Recent progress in perturbative QCD

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Outline

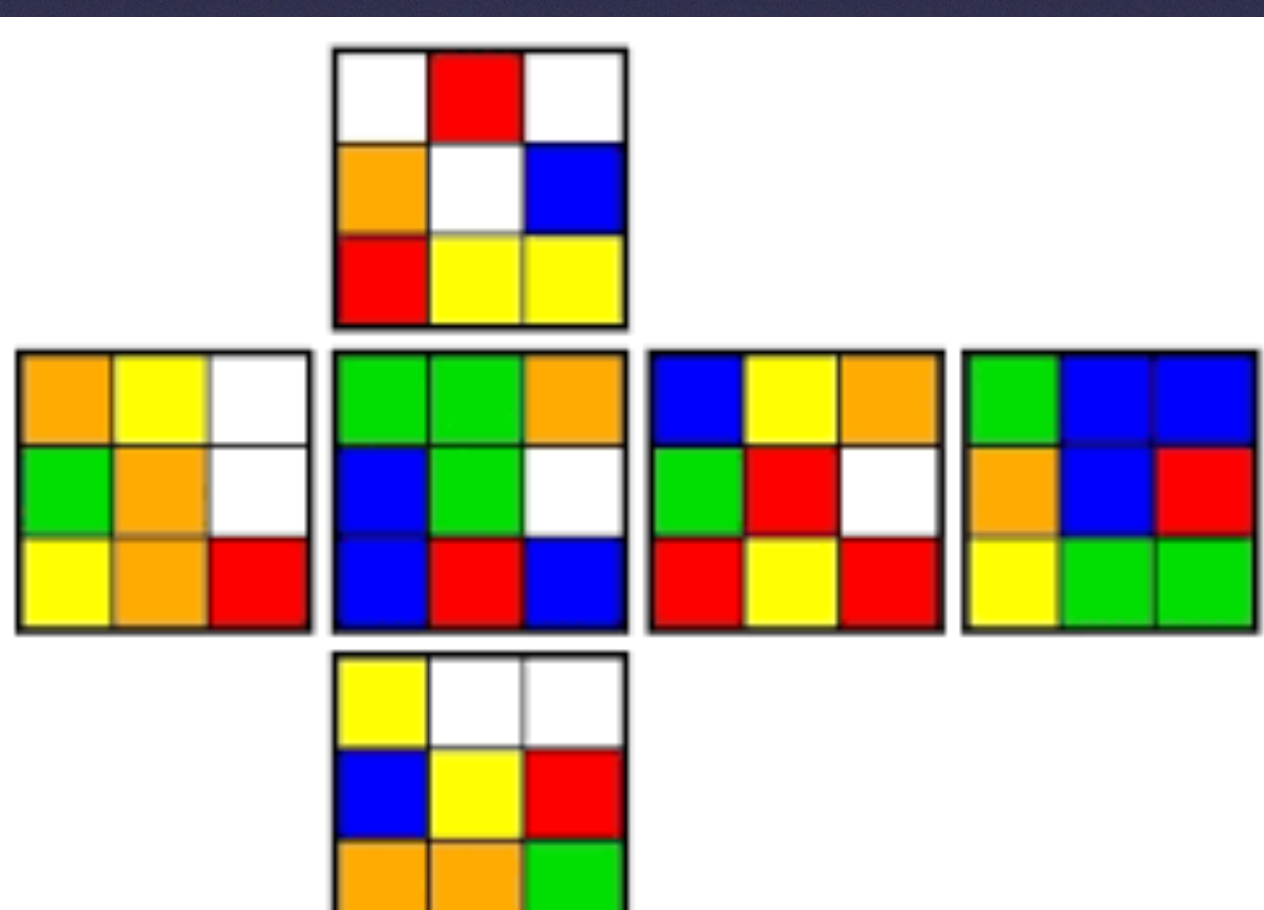
- Motivation
- General state of the art for perturbative QCD
- Examples for phenomenology
- Mathematical aspects

  - Functions in loop amplitudes
  - Reduction to master integrals
  - Numerical methods for loop amplitudes
- Wish lists?

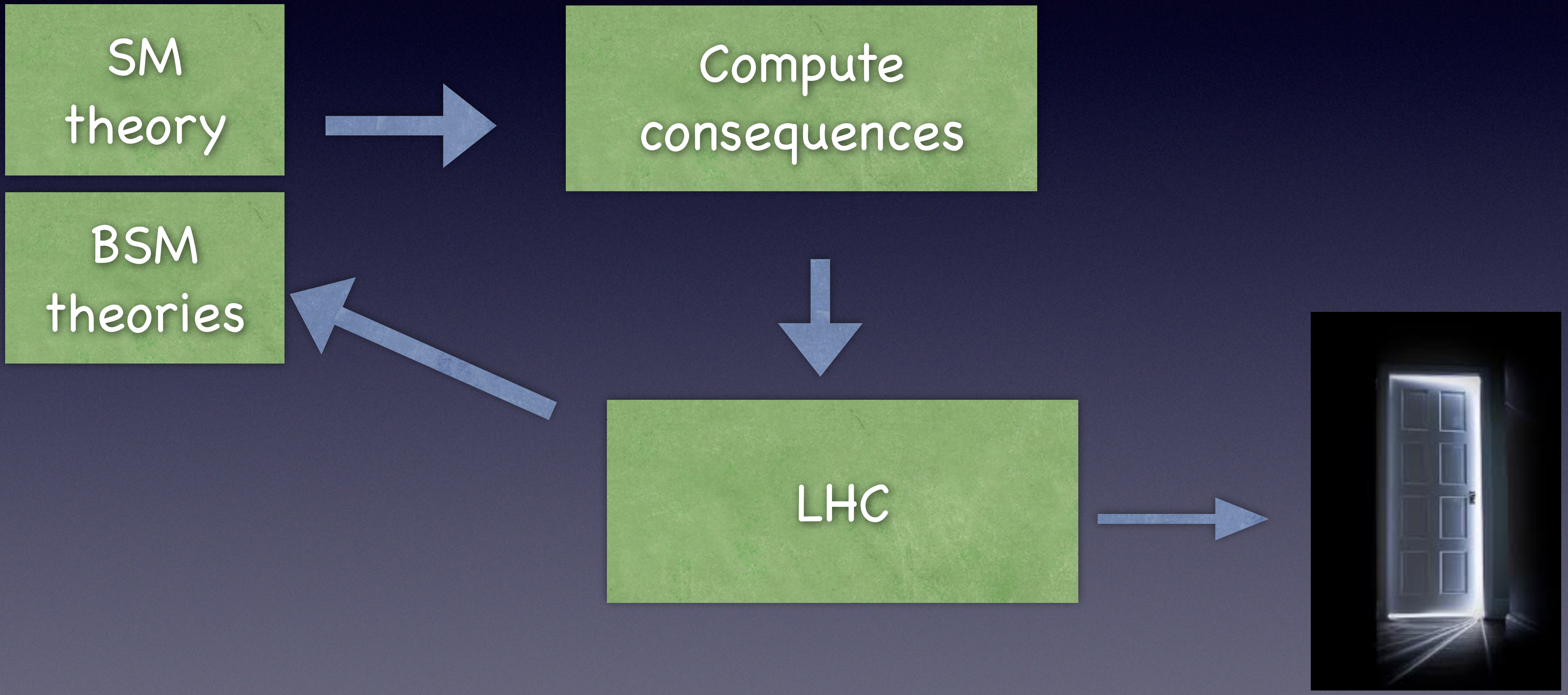
What motivates the field of perturbative QCD?

