 \text{GeV} & \\
N & \begin{cases}
30 & \text{m}=1 \text{TeV} \\
25 & \text{m}=10 \text{TeV} \\
20 & \text{m}=100 \text{TeV}
\end{cases}
\end{align*}

Belanger, Boudjema, Pukhov, Goudelis, Zaldivar: Micromegas 5.0
Phenomenology of the scalar FIMP

Tiny Invisible Width

Produce the higher gears with Order(\(\tau\)) coupling via off-shell Higgs

Displaced b-jets
A fermionic clockwork

\[ \mathcal{L}_{FCW} = \mathcal{L}_{kin} - m \sum_{i=0}^{N-1} (\bar{\psi}_{L,i} \psi_{R,i} - q \bar{\psi}_{L,i} \psi_{R,i+1} + h.c) - \frac{M_L}{2} \sum_{i=0}^{N-1} (\bar{\psi}_{L,i}^c \psi_{L,i}) - \frac{M_R}{2} \sum_{i=0}^{N} (\bar{\psi}_{R,i}^c \psi_{R,i}) \]

\[ = \mathcal{L}_{kin} - \frac{1}{2} (\bar{\psi}^c \mathcal{M} \psi + h.c) \] (12)

\[ \Psi_{2N+1} = (\psi_{L,0}, \ldots, \psi_{L,n-1}, \psi_{R,0}^c, \ldots, \psi_{R,n}^c) \]

\[ \mathcal{M} = m \cdot \]

\[ = \begin{bmatrix} \tilde{q} & 0 & \ldots & 0 & 1 & -q & 0 & \ldots & 0 \\ 0 & \tilde{q} & \ldots & 0 & 0 & 1 & -q & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \tilde{q} & 0 & 0 & 0 & \ldots & -q \\ 1 & 0 & \ldots & 0 & \tilde{q} & 0 & 0 & \ldots & 0 \\ -q & 1 & \ldots & 0 & 0 & \tilde{q} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -q & 0 & 0 & 0 & \ldots & \tilde{q} \end{bmatrix} _{(2N+1)\times(2N+1)} \]

\[ m_0 = m\tilde{q}, \]

\[ m_k = m(\tilde{q} - \sqrt{\lambda_k}), \ k = 1, \ldots, N \]

\[ m_{n+k} = m(\tilde{q} + \sqrt{\lambda_k}), \ k = 1, \ldots, N \]

\[ \mathcal{U} = \begin{bmatrix} \tilde{0} & \frac{1}{\sqrt{2}} U_L & -\frac{1}{\sqrt{2}} U_L \\ \tilde{u}_R & \frac{1}{\sqrt{2}} U_R & \frac{1}{\sqrt{2}} U_L \end{bmatrix} _{(2N+1)\times(2N+1)} \]

\[ \tilde{0}_i = 0, \ i = 1, \ldots, N \]

\[ (\tilde{u}_R)_i = \frac{1}{q^\gamma} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2\gamma}}}, \ i = 0, \ldots, N \]

\[ (U_L)_{ij} = \sqrt{\frac{2}{N+1}} \sin \frac{ij\pi}{N+1}, \ i,j = 1, \ldots, N \]

\[ (U_R)_{ij} = \sqrt{\frac{2}{(N+1)\lambda_k}} \left[ \frac{q \sin \frac{ij\pi}{N+1} - \sin \frac{(i+1)j\pi}{N+1}}{N+1} \right] i = 0, \ldots, N, \ j = 1, \ldots, N \]
We instead set gauge bosons. In this limit the only interaction of the clockwork sector to SM particles will be

\[ \mathcal{L}_{fFIMP} = \mathcal{L}_{fcw} + i \bar{L} \not{\partial} L + i \bar{R} \not{\partial} R + M_D (\bar{L} R) + Y \bar{L} H \psi_{R,n} + \text{h.c} \]

\[ L' = (l_1, l_2), \quad R' = (r_1, r_2) \]

\[ (SU(3), SU(2))_Y = (1, 2)_{-1/2} \]

Work in the limit of \( v/m \to 0 \) set \( m = 100 \text{ TeV} \)

All off-diagonal entries except the Dirac mass vanish
A fermionic clockwork

FIMP production

LHC pheno

Fig. 2: Contours denoting the values of $q$ and $N$ that produce the observed relic abundance for the fermionic clockwork model. Here $m = 100$ TeV and the three contours are shown for three choices of $\tilde{q} = \{0.01, 1, 100\}$ GeV.

0 as well as with the remaining clockwork fermions $k$. Overproduction of the FIMP may result if the mixing and hence the coupling to gauge bosons is too large. The extent of mixing of the fermions is governed by the fraction $v/m$. In the limit that $v/m \to 0$, all off-diagonal entries (except for $M_D$) will vanish, as will couplings of the clockwork sector to gauge bosons. In this limit the only interaction of the clockwork sector to SM particles will be through the higgs yukawa coupling which is suppressed by $1/qN$. FIMP production will proceed through decays of heavier clockwork gears or via SM scattering processes mediated by the Higgs.

Although it is possible to evaluate the relic density when we take into account mixing of the clockwork fermions with the heavy neutrino, this is a numerically challenging task. We instead set $m = 100$ TeV and choose of $q$ and $N$ that this approximately reproduces our results when we assume $v/m \to 0$.

KM: Further, we neglect any contributions from other particles (for example $SU_2(R)$ gauge bosons), i.e. we set their masses to be much larger than the reheating temperature.
Conclusion

- Freeze-in dark matter scenarios are alternatives to WIMP freeze outs.
- FIMPs are out of thermal equilibrium and require very small couplings.
- A “natural” way to generate exponentially small couplings are Clockwork mechanisms.
- Scalar and Fermionic clockwork freeze-in models are constructed and shown to be a viable set up.
- Such set ups also have long lived particles as a viable signature.
Conclusion

- Freeze-in dark matter scenarios are alternatives to WIMP freeze outs
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The hierarchy of scales

Masses and scales are not equivalent quantities

Four Fermi theory:

\[ [\Lambda] = [G_F^{-1/2}] = \frac{[M_W]}{[g]} \]

\[ E \sim G_F^{-1/2} \sim v \]

New degrees/states occur at \( E \sim M_W \)

Scale at which perturbative unitarity breaks down

Weinberg Operator

\[ \ell \ell H H / \Lambda_\nu \]

\[ [\Lambda_\nu] = \frac{[M_R]}{[\lambda_\nu^2]} \]

\[ m_\nu = v^2 / \Lambda_\nu = \lambda_\nu^2 v^2 / M_R \]

Small mass \quad A very large scale

How to dynamically generate a large separation between the UV theory and an interaction scale with order one parameters?