J-Factors for Velocity-Dependent Dark Matter Annihilation

Jason Kumar

University of Hawaii

(PRD **95**, 123008 (2017) [1702.00408], 180x.xxxxx)

collaborators

- Kimberly Boddy
- Louie Strigari
- Mei-Yu Wang

effective J-factors and v-dependence

- prompt photon flux from dark matter annihilation can be factorized into two pieces....
- ... a particle physics factor
  - depends on annihilation cross section, annihilation channel, particle mass
- and an astrophysics factor
  - depends on the dark matter density profile of the target
  - encoded in the J-factor
- but if dark matter annihilation is velocity-dependent
  - then velocity-distribution also come into play

- goal is to compute the effective J-factor (J_S) ...
- ... and see impact on gamma-ray searches for dark matter

main features

- Sommerfeld-enhanced annihilation → $\sigma_A v \propto 1/v$ (Coulomb limit)
  - relative velocities in dSphs tend to be much smaller than in Milky Way halo
  - can get a larger enhancement to annihilation cross section
  - considerable variation in velocity distribution between dSphs, and for different choices of density profile
  - affects which dwarfs are most important for dark matter search
  - see also 1804.05052 (Patec, Ullio, Valli), 1712.03188 (Bergstrom, et al.)

- p-wave or d-wave annihilation suppresses dSph relative to GC
  - weakens dSph constraints on dark matter explanation GC excess
  - changes morphology of expected GC dark matter signal

- important implications for indirect detection searches

what is the J-factor?

- the photon flux depends on ....
  - particle physics of the dark matter model
    - independent of target
  - astrophysics of the target
    - mostly independent of dark matter model
- J-factor is the astrophysics factor
  - larger J = larger flux, regardless of particles physics model
- but factorization based on an assumption
  - $\sigma_A v$ independent of v
- what happens for v-dependent annihilation?

$$\frac{d\Phi}{dE} = \frac{1}{4\pi}\frac{dN}{dE}\int_{\Delta\Omega} d\Omega \int d\ell$$

$$\int d^3v_1 \frac{f(\vec{r}(\ell,\Omega),\vec{v}_1)}{m_X}\int d^3v_2 \frac{f(\vec{r}(\ell,\Omega),\vec{v}_2)}{m_X}$$

$$\times \frac{\sigma_A |\vec{v}_1 - \vec{v}_2|}{2}$$

$$= \frac{\langle \sigma_A v \rangle}{8\pi m_X^2}\frac{dN}{dE} \times J$$

$$J \equiv \int_{\Delta\Omega} d\Omega \int d\ell \left[\rho(\vec{r}(\ell,\Omega))\right]^2$$

$$\rho(\vec{r}) = \int d^3v\, f(\vec{r},\vec{v})$$

f = dark matter velocity distribution

defining J_S

- just need to absorb S(v) into definition of astrophysical factor
- new factor, J_S, encodes astro. info needed to determine dΦ/dE for velocity-dependent case
- need the DM velocity distribution
  - get it from density distribution, using Eddington formula
- what it amounts to:
  - assume f(r,v) spherically-sym., isotropic
  - then f depends only on $\varepsilon = v^2/2 + \Psi(r)$
  - ρ(r) determines f(r,v)

$$\sigma_A v = (\sigma_A v)_0 \times S(v)$$

$$J_S \equiv \int_{\Delta\Omega} d\Omega \int d\ell \int d^3v_1 f(\vec{r}(\ell,\Omega),\vec{v}_1) \int d^3v_2 f(\vec{r}(\ell,\Omega),\vec{v}_2) \times S(|\vec{v}_1 - \vec{v}_2|)$$

$$\frac{d\Phi}{dE} = \frac{(\sigma_A v)_0}{8\pi m_X^2}\frac{dN}{dE} \times J_S$$

Ψ (r) = gravitational potential

determining f(r,v)

- strategy
  - ansatz for DM density distrib.
  - fixes gravitational potential $\Psi(r)$
    - assume a spherically symmetric effective potential from GC bulge, disk (1211.7063, Pato, Strigari, Trotta, Bertone)
  - now Eddington formula determines velocity distribution
  - for GC, we pick some generalized NFW profiles
  - for dSph, assume NFW, but need to fix two parameters
  - fix one by matching stellar velocity dispersion, the other with Aquarius V_{max}-r_{max} relation

$$f(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\int_\varepsilon^0 \frac{d\psi}{\sqrt{\varepsilon-\psi}}\frac{d^2\rho}{d\psi^2}$$

$$\varepsilon \equiv \frac{v^2}{2} + \psi(r) < 0$$

$$\rho(r) = 4\pi\int_0^{\sqrt{-2\psi(r)}} dv\, v^2 f(r,v)$$

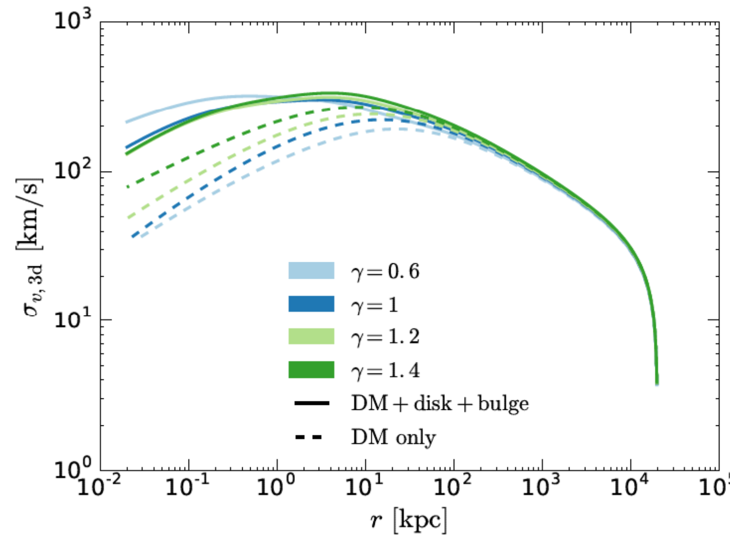
$$\Psi_{bulge} = -\frac{G_N M_b}{r+c_0}$$

