Quantum Critical Higgs Models

in the $gg \rightarrow ZZ \rightarrow 4\ell$ channel

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May 6, 2018
Why QCH?

- Hierarchy problem + LHC: we need to explore new directions.
- Georgi on unparticles: “it’s different; not just more particles”
- What is QCH?
  - Non-trivial critical point: Higgs sector is described by a (strongly coupled) CFT

The strongly coupled theory produces a weakly coupled effective theory
- Scaling dimension is $1 < \Delta < 2$ (In SM $\Delta = 1 + \mathcal{O}(\alpha)$)
- The CFT is softly broken at $\mu > m_h$
- No new massless particles (dilaton)
The 1PI Effective Action

\[ S = S_{CFT} + S_{mix} + S_{elem}. \]

\[ S \rightarrow S_{1PI} = \frac{1}{2Z_h} \int \frac{d^4p}{(2\pi)^4} h(p) \Sigma(p^2, \mu, \Delta) h(-p) + \ldots \]

- \( S_{1PI} \) is weakly coupled
- Higgs observed pole:
- \( S_{CFT} \rightarrow S_{1PI} \) : hierarchy solved.
  - or
- \( S_{CFT} + S_{mix} \rightarrow S_{1PI} \): the case in our models.
At high energies the CFT is restored:

\[ \langle h(p)h(-p) \rangle = \frac{-i}{p^{2(2-\Delta)}}. \]

- consequently

\[ \Sigma(p^2) = p^{2(2-\Delta)} \quad \text{as} \quad p^2 \to \infty \]

- Unitarity requires: \( 1 < \Delta \)

- In QCH we have an action: \( \Delta < 2 \) (unparticles)
unHiggs
A 4D example: unparticles model with mass gap

- We break the CFT using the ansatz:
  \[ \langle h(p)h(-p) \rangle \propto \int_{\mu^2}^{\infty} (M^2 - \mu^2)^{\Delta-2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2, \]

- The kinetic term is
  \[ \Sigma(p^2) = -(\mu^2 - p^2)^{2-\Delta} + (\mu^2 - m_h^2)^{2-\Delta} \]

- Unitarizes \( WW \rightarrow WW \)
Higgs from AdS

- n-point functions can be calculated from:

\[ \langle e^{-\int d^4x \phi_0 \mathcal{O}} \rangle_{CFT} = e^{-S_{gravity}[\phi]}, \]

- The dual field has the scaling dimension

\[ ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2) \]

\[ \Delta = 2 - \nu, \]

\[ \nu = \sqrt{m^2 R^2 + 4}, \quad M^2(z) = m^2 + f(z) \]

\[ \Sigma(p^2) = \frac{R^3}{\epsilon^{3-2\Delta}} \partial_z K(p^2, z = \epsilon) - m_{UV}^2 \]

\[ h_\epsilon = \epsilon^\Delta h_{QCH}, \quad p\epsilon \to 0 \]
Model I

“Holographic unHiggs”

hep-ph/0810.4940

- The background is assumed:

\[
\begin{align*}
A(z) &= \log\left(\frac{z}{R}\right) + \frac{2}{3}\mu(z - R), \\
M^2(z) &= e^{\frac{4}{3}\mu(z-R)}\left(m^2 - \frac{3z}{R^2}\right).
\end{align*}
\]

- After redefining the field and removing the regulating brane

\[
h_{\epsilon}^2 = \frac{\epsilon^{4-2\nu}2^{2\nu}\Gamma(\nu)}{R^3\Gamma(1-\nu)}h^2
\]

\[
\Sigma(p^2) = -(\mu^2 - p^2)\nu + (\mu^2 - m_h^2)\nu
\]
Model II

“Dynamical Soft-Wall AdS/QCD”

- Constructed to study AdS/QCD
- Background is solved in the presence of a dilaton, $\Phi$, and a Tachyonic mode.

\[
\begin{align*}
A(z) &= \log \left( \frac{z}{R} \right) \\
M^2(z) &= m^2 \\
\Phi(z) &= (2\mu z)^\alpha
\end{align*}
\]