Precision Phenomenology

Mathematical Challenge

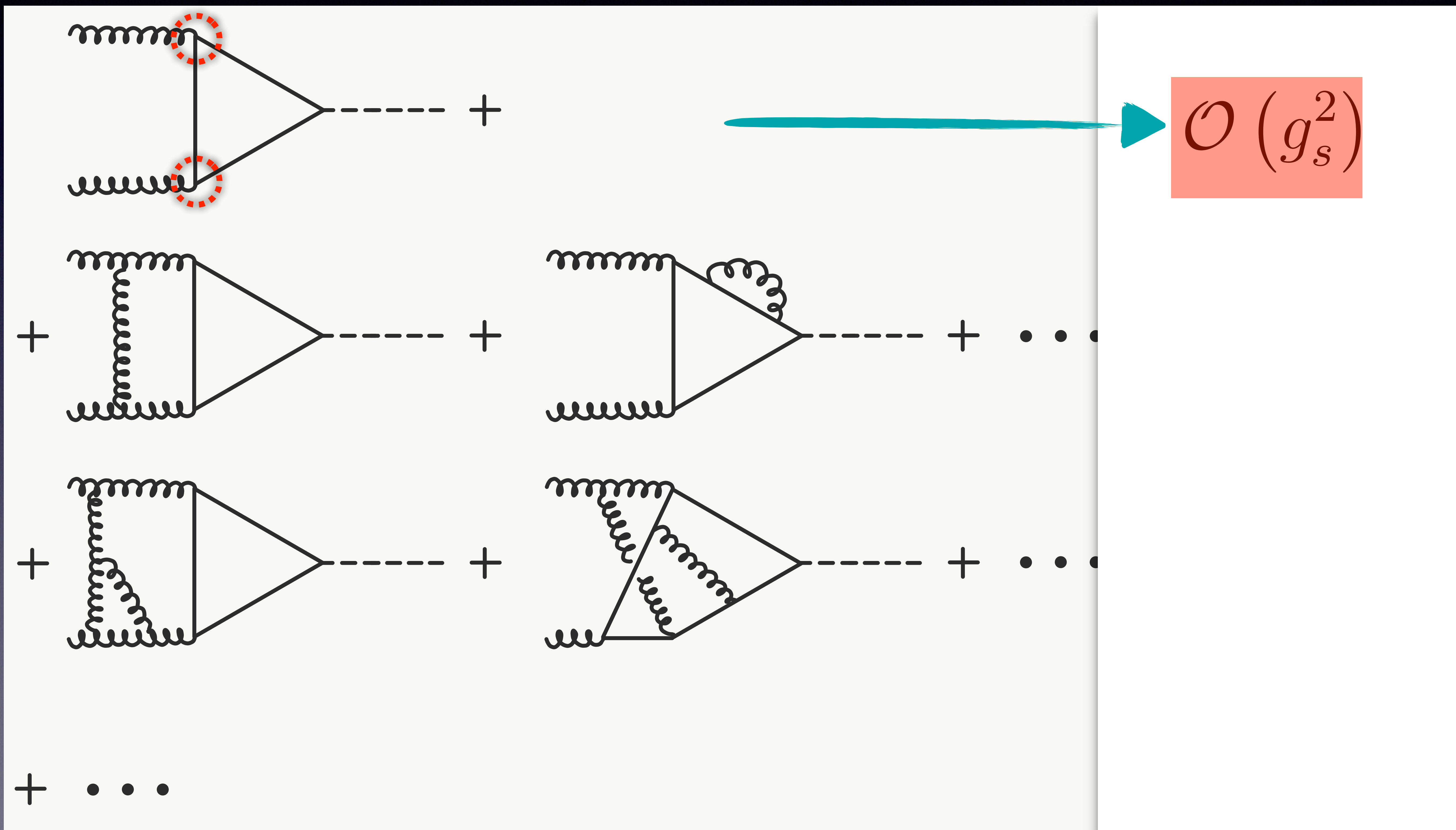


LHC precision physics

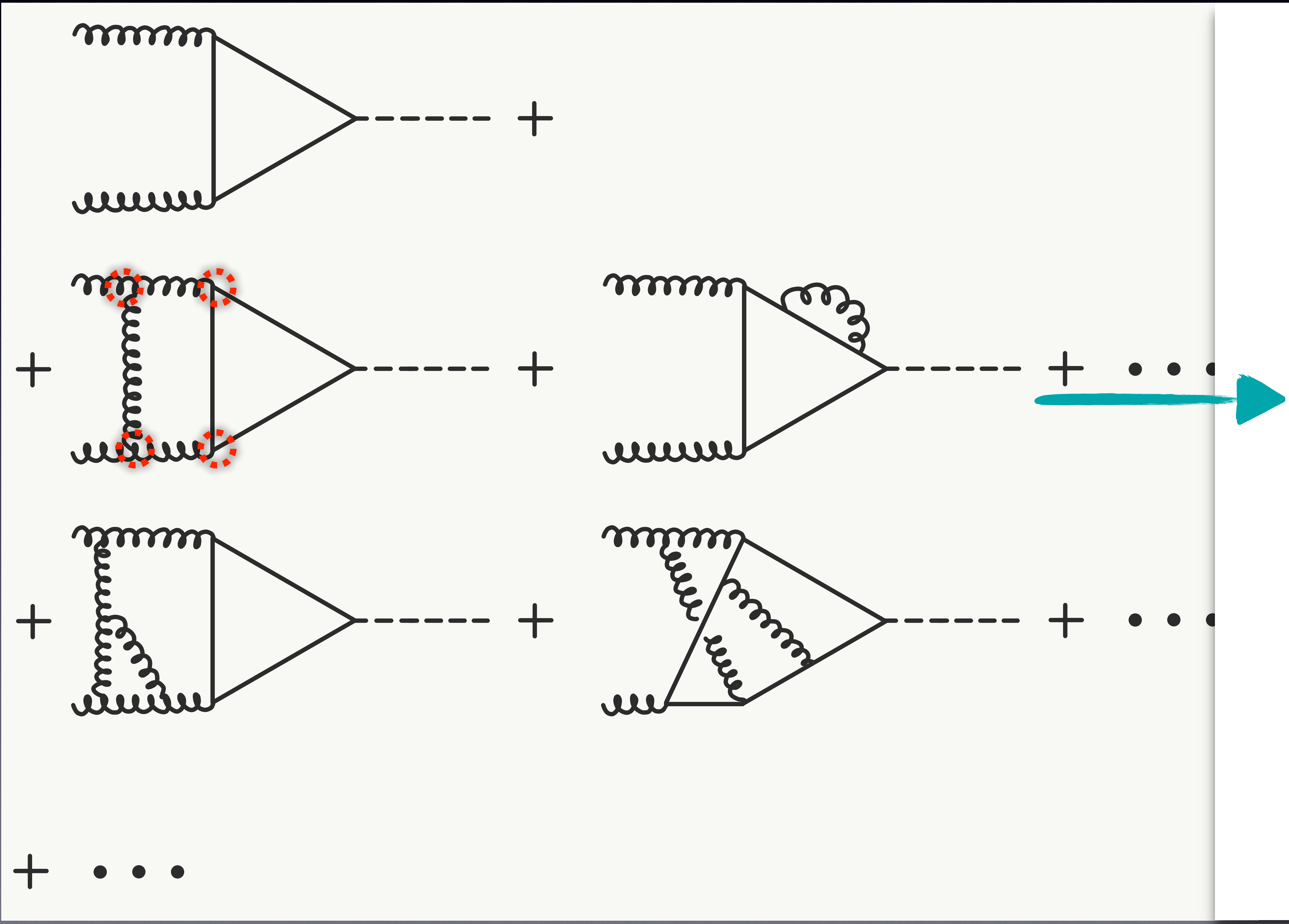


A “Feynman diagram” of finding New Laws

Perturbative expansion



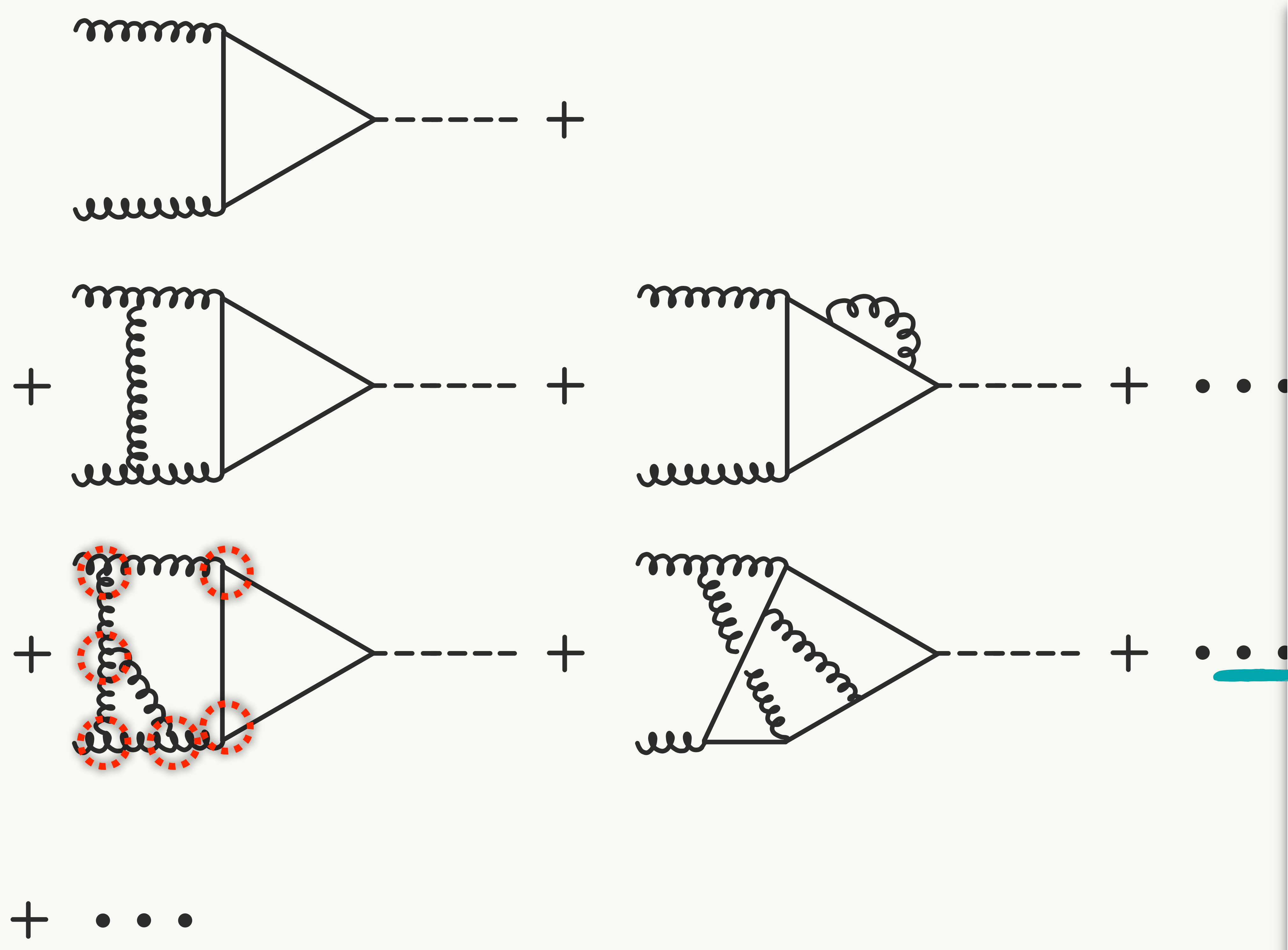
Perturbative expansion



$\mathcal{O}\left(g_s^2\right)$

$\mathcal{O}\left(g_s^4\right)$

Perturbative expansion

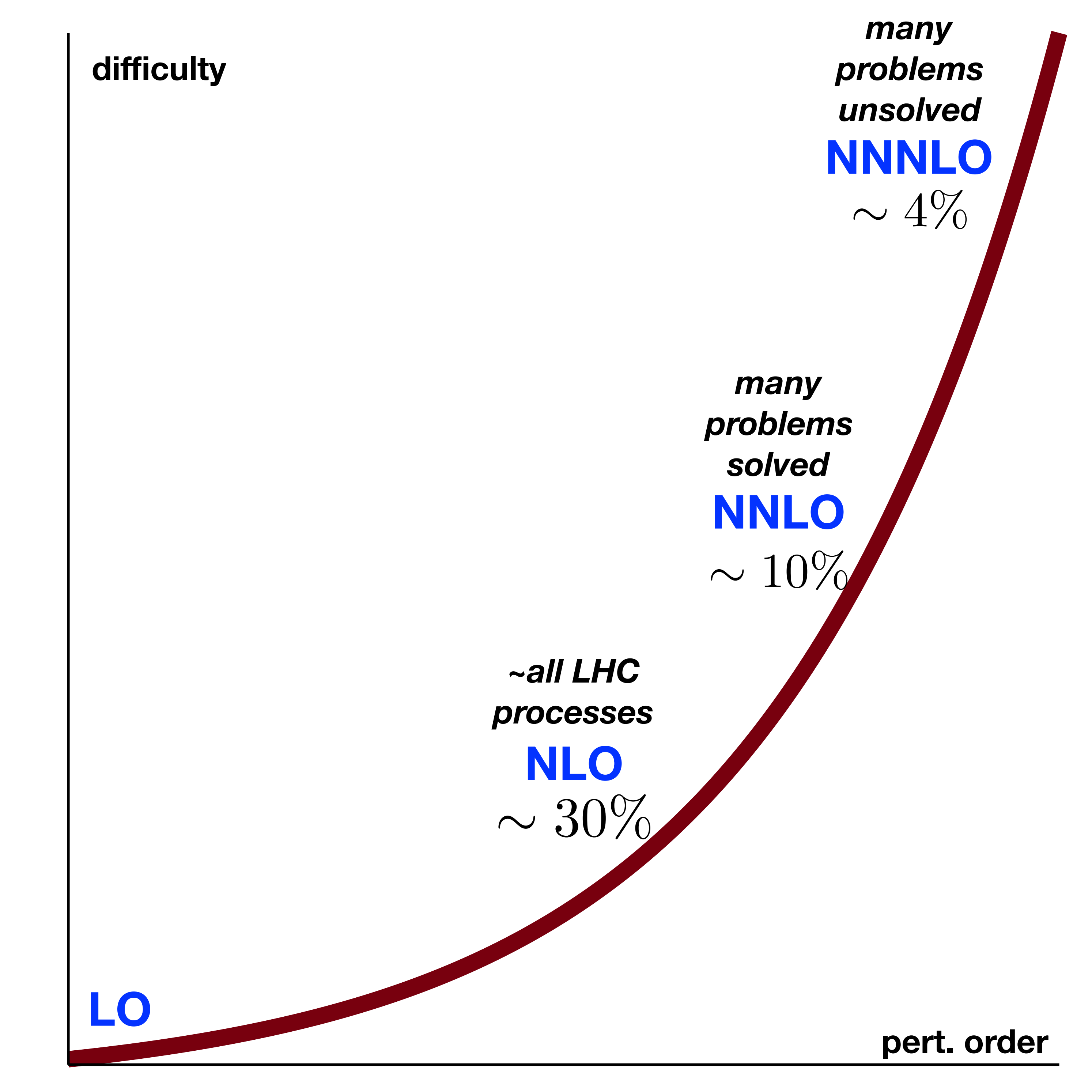


$$\mathcal{O}\left(g_s^2\right)$$

$$\mathcal{O}\left(g_s^4\right)$$

$$\mathcal{O}\left(g_s^6\right)$$

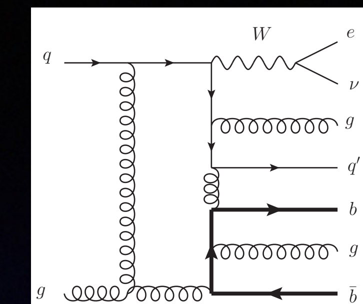
What has been achieved for the LHC?



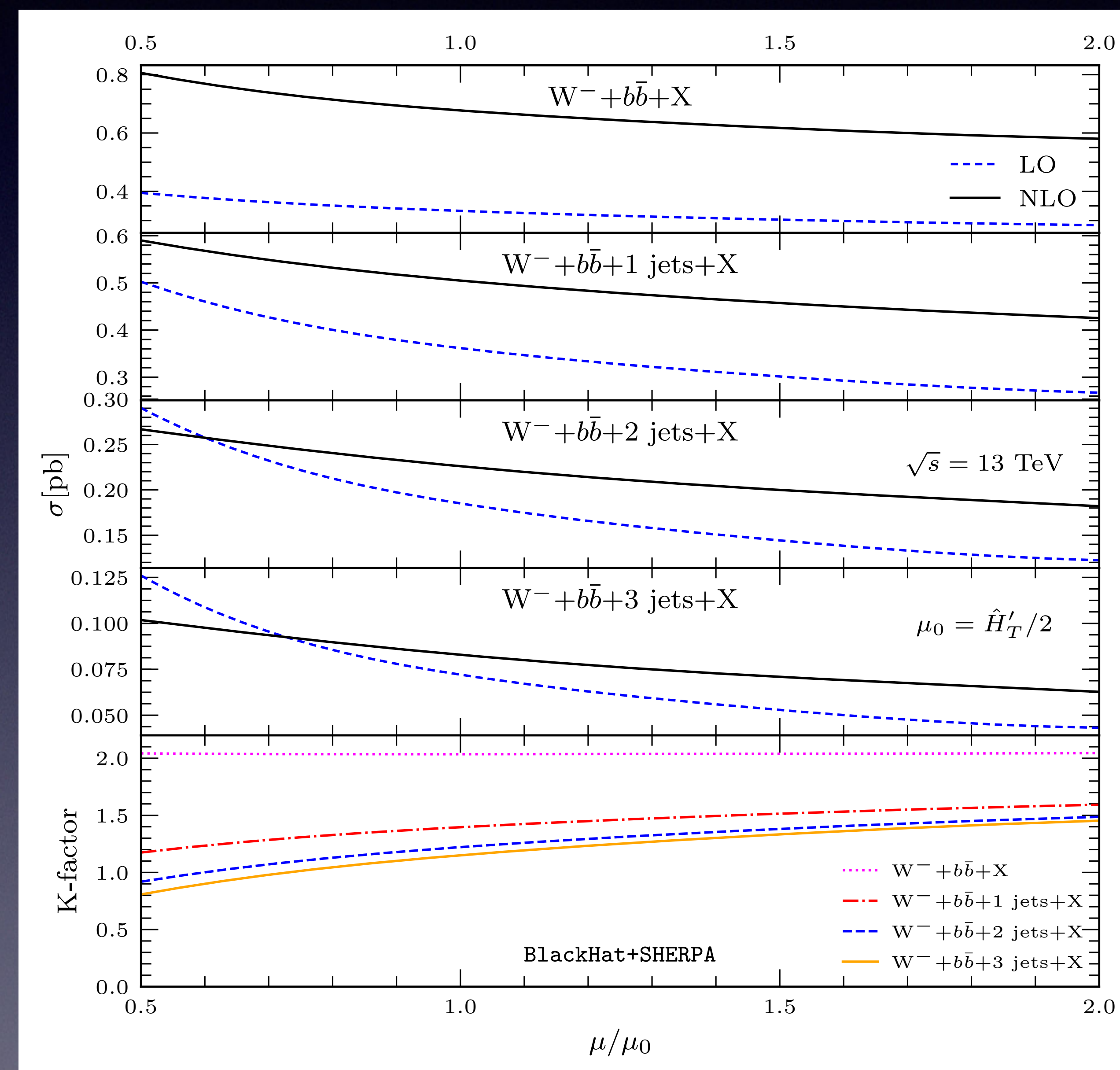
2

$$\sigma = \sigma_0 \alpha_s^n + \sigma_1 \alpha_s^{n+1} + \sigma_2 \alpha_s^{n+2} + \ldots$$

Wbb+multijet production at NLO



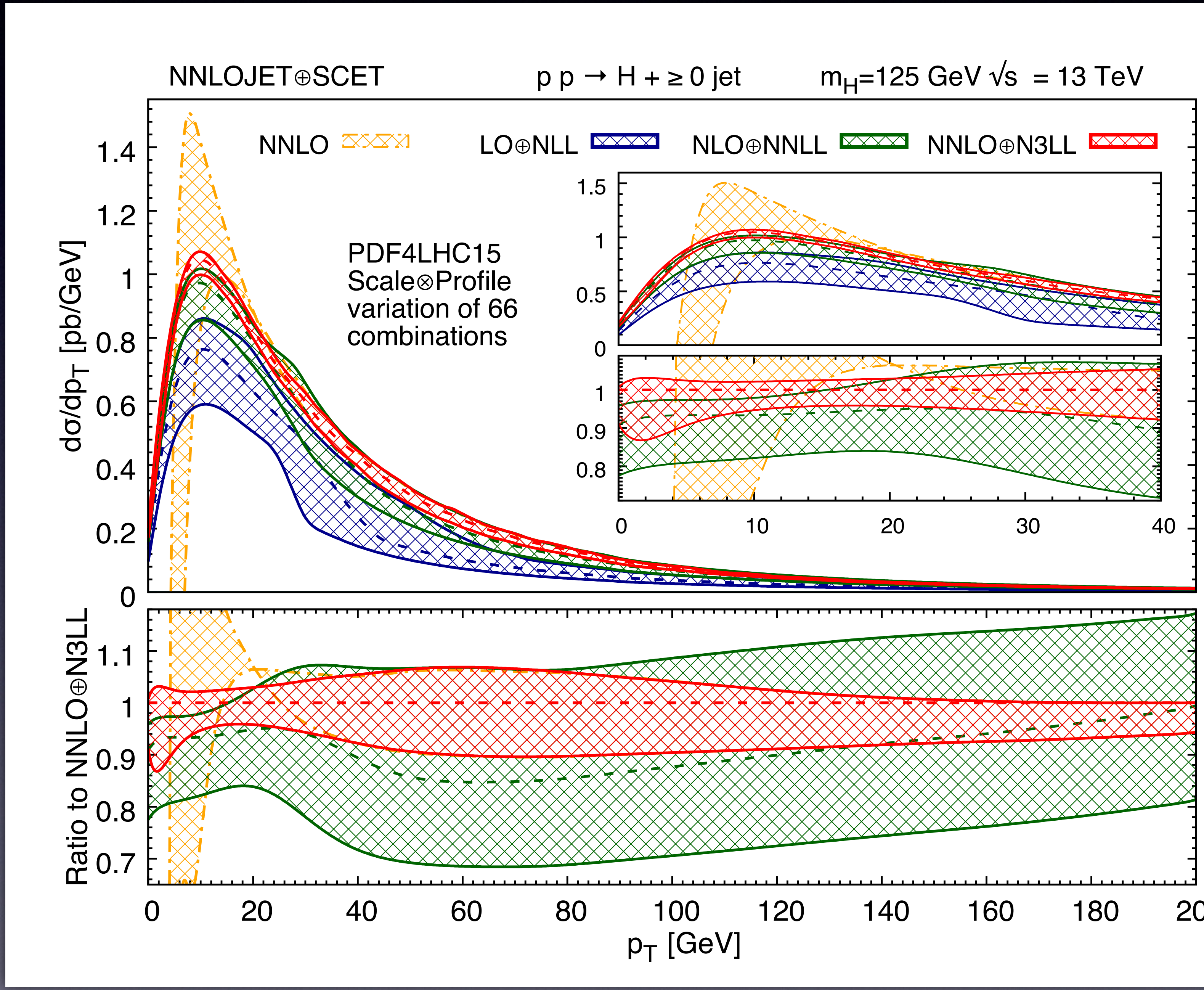
- An example of what can be done
- Such results were unimaginable one decade ago
- NLO at LHC is fully automated: aMC@NLO, BlackHat, GoSam, Helac-NLO, OpenLoops, Recola, Sherpa, NJet, …



Anger, Cordero, Ita, Sotnikov in Dec. 2017

Higgs boson transverse momentum spectrum at NNLO + N3LL

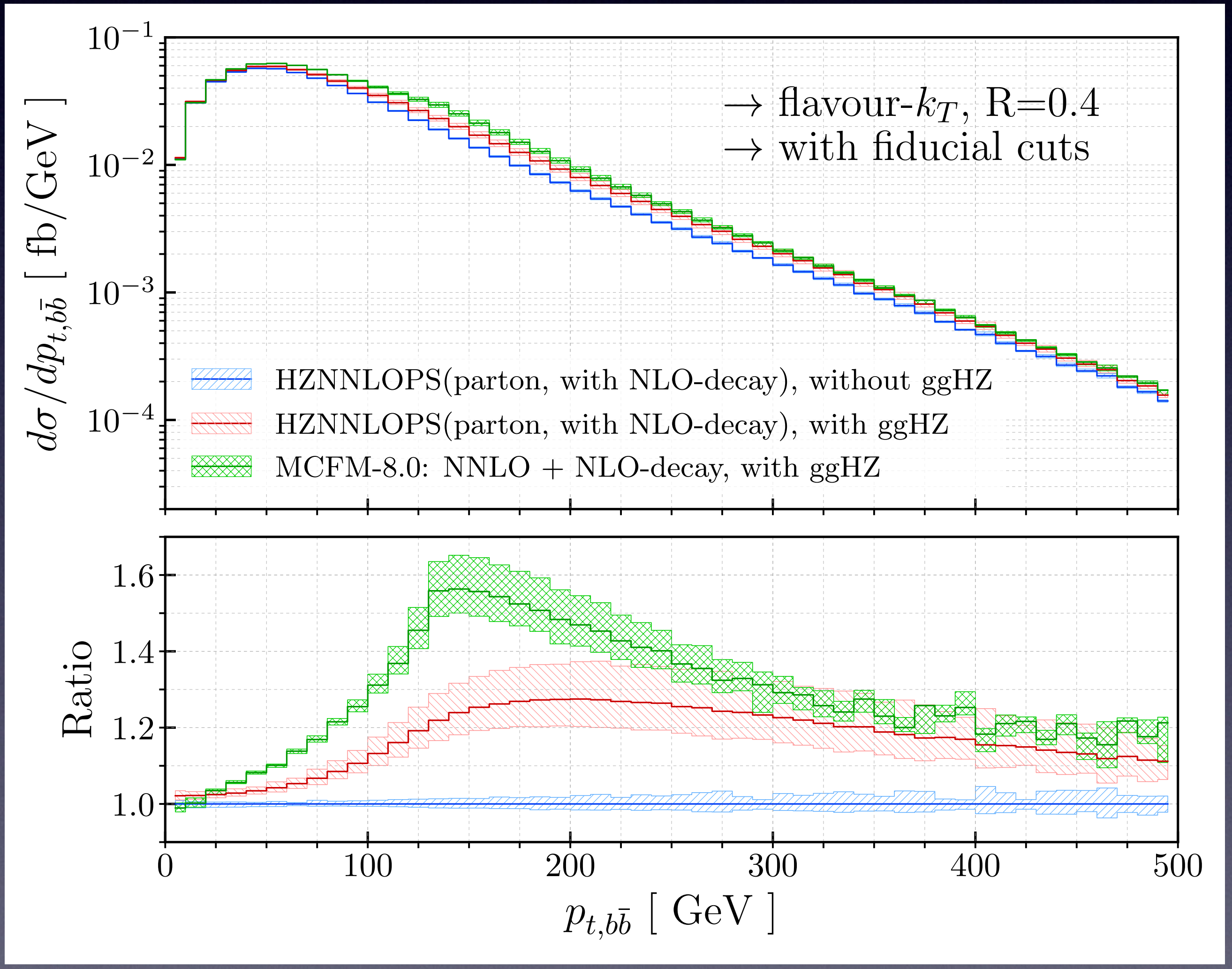
- Combines recent breakthrough computations at NNLO for H+1 jet
- and a profound knowledge of QCD factorisation in the soft and collinear regime.
- A ~6% precision for the transverse momentum spectrum (below the top-quark threshold).