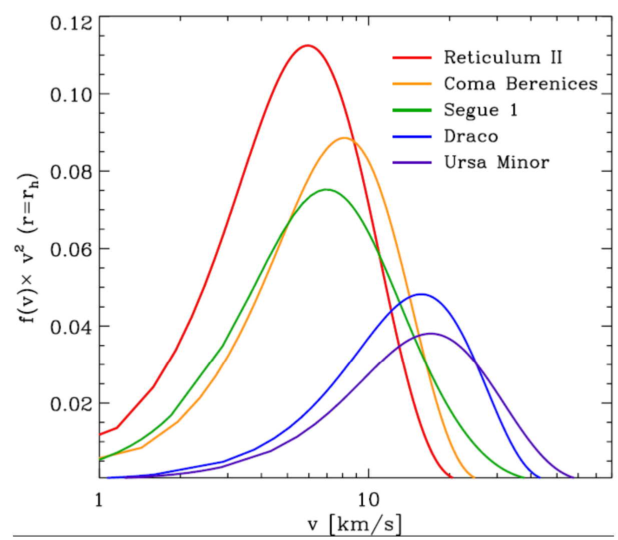
$$\Psi_{disk} = -\frac{G_N M_d}{r}\left[1-\exp(r/b_d)\right]$$

GC parameters fixed to standard values, but NFW slope varied

what do we care about?

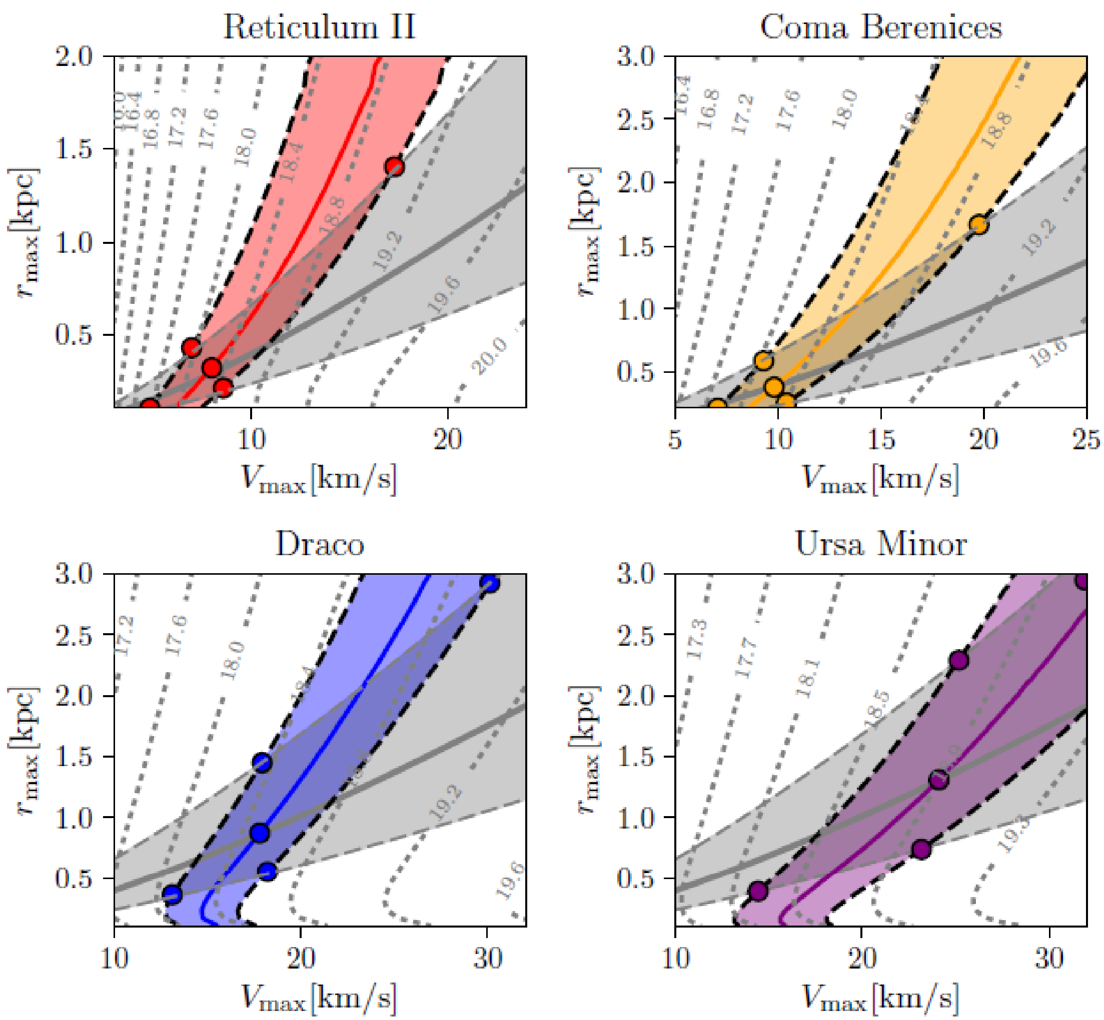
- basic comparison → GC excess signal is near dSph exclusion
- DM velocities in dSphs about 10× smaller than in GC
- dSph
  - signal enhanced significantly for Sommerfeld-enhancement
  - different for different dSphs
- Galactic Center
  - p-wave, d-wave will weaken signal from dSph relative to GC
  - morphology
    - v smaller near center due to angular momentum barrier