$\alpha = 1$, continuum

$\alpha = 2$, Regge

\[
S_5 = \int d^5x \sqrt{g} e^{-\Phi} \mathcal{L}
\]

\[
\Sigma(p^2) = (\mu^2 - p^2)^\nu \frac{\Gamma\left(\frac{1}{2} + \frac{3\mu}{2\sqrt{\mu^2 - p^2}} + \nu\right)}{\Gamma\left(\frac{1}{2} + \frac{3\mu}{2\sqrt{\mu^2 - p^2}} - \nu\right)} - (p^2 \to m_h^2)
\]
Model III

“ The AdS/CFT/Unparticle Correspondence”

- An additional scalar field was added in the bulk
- The scalar acquires a $z$-dependent VEV
  \[
  \begin{align*}
  A(z) &= \log\left(\frac{z}{R}\right) \\
  \phi(z) &= \mu^2 R^{1/2} \left(\frac{z}{R}\right)^2 \\
  M^2(z) &= m^2 + R^{1/2} \phi(z)
  \end{align*}
  \]
- We further investigate the back-reaction of the scalar field on the geometry
- There is a singularity at $z_s \approx (R M_5)^{3/4} / \mu$

\[
\Sigma(p^2) = (\mu^2 - p^2)\nu \left(1 + \frac{2}{\Gamma(1 - \nu) \Gamma(\nu)} \frac{K_{\nu}(\sqrt{\mu^2 - p^2 z_s})}{I_{\nu}(\sqrt{\mu^2 - p^2 z_s})} \right) - (p^2 \to m_h^2)
\]
The generic behavior of the spectral density functions

\[
\langle h(p)h(-p) \rangle \propto \int_{\mu^2}^{\infty} \rho(M^2) \frac{i}{p^2 - M^2 + i\epsilon} dM^2,
\]

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- $gg \rightarrow ZZ$:

- We will focus on the $hZZ$ coupling.
Gauging the non-local action

- We can gauge the non-local action using Mandelstam’s method (Wilson line)

\[
S = \int d^4x \int d^4y \ h(x) \Sigma(x - y) P \cdot e^{-ig \int_x^y A_\mu du^\mu} h(y)
\]

- \( hZZ \) (Stancato’s PhD thesis)

\[
\begin{align*}
q_2 & \quad \nu \\
q_1 & \quad \mu \\
\rightarrow & \quad p
\end{align*}
\]

\[
= -i \frac{gM_Z}{2c_w Z_h} \left[ 2\eta^{\mu\nu} \Sigma(p) - \Sigma(0) \right. \\
& \quad \left. \frac{\Sigma(p) - \Sigma(0)}{p^2} \right. \\
& \quad + (2q_2 + q_1)^\mu q_2^\nu * \# + \ldots
\]

Results

- Plot has been generated using $ggZZ$ program
- The Zs are on-shell
- No background
AdS/Broken-CFT

\[ ds^2 = e^{-2A(z)} \left( \eta^{\mu\nu} dx_\mu dx_\nu - dz^2 \right) \]

AdS ⇔ CFT \quad \text{energy scale} ⇔ \frac{1}{z}

5D

4D dual

- UV cut-off = $1/\epsilon$
- CFT not broken in IR
- e.g. RS II model

\[ A(z)_{\text{AdS}} = \log \left( \frac{z}{R} \right) \quad , \quad R : \text{radius of AdS} \]
AdS/Broken-CFT

\[ ds^2 = e^{-2A(z)} (\eta_{\mu\nu} dx_\mu dx_\nu - dz^2) \]

AdS ⇔ CFT \hspace{0.5cm} \text{energy scale} \leftrightarrow 1/z

5D

4D dual

- UV cut-off = 1/\epsilon
- hard breaking of CFT
  - K.K. modes
- e.g. RS I model

\[ \text{IR brane} \]
$ds^2 = e^{-2A(z)}(\eta^{\mu\nu} dx_{\mu} dx_{\nu} - dz^2)$