Chen, Gehrmann, Glover, Huss, Li, Neill, Schulze, Stewart, Zhu in May 2017

Associated Higgs production at NNLO merged with parton-shower

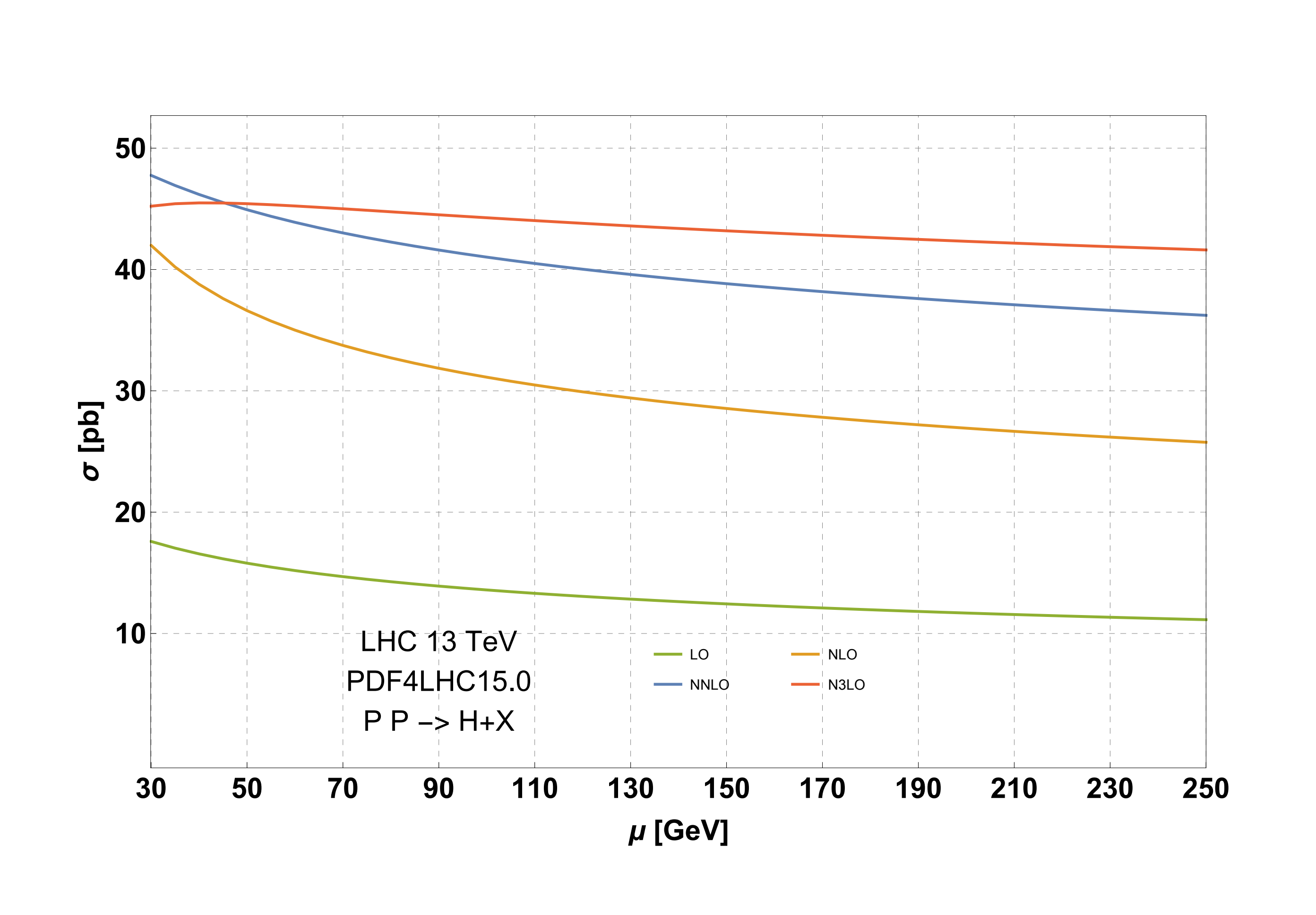
- The interplay of radiation processes described by parton-showers and fixed orders is getting better understood.
- The merging of these two approaches has been achieved through NNLO for simple (but crucial) processes.
- Important impact on phenomenology and a precise description of realistic processes.



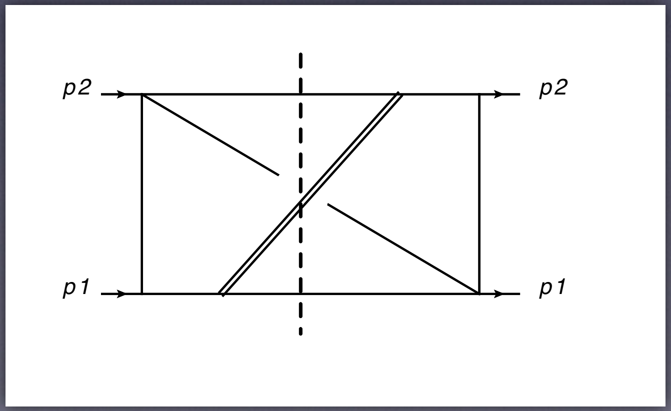
Astill, Bizon, Re, Zanderighi in 2018

Higgs cross-section at N3LO

- First, as a deep expansion around threshold
- Now, in a closed complete form ***(Mistlberger in 2018)***
- Reflects a big progress since 2016 in our mathematical understanding (elliptic Feynman integrals)

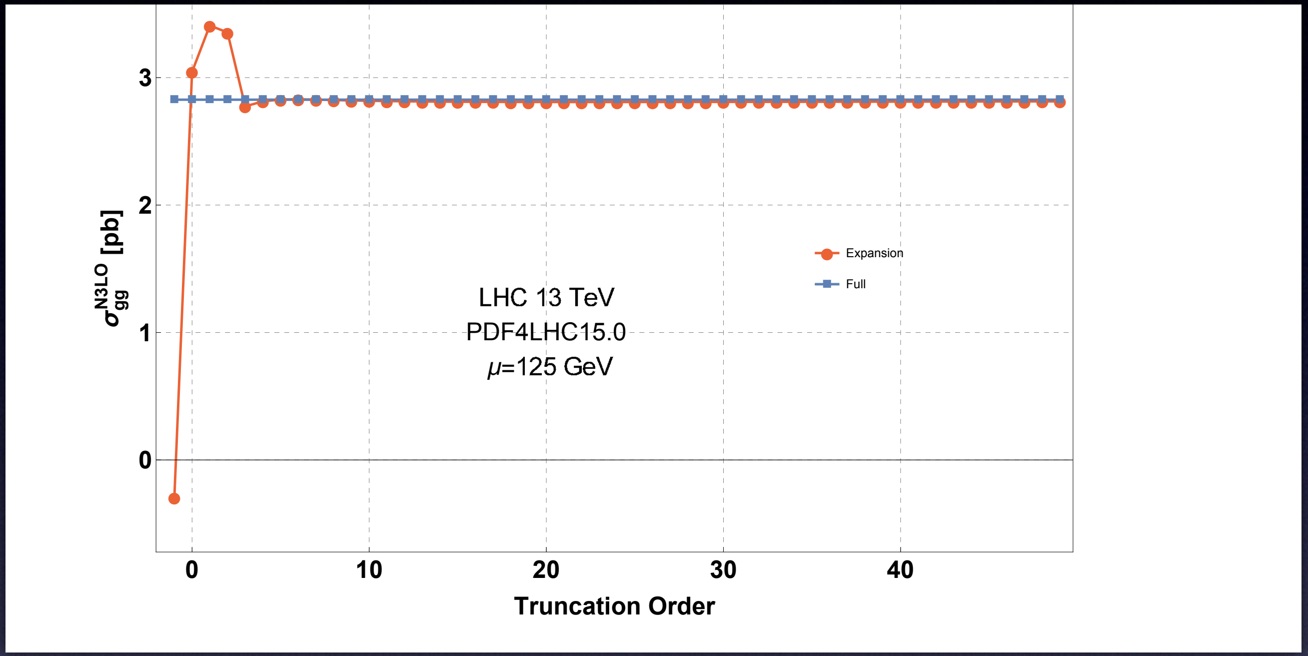


CA, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger in 2016, Mistlberger in 2018

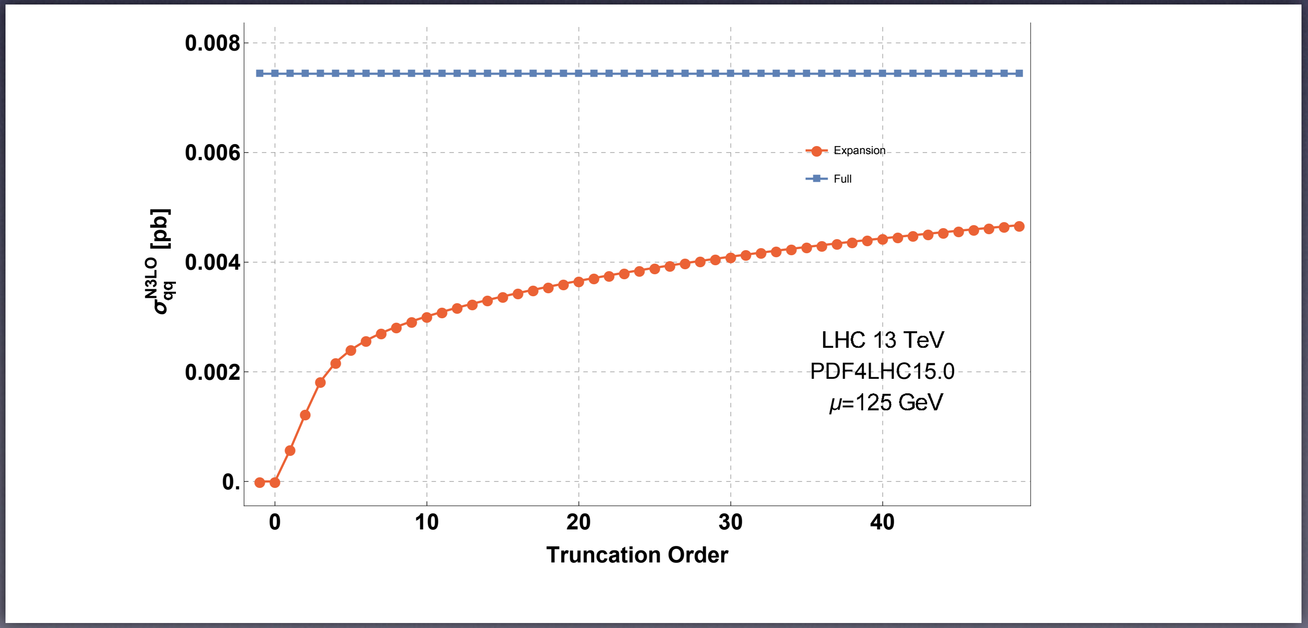


Complete Higgs cross-section at N3LO

- Threshold expansion was reliable for the gluon-gluon initial-state channel.
- Unreliable for quark-initiated channels.
- Now all integrals are known for the computation of all 2 -> 1 processes in N3LO massless QCD.
- Could eventually bring theoretical predictions for Drell-Yan production to a precision of 0.1%