dSph velocity profiles

V_{max} = max. circ velocity, at radius r_{max}



colored bands = fit from stellar velocity dispersion

$\sim V_{max} \propto r_{max}^{1/2}$

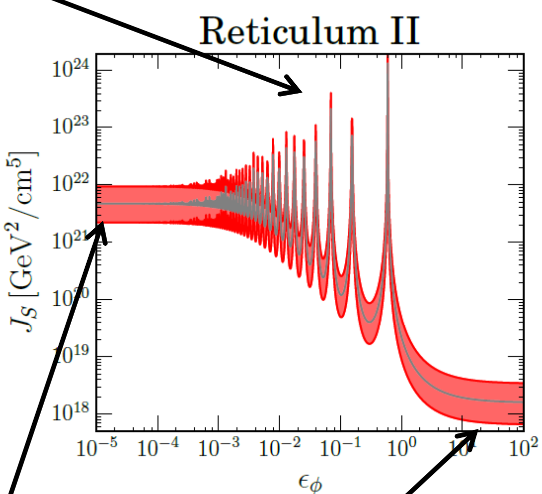
gray bands = fit to Aquarius

(Martinez, Bullock, Kaplinghat, Strigari, Trotta 0902.4715)

dSph J_S

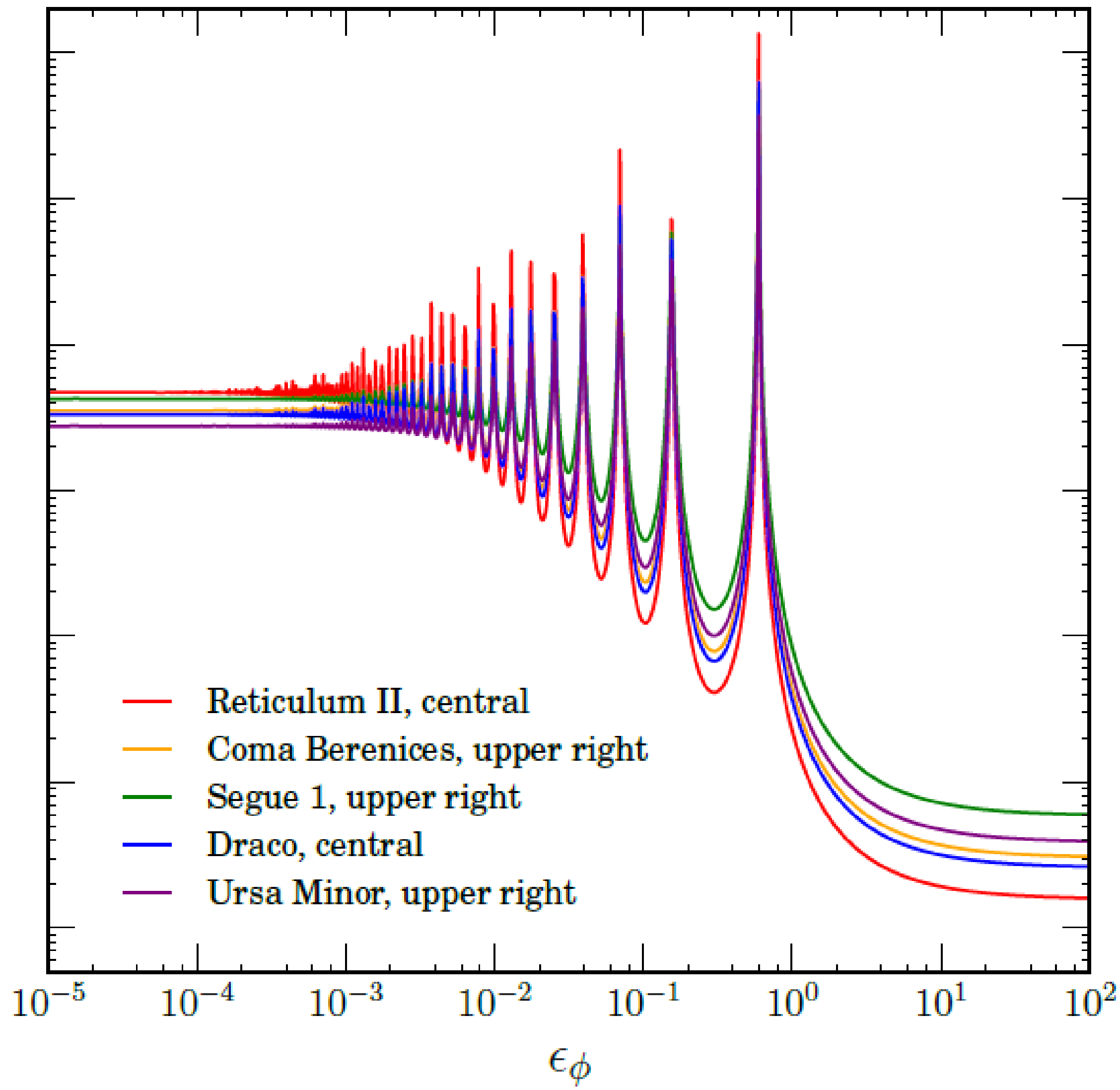
$\epsilon_\phi \equiv m_\phi / \alpha_\chi m_\chi$