AdS$\iff$CFT   energy scale $\iff 1/z$

5D

4D dual

- UV cut-off $= 1/\epsilon$
- soft breaking of CFT
  - mass gap + no K.K. modes
- e.g. models I & II
\[ ds^2 = e^{-2A(z)} (\eta^{\mu\nu} dx_\mu dx_\nu - dz^2) \]

AdS \iff CFT \quad \text{energy scale} \iff \frac{1}{z}

5D

4D dual

- UV cut-off \( = \frac{1}{\epsilon} \)
- soft + hard breaking of CFT
  - mass gap + K.K. modes
- e.g. model III
We need to check for further contributions in AdS picture

We remove the unwanted gauges by b.c.:

\[ SU(2)_L \times SU(2)_R \rightarrow SU(2)_L \times U(1)_Y \]

Boundary potential for the Higgs ignites SB in the bulk

\[ \mathcal{L}_H = \frac{1}{2} \partial_M h \partial^M h - \frac{1}{2} M(z)^2 h^2 + \frac{1}{2} (V(z) + h)^2 (L^a_M - R^a_M)^2 + \ldots \]

where

\[ L^3_{\mu \nu} - R^3_{\mu \nu} = \frac{g}{c_w} Z_\mu \]
Interactions from the bulk

\[ \nu_{\text{eff}} = \nu_{\text{bulk}} + \pi_{\text{bulk}} + \text{permutations} \]

- These will give what we already have found by imposing gauge invariance
- We can add gauge invariant coupling that further contribute to \( hZZ \):

\[ \mathcal{L}_{\text{int}} \propto H^2 F^2 \]

- Our philosophy is to keep few free parameters and so will ignore these
- No further contribution at tree level
Unitarity of $t\bar{t}WW$ scattering

- Non conformal case
  \[ \mathcal{L}_{Y, NC} = - \frac{m_t}{\mathcal{V}} h t \bar{t} \]
  \[ \mathcal{M}(t\bar{t} \to WW) = \sqrt{2} G_F m_t m_h^{4-2\Delta} (-1)^{\Delta-2} s^{\Delta-3/2} \]
  \[ \lim_{p^2 \to \infty} F(p^2) \propto p^{1-\Delta} \]

- The amplitude diverges for $\Delta > 3/2$
- Propagating the top into the bulk we find
unitarity of $WW$ scattering

- vertices with arbitrary number of gauge bosons

- The crosses indicate insertion of Higgs VEV
SB in the Bulk

\[ S = \int d^4x dz \sqrt{g} \left[ |D_M H|^2 - \frac{1}{4g_L^2} L_{MN}^a L_{MN}^a - \frac{1}{4g_R^2} R_{MN}^a R_{MN}^a \right. \]

\[ - M(z)^2 + L_{\text{int}}(H) \bigg] + \int d^4x L_{\text{boundary}} \]

\[ L_{\text{boundary}} \supset - m_{\text{UV}}^2 H_0^2 + \lambda/4H_0^4 \]

\[ \mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + V \end{pmatrix} + \Pi \]
Peskin-Takeuchi S and T

\[ T = 0 \]

\[ \alpha S = 4 s_w^2 g^2 \partial_p^2 \Pi_{WB}(p^2 = 0) \]

\[ \Pi_{WB} = \frac{1}{g_L^2 + g_R^2} \left( \frac{v'(p^2, z = z_{UV})}{v(p^2, z = z_{UV})} - \frac{a'(p^2, z = z_{UV})}{a(p^2, z = z_{UV})} \right) \]

- \( a \) and \( v \) are the solution to the axial and vector 5D fields

\[ SU(2)_L \times SU(2)_R \rightarrow SU(2)_V \]
“The Schrödinger potential”

- After a field re-definition

\[-\dddot{h} + V\ddot{h} = p^2 \dot{h}\]

\[V = \frac{9}{4} A'^2 - \frac{3}{2} A'' + e^{-2A} M^2\]

- as \(z \to \infty:\)

\[V \to \infty \text{ discretuum}\]

\[V \to 0 \text{ continuum w/o mass gap}\]

\[V \to \mu \text{ continuum w/ mass gap}\]