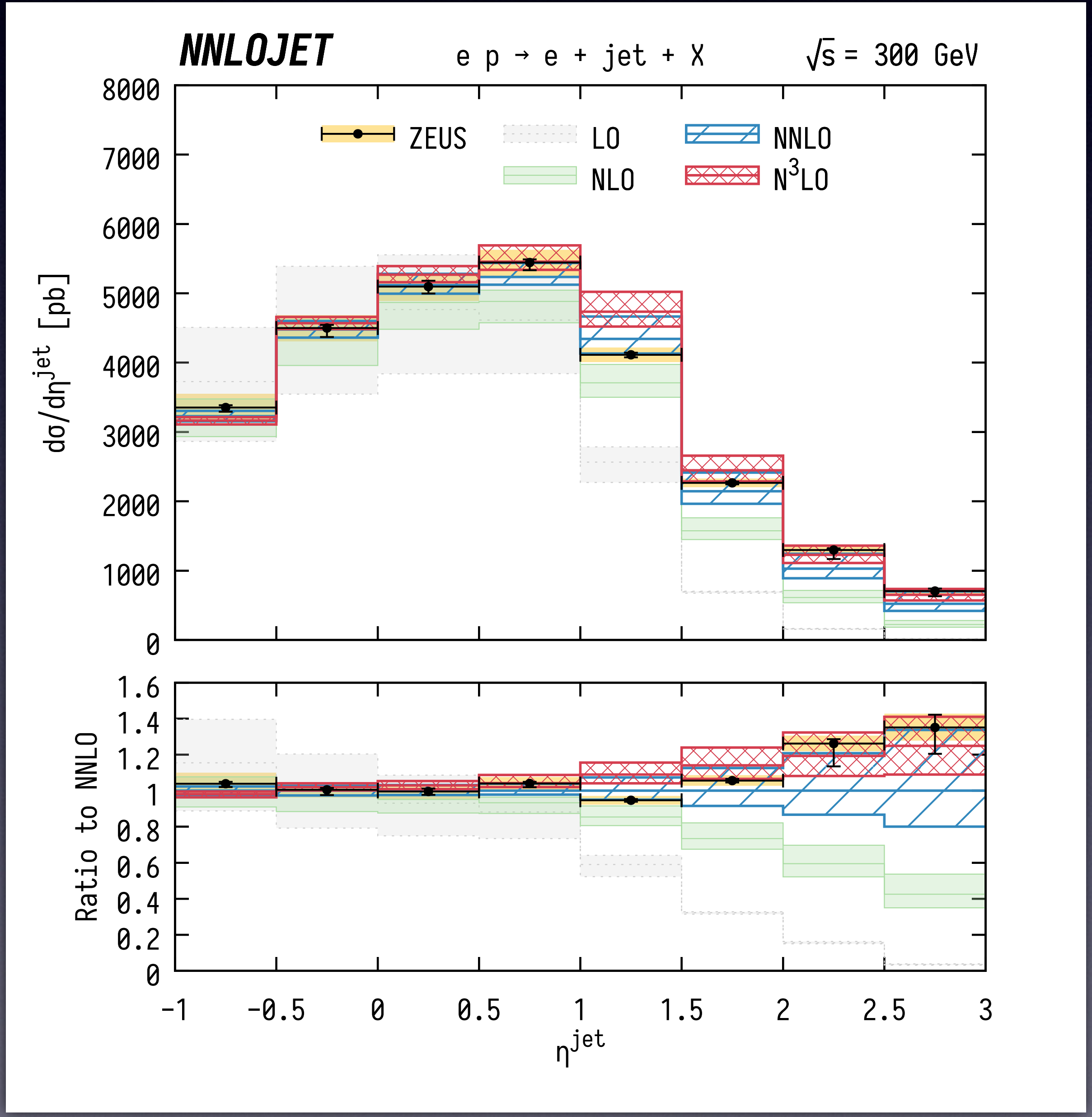
Mistlberger in 2018



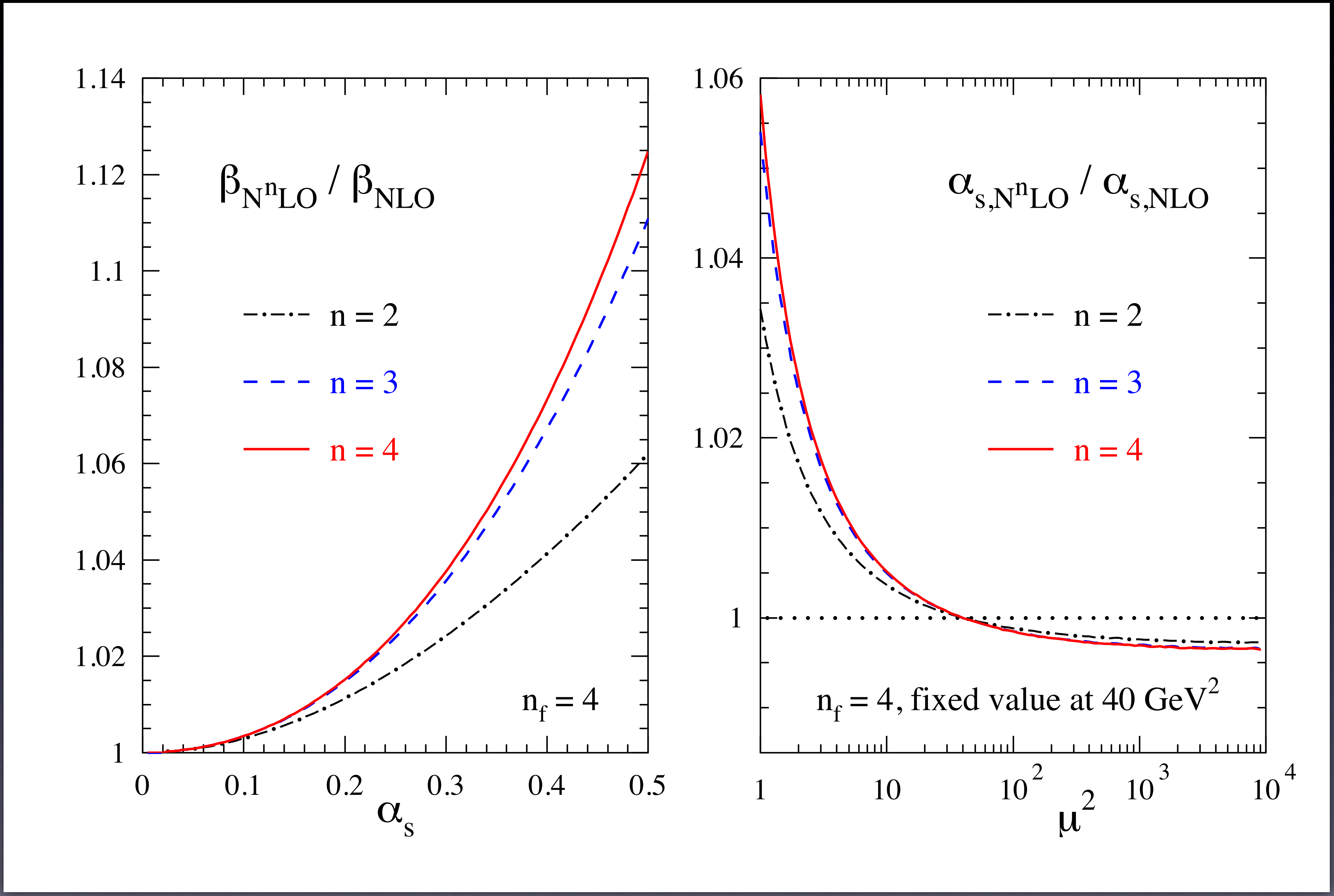
Jet production at N3LO i n DIS

- A first of a kind achievement.
- fully differential cross-section at N3LO.
- It exploits NNLO computations and knowledge of an inclusive cross-section
- Used a “projection-to-Born method”, first developed at NNLO ***(Cacciari, Dreyer, Karlberg, Salam, Zanderighi in 2015)***

Currie, Gehrmann, Glover, Huss, Nietes in 2018



5-loop QCD β-function



Herzog, Ruijl, Uefa, Vermaseren, Vogt; also by Baikov, Chetyrkin, Kuehn ***(in 2017)***

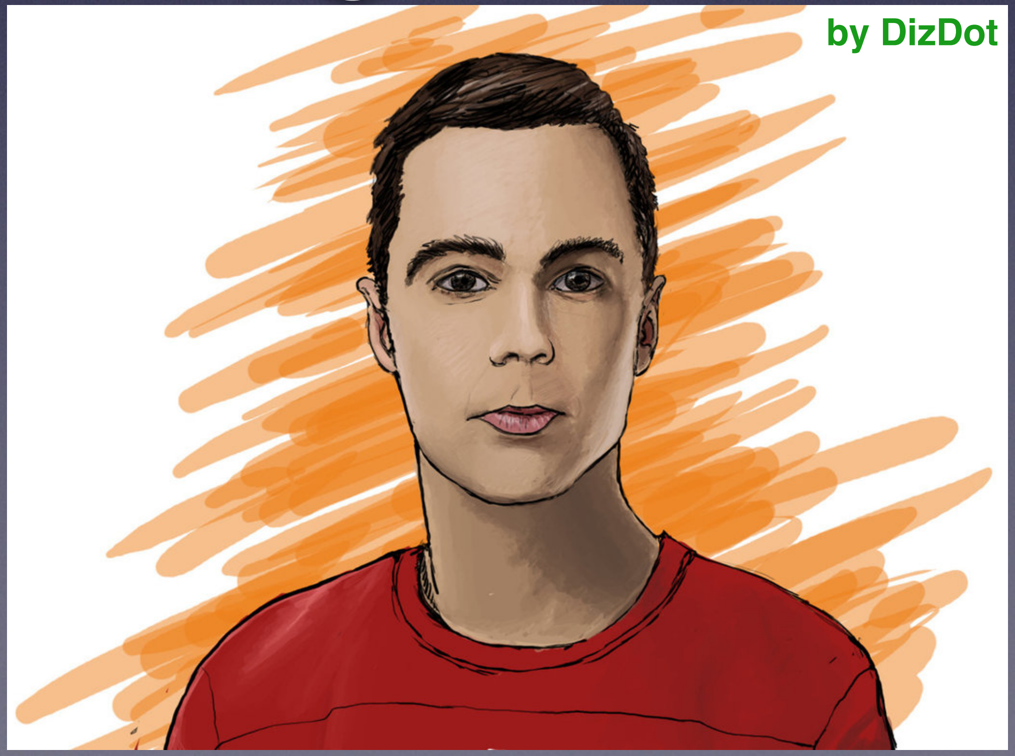
A day at work…

Compute:

$$\int_0^1 \frac{dt}{t} \log(1-tx) \log(1-ty)$$

$$= \sum_{n,m=1}^{\infty} \frac{x^n y^m}{nm(n+m)}$$

$x, y < 1$



A day at work…

Guess:

$$\int_0^1 \frac{dt}{t} \log(1-tx)\log(1-ty) = \sum_i c_i f_i\left(z_i(x,y)\right)$$

- In the rhs, known functions
- with constant coefficients
- and a single argument

the ubiquitous polylogarithms…

- All one-loop amplitudes in 4d QFT can be written in terms of two functions only!

$$log(z) \qquad Li_2(z) = -\int_0^1 \frac{\log(1-tx)}{t} = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

- More "polylogarithms" show up at two and higher loops… but not many

$$Li_3(z), S_{12}(z), Li_4(z), S_{22}(z), \ldots$$

A day at work…

Guess:

$$\int_0^1 \frac{dt}{t} \log(1-tx)\log(1-ty)$$
$$= \sum_i c_i \mathrm{PolyLog}_i\left(z_i(x,y)\right)$$

- In the rhs, known polylogarithms
- with constant coefficients
- and a single argument which captures the singularities of the lhs.

- polylogarithms have a nice behaviour under differentiation

$$\frac{d}{dz}Li_4(z) = \frac{1}{z}Li_3(z), \frac{d}{dz}Li_3(z) = \frac{1}{z}Li_2(z), \ldots$$

- We can act on our ansatz with derivatives, simplifying both sides to simpler functions, all the way down to logarithms

$$\frac{d^{l+m}}{dx^l dy^m}\int_0^1 \frac{dt}{t}\log(1-tx)\log(1-ty) = \sum_i c_i \frac{d^{l+m}}{dx^l dy^m}\mathrm{PolyLog}_i\left(z_i(x,y)\right)$$

- We can then get easily a system of equations for the coefficients.

INTEGRATION → ***LINEAR ALGEBRA***

Multiple polylogarithms formally

- There is a vector space of "multiple polylogarithms"

$$G(a_1, a_2, \ldots, a_n, z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n, t)$$

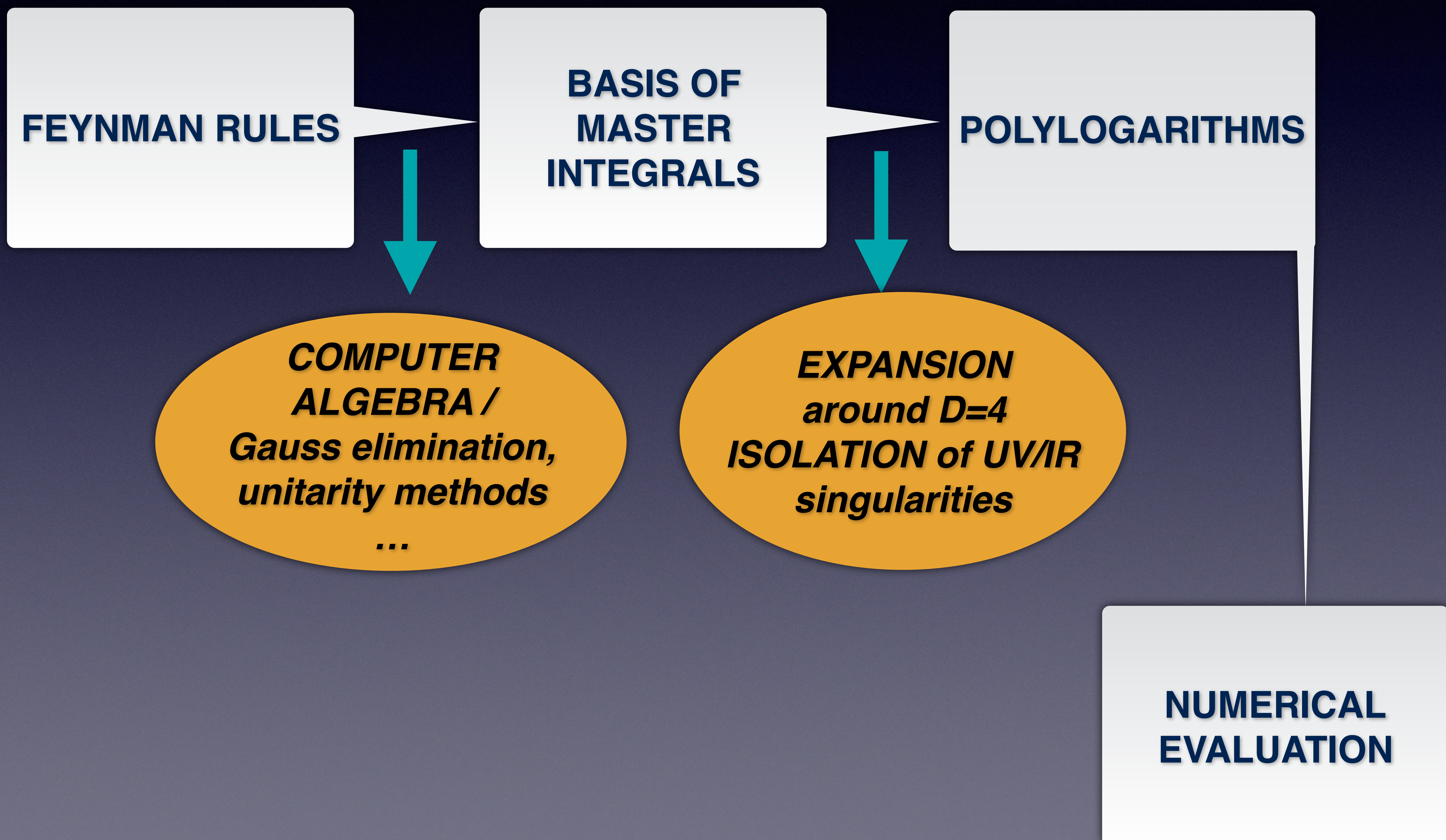
- Functional relations among polylogarithms or their derivatives, take the form of an algebra (multiplication rules)
  - shuffle, stuffle, Hopf algebras

(Vermaseren; Gehrmann, Remiddi; Brown; Goncharov; Duhr; …)

- Enormous breakthroughs in uncovering the rules (symbol, coproduct)

- Computing multidimensional Feynman integrals in terms of such polylogarithms is now "targeted" research with powerful theorems which restrict every step of the integration within the vector space of polylogarithms.

How to compute a multi - loop amplitude



Fully known (analytically) 2-loop amplitudes for LHC processes

• parton+parton -> parton+parton (~2000)	*2 scales, massless propagators*
• parton+parton -> (H,V) + parton (~2003)	*3 scales, massless propagators*
• gluon-gluon -> Higgs via quarks (~2007) • gluon-gluon -> Higgs via electroweak (~2006)	*2 scales, massive propagators*
• parton-parton -> diboson (~2014)	*4 scales, massless propagators*

Fully known (analytically) 2-loop amplitudes for LHC processes

• parton+parton -> parton+parton (~2000)	*2 scales, massless propagators*
• parton+parton -> (H,V) + parton (~2003)	*...es, ...propagators*
• gluon-gluon -> Hi... (~2007) • gluon-gluon -> Higgs via electroweak (~2006)	*2 scales, massive propagators*
• parton-parton -> diboson (~2014)	*4 scales, massless propagators*

POLYLOGARITHMS

Where do polylogarithms come from?