resonances



Coulomb
$\propto \alpha_\chi = 10^{-2}$

non-enhanced



$\Delta\Omega = 2.4 \times 10^{-4}$

upshot

- ordering of J_S-factors can change between “ordinary” s-wave limit and Sommerfeld-enhanced Coulomb limit
- affects how we would interpret any gamma-ray excess
- suppose we see an excess in a dwarf
  - ask if an excess is seen in other dwarfs with larger J-factors, where you expect a larger flux
  - if not, would call into question the dark matter interpretation
  - but using J_S-factor may resolve the tension

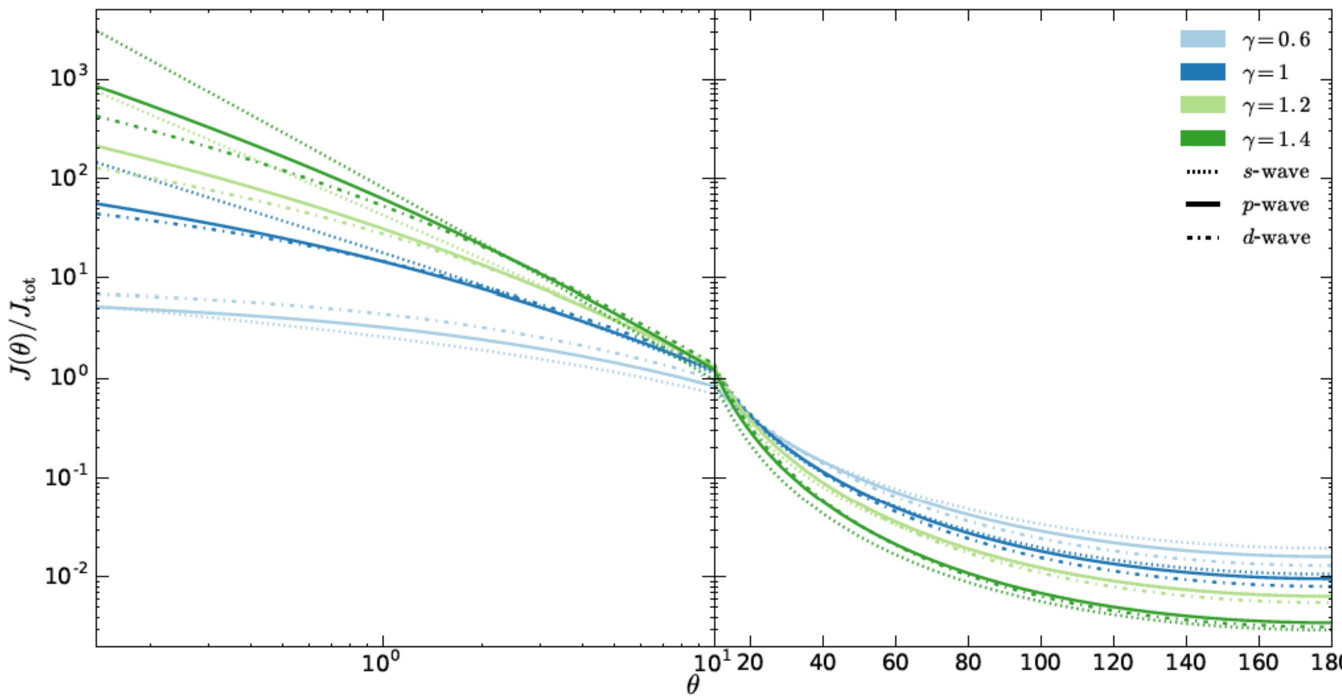
- applications extend to any new dwarfs which are found
  - potential to find excesses in new dwarfs
  - important part of analysis of dark matter interpretation

Galactic Center J_S

- velocity-suppressed cross sections decrease angular distribution within inner 1°, and at large angle
- increase at ~ $\mathcal{O}$(10°) (~ 10-15% effect)
- not degenerate with changes to the inner slope
- upshot – morphology can constrain velocity-dependence of signal from GC

$\rho_s^{NFW} = 8 \times 10^6 \, M_\odot / kpc^3$
$r_s^{NFW} = 20$ kpc
$\gamma = 0.6, 1, 1.2$

fraction of flux from inner 1° (NFW)
s-wave → ~ 3.1%
p-/d-wave → ~ 2%
s-wave ($\gamma = 1.2$)→~10%
p-wave ($\gamma = 1.4$)→~14%
but steeper profiles suppressed far away



conclusion

- J-factors of astrophysical objects change dramatically if dark matter annihilation is velocity-dependent
- relative importance of dSphs can change
- modifies standard consistency check for dark matter interpretation of an excess

- for GC, morphology of dark matter signal changes
- Reticulum II? new dSphs? Galactic Center excess?

