Baryon spectroscopy and structure with Dyson-Schwinger equations

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Many manifestations of nonperturbative QCD
Camburí, Sao Paulo, Brazil

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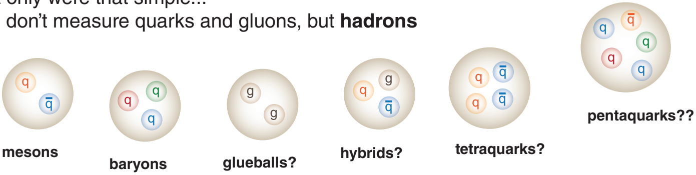
Why?

QCD Lagrangian: $\mathcal{L} = \bar{\psi}\,(\not{\partial} + ig\not{A} + m)\,\psi + \frac{1}{4}\,F^{a}_{\mu\nu}\,F^{\mu\nu}_{a}$

- if it only were that simple...
  we don't measure quarks and gluons, but **hadrons**



- origin of **mass generation** and **confinement?**

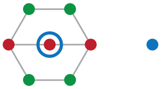
	u	d	s	c	b	t
Current mass [GeV]	**0.003**	**0.005**	**0.1**	**1**	**4**	**175**
„Constituent“ mass [GeV]	**0.35**	**0.35**	**0.5**	**1.5**	**4.5**	**175**

- need to understand **spectrum and interactions!**

The hadron zoo

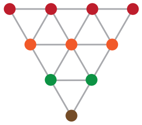
Mesons

0^{-+}	0^{++}	1^{-+}	1^{--}	1^{++}	1^{+-}	2^{-+}	2^{++}	3^{--}
$\pi(140)$	$a_0(980)$	$\pi_1(1400)$	$\rho(770)$	$a_1(1260)$	$b_1(1235)$	$\pi_2(1670)$	$a_2(1320)$	$\rho_3(1690)$
$\pi(1300)$	$a_0(1450)$	$\pi_1(1600)$	$\rho(1450)$	$a_1(1420)$		$\pi_2(1880)$	$a_2(1700)$	$\rho_3(1990)$
$\pi(1800)$	$a_0(1950)$		$\rho(1570)$	$a_1(1640)$				
			$\rho(1700)$					
			$\rho(1900)$					
$K(494)$	$K_0^*(800)$		$K^*(892)$	$K_1(1400)$	$K_1(1270)$	$K_2(1580)$	$K_2^*(1430)$	$K_3^*(1780)$
$K(1460)$	$K_0^*(1430)$		$K^*(1410)$	$K_1(1650)$		$K_2(1770)$	$K_2^*(1980)$	
$K(1830)$	$K_0^*(1950)$		$K^*(1680)$			$K_2(1820)$		
$\eta(548)$	$f_0(500)$		$\omega(782)$	$f_1(1285)$	$h_1(1170)$	$\eta_2(1645)$	$f_2(1270)$	$\omega_3(1670)$
$\eta'(958)$	$f_0(980)$		$\phi(1020)$	$f_1(1420)$	$h_1(1380)$	$\eta_2(1870)$	$f_2(1430)$	$\phi_3(1850)$
$\eta(1295)$	$f_0(1370)$		$\omega(1420)$	$f_1(1510)$	$h_1(1595)$		$f_2'(1525)$	
$\eta(1405)$	$f_0(1500)$		$\omega(1650)$				$f_2(1565)$	
$\eta(1475)$	$f_0(1710)$		$\phi(1680)$				$f_2(1640)$	
$\eta(1760)$							$f_2(1810)$	
							$f_2(1910)$	
							$f_2(1950)$	

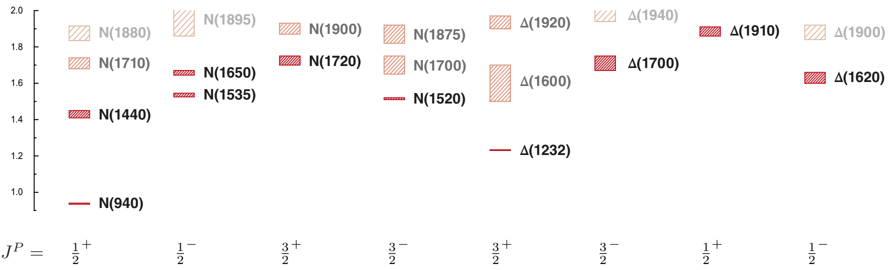


Baryons

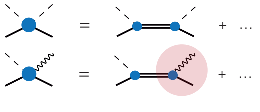
$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$	$N(1535)$	$N(1720)$	$N(1520)$	$N(1680)$	$N(1675)$	$N(1990)$
$N(1440)$	$N(1650)$	$N(1900)$	$N(1700)$	$N(1860)$		
$N(1710)$	$N(1895)$		$N(1875)$	$N(2000)$		
$N(1880)$						
$\Delta(1910)$	$\Delta(1620)$	$\Delta(1232)$	$\Delta(1700)$	$\Delta(1905)$	$\Delta(1930)$	$\Delta(1950)$
	$\Delta(1900)$	$\Delta(1600)$	$\Delta(1940)$	$\Delta(2000)$		
		$\Delta(1920)$				
$\Lambda(1116)$	$\Lambda(1405)$	$\Lambda(1890)$	$\Lambda(1520)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Lambda(1600)$	$\Lambda(1670)$		$\Lambda(1690)$			
$\Lambda(1810)$	$\Lambda(1800)$					
$\Sigma(1189)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Sigma(1660)$			$\Sigma(1940)$			
$\Sigma(1880)$						
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



Light baryons



- Extraction of resonances?



- **Gluon exchange** vs. flavor dependence?
- Nature of **Roper?**
- qqq vs. **quark-diquark?**

- "Quark core" vs. **chiral dynamics?**
- Admixture of **multiquarks?**
- **Hybrid baryons?**

Outline

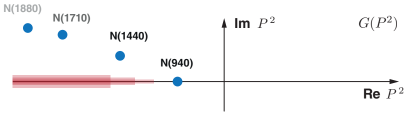
- **DSEs, BSEs** and their applications to mesons & baryons
- **Baryon spectrum:** light and strange baryons, quark-diquark vs. three-quark structure
- **Nucleon resonances** in **Compton scattering,** transition form factors
- **Outlook:** resonances & multiquark states

Hadrons in QCD

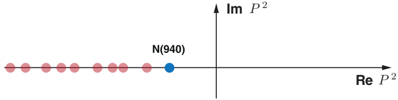
Lattice: extract baryon poles from (gauge-invariant) two-point correlators:

$$G(x-y) = \langle 0 \,|\, T \, \underbrace{[\Gamma_{\alpha\beta\gamma}\, \psi_\alpha\, \psi_\beta\, \psi_\gamma]\,(x)}_{B(x)} \, \underbrace{[\bar{\Gamma}_{\rho\sigma\tau}\, \bar{\psi}_\rho\, \bar{\psi}_\sigma\, \bar{\psi}_\tau]\,(y)}_{\overline{B}(y)} \,|\, 0 \rangle \;=\; \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S}\, B(x)\, \overline{B}(y)$$

$$G(\tau) \sim e^{-m\tau} \qquad \Leftrightarrow \qquad G(P^2) \sim \frac{1}{P^2+m^2}$$



- **Infinite volume:**
  Bound states, resonances, branch cuts



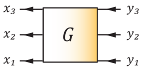
- **Finite volume:**
  bound states & scattering states

Hadrons in QCD

Lattice: extract baryon poles from (gauge-invariant) two-point correlators:

$$G(x-y) = \langle 0 \,|\, T \underbrace{\left[\Gamma_{\alpha\beta\gamma}\, \psi_\alpha\, \psi_\beta\, \psi_\gamma\right](x)}_{B(x)}\, \underbrace{\left[\bar{\Gamma}_{\rho\sigma\tau}\, \bar{\psi}_\rho\, \bar{\psi}_\sigma\, \bar{\psi}_\tau\right](y)}_{\bar{B}(y)} \,|\, 0 \rangle \;=\; \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S}\, B(x)\, \bar{B}(y)$$

$$= \lim_{\substack{x_i \to x \\ y_i \to y}} \Gamma_{\alpha\beta\gamma}\, \bar{\Gamma}_{\rho\sigma\tau}\, \boxed{\langle 0 \,|\, T\, \psi_\alpha(x_1)\, \psi_\beta(x_2)\, \psi_\gamma(x_3)\, \bar{\psi}_\rho(y_1)\, \bar{\psi}_\sigma(y_2)\, \bar{\psi}_\tau(y_3) \,|\, 0 \rangle}$$



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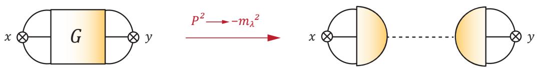
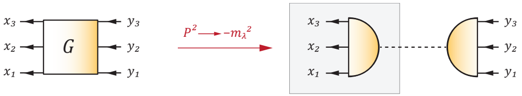


Alternative: extract **gauge-invariant** baryon poles from **gauge-fixed** quark 6-point function:



Bethe-Salpeter wave function:
residue at pole, contains all information about baryon

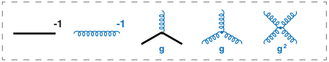
$$\langle 0 \,|\, T\, \psi_\alpha(x_1)\, \psi_\beta(x_2)\, \psi_\gamma(x_3) \,|\, \lambda \rangle$$

QCD's n-point functions

QCD's classical action:

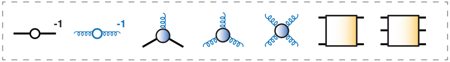
$$S \;=\; \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + ig A\!\!\!/ + m \right) \psi + \tfrac{1}{4}\, F^{\,a}_{\mu\nu}\, F^{\mu\nu}_{a} \right]$$

$=$

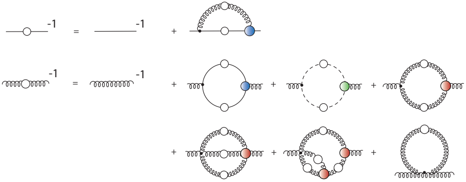


Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} \;=\; e^{-\Gamma}$$



DSEs = quantum equations of motion:
derived from path integral, relate n-point functions

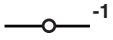


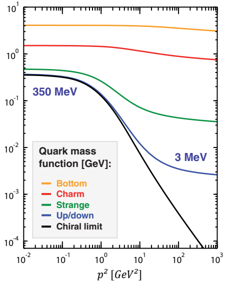
- infinitely many coupled equations
- reproduce perturbation theory, but **nonperturbative**
- systematic truncations: neglect higher n-point functions to obtain **closed system**

Some Reviews:
Roberts, Williams, Prog. Part. Nucl. Phys. 33 (1994),
Alkofer, von Smekal, Phys. Rept. 353 (2001)
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602 [hep-ph]

QCD's n-point functions

- **Quark propagator**

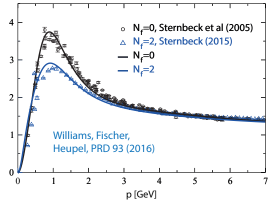
$A(p^2)\,(i\not{p} + M(p^2))$ 



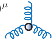
Dynamical chiral symmetry breaking generates 'constituent-quark masses'

- **Gluon propagator**

$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$ 



- **Three-gluon vertex**

$F_1\,[\,\delta^{\mu\nu}(p_1-p_2)^\rho + \delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu] + \dots$ 

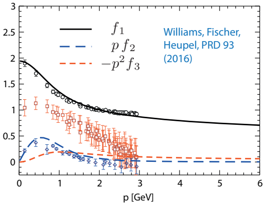
Agreement between lattice, DSE & FRG within reach

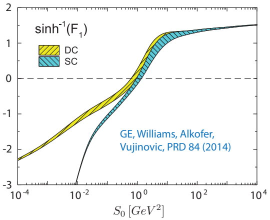
Huber, EPJ C77 (2017),
Cyrol, Mitter, Pawlowski, PRD 97 (2018), . . .