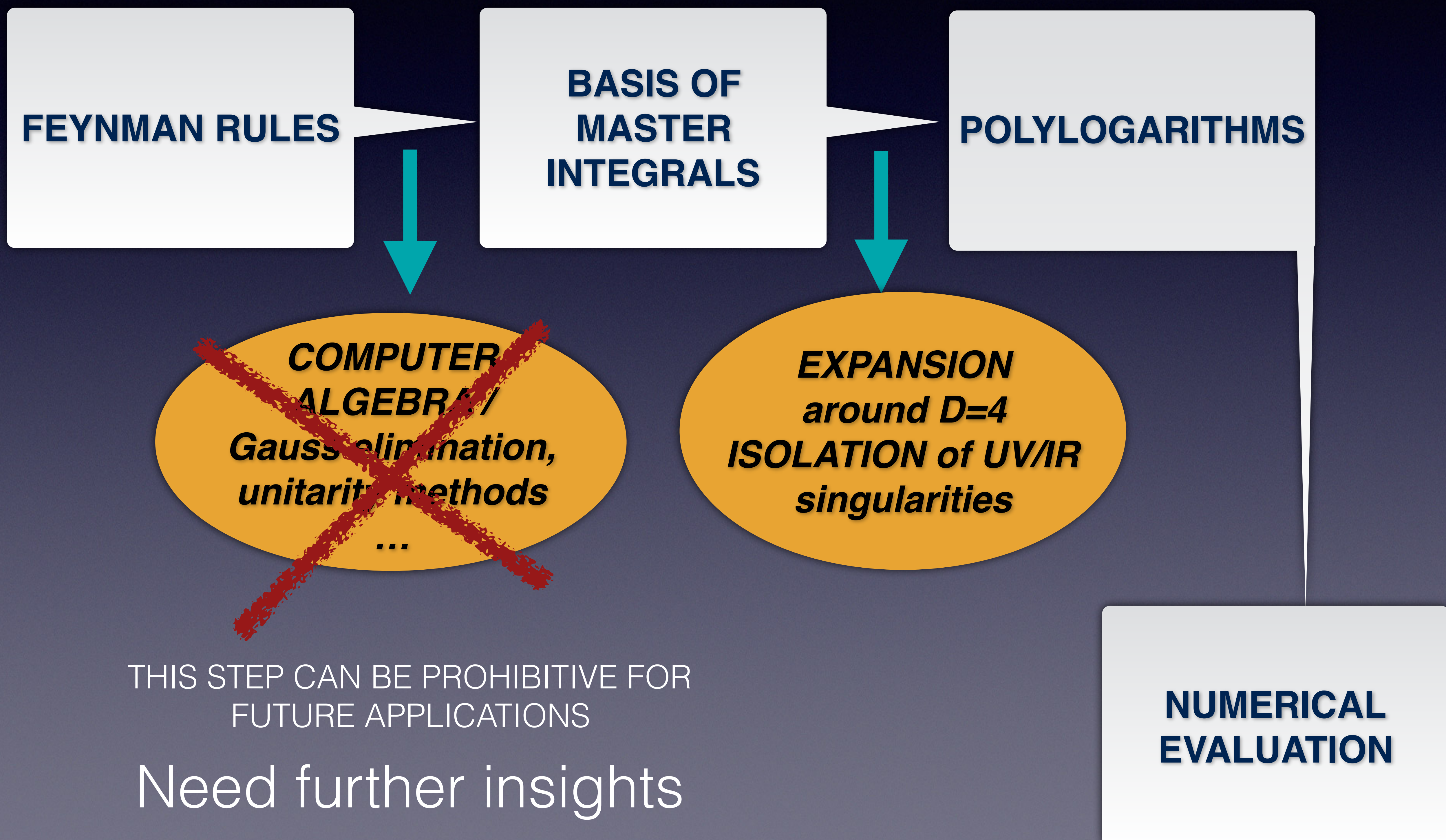
- If an amplitude can be written in terms of polylogarithms exclusively, …
- … what does this imply for the basis of master integrals?
- Conjecture: There is a ”canonical” basis of master integrals…
- …which satisfies simple differential equations

$$\frac{\partial I_i}{\partial x_m} = (d-4) A_{ij}^{(m)}(\vec{x}) I_j$$

Johannes Henn

INTEGRATION → ***LINEAR ALGEBRA***

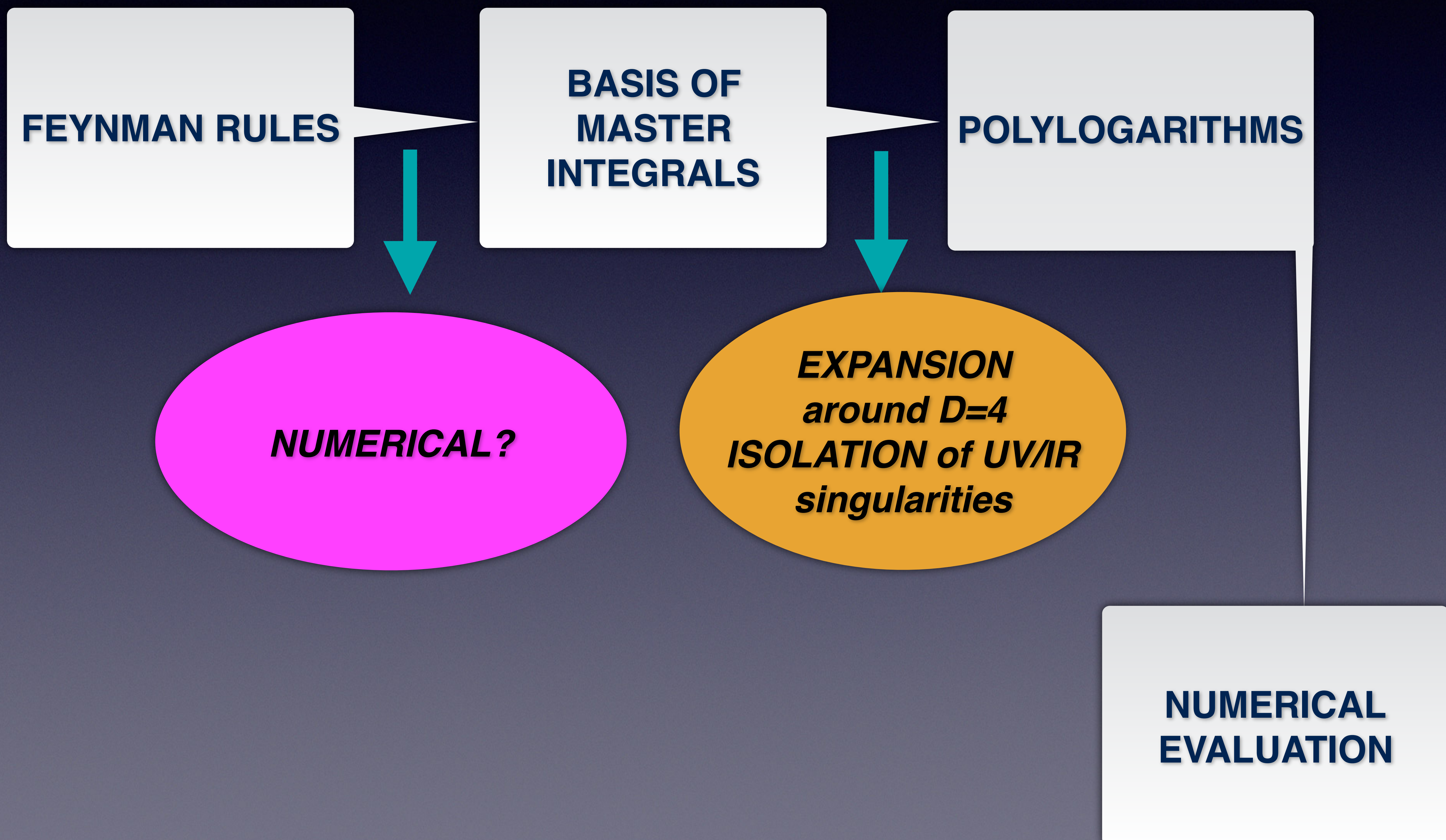
IS SYMBOLIC COMPUTER ALGEBRA ALWAYS SIMPLE?



THIS STEP CAN BE PROHIBITIVE FOR FUTURE APPLICATIONS

Need further insights

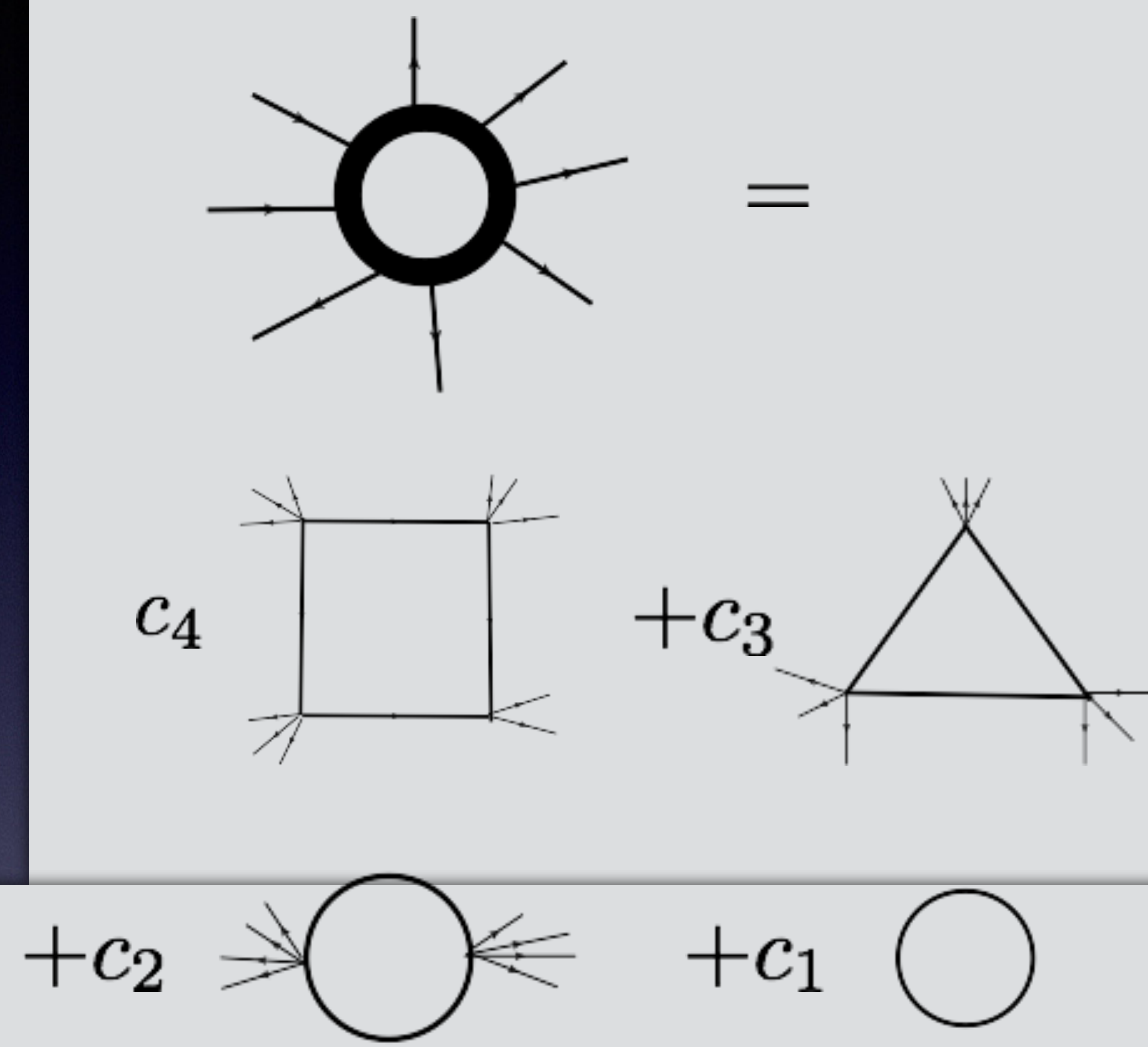
NUMERICAL REDUCTION

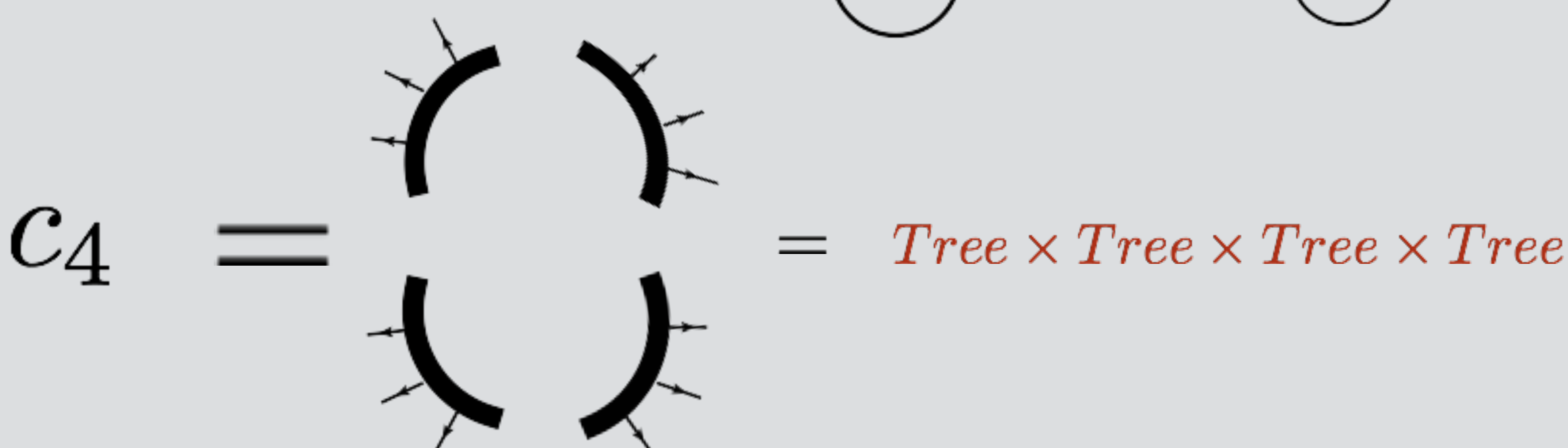


NLO:solved with numerical reduction to master integrals

- Reduction of amplitudes to master integrals has been understood physically very well, in multiple ways.
- One-loop amplitudes in gauge theories = (Tree-amplitudes in gauge-theories) and (Integrals in scalar field theories)

Ossola, Pittau, Papadopoulos;
del Aguila, Pittau;
Ellis, Giele, Kunszt;
Ellis, Giele, Kunszt, Melnikov;
...



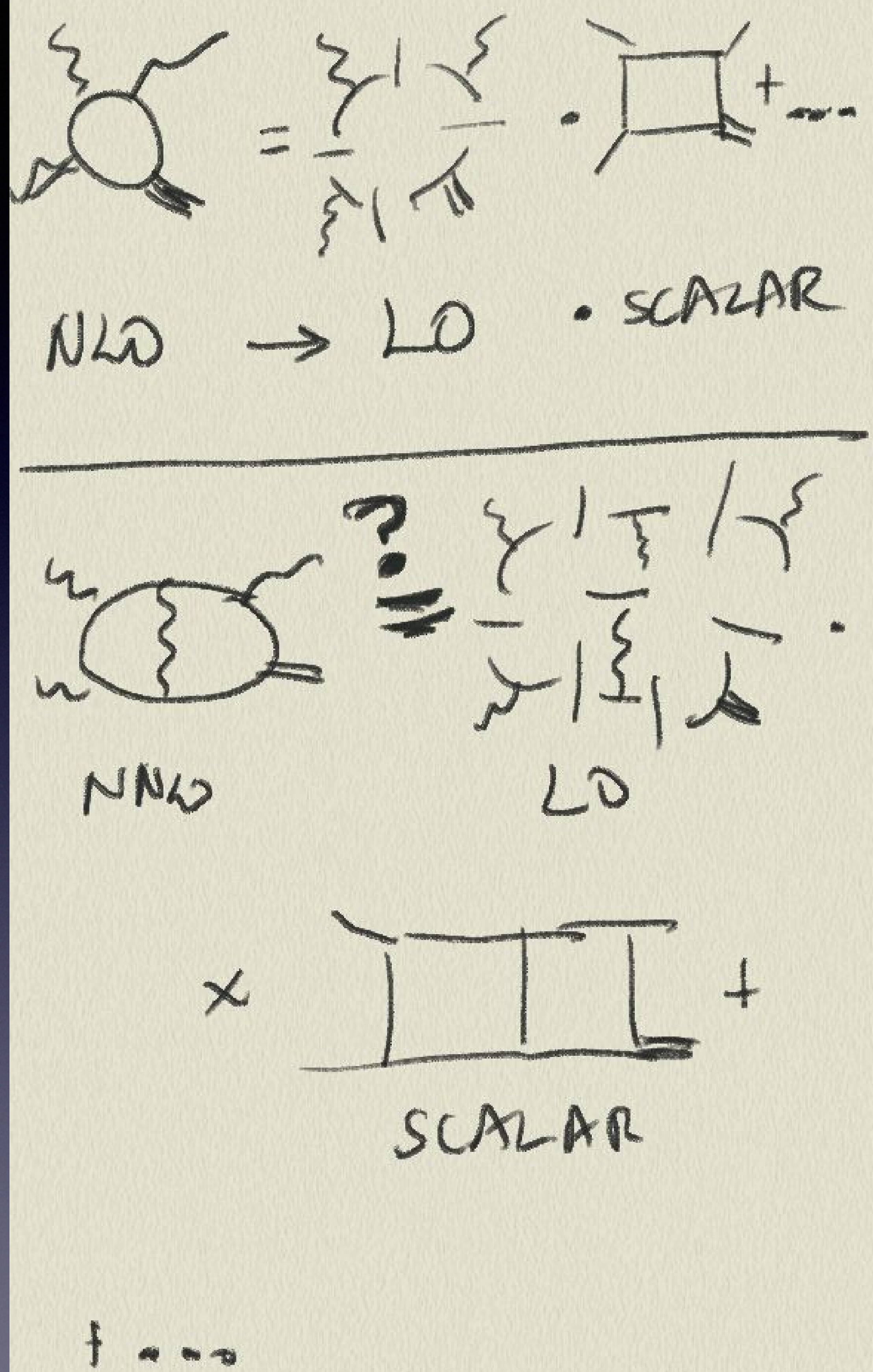


From NLO to NNLO

- A very beautiful structure of perturbation theory at NLO, where we can reduce the cross-section calculations to a few master integrals and tree-amplitudes.