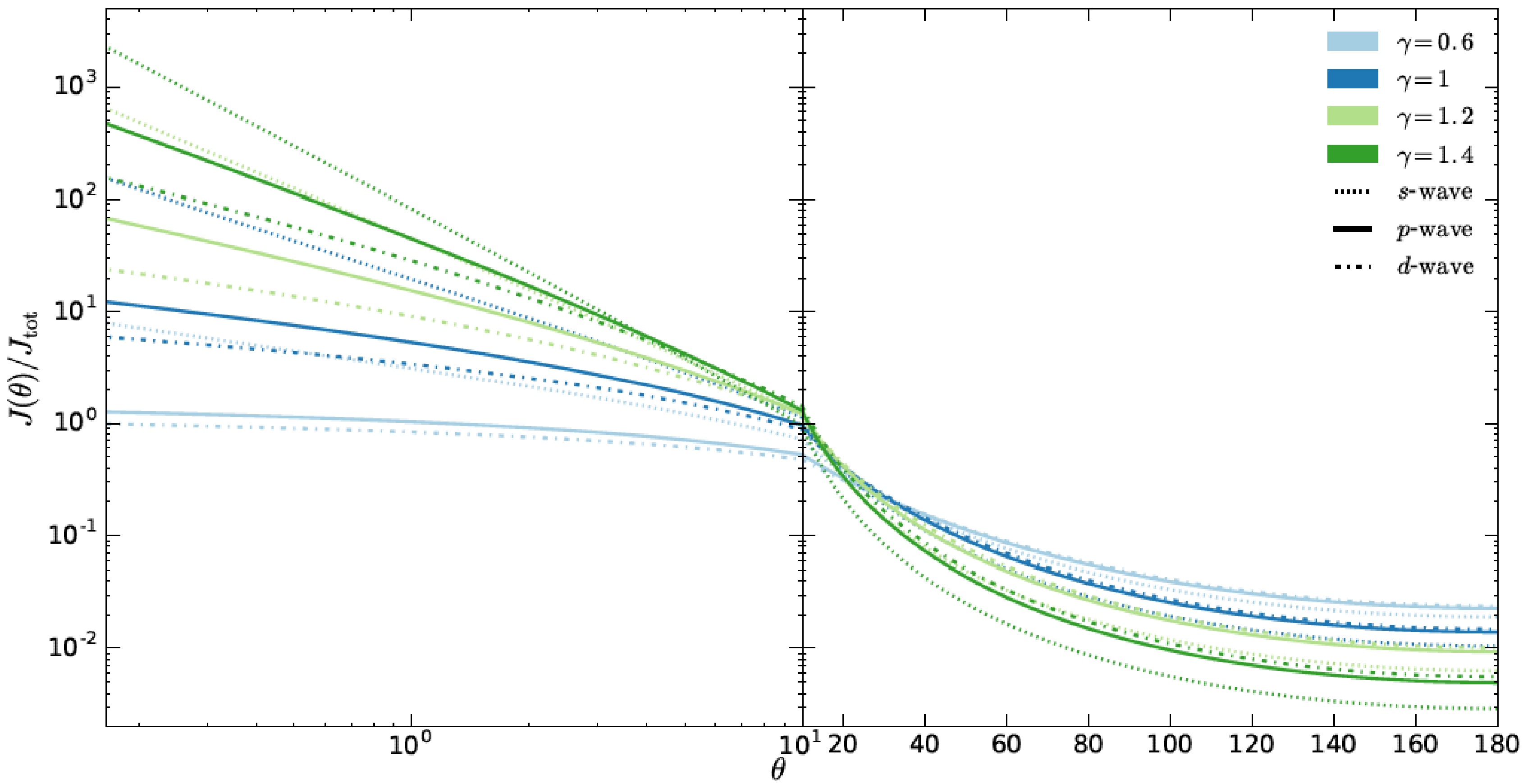
Mahalo!