→ see talks by **Cristina & Richard**

- **Quark-gluon vertex**

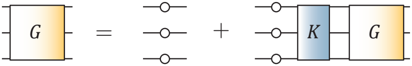
$f_1\,\gamma^\mu + f_2\,ip^\mu + f_3\,p^\mu\not{p} + \dots$ 





DSEs $\longrightarrow$ Hadrons?

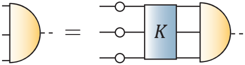
Bethe-Salpeter approach:
use scattering equation $G = G_0 + G_0\, K\, G$



- still exact - to begin with, kernel is black box
- but can be derived together with QCD's n-point functions.
  Important to preserve symmetries!

$P^2 \longrightarrow -m^2$

Homogeneous BSE for **BS wave function:**



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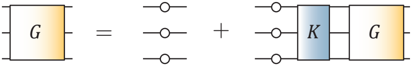

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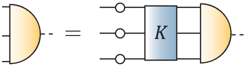
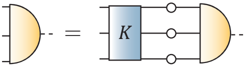
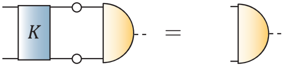


. . . or **BS amplitude:**



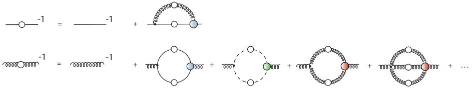
Mesons

- Meson **Bethe-Salpeter equation** in QCD:

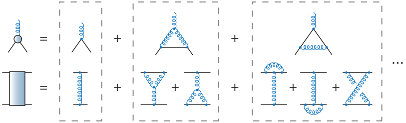


$$K(M)\; G(M)\; \phi_i(M) \quad = \quad \lambda_i(M)\; \phi_i(M)$$

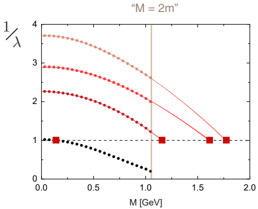
- Depends on QCD's n-point functions, satisfy **DSEs:**



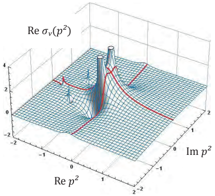
- Kernel derived in accordance with **chiral symmetry:**



Eigenvalues in **pion** channel:



Quark propagator has **complex singularities:** no physical threshold



Mesons

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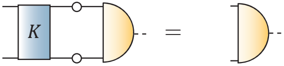

$$K(M)\; G(M)\; \phi_i(M) \;=\; \lambda_i(M)\; \phi_i(M)$$

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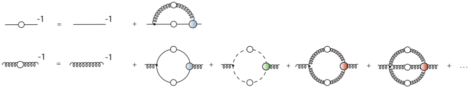

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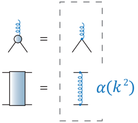
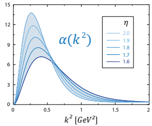




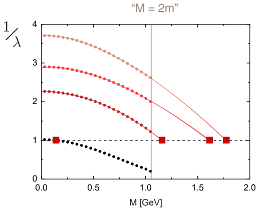
Rainbow-ladder:
effective gluon exchange

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

Eigenvalues in **pion** channel:



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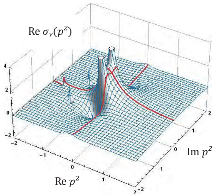
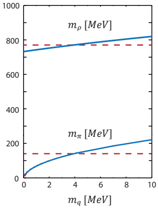
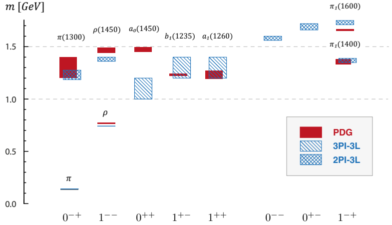


Mesons

- Pion is **Goldstone boson:** $m_\pi^{\,2} \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder



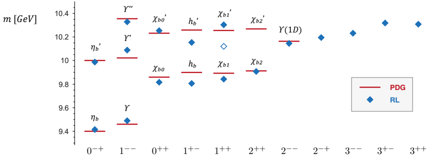
Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

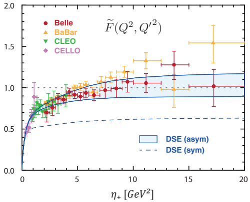
see also Chang, Roberts, PRL 103 (2009), PRC 85 (2012)

- **Charmonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



- **Pion transition form factor**

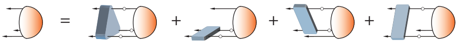


GE, Fischer, Weil, Williams, PLB 774 (2017)

Baryons

Covariant Faddeev equation for baryons:

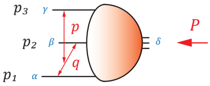
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)



- 3-gluon diagram vanishes ⇒ **3-body effects small?**

  Sanchis-Alepuz, Williams, PLB 749 (2015)

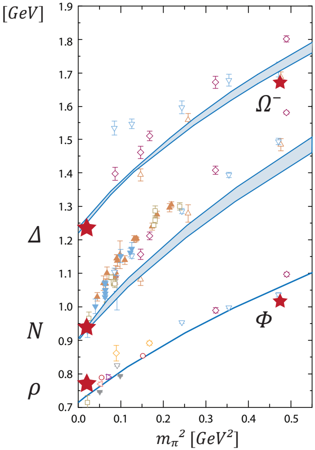
- 2-body kernels same as for mesons, no further approximations:



$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\ \tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

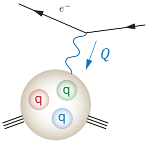
Lorentz-invariant dressing functions

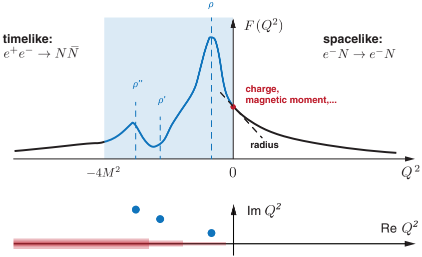
Dirac-Lorentz tensors carry OAM: s, p, d,...



Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

Form factors

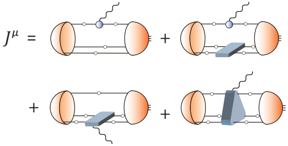




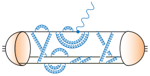
$$J^\mu = e\,\bar{u}(p_f)\left(F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i}{4m}\,[\gamma^\mu, \not{Q}]\right) u(p_i)$$

Consistent derivation of **current matrix elements & scattering amplitudes**

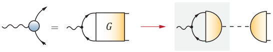
Kvinikhidze, Blankleider, PRC 60 (1999),
GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



- **rainbow-ladder** topologies (1st line):

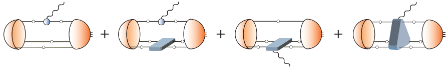


- **quark-photon vertex** preserves em. gauge invariance, dynamically generates **VM poles:**

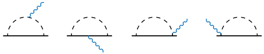


Form factors

Nucleon em. form factors
from three-quark equation
GE, PRD 84 (2011)

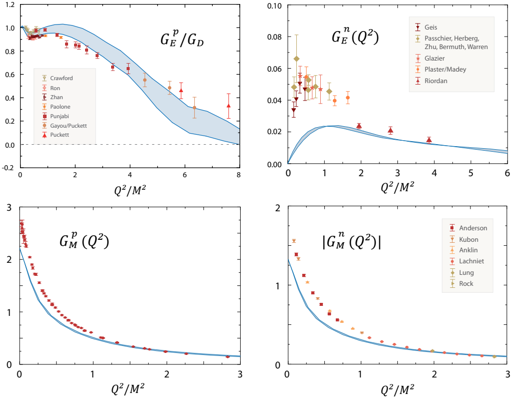
$J^\mu =$ 

- **"Quark core without pion cloud"**



- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602



Scattering amplitudes

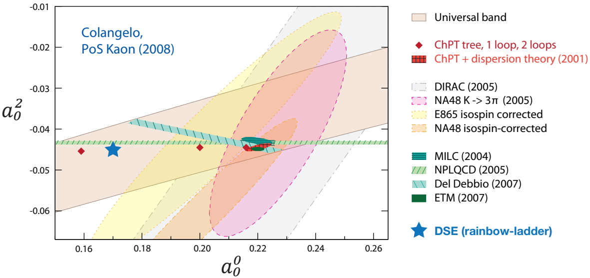
Scattering amplitudes from quark level:

- **$\pi\pi$ scattering**

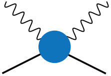
  DSE: Bicudo, Cotanch, Llanes-Estrada, Maris, Ribeiro, Szczepaniak, PRD 65 (2002),

  Cotanch, Maris, PRD 66 (2002)

  CST: Biernat, Pena, Ribeiro, Stadler, Gross, PRD 90 (2014)



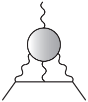
- **Nucleon Compton scattering**



  GE, Fischer, PRD 85 (2012) & PRD 87 (2013), GE, FBS 57 (2016)

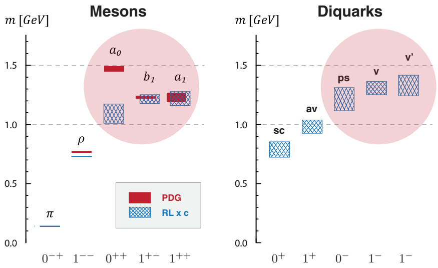
- **Hadronic light-by-light scattering**

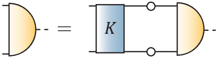
  Goecke, Fischer, Williams, PLB 704 (2011), GE, Fischer, Heupel, PRD 92 (2015)

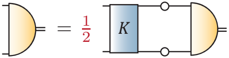


The role of diquarks

Mesons and 'diquarks' closely related:
after taking traces, only factor 1/2 remains
$\Rightarrow$ **diquarks 'less bound' than mesons**