- It makes one dream that also higher orders NNLO, NNNLO, etc can be reduced to master integrals in a similarly physical and efficient eay.

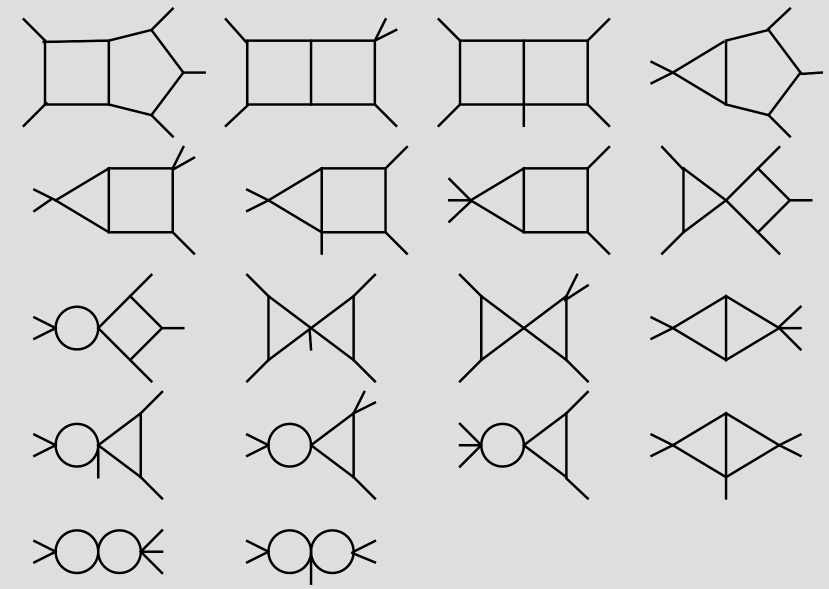
- This dream is becoming real!



5-gluon amplitudes at leading colour

Badger, Bronunm-Hansen, Hartanto, Peraro in 2017

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	27.7526	-23.773	-168.117	-175.207±0.004
$P^{(2),[0]}_{--+++}$	12.5	27.7526	-23.773	-168.116	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5029	-35.8094	69.661±0.009
$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

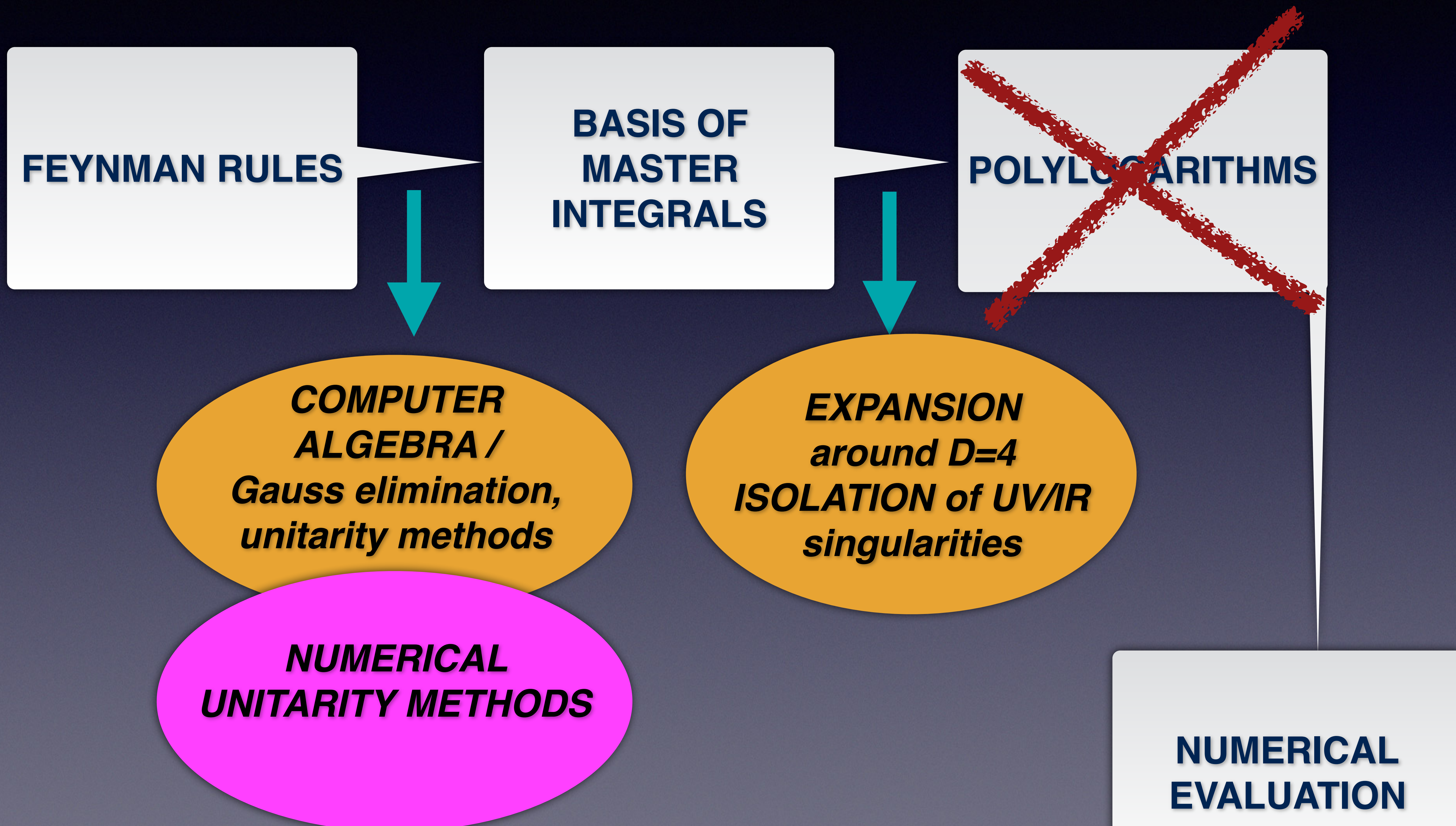


Abreu, Cordero, Ita, Page, Zeng in 2017

$\mathcal{A}^{(2)}/\mathcal{A}^{(0)}$	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$(1^-,2^-,3^+,4^+,5^+)$	12.5000000	25.46246919	-1152.843107	-4072.938337	-3637.249567
$(1^-,2^+,3^-,4^+,5^+)$	12.5000000	25.46246919	-6.121629624	-90.22184215	-115.7836685

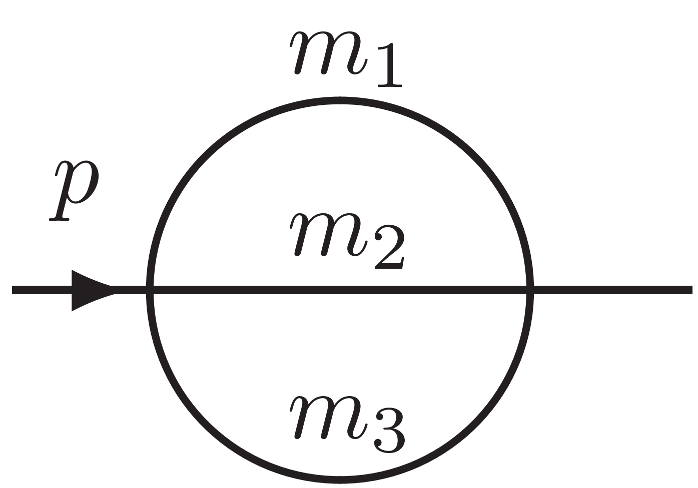
With unitarity based (semi-)numerical methods

ARE MASTER INTEGRALS SIMPLE?



Are always master integrals simple?

- The canonical basis of master integrals does not always exist…
- or, if it does, we do not know how to find it.
- Not all amplitudes can be written in terms of polylogarithms exclusively
- New functions (elliptic integrals) have emerged in two-loop amplitudes with massive propagators and in Higgs production at N3LO.
- Fast-paced progress in figuring out their systematics.

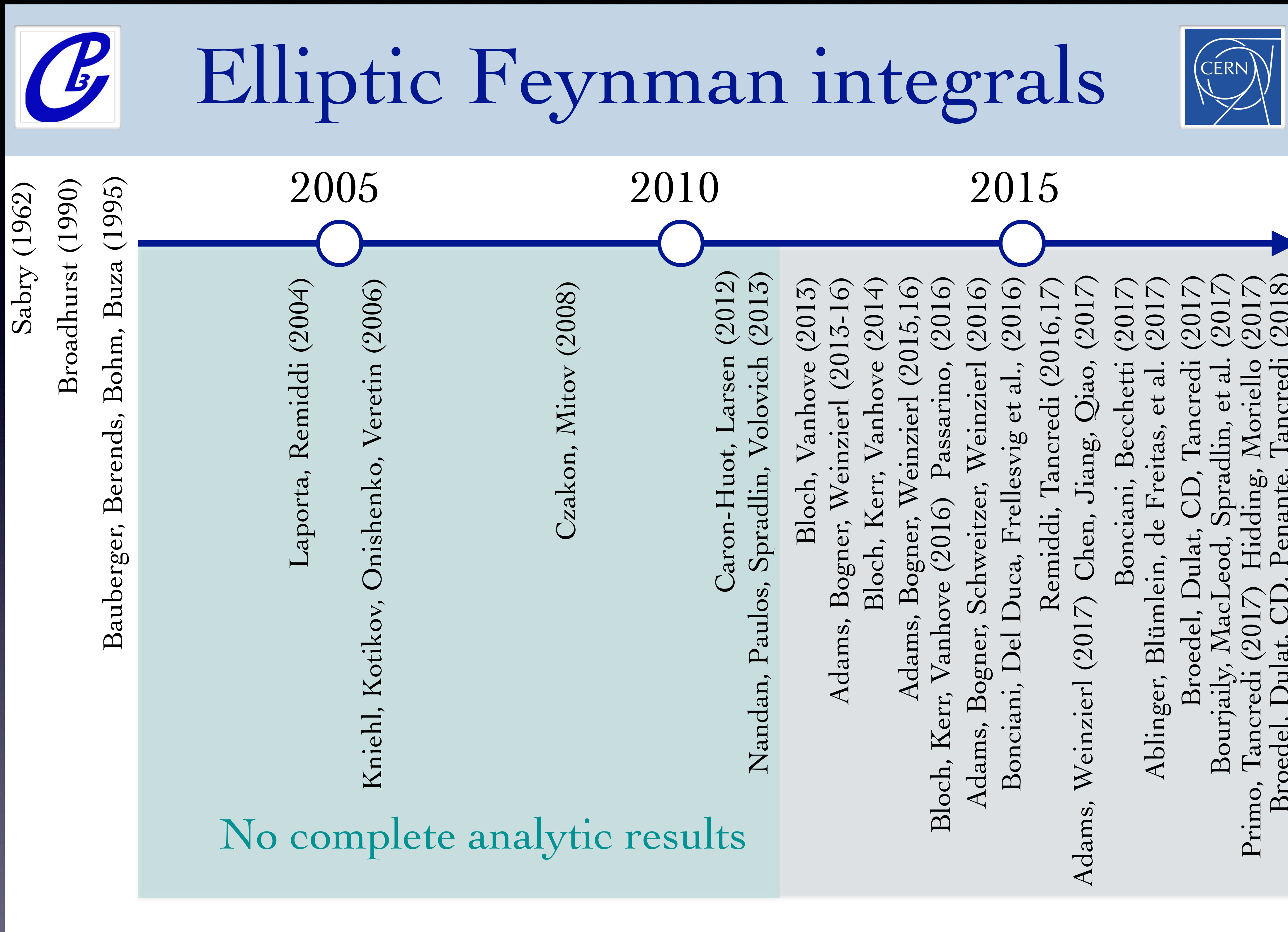


$$S = \frac{\psi_1(q)}{\pi}\left[\mathrm{E}_{2;0}(w_1(q);-1;-q)+\mathrm{E}_{2;0}(w_2(q);-1;-q)+\mathrm{E}_{2;0}(w_3(q);-1;-q)\right]$$

$$w_i = e^{i\beta_i}, \quad \beta_i = \pi\frac{F(u_i,k)}{K(k)}, \quad u_i = \sqrt{\frac{e_1-e_2}{x_{j,k}-e_2}}, \quad x_{j,k} = e_3+\frac{m_j^2 m_k^2}{\mu^4}.$$

$$F(z,x) = \int_0^z \frac{dt}{\sqrt{(1-t^2)(1-x^2t^2)}}.$$

Adams, Bogner, Weinzierl



from the presentation of Claude Duhr at Loops and Legs 2018

NUMERICAL EVALUATION OF MASTER INTEGRALS

FEYNMAN RULES

BASIS OF MASTER INTEGRALS

NUMERICAL INTEGRATION OF MASTER INTEGRALS

COMPUTER ALGEBRA / Gauss elimination, unitarity methods

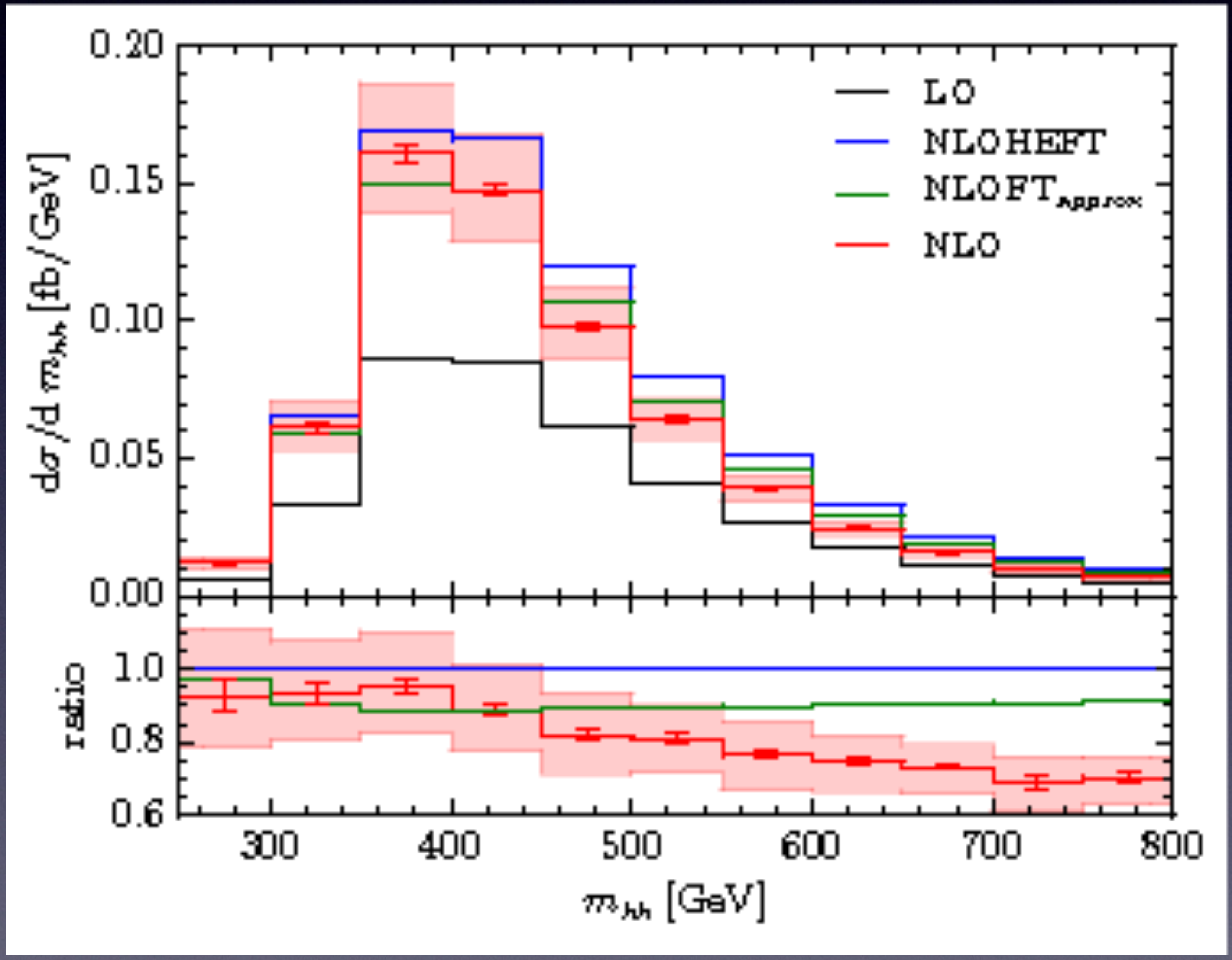
EXPANSION around D=4 ISOLATION of UV/IR singularities

