J-Factors for Velocity-Dependent Dark Matter Annihilation

Jason Kumar

University of Hawaii

(PRD 95, 123008 (2017) [1702.00408], 180x.xxxxx)
collaborators

• Kimberly Boddy
• Louie Strigari
• Mei-Yu Wang
effective J-factors and v-dependence

- prompt photon flux from dark matter annihilation can be factorized into two pieces....
- ... a particle physics factor
  - depends on annihilation cross section, annihilation channel, particle mass
- and an astrophysics factor
  - depends on the dark matter density profile of the target
  - encoded in the J-factor
- but if dark matter annihilation is velocity-dependent
  - then velocity-distribution also come into play

- goal is to compute the effective J-factor ($J_\text{s}$) ...
- ... and see impact on gamma-ray searches for dark matter
main features

• Sommerfeld-enhanced annihilation $\rightarrow \sigma_A v \propto 1/v$ (Coulomb limit)
  – relative velocities in dSphs tend to be much smaller than in Milky Way halo
  – can get a larger enhancement to annihilation cross section
  – considerable variation in velocity distribution between dSphs, and for different choices of density profile
  – affects which dwarfs are most important for dark matter search
  – see also 1804.05052 (Patec, Ullio, Valli), 1712.03188 (Bergstrom, et al.)

• p-wave or d-wave annihilation suppresses dSph relative to GC
  – weakens dSph constraints on dark matter explanation GC excess
  – changes morphology of expected GC dark matter signal

• important implications for indirect detection searches
what is the J-factor?

• the photon flux depends on ....  
  – particle physics of the dark matter model  
    • independent of target  
    – astrophysics of the target  
      • mostly independent of dark matter model  
  • J-factor is the astrophysics factor  
    – larger J = larger flux, regardless of particles physics model  
  • but factorization based on an assumption  
    – $\sigma v$ independent of $v$  
  • what happens for $v$-dependent annihilation?

\[
\frac{d\Phi}{dE} = \frac{1}{4\pi} \frac{dN}{dE} \int_{\Delta \Omega} d\omega \int d\ell \\
\int d^3v_1 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_1)}{m_x} \int d^3v_2 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_2)}{m_x} \\
\times \frac{\sigma_A |\vec{v}_1 - \vec{v}_2|}{2} \\
= \langle \sigma_A v \rangle \frac{dN}{8\pi m_x^2 dE} \times J \\
J \equiv \int_{\Delta \Omega} d\omega \int d\ell \left[ \rho(\vec{r}(\ell, \Omega)) \right]^2 \\
\rho(\vec{r}) = \int d^3v f(\vec{r}, \vec{v})
\]

$f =$ dark matter velocity distribution
defining $J_S$

- just need to absorb $S(v)$ into definition of astrophysical factor
- new factor, $J_S$, encodes astro. info needed to determine $\frac{d\Phi}{dE}$ for velocity-dependent case
- need the DM velocity distribution
  - get it from density distribution, using Eddington formula
- what it amounts to:
  - assume $f(r,v)$ spherically-sym., isotropic
  - then $f$ depends only on $\varepsilon = v^2/2 + \Psi(r)$
  - $\rho(r)$ determines $f(r,v)$

\[
\sigma_A v = (\sigma_A v)_0 \times S(v)
\]

\[
J_S \equiv \int_{\Delta \Omega} d\Omega \int d\ell
\int d^3v_1 f(\vec{r}(\ell,\Omega), \vec{v}_1) \int d^3v_2 f(\vec{r}(\ell,\Omega), \vec{v}_2)
\times S(|\vec{v}_1 - \vec{v}_2|)
\]

\[
\frac{d\Phi}{dE} = \frac{(\sigma_A v)_0 dN}{8\pi m_X^2 dE} \times J_S
\]

$\Psi(r) = \text{gravitational potential}$
determining $f(r,v)$

- **strategy**
  - ansatz for DM density distrib.
  - fixes gravitational potential $\Psi(r)$
    - assume a spherically symmetric effective potential from GC bulge, disk (1211.7063, Pato, Strigari, Trotta, Bertone)
  - now **Eddington formula** determines velocity distribution
  - for GC, we pick some **generalized NFW profiles**
  - for dSph, assume **NFW**, but need to fix **two parameters**
  - fix one by matching **stellar velocity dispersion**, the other with Aquarius $V_{\text{max}}$-$r_{\text{max}}$ relation

$$f(\varepsilon) = \frac{1}{\sqrt{8\pi^2}} \int_{\varepsilon}^{0} \frac{d\psi}{\sqrt{\varepsilon - \psi}} \frac{d^2\rho}{d\psi^2}$$

$$\varepsilon \equiv \frac{v^2}{2} + \psi(r) < 0$$

$$\rho(r) = 4\pi \int_0^{\sqrt{-2\psi(r)}} dv \ v^2 f(r, v)$$

$$\Psi_{\text{bulge}} = \frac{G_N M_b}{r + c_0}$$

$$\Psi_{\text{disk}} = -\frac{G_N M_d}{r} \left[1 - \exp(r / b_d)\right]$$

GC parameters fixed to standard values, but NFW slope varied
what do we care about?