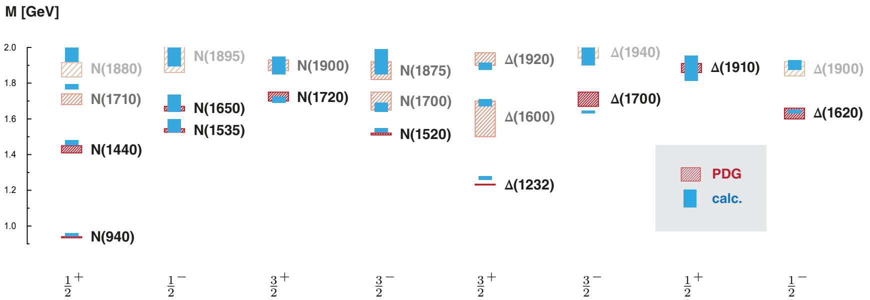


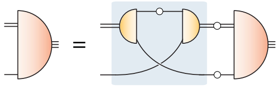
Pseudoscalar & vector mesons $\Leftrightarrow$ **Scalar & axialvector diquarks**
already good in rainbow-ladder — sufficient for nucleon and Δ

Scalar & axialvector mesons $\Leftrightarrow$ **Pseudoscalar & vector diquarks**
too light, repulsion beyond RL — important for remaining channels

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

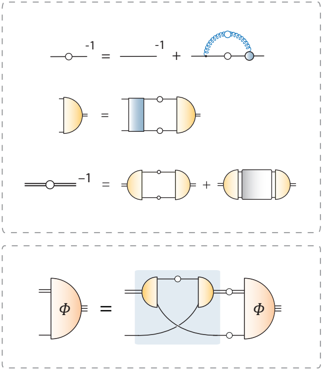




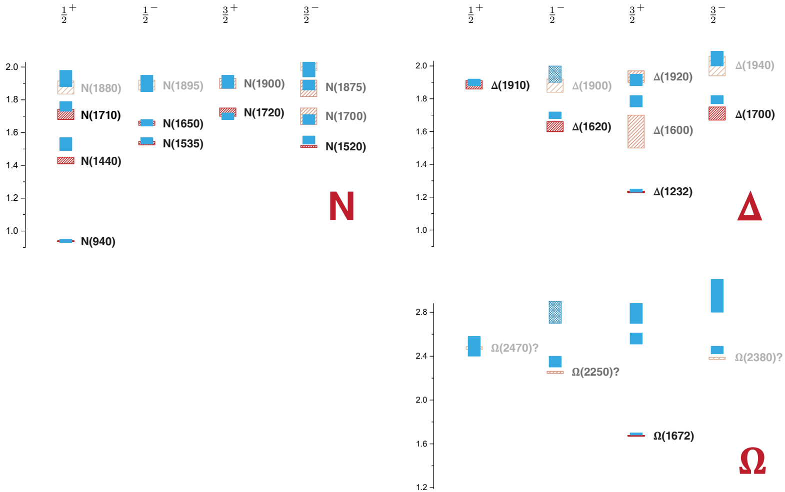
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

Strange baryons

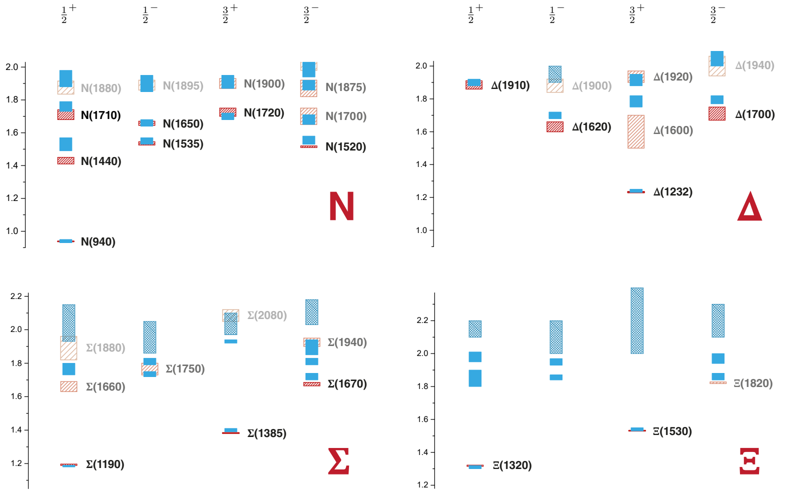
	[nn]	{nn}	[ns]	{ns}	{ss}
N	●	●			
Δ		●			
Λ	●		●	●	
Σ		●	●	●	
Ξ			●	●	●
Ω					●



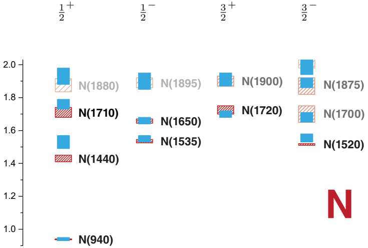
Strange baryons

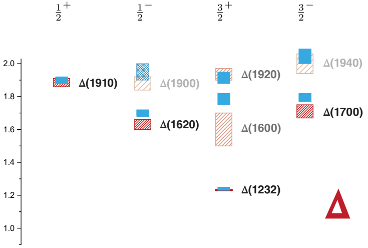


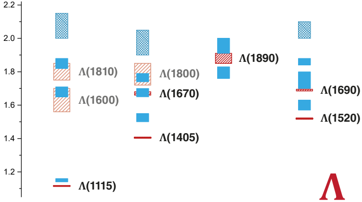
Strange baryons



Strange baryons







- Strange baryons similar to **light baryons:**

$$\Omega \to \Delta$$
$$\Sigma, \Xi \to \mathbf{N} + \Delta \qquad \to \text{rich spectrum!}$$
$$\Lambda \to \mathbf{N} + \text{singlets}$$

- Roper, Δ(1600), Λ(1405), Λ(1520): additional dynamics?

GE, Fischer, in preparation

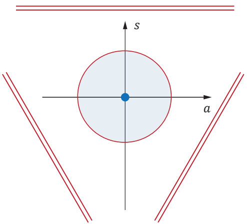
The role of diquarks?

- **Singlet:** symmetric variable, carries overall scale:

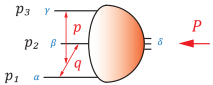
$$\mathcal{S}_0 \sim p_1^2 + p_2^2 + p_3^2 + \frac{M^2}{3}$$

- **Doublet:** $\mathcal{D}_0 \sim \frac{1}{\mathcal{S}_0} \begin{bmatrix} -\sqrt{3}\,(\delta x + 2\delta\omega) \\ x + 2\omega \end{bmatrix}$

Mandelstam plane, outside: **diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S3:**
GE, Fischer, Heupel, PRD 92 (2015)



- Second **doublet:** $\mathcal{D}_1 \sim \frac{1}{\sqrt{\mathcal{S}_0}} \begin{bmatrix} -\sqrt{3}\,(\delta x - \delta\omega) \\ x - \omega \end{bmatrix}$

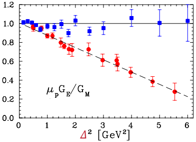
$f_i\,(\mathcal{S}_0, \bigcirc, \bigcirc)$ → full result as before

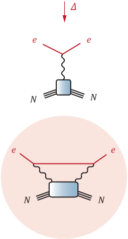
$f_i\,(\mathcal{S}_0, \bigcirc, \bigcirc)$ → **same ground-state spectrum,** but diquark poles switched off!

Scattering amplitudes

TPE corrections to form factors

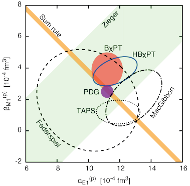
Guichon, Vanderhaeghen, PRL 91 (2003)





Proton radius puzzle?

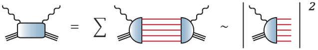
Antonigni et al., 2013, Pohl et al. 2013,
Birse, McGovern 2012, Carlson 2015



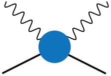
Nucleon polarizabilities

Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)

Structure functions & PDFs in forward limit

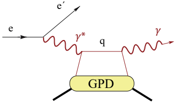


- **Nucleon Compton scattering**



GE, Fischer, PRD 85 (2012) &
PRD 87 (2013), GE, FBS 57 (2016)

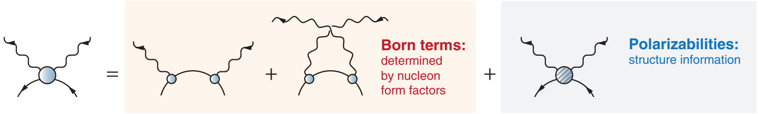
Handbag dominance & GPDs in DVCS

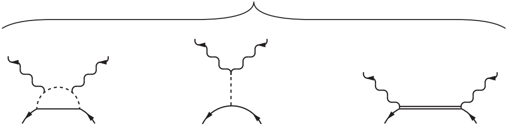


$p\bar{p}$ annihilation @ PANDA

Compton scattering

Compton amplitude = sum of **Born terms + 1PI structure part:**

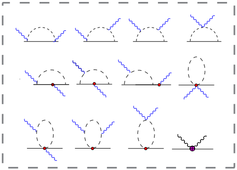




„Pion cloud“
(ChPT)

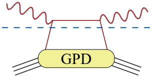
t-channel
meson exchange
$(\pi, \sigma, a_1, \dots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \dots)$



Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

but also:

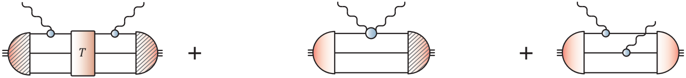


$\Rightarrow$ is there a common underlying **quark-level description?**

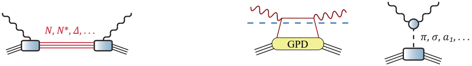
Compton scattering

Scattering amplitude: GE, Fischer, PRD 85 (2012) & PRD 87 (2013)

$\mathcal{M}^{\mu\nu} =$



Nucleon resonances | quark **Compton vertex:** reproduces perturbative handbag & t-channel meson poles | **cat's ears** diagrams



- **Poincaré covariance** and **crossing symmetry** automatic
- **em. gauge invariance** and **chiral symmetry** automatic
  as long as all ingredients calculated from symmetry-preserving kernel
- **perturbative processes** included
- **s, t, u channel poles** dynamically generated,
  no need for "offshell hadrons"

Nucleon resonances

$\sum$ 

$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940)	**N(1720)**	**N(1535)**	**N(1520)**
N(1440)	$N(1900)$	**N(1650)**	$N(1700)$
$N(1710)$		$N(1895)$	$N(1875)$
$N(1880)$			
$\boldsymbol{\Delta}$**(1910)**	$\boldsymbol{\Delta}$**(1232)**	$\boldsymbol{\Delta}$**(1620)**	$\boldsymbol{\Delta}$**(1700)**
	$\Delta(1600)$	$\Delta(1900)$	$\Delta(1940)$
	$\Delta(1920)$		