Back-up slides

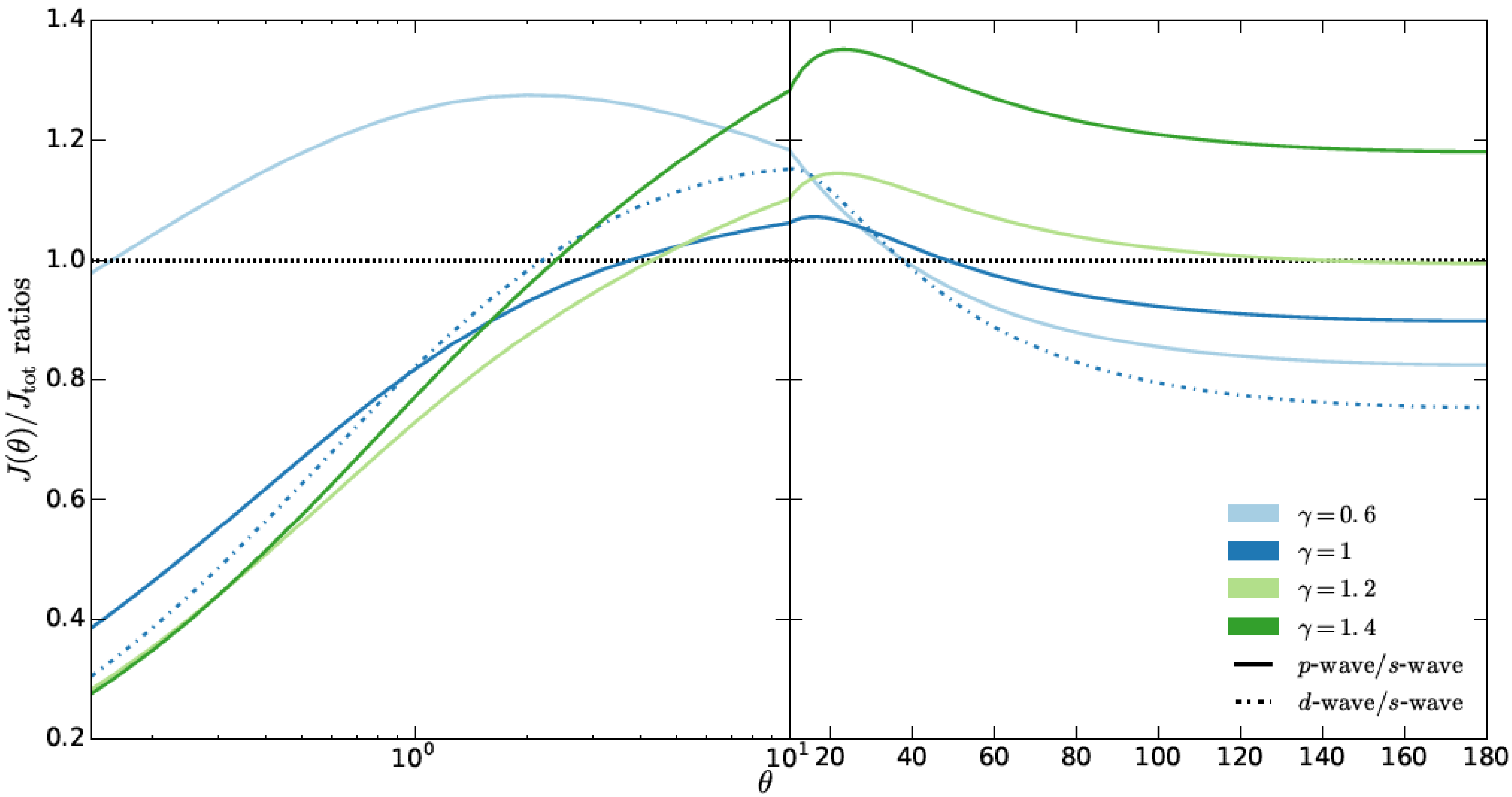
idea behind Eddington formalism

- velocity distribution f(**r**,**v**) is essentially the phase space density
- assume particles move only under a collective gravitational central potential (not two-body scattering)
- classical path depends only on integrals of motion, E and **L**
- Jean's Theorem – phase space distribution depends only on integrals of motion --> why?
  – if two phase space points have the same integrals of motion, any particles at one point will be (or once were) at the other
  – phase space density along path is constant (Liouville's Theorem)
  – so time-averaged phase space density has to be a function on only the integrals of motion
- if velocity distribution is spherically symmetric (depends on r, not **r**) and isotropic (depends on v, not **v**), then velocity distribution depends only on E, not **L**

J-factors - DM only



Ratios



1.4
1.2
1.0
0.8
0.6
0.4
0.2
$J(\theta)/J_{\rm tot}$ ratios
10^0
10^1
20
40
60
80
100
120
140
160
180
θ
$\gamma = 0.6$
$\gamma = 1$
$\gamma = 1.2$
$\gamma = 1.4$
p-wave/s-wave
d-wave/s-wave

GC parameters

- NFW
  - γ=0.6, 1, 1.2, 1.4
  - b=1, c=3

ρ_s^{NFW}	$8 \times 10\ M_\odot / kpc^3$
r_s^{NFW}	20 kpc
M_b	$1.5 \times 10^{10}\ M_\odot$
c_0	0.6 kpc
M_d	$7 \times 10^{10}\ M_\odot$
b_d	4 kpc

$$\rho^{NFW}(r) = \frac{\rho_s^{NFW}}{\left(\frac{r}{r_s^{NFW}}\right)^{\gamma} \left(1 + \left(\frac{r}{r_s^{NFW}}\right)^{b}\right)^{\frac{c-\gamma}{b}}}$$

Plummer profile

- fit M and a from stellar data
- $r_h \sim 1.3\ a$ = half-light radius
- from Eddington formula, stellar velocity dispersion now depends on NFW parameters, ρ_s and r_s
- matching to stellar velocity dispersion to data determines an allowed band for r_{max}, V_{max}

$$\rho_P(r) = \left(\frac{3M}{4\pi a^3}\right)\left(1 + \frac{r^2}{a^2}\right)^{-\frac{5}{2}}$$

Hulthen potential

$$V_H(r) = -\frac{\alpha_X \left(\pi^2 m_\phi / 6\right) e^{-\left(\pi^2 m_\phi / 6\right) r}}{1 - e^{-\left(\pi^2 m_\phi / 6\right) r}} \simeq -\frac{\alpha_X e^{-m_\phi r}}{r}$$

$$S(v) = \frac{\pi}{\varepsilon_v} \frac{\sinh\left(\dfrac{2\pi\varepsilon_v}{\pi^2 \varepsilon_\phi / 6}\right)}{\cosh\left(\dfrac{2\pi\varepsilon_v}{\pi^2 \varepsilon_\phi / 6}\right) - \cos\left(2\pi \sqrt{\dfrac{1}{\pi^2 \varepsilon_\phi / 6} - \dfrac{\varepsilon_v^2}{\left(\pi^2 \varepsilon_\phi / 6\right)^2}}\right)}$$