• basic comparison → GC excess signal is near dSph exclusion
• DM velocities in dSphs about 10× smaller than in GC
• dSph
  – signal enhanced significantly for Sommerfeld-enhancement
  – different for different dSphs
• Galactic Center
  – p-wave, d-wave will weaken signal from dSph relative to GC
  – morphology
    • v smaller near center due to angular momentum barrier
dSph velocity profiles

\[ V_{\text{max}} = \text{max. circ velocity, at radius } r_{\text{max}} \]

colored bands = fit from stellar velocity dispersion
\[ \sim V_{\text{max}} \propto r_{\text{max}}^{1/2} \]

gray bands = fit to Aquarius (Martinez, Bullock, Kaplinghat, Strigari, Trotta 0902.4715)
dSph $J_S$

$\varepsilon_\phi \equiv m_\phi / \alpha_\chi m_\chi$

Resonances

Reticulum II

Coulomb $\propto \alpha_\chi = 10^{-2}$

Non-enhanced

$\Delta \Omega = 2.4 \times 10^{-4}$

$J_S [\text{GeV}^2/\text{cm}^3]$ vs $\varepsilon_\phi$ for different regions.
upshot

• ordering of $J_S$-factors can change between “ordinary” s-wave limit and Sommerfeld-enhanced Coulomb limit
• affects how we would interpret any gamma-ray excess
• suppose we see an excess in a dwarf
  – ask if an excess is seen in other dwarfs with larger $J$-factors, where you expect a larger flux
  – if not, would call into question the dark matter interpretation
  – but using $J_S$-factor may resolve the tension

• applications extend to any new dwarfs which are found
  – potential to find excesses in new dwarfs
  – important part of analysis of dark matter interpretation
Galactic Center $J_S$

- velocity-suppressed cross sections decrease angular distribution within inner $1^\circ$, and at large angle
- increase at $\sim O(10^\circ)$ ($\sim 10$-$15\%$ effect)
- not degenerate with changes to the inner slope
- upshot – morphology can constrain velocity-dependence of signal from GC

$\rho_s^{NFW} = 8 \times 10^6 M_\odot / \text{kpc}^3$
$\gamma = 0.6, 1, 1.2$
$r_s^{NFW} = 20 \text{kpc}$

fraction of flux from inner $1^\circ$ (NFW)
- s-wave $\rightarrow \sim 3.1\%$
- p-/d-wave $\rightarrow \sim 2\%$
- s-wave ($\gamma = 1.2$) $\rightarrow \sim 10\%$
- p-wave ($\gamma = 1.4$) $\rightarrow \sim 14\%$

but steeper profiles suppressed far away
conclusion

• J-factors of astrophysical objects change dramatically if dark matter annihilation is velocity-dependent
• relative importance of dSphs can change
• modifies standard consistency check for dark matter interpretation of an excess

• for GC, morphology of dark matter signal changes
• Reticulum II? new dSphs? Galactic Center excess?
Back-up slides
idea behind Eddington formalism

• velocity distribution \( f(r,v) \) is essentially the phase space density
• assume particles move only under a collective gravitational central potential (not two-body scattering)
• classical path depends only on integrals of motion, \( E \) and \( L \)
• Jean’s Theorem – phase space distribution depends only on integrals of motion --> why?
  – if two phase space points have the same integrals of motion, any particles at one point will be (or once were) at the other
  – phase space density along path is constant (Liouville’s Theorem)
  – so time-averaged phase space density has to be a function on only the integrals of motion
• if velocity distribution is spherically symmetric (depends on \( r \), not \( r \)) and isotropic (depends on \( v \), not \( v \)), then velocity distribution depends only on \( E \), not \( L \)
J-factors - DM only
Ratios
GC parameters

- NFW
  - $\gamma = 0.6, 1, 1.2, 1.4$
  - $b = 1, c = 3$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_s^\text{NFW}$</td>
<td>$8 \times 10\ M_\odot / \text{kpc}^3$</td>
</tr>
<tr>
<td>$r_s^\text{NFW}$</td>
<td>20 kpc</td>
</tr>
<tr>
<td>$M_b$</td>
<td>$1.5 \times 10^{10} \ M_\odot$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>0.6 kpc</td>
</tr>
<tr>
<td>$M_d$</td>
<td>$7 \times 10^{10} \ M_\odot$</td>
</tr>
<tr>
<td>$b_d$</td>
<td>4 kpc</td>
</tr>
</tbody>
</table>

\[
\rho^\text{NFW}(r) = \frac{\rho_s^\text{NFW}}{\left( \frac{r}{r_s^\text{NFW}} \right)^\gamma \left( 1 + \left( \frac{r}{r_s^\text{NFW}} \right)^b \right)^{\frac{c-\gamma}{b}}} \]
Plummer profile