Need **em. transition FFs**

But vertices are half offshell:
need 'consistent couplings'
Pascalutsa, Timmermans, PRC 60 (1999)

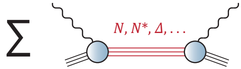
- **em gauge invariance:** $Q^\mu \, \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \, \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \, \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

E.g. Jones-Scadron current **cannot be used offshell:**

$$\Gamma^{\alpha\mu} \sim \bar{u}^\alpha(k) \Big[m^2 \lambda_- (G_M^* - G_E^*) \, \varepsilon^{\alpha\mu}_{kQ} - G_E^* \, \varepsilon^{\alpha\beta}_{kQ} \, \varepsilon^{\beta\mu}_{kQ} - \tfrac{1}{2} \, G_C^* \, Q^\alpha k^\beta t^{\beta\mu}_{QQ} \Big] \, u(k')$$

$$t^{\alpha\beta}_{AB} = A \cdot B \, \delta^{\alpha\beta} - B^\alpha A^\beta$$
$$\varepsilon^{\alpha\beta}_{AB} = \gamma_5 \, \varepsilon^{\alpha\beta\gamma\delta} A^\gamma B^\delta$$

Nucleon resonances



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N(1440)	N(1900)	**N(1650)**	N(1700)
N(1710)		N(1895)	N(1875)
N(1880)			
Δ(1910)	**Δ(1232)**	**Δ(1620)**	**Δ(1700)**
	Δ(1600)	Δ(1900)	Δ(1940)
	Δ(1920)		

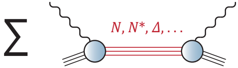
Most general **offshell vertices** satisfying these constraints:

GE, Ramalho, in preparation

$$\frac{1}{2}^+ \to \frac{1}{2}^\pm : \quad \Gamma^\mu = \begin{bmatrix} \mathbf{1} \\ \gamma_5 \end{bmatrix} \sum_{i=1}^{8} F_i \, T_i^\mu \qquad \left\{ \begin{array}{l} t^{\mu\nu}_{QQ}\gamma^\nu \\ [\gamma^\mu, \not{Q}] \\ \ldots \end{array} \right.$$

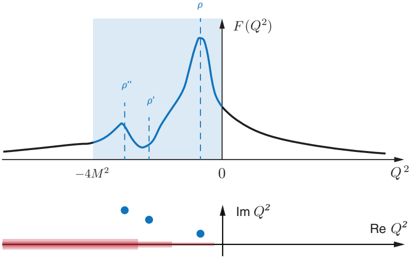
$$\frac{1}{2}^+ \to \frac{3}{2}^\pm : \quad \Gamma^{\alpha\mu} = \begin{bmatrix} \gamma_5 \\ \mathbf{1} \end{bmatrix} \sum_{i=1}^{12} F_i \, T_i^{\alpha\mu} \qquad \left\{ \begin{array}{l} \varepsilon^{\alpha\mu}_{kQ} \\ t^{\alpha\mu}_{kQ} \\ i t^{\alpha\beta}_{k\gamma} t^{\beta\mu}_{QQ} \\ \ldots \end{array} \right.$$

Nucleon resonances

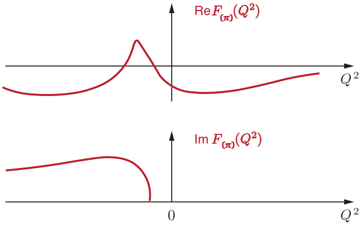


Constraint-free transition FFs:
only physical poles and cuts

- ρ **poles** $\rightsquigarrow$ monotonous behavior
  (+ zero crossings for excited states)



- Non-monotonicity at low Q2
  $\rightsquigarrow$ signature for cuts ($\rho \to \pi\pi$, etc.):
  meson cloud



Nucleon resonances

$\sum$ 

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N(940)	**N(1720)**	**N(1535)**	**N(1520)**
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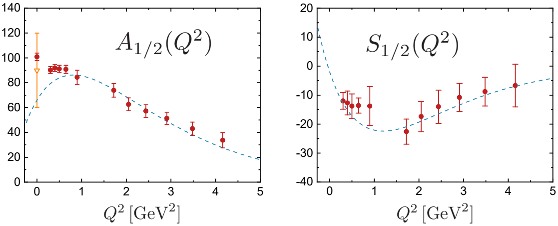


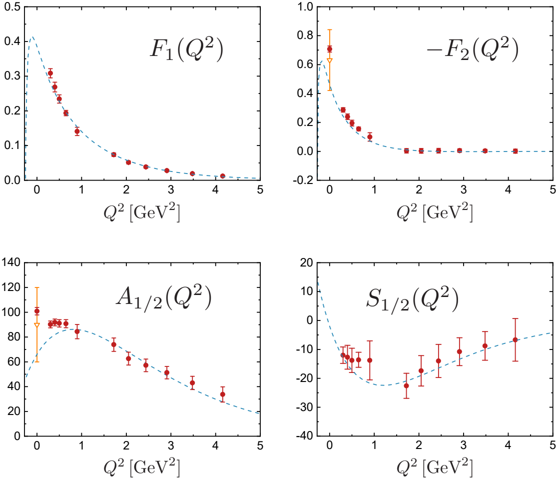
Example:
N(1535) helicity amplitudes

PDG

CLAS data
userweb.jlab.org/~mokeev/resonance_electrocouplings

MAID
Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

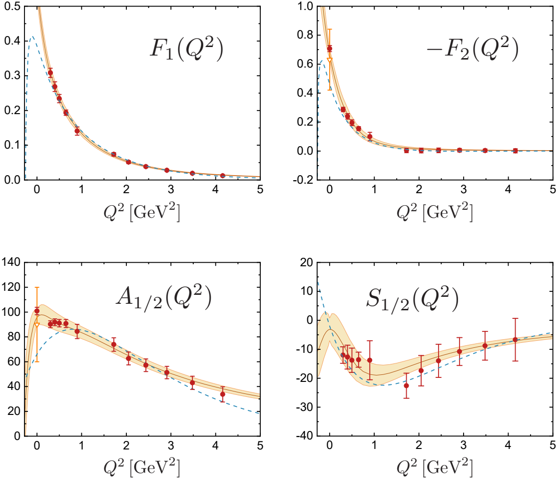
Nucleon resonances



N(1535) transition FFs:
no kinematic constraints

Example:
N(1535) helicity amplitudes

Nucleon resonances



N(1535) transition FFs:
no kinematic constraints

Fit

Example:
N(1535) helicity amplitudes

PDG

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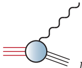
Nucleon resonances



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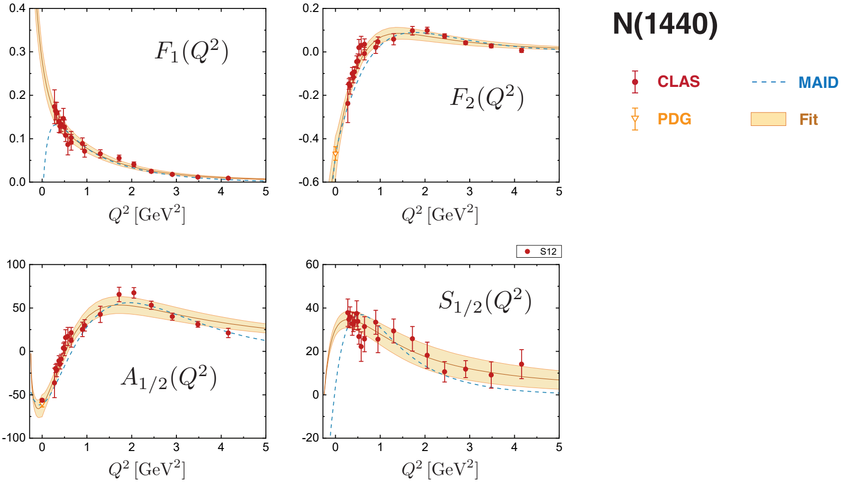
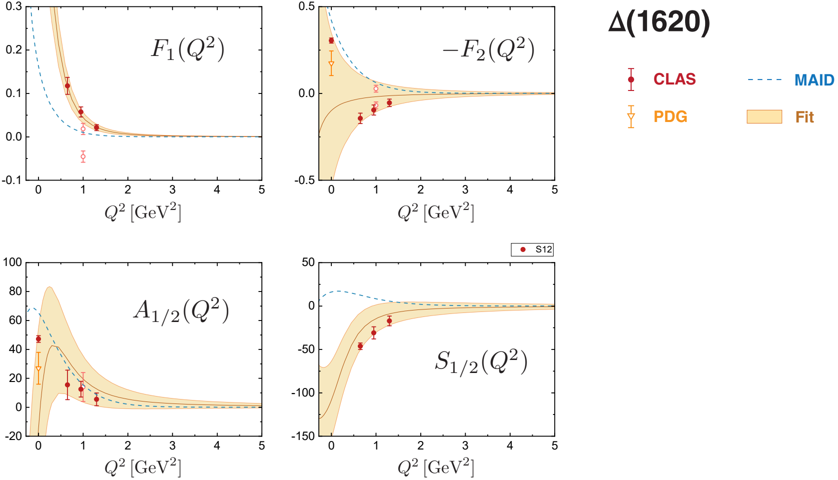
	$\Delta(1600)$	$\Delta(1900)$	$\Delta(1940)$
	$\Delta(1920)$		

Nucleon resonances

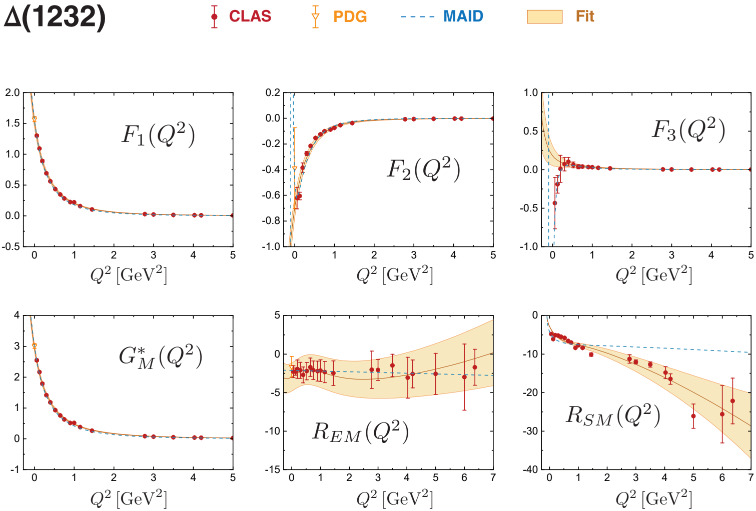


Nucleon resonances



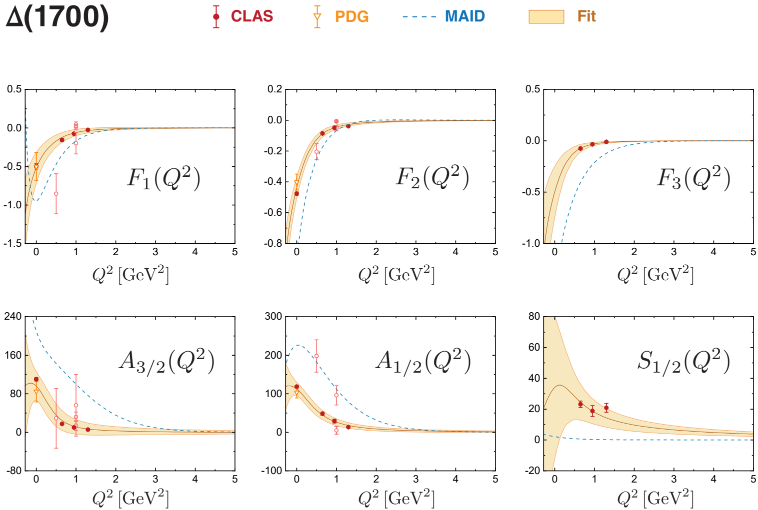
Nucleon resonances

Δ(1232)