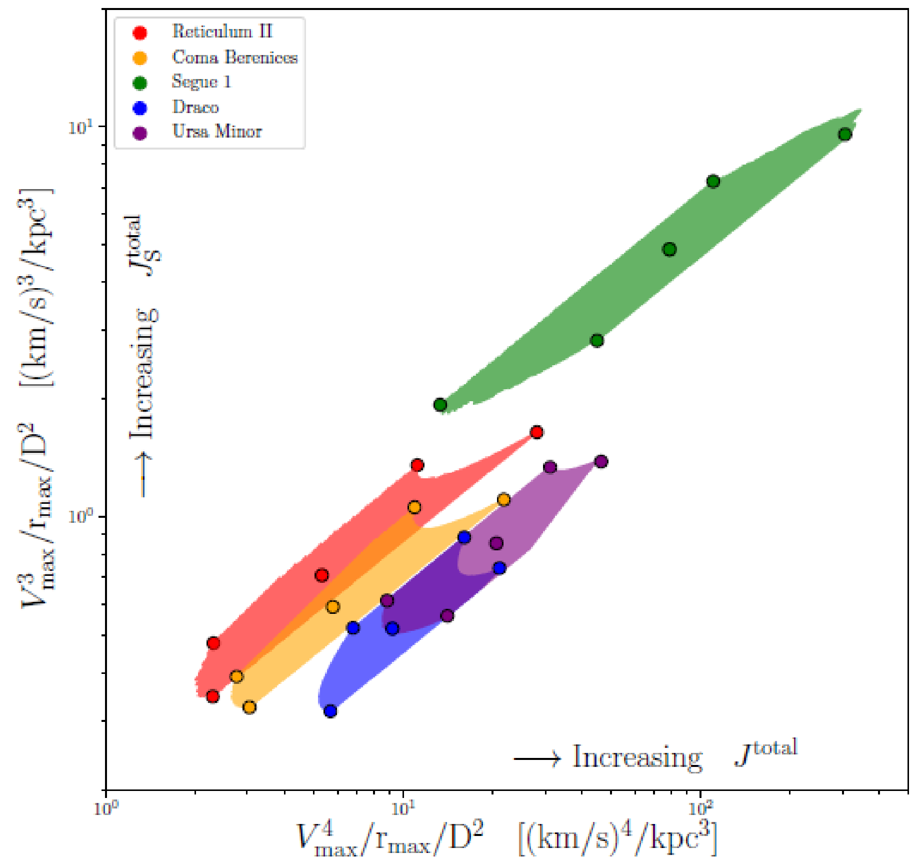
$$\varepsilon_v \equiv \frac{v}{2\alpha_X}$$

$$\varepsilon_\phi \equiv \frac{m_\phi}{\alpha_X m_X}$$

Cassel, 0903.5307

a general analysis

- say $\rho(r) = \rho_S\, \tilde{\rho}(r / r_s)$
- say we integrate J-factor over essentially entire dwarf
- J, J_S-factors parametrically determined by dimen. analysis
  - $V_{max} \propto (G_N\, \rho_s)^{1/2}\, r_s$ (Virial Thm)
- $J \propto \rho_s^2\, r_s^3 / D^2 \propto V_{max}^4 / r_{max} D^2$
- $J_S \propto \rho_s^{3/2}\, r_s^2 / D^2 \propto V_{max}^3 / r_{max} D^2$
  - Coulomb limit
- if one point is to upper left of another, J-J_S ordering changes
- valid in the large angle limit, but instructive even for fixed angle



D = distance to dwarf

determining f(r,v)

- strategy
  - assume NFW profile
    - just need parameters
    - fixes gravitational potential
  - assume Plummer stellar profile
    - find stellar velocity dispersion using Eddington formula and NFW gravitational potential
  - find NFW parameters by matching stellar velocity relation, Aquarius V_{max} -r_{max} relation
  - now Eddington formula determines DM velocity distribution

$$\rho_{NFW}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$$

$$\psi_{NFW}(r) = -4\pi G_N \rho_s \frac{r_s^3}{r} \ln\left(1+\frac{r}{r_s}\right)$$

$$f(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\int_\varepsilon^0 \frac{d\psi}{\sqrt{\varepsilon-\psi}} \frac{d^2\rho}{d\psi^2}$$