- fit $M$ and $a$ from stellar data
- $r_h \sim 1.3 \ a = \text{half-light radius}$
- from Eddington formula, stellar velocity dispersion now depends on NFW parameters, $\rho_s$ and $r_s$
- matching to stellar velocity dispersion to data determines an allowed band for $r_{\text{max}}, V_{\text{max}}$

$$\rho_p(r) = \left( \frac{3M}{4\pi a^3} \right) \left( 1 + \frac{r^2}{a^2} \right)^{-\frac{5}{2}}$$
Hulthen potential

\[ V_H(r) = -\frac{\alpha_x \left( \pi^2 m_\phi / 6 \right) e^{-(\pi^2 m_\phi / 6)r}}{1 - e^{-(\pi^2 m_\phi / 6)r}} - \alpha_x e^{-m_\phi r} \]

\[ S(v) = \frac{\pi}{\varepsilon_v} \frac{\sinh \left( \frac{2\pi \varepsilon_v}{\pi^2 \varepsilon_\phi / 6} \right)}{\cosh \left( \frac{2\pi \varepsilon_v}{\pi^2 \varepsilon_\phi / 6} \right) - \cos \left( 2\pi \sqrt{\frac{1}{\pi^2 \varepsilon_\phi / 6} - \frac{\varepsilon_v^2}{(\pi^2 \varepsilon_\phi / 6)^2}} \right)} \]

\[ \varepsilon_v \equiv \frac{v}{2\alpha_x} \]

\[ \varepsilon_\phi \equiv \frac{m_\phi}{\alpha_x m_x} \]

Cassel, 0903.5307
a general analysis

- say \( \rho(r) = \rho_s \tilde{\rho}(r/r_s) \)
- say we integrate J-factor over essentially **entire** dwarf
- \( J, J_s \)-factors **parametrically** determined by dimen. analysis
  - \( V_{\text{max}} \propto (G_N \rho_s)^{1/2} r_s \) (Virial Thm)
- \( J \propto \rho_s^2 r_s^3 / D^2 \propto V_{\text{max}}^4 / r_{\text{max}} D^2 \)
- \( J_s \propto \rho_s^{3/2} r_s^2 / D^2 \propto V_{\text{max}}^3 / r_{\text{max}} D^2 \)
  - Coulomb limit
- if one point is to upper left of another, \( J-J_s \) ordering changes
- valid in the large angle limit, but instructive even for fixed angle

\[ D = \text{distance to dwarf} \]
determining $f(r,v)$

- **strategy**
  - assume NFW profile
    - just need parameters
    - fixes gravitational potential
  - assume Plummer stellar profile
    - find stellar velocity dispersion using Eddington formula and NFW gravitational potential
  - find NFW parameters by matching stellar velocity relation, Aquarius $V_{\text{max}}-r_{\text{max}}$ relation
  - now Eddington formula determines DM velocity distribution

\[
\rho_{\text{NFW}}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1 + \frac{r}{r_s}\right)^2}
\]

\[
\psi_{\text{NFW}}(r) = -4\pi G r^3 \rho_s \frac{r_s}{r} \ln\left(1 + \frac{r}{r_s}\right)
\]

\[
f(\varepsilon) = \frac{1}{\sqrt{8\pi^2}} \int_{\varepsilon}^{0} \frac{d\psi}{\sqrt{\varepsilon - \psi}} \frac{d^2\rho}{d\psi^2}
\]

\[
\varepsilon \equiv \frac{v^2}{2} + \psi(r) < 0
\]

\[
\rho(r) = 4\pi \int_{0}^{\sqrt{-2\psi(r)}} dv \ v^2 f(r,v)
\]
Sommerfeld-enhancement

• essential setup
  – dark matter annihilation is a contact interaction
  – but dark matter self-interacts through a long range force
    • mediator mass = $m_\Phi$
  – so have to rescale matrix element by wavefunction at the origin

• actual potential is Yukawa
  – can solve numerically
  – but can solve analytically if we approximate it with a Hulthén potential (within 10%)

• $\langle \sigma_A v \rangle \equiv \langle \sigma_A v \rangle_0 \times S(v)$
• $V(r) = -(\alpha_X / r) \exp(-m_\Phi r)$
• four regimes for Hulthén
• $m_\Phi \gg \alpha_X m_X$ : non-enhanced
  – $S = 1$
• $m_\Phi \ll \alpha_X m_X : \text{Coulomb limit}$
  – $S(v) = 2\pi\alpha_X / v$
• $\alpha_X m_X \ll m_\Phi \ll \alpha_X m_X : \text{saturation}$
  – $S(v) = 16 \alpha_X m_X / m_\Phi$
• $m_\Phi = 6\alpha_X m_X/(\pi^2 n^2) \ll \alpha_X m_X : \text{resonance}$
  – $S = 4\alpha_X^2 / v^2 n^2$ (cutoff at small v)
• focus: non-enhanced v. Coulomb
resonances

Coulomb $\propto \alpha_x=10^{-2}$

non-enhanced

$\Delta \Omega=2.4 \times 10^{-4}$

dSph $J_S$

$\varepsilon_\phi \equiv m_\phi/\alpha_x m_x$

Coulomb $\propto \alpha_x=10^{-2}$

non-enhanced

$\Delta \Omega=2.4 \times 10^{-4}$