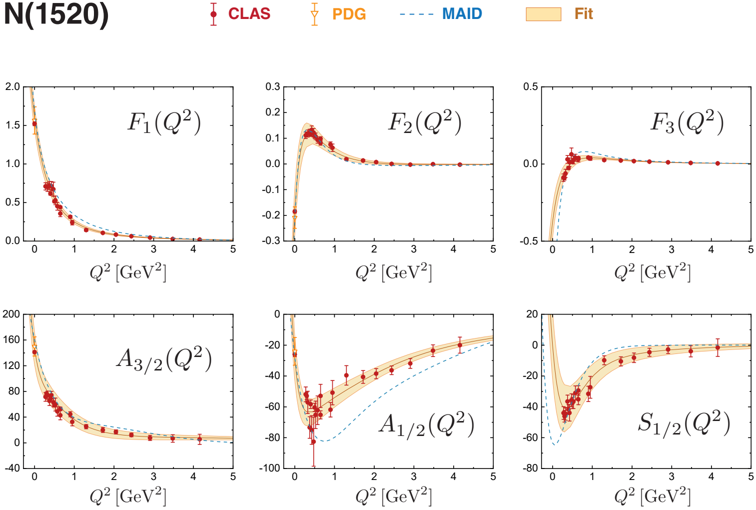
Nucleon resonances

$\Delta(1700)$

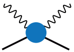


Nucleon resonances

N(1520)



Kinematics



$$= \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda)\, \bar{u}(p_f)\, X_i^{\mu\nu}(p, Q, Q')\, u(p_i)$$

18 CFFs

4 kinematic variables:

$$\eta_+ = \frac{Q^2+Q'^2}{2m^2}$$
$$\eta_- = \frac{Q\cdot Q'}{m^2}$$
$$\omega = \frac{Q^2-Q'^2}{2m^2}$$
$$\lambda = -\frac{p\cdot Q}{m^2}$$

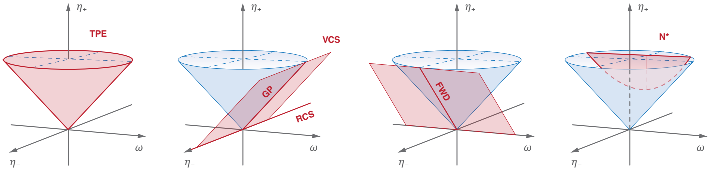
18 Compton tensors,
form minimal basis

- systematic derivation
- similar to Tarrach basis
  Tarrach, Nuovo Cim. A28 (1975)

$$X_i' = U_{ij}\, X_j\,, \qquad \det U = const.$$

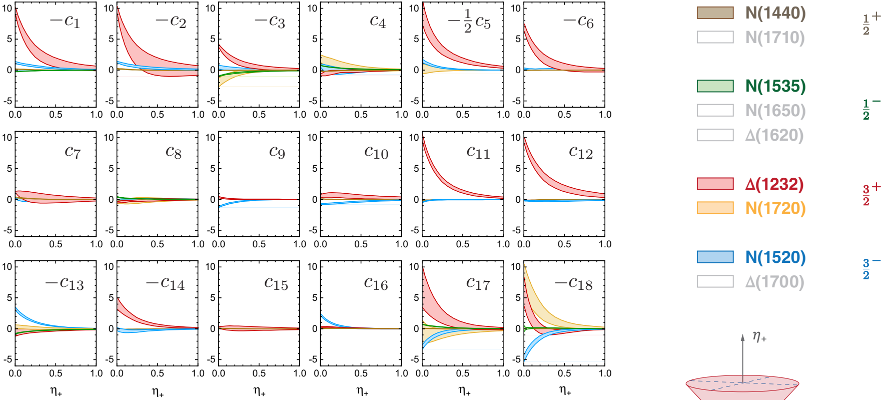
- CFFs free of kinematics

$$X_1^{\mu\nu} = \frac{1}{m^4}\, t^{\mu\alpha}_{Q'p}\, t^{\alpha\nu}_{pQ}\,,$$
$$X_2^{\mu\nu} = \frac{1}{m^2}\, t^{\mu\nu}_{Q'Q}\,,$$
$$X_3^{\mu\nu} = \frac{1}{m^4}\, t^{\mu\alpha}_{Q'Q'}\, t^{\alpha\nu}_{QQ}\,,$$
$$X_4^{\mu\nu} = \frac{1}{m^6}\, t^{\mu\alpha}_{Q'Q'}\, p^\alpha p^\beta\, t^{\beta\nu}_{QQ}\,,$$
$$X_5^{\mu\nu} = \frac{\lambda}{m^4}\left(t^{\mu\alpha}_{Q'Q'}\, t^{\alpha\nu}_{pQ} + t^{\mu\alpha}_{Q'p}\, t^{\alpha\nu}_{QQ}\right),$$
$$X_6^{\mu\nu} = \frac{1}{m^2}\, \varepsilon^{\mu\nu}_{Q'Q}\,,$$
$$X_7^{\mu\nu} = \frac{1}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\, \varepsilon^{\alpha\nu}_{\gamma Q} - \varepsilon^{\mu\alpha}_{Q'\gamma}\, t^{\alpha\nu}_{QQ}\right),$$
$$X_8^{\mu\nu} = \frac{\omega}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\, \varepsilon^{\alpha\nu}_{\gamma Q} + \varepsilon^{\mu\alpha}_{Q'\gamma}\, t^{\alpha\nu}_{QQ}\right),$$
$$\vdots$$

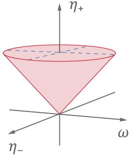


GE, Ramalho, in preparation

Compton form factors



To be multiplied with $\dfrac{(m_R^2-m^2)^2}{(s-m_R^2)(u-m_R^2)} = \dfrac{\delta^2}{(\eta_-+\delta)^2-4\lambda^2}$



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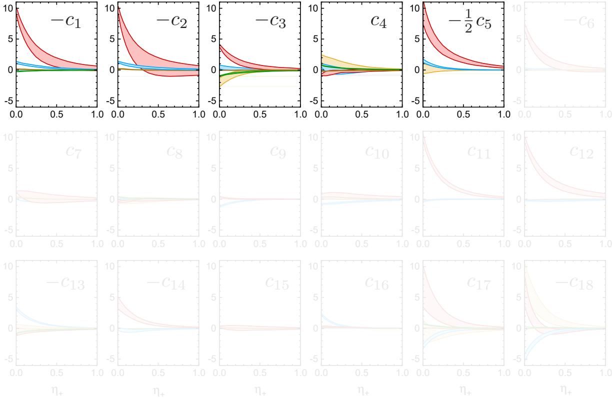
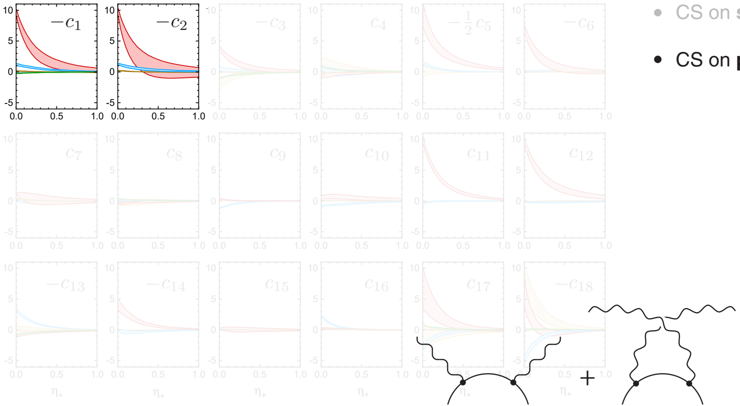


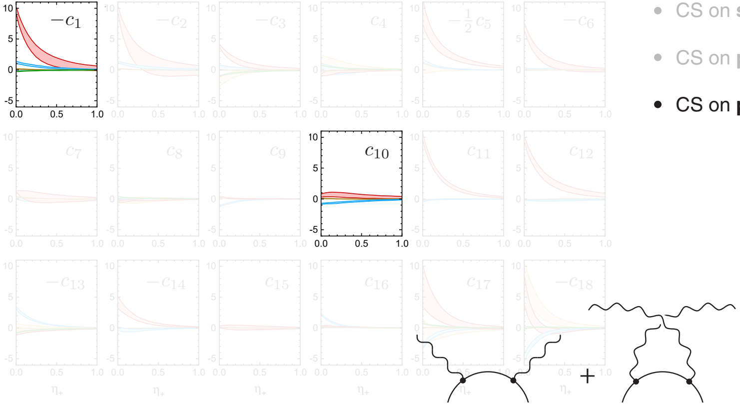
- CS on **scalar particle**

Compton form factors



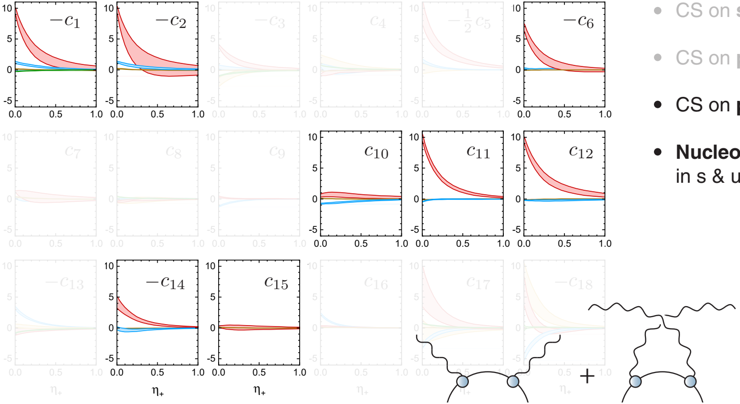
- CS on **scalar particle**
- CS on **pointlike scalar**