$$\varepsilon \equiv \frac{v^2}{2} + \psi(r) < 0$$

$$\rho(r) = 4\pi\int_0^{\sqrt{-2\psi(r)}} dv\, v^2 f(r,v)$$

Sommerfeld-enhancement

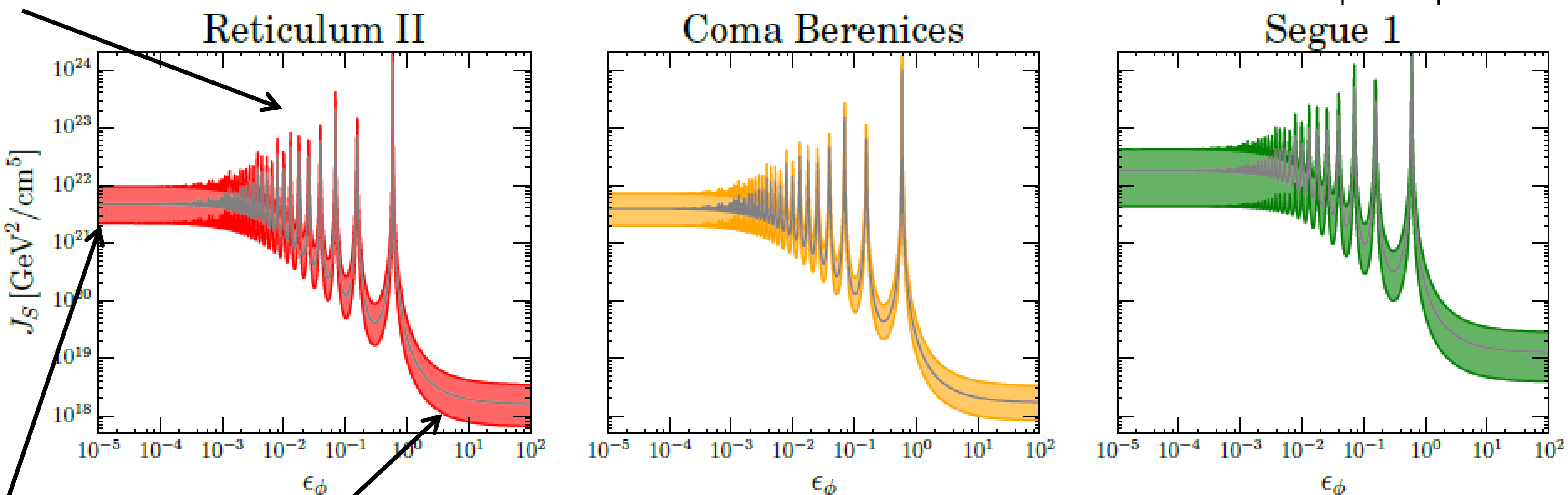
- essential setup
  - dark matter annihilation is a contact interaction
  - but dark matter self-interacts through a long range force
    - mediator mass = m_ϕ
  - so have to rescale matrix element by wavefunction at the origin
- actual potential is Yukawa
  - can solve numerically
  - but can solve analytically if we approximate it with a Hulthén potential (within 10%)

- $\langle \sigma_A v \rangle \equiv \langle \sigma_A v \rangle_0 \times S(v)$
- $V(r) = -(\alpha_X / r) \exp(-m_\phi r)$
- four regimes for Hulthén
- $m_\Phi \gg \alpha_X m_X$: non-enhanced
  - $S = 1$
- $m_\Phi \ll v m_X \ll \alpha_X m_X$:Coulomb limit
  - $S(v) = 2\pi\alpha_X / v$
- $v m_X \ll m_\Phi \ll \alpha_X m_X$:saturation
  - $S(v) = 16\, \alpha_X\, m_X / m_\phi$
- $m_\Phi = 6\alpha_X m_X/(\pi^2 n^2) \ll \alpha_X m_X$: resonance
  - $S = 4\alpha_X^2 / v^2 n^2$ (cutoff at small v)
- focus: non-enhanced v. Coulomb

dSph J_S

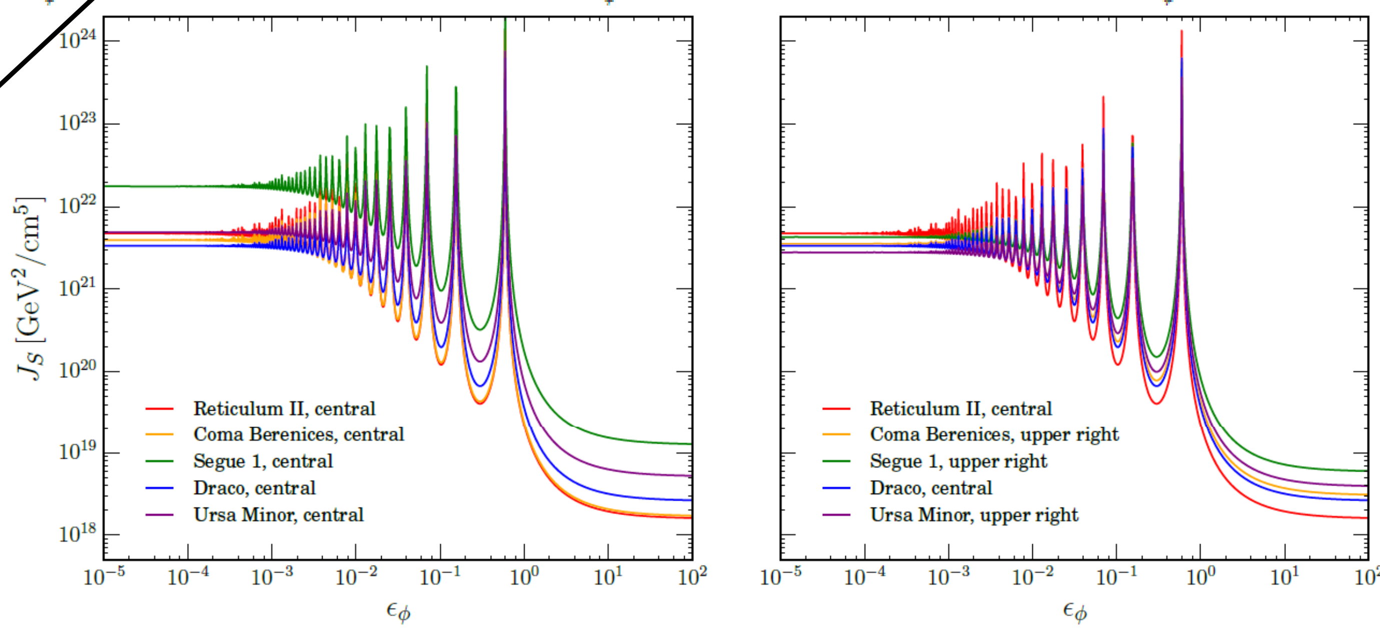
$\epsilon_\phi \equiv m_\phi/\alpha_\chi m_\chi$

resonances



Coulomb
$\propto \alpha_\chi = 10^{-2}$

non-enhanced



$\Delta\Omega = 2.4 \times 10^{-4}$