NUMERICAL UNITARITY METHODS

Bypasses the problem of elliptic + other unknown functions that may emerge

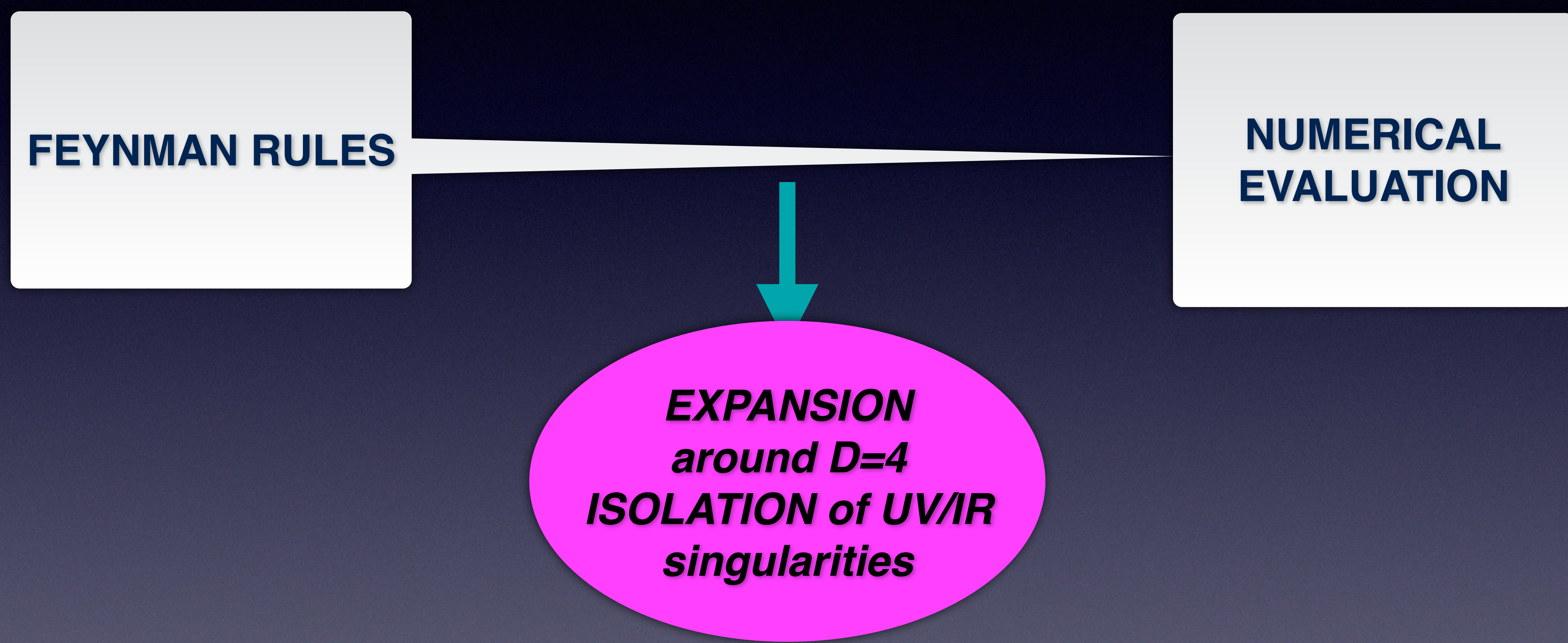
Higgs pair production at NLO (2-loop)

- Two-loop amplitude is required.
- Calculated with numerical methods.
- A promising avenue when analytic methods fail for computing multi-scale amplitudes.



Borowka, Greiner, Heinrich, Jones, Kemer, Schienk, Schubert, Zirke

A PURELY NUMERICAL APPROACH?

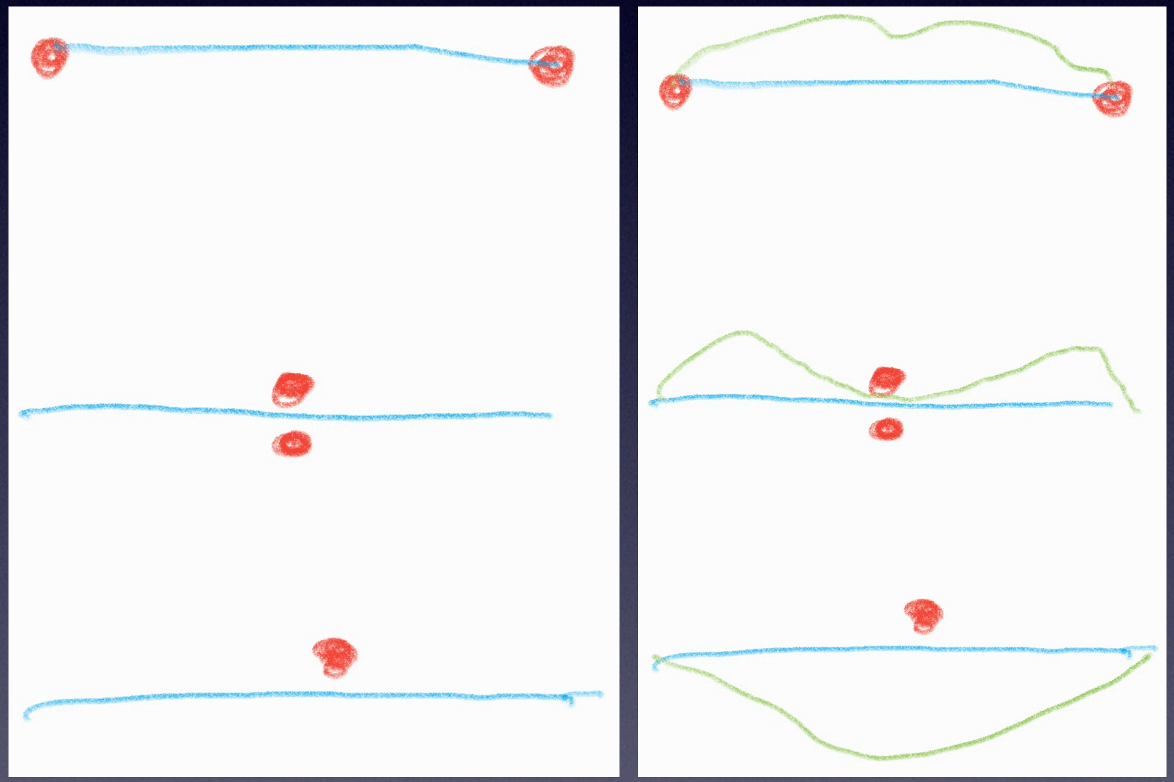


Bypasses the problem of algebraically demanding reductions to master integrals

Bypasses the problem of elliptic + other unknown functions that may emerge

Numerical approaches

- Direct numerical integrations of Feynman integrals are complicated due to IR/UV and thresholds.
- UV/IR need to be subtracted away before integration.
- Integrable singularities require a deformation of the contour of integration.



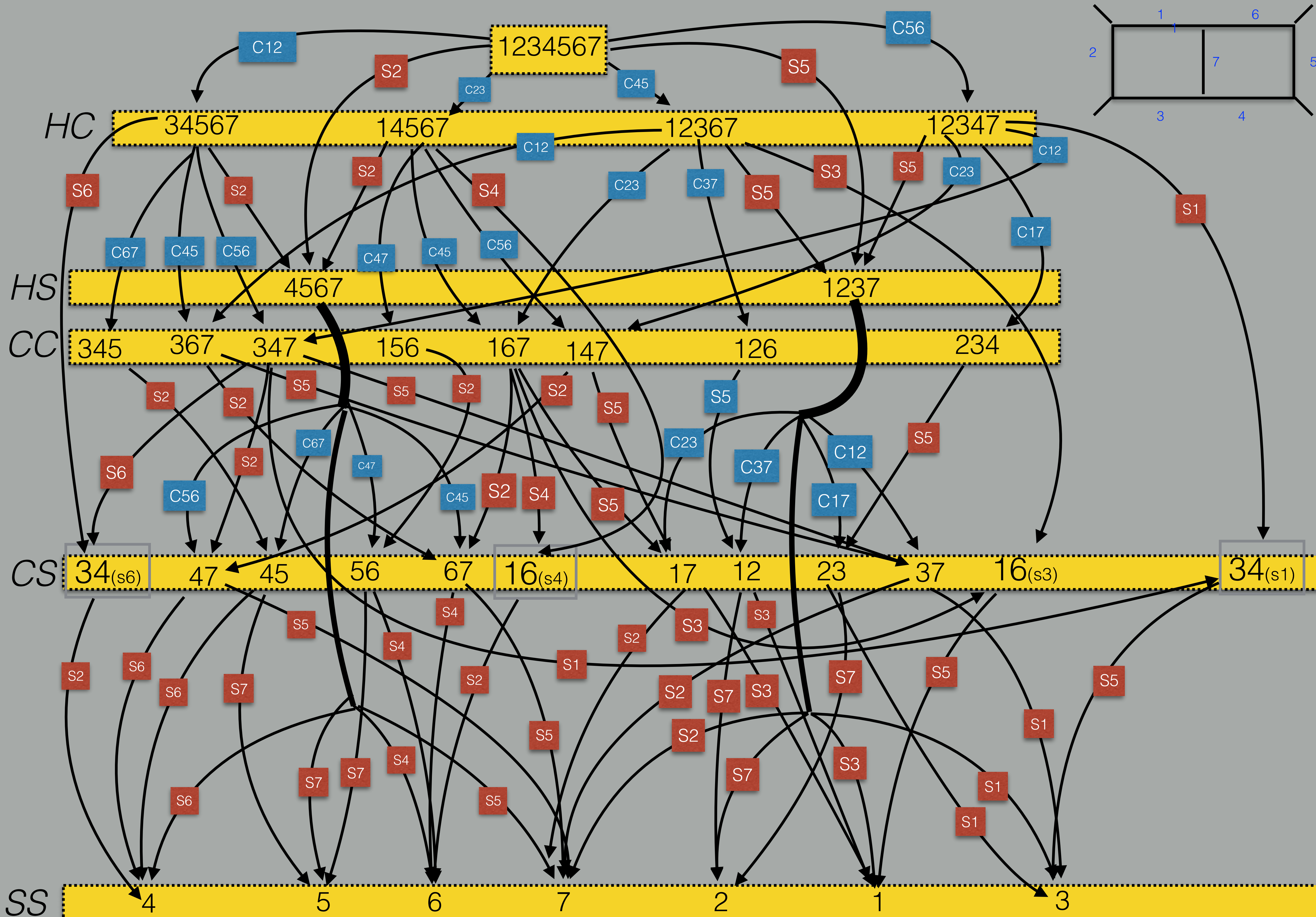
Soft+collinear+threshold singularities

Numerical approaches

Feynman parameter space	Momentum space
• IR/UV counterterms can be found algorithmically for arbitrary loops • A sector-decomposition algorithm can disentangle overlapping singularities (*Binoth, Heinrich; …)* • Contour deformations can be produced algorithmically for arbitrary loops *(Nagy, Soper; …)*	• IR/UV counterterms have been found only at one-loop *(Nagy, Soper)* • Contour deformations are known at one-loop and beyond for processes with massless propagators. *(Nagy, Soper; Becker, Weinzierl), But not efficient!* • A promising field of research with space for new ideas

RENDERING 2-LOOP FEYNMAN AMPITUDES FINITE

- Two-loop integrals become divergent when internal particles in the loop become collinear to external particles, or they are soft.
- Removing these singularities of two-loop is complicated.
- Singularities are many and highly entangled!



1234567
HC
34567
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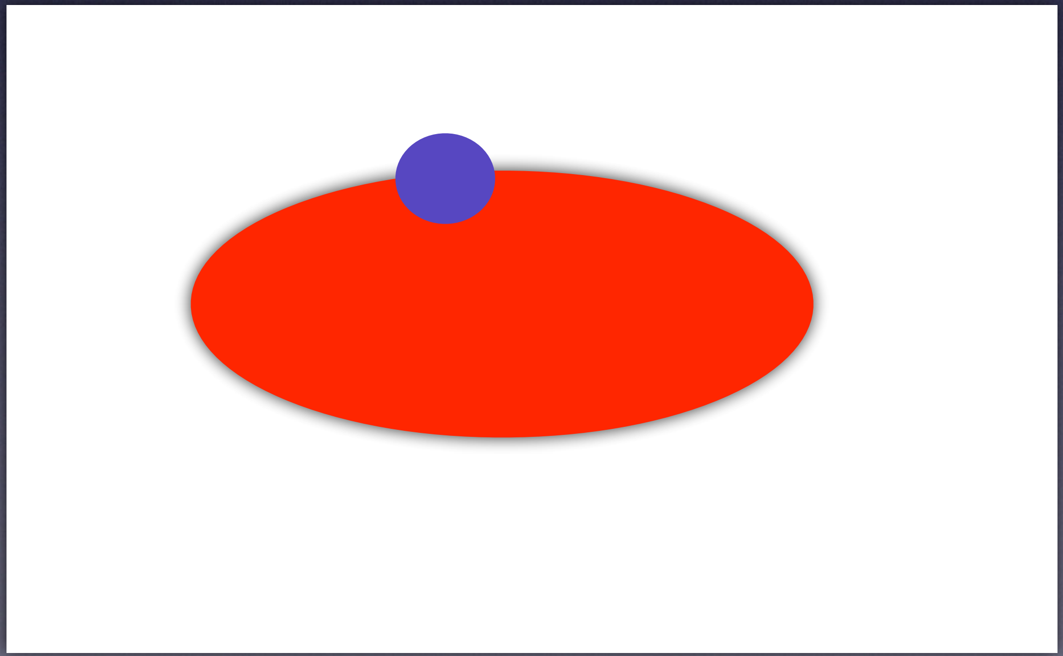
12367
12347
HS
4567
1237
CC
345
367
347
156
167
147
126
234
CS
34(s6)
47
45
56
67
16(s4)
17
12
23
37
16(s3)
34(s1)
SS
4
5
6
7
2
1
3

Nested subtractions

Ozan Erdogan, George Sterman

- Order the singular regions by their "volume"
- Subtract an approximation of the integrand in the smallest volume
- Then, proceed to the next volume and repeat until there are no more singularities to remove.

$$R^{(n)}\,\gamma^{(n)} \;=\; \gamma^{(n)} + \sum_{N\in\mathcal{N}[\gamma^{(n)}]} \prod_{\rho\in N} (-t_\rho)\,\gamma^{(n)}\,,$$