Compton form factors



- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**

Compton form factors



- CS on **scalar particle**
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- **Nucleon Born poles** in s & u channel

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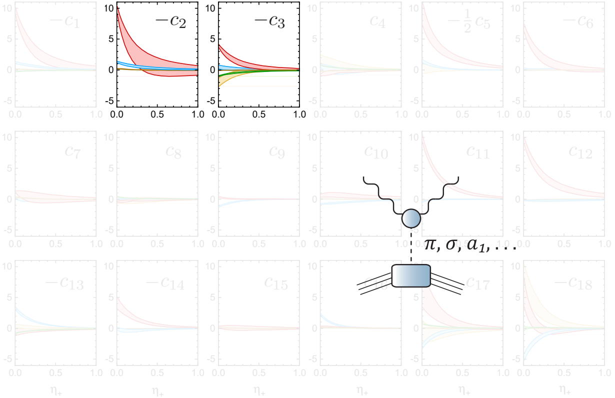

- CS on **scalar particle**
- CS on **pointlike scalar**
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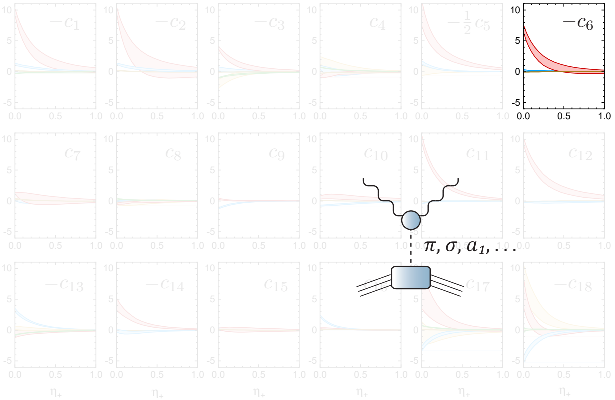
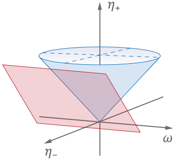
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel

Compton form factors

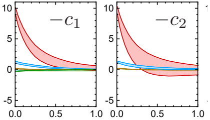


- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel
- **Pion pole** in t channel ($\pi^0 \to \gamma^* \gamma^*$)

GE, Fischer, Weil, Williams, PLB 774 (2017)

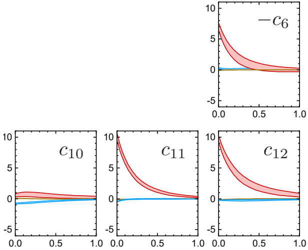


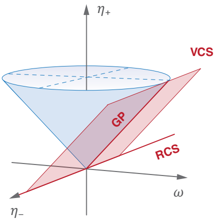
Polarizabilities



Scalar polarizabilities:

$$\begin{bmatrix} \alpha+\beta \\ \beta \end{bmatrix} = -\frac{\alpha_{\text{em}}}{m^3} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$



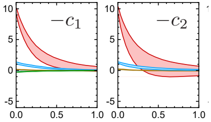


Spin polarizabilities:

$$\begin{bmatrix} \gamma_{E1E1} \\ \gamma_{M1M1} \\ \gamma_{E1M2} \\ \gamma_{M1E2} \end{bmatrix} = \frac{\alpha_{\text{em}}}{2m^4} \begin{bmatrix} c_6 + 4c_{11} - 4c_{12} \\ -c_6 - 2c_{10} + 4c_{12} \\ c_6 + 2c_{10} \\ -c_6 \end{bmatrix}$$

$$\begin{bmatrix} \gamma_0 \\ \gamma_\pi \end{bmatrix} = -\frac{2\alpha_{\text{em}}}{m^4} \begin{bmatrix} c_{11} \\ c_6 + c_{10} + c_{11} - 2c_{12} \end{bmatrix}$$

Polarizabilities



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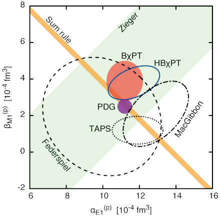
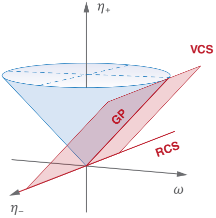


Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)

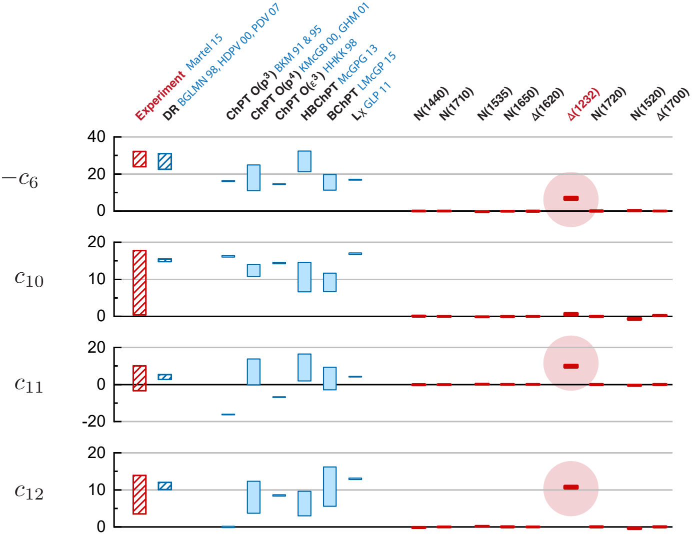


PDG:

$-c_1 = 20.3(4)$
$-c_2 = \ \ 3.7(6)$

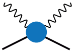
Large **Δ(1232)** contribution,
but also **N(1520)** non-negligible

Spin polarizabilities



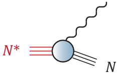
Only $\Delta(1232)$ important

Compton scattering



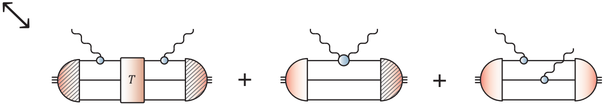
- kinematic variables
- tensor basis
- constraint-free **Compton FFs**

GE, Ramalho, in preparation

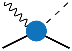


- general offshell transition vertices
- constraint-free **transition FFs**
- fits for transition FFs

- impact of higher resonances on Compton FFs
- only **Δ(1232)** and **N(1520)** relevant for polarizabilities

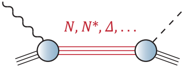


Meson electroproduction?

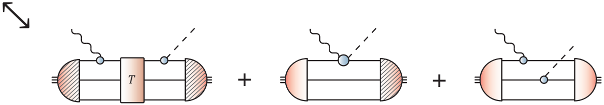


- kinematic variables
- tensor basis
- constraint-free **electroproduction amplitudes**

  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

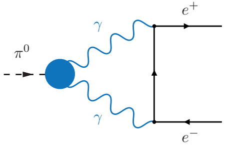


How important is the **“QCD background”?**



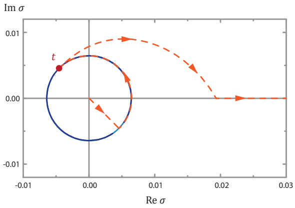
Developing numerical tools

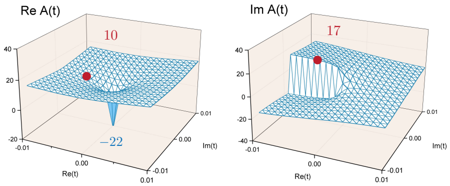
Rare pion decay $\pi^0 \to e^+e^-$:



$$A(t) = \int d\sigma \int dz \; \cdots \; \frac{1}{k^2+m^2} \, \frac{1}{Q^2} \, \frac{1}{Q'^2}$$

Photon and lepton poles produce branch cuts in complex plane: **deform integration contour!**





- Result agrees with dispersion relations
- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known

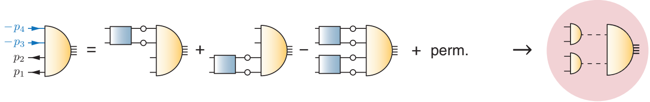
Weil, GE, Fischer, Williams, PRD 96 (2017)

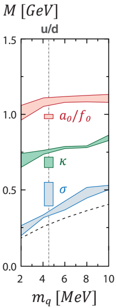
→ talk by **Richard Williams**

Tetraquarks

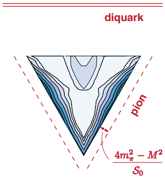
→ Christian Fischer

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)



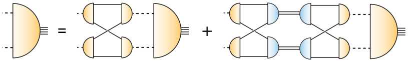


BSE dynamically generates **meson poles** in wave function:



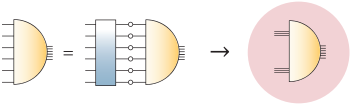
Four quarks rearrange to **"meson molecule"**

- Similar in **meson-meson / diquark-antidiquark** approximation (analogue of quark-diquark for baryons)
  Heupel, GE, Fischer, PLB 718 (2012)



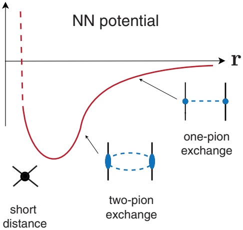
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

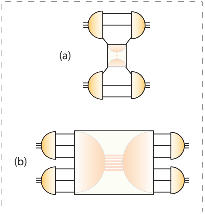


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



Weise, Nucl. Phys. A805 (2008)

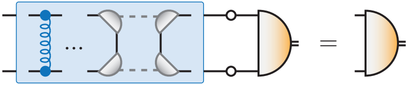


- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

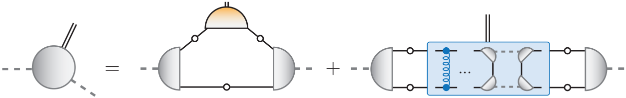
Backup slides

Resonances?

$\rho \to \pi\pi$: **resonance dynamics**
only beyond rainbow-ladder,
would shift ρ pole into complex plane
(above $\pi\pi$ threshold)

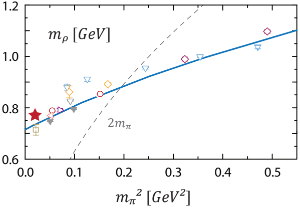


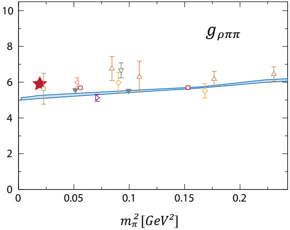
But ρ **decay width**
already calculable
in rainbow-ladder



Rainbow-ladder vs. lattice:

References: GE et al., PPNP 91 (2016) 1606.09602





actual resonance dynamics
subleading effect?

ρ may just be a special case,
but baryon spectrum?

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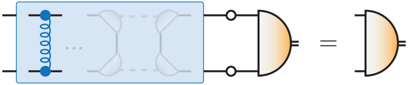

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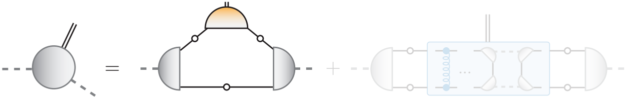

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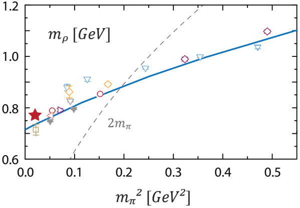

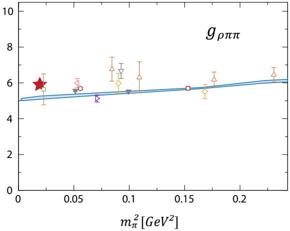

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Bethe-Salpeter equations

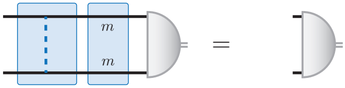
Simplest: **Wick-Cutkosky model**

Wick 1954, Cutkosky 1954, Nakanishi 1969, . . .

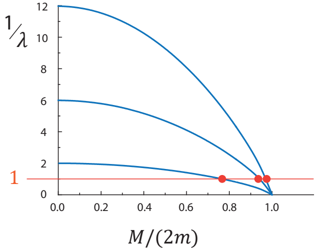
- scalar tree-level propagators, scalar exchange particle
- bound states for $M < 2m$

But:

- no confinement: threshold $2m$
- not a consistent QFT: would need to solve DSEs for propagators, vertices etc.



$$K(M)\; G(M)\; \phi_i(M) \;=\; \lambda_i(M)\, \phi_i(M)$$



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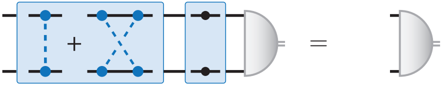

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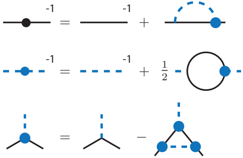

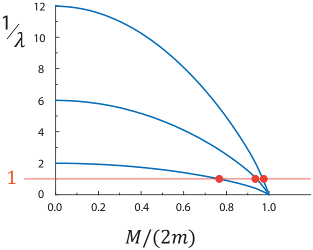

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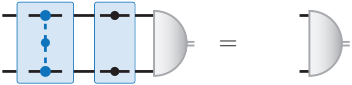
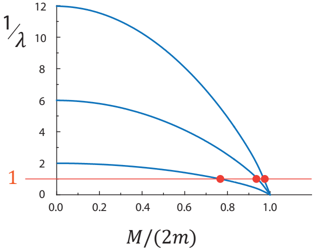
- scalar tree-level propagators, scalar exchange particle
- bound states for $M < 2m$



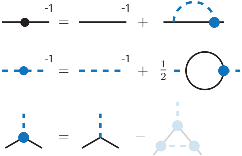
$$K(M) \quad G(M) \quad \phi_i(M) \quad = \quad \lambda_i(M)\, \phi_i(M)$$

But:

- no confinement: threshold $2m$
- not a consistent QFT: would need to solve DSEs for propagators, vertices etc.



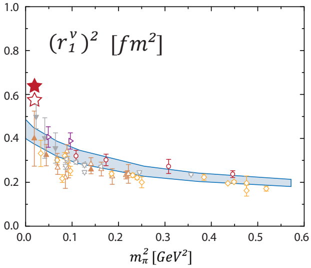
Ahlig, Alkofer, Ann. Phys. 275 (1999)



Form factors

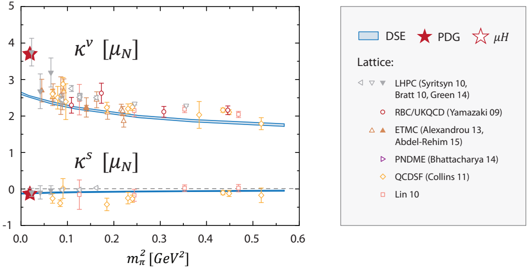
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

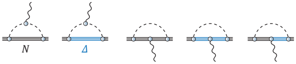


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

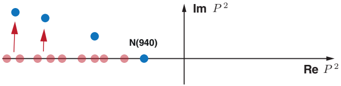
Exp: $\kappa^s = -0.12$

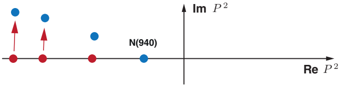
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

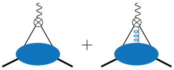
Lattice vs. DSE / BSE

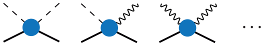
Lattice	DSE / BSE
Full dynamics contained in path integral	Dynamics constructed from underlying **n-point functions**
Proper treatment of **resonances** essential	Resonance dynamics "on top of" **quark-gluon dynamics**





Lattice	DSE / BSE
Simpler access to **position-space** and **gluonic operators**	Simpler access to multi-scale problems and higher n-point functions

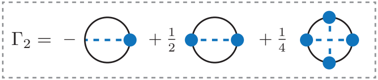
$\langle N|\bar{\psi}\,\not{D}\,\psi|N\rangle \sim$ 



Lattice	DSE / BSE
Precision!	Can tell us about underlying dynamics!

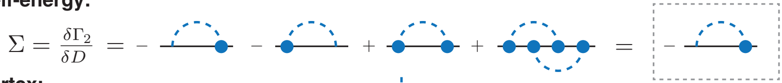
nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
3PI at 3-loop: **all two- and three-point functions are dressed;** 4, 5, ... do not appear.

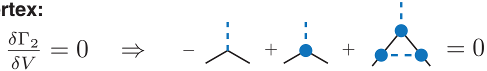


see: Sanchis-Alepuz & Williams,
J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

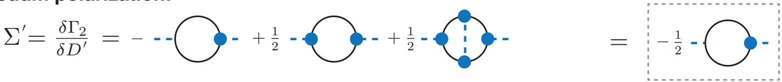
Self-energy:



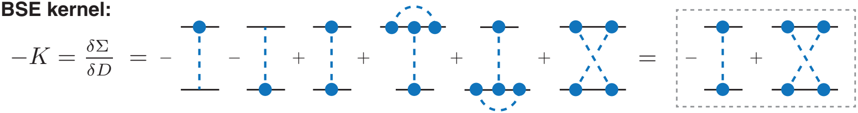
Vertex:



Vacuum polarization:



BSE kernel:



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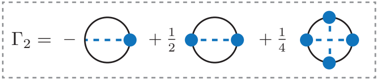

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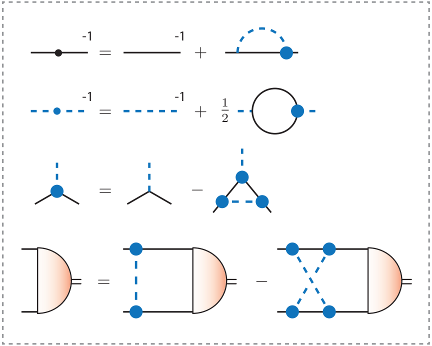
So we arrive at a closed system of equations:



- Crossed ladder
  cannot be added by hand,
  requires **vertex correction!**

nPI effective action

nPI effective actions provide **symmetry-preserving closed truncations.**
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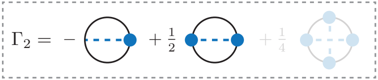

see: Sanchis-Alepuz & Williams, J. Phys. Conf. Ser. 631 (2015), arXiv:1503.05896 and refs therein

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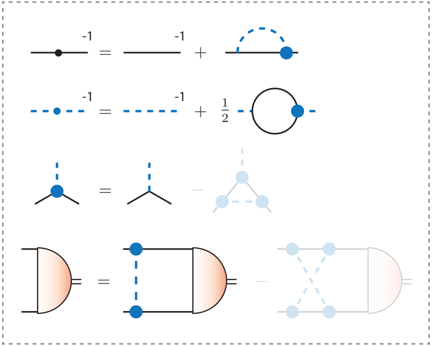
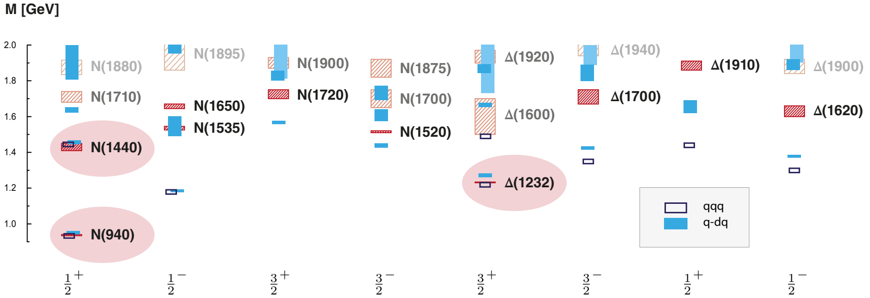


- Crossed ladder cannot be added by hand, requires **vertex correction!**
- without 3-loop term: **rainbow-ladder** with tree-level vertex ⇒ 2PI
- but still requires **DSE solutions** for propagators!
- Similar in QCD. nPI truncation guarantees chiral symmetry, massless pion in chiral limit, etc.

Baryon spectrum I

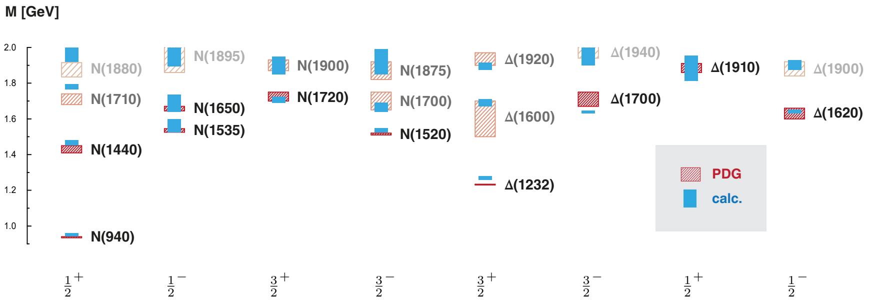
Three-quark vs. quark-diquark in rainbow-ladder: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

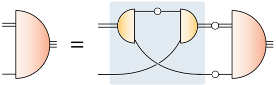


- **qqq** and **q-dq** agrees: N, Δ, Roper, N(1535)
- # levels compatible with experiment: **no states missing**
- N, Δ and their 1st excitations (including **Roper**) agree with experiment
- But remaining states too low ⇒ wrong level ordering between Roper and N(1535)

Baryon spectrum

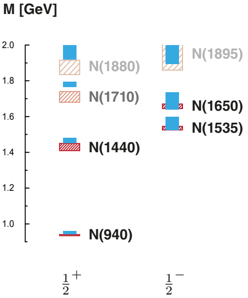
Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)