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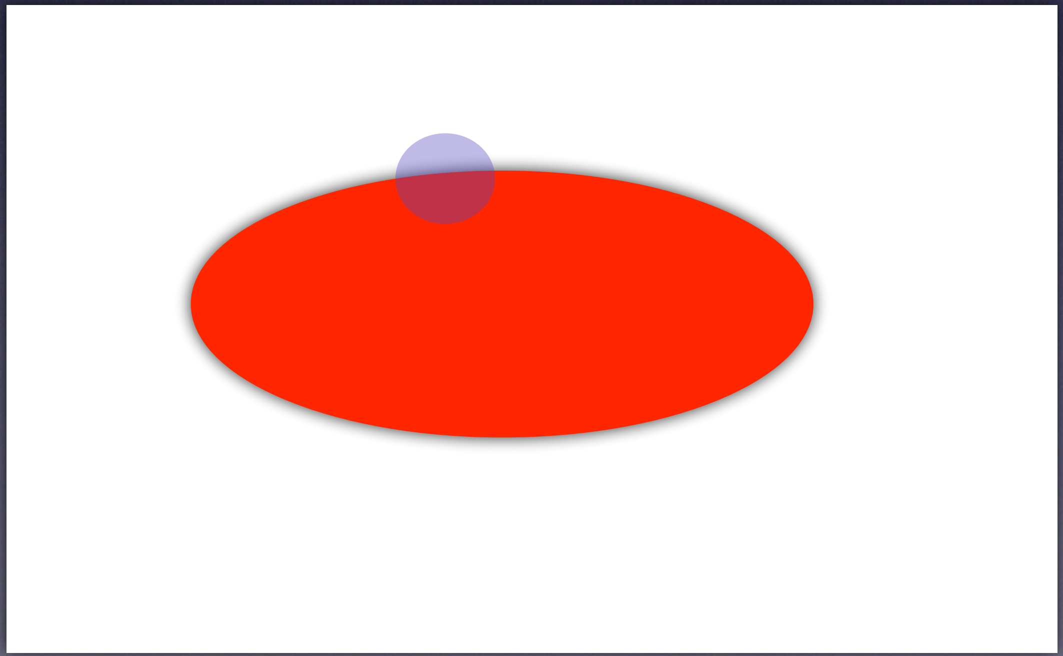
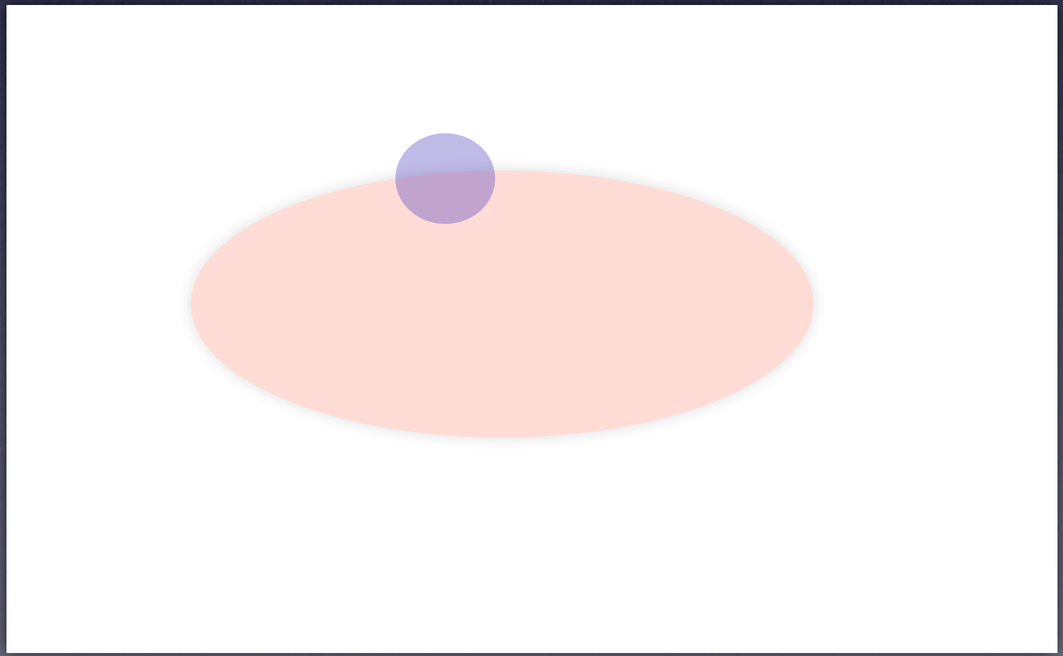


Nested subtractions

Ozan Erdogan, George Sterman

- Order the singular regions by their "volume"
- Subtract an approximation of the integrand in the smallest volume
- Then, proceed to the next volume and repeat until there are no more singularities to remove.
- Method should work at all orders in perturbation theory.
- This structure gives rise to factorisation into Jet, Soft and Hard functions for scattering amplitudes.

$$R^{(n)}\,\gamma^{(n)} \;=\; \gamma^{(n)} + \sum_{N \in \mathcal{N}[\gamma^{(n)}]} \prod_{\rho \in N} (-t_\rho)\, \gamma^{(n)}\,,$$



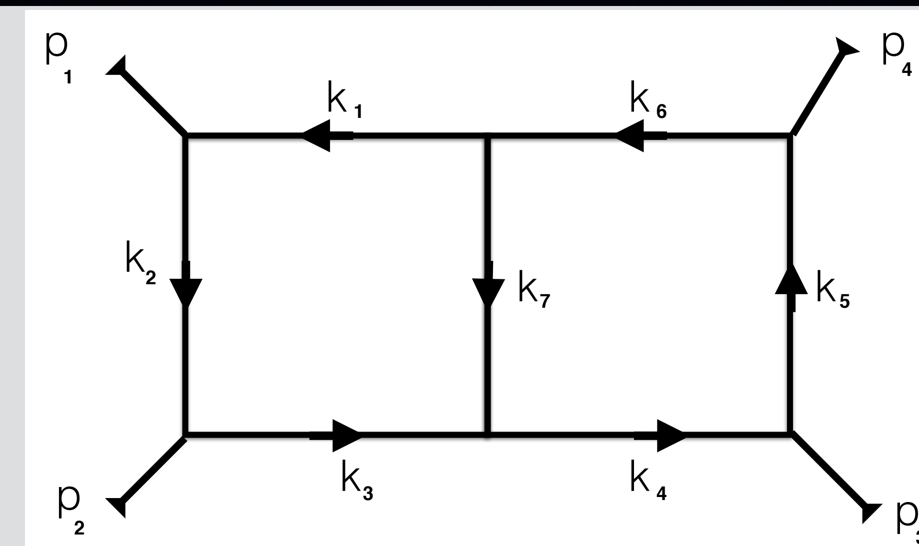
Nested subtractions at 2-loops

CA, George Sterman

- Order of subtractions:
  - double-soft
  - soft-collinear
  - double-collinear
  - single-soft
  - single-collinear
- Approximations in singular regions do not need to be strict limits!
- Good approximations should not introduce ultraviolet divergences
- Good approximations should be easy to integrate exactly.

Example: planar double-box

CA, George Sterman



$$F_{Pbox} = \frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_4P_5P_6P_7} + F^{(1s)}_{Pbox} + F^{(1c)}_{Pbox},$$

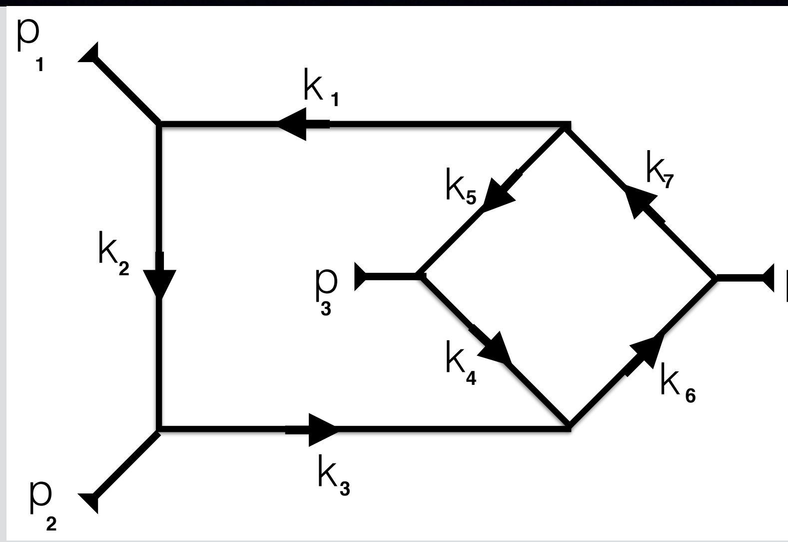
$$F^{(2)}_{Pbox} = 1 - \frac{P_{257}}{t} - \frac{P_{1346}}{s} + \frac{P_1P_6 + P_3P_4}{s^2} + \frac{P_{13}P_5 + P_{46}P_2}{st} + \frac{s+t}{s^2 t}\left(P_1P_4 + P_3P_6\right)$$

$$F^{(1c)}_{Pbox} = -\frac{\frac{\mu^2}{\mu^2-P_1}}{P_1P_2s(1-x_1)}\left\{\left[\frac{F^{(2)}_{Pbox}}{P_4P_5P_6P_7}\right]_{k_1=-x_1p_1} - \left[\frac{F^{(2)}_{Pbox}}{P_4P_5P_6P_7}\right]_{k_2=0}\right\}$$
$$-\frac{\frac{\mu^2}{\mu^2-P_3}}{P_3P_2s(1-x_2)}\left\{\left[\frac{F^{(2)}_{Pbox}}{P_4P_5P_6P_7}\right]_{k_3=-x_2p_2} - \left[\frac{F^{(2)}_{Pbox}}{P_4P_5P_6P_7}\right]_{k_2=0}\right\}$$
$$-\frac{\frac{\mu^2}{\mu^2-P_6}}{P_6P_5s(1-x_4)}\left\{\left[\frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_7}\right]_{k_6=x_4p_4} - \left[\frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_7}\right]_{k_5=0}\right\}$$
$$-\frac{\frac{\mu^2}{\mu^2-P_4}}{P_4P_5s(1-x_3)}\left\{\left[\frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_7}\right]_{k_4=-x_3p_3} - \left[\frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_7}\right]_{k_5=0}\right\}$$

$$F^{(1s)}_{Pbox} = -\frac{1}{P_1P_2P_3}\left[\frac{F^{(2)}_{Pbox}}{P_4P_5P_6P_7}\right]_{k_2=0} - \frac{1}{P_4P_5P_6}\left[\frac{F^{(2)}_{Pbox}}{P_1P_2P_3P_7}\right]_{k_5=0}$$

Example:cross-box

CA, George Sterman



$$F_{Xbox} = \frac{F^{(2)}_{Xbox}}{P_1P_2P_3P_4P_5P_6P_7} + F^{(1s)}_{Xbox} + F^{(1c)}_{Xbox},$$

$$F^{(2)}_{Xbox} = \left(1 - \frac{P_{13}}{s}\right)^2 + \frac{P_2}{tu}\left(P_2 + s - P_{13}\right)$$
$$- \left(1 - \frac{P_1}{s}\right)\left(\frac{P_5}{t} + \frac{P_7}{u}\right) - \left(1 - \frac{P_3}{s}\right)\left(\frac{P_4}{u} + \frac{P_6}{t}\right) + \frac{P_2P_{4567}}{tu}$$
$$- \frac{P_3}{s}\left(\frac{P_7}{t} + \frac{P_5}{u}\right) - \frac{P_1}{s}\left(\frac{P_6}{u} + \frac{P_4}{t}\right) + \frac{(t-u)^2}{s^2}\frac{P_1P_3}{tu}.$$

$$F^{(1c)}_{Xbox} = -\frac{\frac{\mu^2}{\mu^2 - P_1}}{P_1P_2s(1-x_1)}\left\{\left[\frac{F^{(2)}_{Xbox}}{P_4P_5P_6P_7}\right]_{k_1=-x_1p_1} - \left[\frac{F^{(2)}_{Xbox}}{P_4P_5P_6P_7}\right]_{k_2=0}\right\}$$
$$-\frac{\frac{\mu^2}{\mu^2 - P_1}}{P_2P_3s(1-x_3)}\left\{\left[\frac{F^{(2)}_{Xbox}}{P_4P_5P_6P_7}\right]_{k_3=-x_2p_2} - \left[\frac{F^{(2)}_{Xbox}}{P_4P_5P_6P_7}\right]_{k_2=0}\right\}$$
$$-\frac{\frac{\mu^2}{\mu^2 - P_4}}{P_4P_5}\left[\frac{F^{(2)}_{Xbox}}{P_1P_2P_3P_6P_7}\right]_{k_5=-x_3p_3} - \frac{\frac{\mu^2}{\mu^2 - P_6}}{P_6P_7}\left[\frac{F^{(2)}_{Xbox}}{P_1P_2P_3P_4P_5}\right]_{k_5=-x_4p_4}$$

$$F^{(1s)}_{Xbox} = -\frac{1}{P_1P_2P_3}\left[\frac{F^{(2)}_{Xbox}}{P_4P_5P_6P_7}\right]_{k_2=0}$$

Les Houches wish-list

- There is a long agenda for precision physics at the LHC
- Essential theoretical work which is needed to exploit to its maximum the multi-billion investment for the experiments

Process	State of the Art	Desired
H	$d\sigma$ @ NNLO QCD (expansion in $1/m_t$) full m_t/m_b dependence @ NLO QCD and @ NLO EW NNLO+PS, in the $m_t \to \infty$ limit	$d\sigma$ @ NNNLO QCD (infinite-m_t limit) full m_t/m_b dependence @ NNLO QCD and @ NNLO QCD+EW NNLO+PS with finite top quark mass effects
$H+j$	$d\sigma$ @ NNLO QCD (g only) and finite-quark-mass effects @ LO QCD and LO EW	$d\sigma$ @ NNLO QCD (infinite-m_t limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H+2j$	σ_{tot}(VBF) @ NNLO(DIS) QCD $d\sigma$(VBF) @ NLO EW $d\sigma$(gg) @ NLO QCD (infinite-m_t limit) and finite-quark-mass effects @ LO QCD	$d\sigma$(VBF) @ NNLO QCD + NLO EW $d\sigma$(gg) @ NNLO QCD (infinite-m_t limit) and finite-quark-mass effects @ NLO QCD and NLO EW
$H+V$	$d\sigma$ @ NNLO QCD $d\sigma$ @ NLO EW σ_{tot}(gg) @ NLO QCD (infinite-m_t limit)	with $H \to b\bar{b}$ @ same accuracy $d\sigma$(gg) @ NLO QCD with full m_t/m_b dependence
tH and $\bar{t}$H	$d\sigma$(stable top) @ LO QCD	$d\sigma$(top decays) @ NLO QCD and NLO EW
$t\bar{t}H$	$d\sigma$(stable tops) @ NLO QCD	$d\sigma$(top decays) @ NLO QCD and NLO EW
$gg \to HH$	$d\sigma$ @ NLO QCD (leading m_t dependence) $d\sigma$ @ NNLO QCD (infinite-m_t limit)	$d\sigma$ @ NLO QCD with full m_t/m_b dependence

Table 1: Wishlist part 1 – Higgs ($V = W, Z$)

Process	State of the Art	Desired
$t\bar{t}$	σ_{tot}(stable tops) @ NNLO QCD $d\sigma$(top decays) @ NLO QCD $d\sigma$(stable tops) @ NLO EW	$d\sigma$(top decays) @ NNLO QCD + NLO EW
$t\bar{t}+j(j)$	$d\sigma$(NWA top decays) @ NLO QCD	$d\sigma$(NWA top decays) @ NNLO QCD + NLO EW
$t\bar{t}+Z$	$d\sigma$(stable tops) @ NLO QCD	$d\sigma$(top decays) @ NLO QCD + NLO EW
single-top	$d\sigma$(NWA top decays) @ NLO QCD	$d\sigma$(NWA top decays) @ NNLO QCD + NLO EW
dijet	$d\sigma$ @ NNLO QCD (g only) $d\sigma$ @ NLO EW (weak)	$d\sigma$ @ NNLO QCD + NLO EW
3j	$d\sigma$ @ NLO QCD	$d\sigma$ @ NNLO QCD + NLO EW
$\gamma+j$	$d\sigma$ @ NLO QCD $d\sigma$ @ NLO EW	$d\sigma$ @ NNLO QCD + NLO EW

Table 2: Wishlist part 2 – Jets and Heavy Quarks

Process	State of the Art	Desired
V	$d\sigma$(lept. V decay) @ NNLO QCD $d\sigma$(lept. V decay) @ NLO EW	$d\sigma$(lept. V decay) @ NNNLO QCD and @ NNLO QCD+EW NNLO+PS
$V+j(j)$	$d\sigma$(lept. V decay) @ NLO QCD $d\sigma$(lept. V decay) @ NLO EW	$d\sigma$(lept. V decay) @ NNLO QCD + NLO EW
VV'	$d\sigma$(V decays) @ NLO QCD $d\sigma$(on-shell V decays) @ NLO EW	$d\sigma$(decaying off-shell V) @ NNLO QCD + NLO EW
$gg \to VV$	$d\sigma$(V decays) @ LO QCD	$d\sigma$(V decays) @ NLO QCD
$V\gamma$	$d\sigma$(V decay) @ NLO QCD $d\sigma$(PA, V decay) @ NLO EW	$d\sigma$(V decay) @ NNLO QCD + NLO EW
$Vb\bar{b}$	$d\sigma$(lept. V decay) @ NLO QCD massive b	$d\sigma$(lept. V decay) @ NNLO QCD + NLO EW, massless b
$VV'\gamma$	$d\sigma$(V decays) @ NLO QCD	$d\sigma$(V decays) @ NLO QCD + NLO EW
$VV'V''$	$d\sigma$(V decays) @ NLO QCD	$d\sigma$(V decays) @ NLO QCD + NLO EW
$VV'+j$	$d\sigma$(V decays) @ NLO QCD	$d\sigma$(V decays) @ NLO QCD + NLO EW
$VV'+jj$	$d\sigma$(V decays) @ NLO QCD	$d\sigma$(V decays) @ NLO QCD + NLO EW
$\gamma\gamma$	$d\sigma$ @ NNLO QCD + NLO EW	q_T resummation at NNLL matched to NNLO

Table 3: Wishlist part 3 – Electroweak Gauge Bosons ($V = W, Z$)

Perturbative QCD is not only to be used…
above all, it must be enjoyed!

The math of perturbation theory is full of challenges and surprises.

An alternative wish list

- Discover fully the mathematics of perturbative QCD.
- Automate!