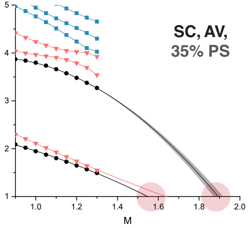
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn't change much

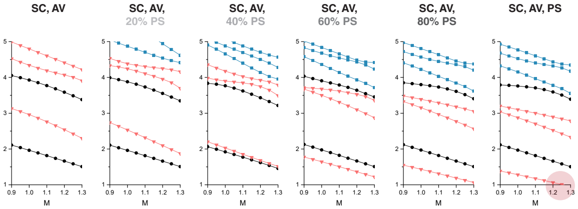
Baryon spectrum



Level ordering between **Roper and N(1535):**

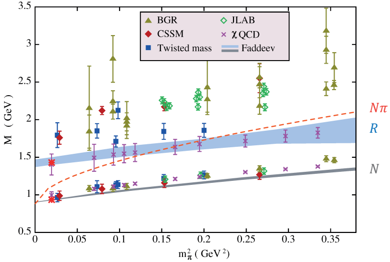
dynamics of ps diquark produces 2 nearby states: **N(1535), N(1650)**



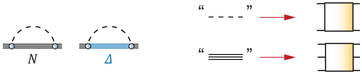


Resonances

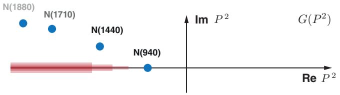
- **Current-mass evolution** of Roper:
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



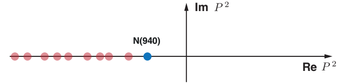
- **'Pion cloud'** effects difficult to implement at **quark-gluon level:**

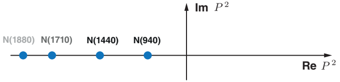


- Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



- **Lattice:** finite volume, **DSE** (so far): bound states





Resonance dynamics
shifts poles into complex plane,
but effects on real parts small?

QED

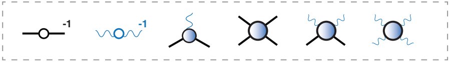
QED's classical action:

$$S = \int d^4x \left[\bar\psi \left(\partial\!\!\!/ + ig A\!\!\!/ + m \right) \psi + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

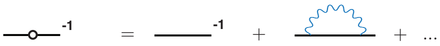
$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar\psi, A]\, e^{-S} = e^{-\Gamma}$$

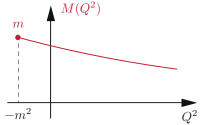


Perturbation theory: expand Green functions in powers of the coupling



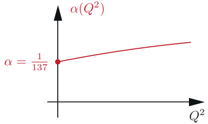
$A(p^2)\left(i p\!\!\!/ + M(p^2)\right) \qquad i p\!\!\!/ + m$

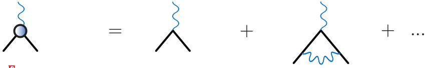
mass function



$D^{-1}(p^2)\left(p^2\,\delta^{\mu\nu} - p^\mu p^\nu\right) \qquad p^2\,\delta^{\mu\nu} - p^\mu p^\nu$

running coupling





$F_1 \gamma^\mu - \frac{F_2}{2m}\,\sigma^{\mu\nu} Q^\nu + \dots \qquad \gamma^\mu$

anomalous magnetic moment

$F_2(0) = \frac{\alpha}{2\pi}$

QED

QED's classical action:

$$S = \int d^4x \left[\bar{\psi} \left(\not{\partial} + ig\not{A} + m \right) \psi + \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

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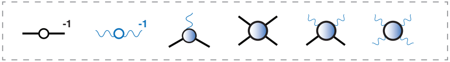
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} = e^{-\Gamma}$$



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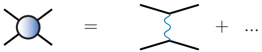
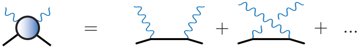


Moller scattering



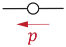
Compton scattering

Light-by-light scattering

$\Longrightarrow$ **extremely precise theory predictions!**

Dynamical quark mass

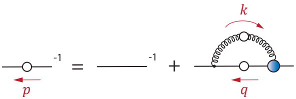
- General form of dressed **quark propagator:**



$$S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

$$S^{-1}(p) = A(p^2)\,(i\not{p}+M(p^2))$$

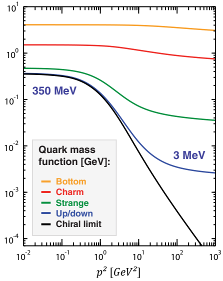
- **Quark DSE:** determines quark propagator, input → gluon propagator, quark-gluon vertex



$$S^{-1}(p) = S_0^{-1}(p) + \int_q \gamma^\mu\, S(q)\, D^{\mu\nu}(k)\, \Gamma^\nu(p,q)$$

- Reproduces **perturbation theory:**

$$\boldsymbol{S}^{-1} = S_0^{-1} - \boldsymbol{\Sigma} \quad \Rightarrow \quad \boldsymbol{S} = S_0 + S_0\,\boldsymbol{\Sigma}\,\boldsymbol{S}$$
$$= S_0 + S_0\,\boldsymbol{\Sigma}\,S_0 + S_0\,\boldsymbol{\Sigma}\,S_0\,\boldsymbol{\Sigma}\,\boldsymbol{S}$$
$$= \dots$$



- If strength large enough ($\alpha > \alpha_{\text{crit}}$), **chiral symmetry is dynamically broken**

- Generates $M(p^2) \neq 0$ even in chiral limit. Cannot happen in perturbation theory!

- Mass function ~ **chiral condensate:**

$$-\langle \bar{q}q \rangle = N_C \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,S(p)$$

Dynamical quark mass

Simplest example: **Munczek-Nemirovsky model**
Gluon propagator = δ-function, analytically solvable
Munczek, Nemirovsky, PRD 28 (1983)

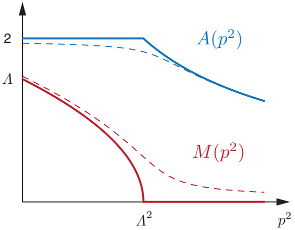
$$S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

$$S^{-1}(p) = A(p^2)\,(i\not{p} + M(p^2))$$

$$D^{\mu\nu}(k)\,\Gamma^\nu(p,q) \;\longrightarrow \sim \Lambda^2\,\delta^4(k)\,\gamma^\mu$$

Quark DSE becomes

$$S^{-1}(p) - S_0^{-1}(p) = \Lambda^2\,\gamma^\mu S(p)\gamma^\mu = \Lambda^2\,\frac{2i\not{p}+4M}{(p^2+M^2)\,A}\;,$$



leads to self-consistent equations for A, M:

$$A = 1 + \frac{2\Lambda^2}{(p^2+M^2)\,A}\,, \qquad AM = m_0 + 2M\,\frac{2\Lambda^2}{(p^2+M^2)\,A}$$

Two solutions in chiral limit: IR + UV

$$M(p^2) = \sqrt{\Lambda^2 - p^2} \qquad\qquad M(p^2) = 0$$
$$A(p^2) = 2 \qquad\qquad A(p^2) = \tfrac{1}{2}\left(1+\sqrt{1+8\,\Lambda^2/p^2}\right)$$

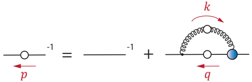
Quark condensate:

$$-\langle\bar{q}q\rangle = N_C \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\,S(p) \;=\; \frac{2}{15}\,\frac{N_C}{(2\pi)^2}\,\Lambda^3$$

Another extreme case: **NJL model,**
gluon propagator = const,
$M(p^2)$ = const, but critical behavior
Nambu, Jona-Lasinio, 1961

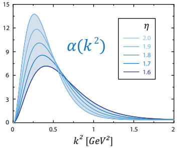
Dynamical quark mass

- Simplest realistic example: **rainbow-ladder**



Tree-level quark-gluon vertex + **effective interaction:**

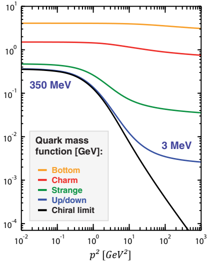
$$D^{\mu\nu}(k)\,\Gamma^{\nu}(p,q) \longrightarrow \sim \frac{\alpha(k^2)}{k^2}\left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\gamma^\nu$$



$$\alpha(k^2) = \alpha_{\text{IR}}\left(\tfrac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\text{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter
Maris, Tandy, PRC 60 (1999)

- If strength is large enough ($\alpha > \alpha_{\text{crit}}$): **DCSB**
- All dimensionful quantities ~ Λ in chiral limit
  ⇒ **mass generation for hadrons!**



Classical PCAC relation for $SU(N_f)_A$:

$$\partial_\mu\; \bar\psi\,\gamma^\mu\gamma_5\,\mathsf{t}_a\,\psi \;=\; i\bar\psi\,\{\mathsf{M},\mathsf{t}_a\}\,\gamma_5\,\psi$$

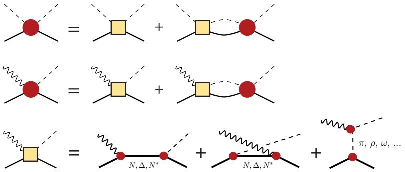
At quantum level:

$$f_\pi\, m_\pi^2 = 2m\, r_\pi$$

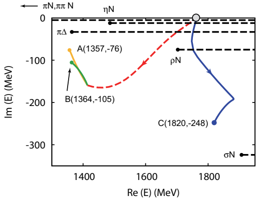
Also $f_\pi \sim \Lambda$ ⇒ $m_\pi = 0$ in chiral limit!
⇒ **massless Goldstone bosons!**

Extracting resonances

Hadronic coupled-channel equations:

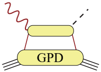


Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI, JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina, JPAC, . . .



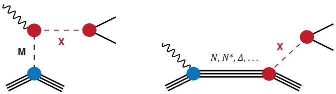
Suzuki et al., PRL 104 (2010)

Microscopic effects?
What is an "offshell hadron"?



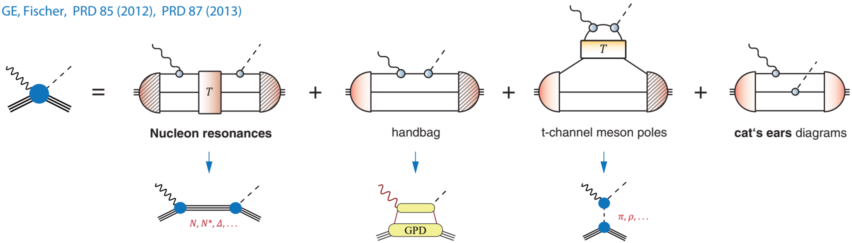
Extracting resonances

Photoproduction of **exotic mesons** at JLab/GlueX:



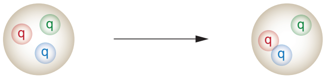
What if exotic mesons are **relativistic $q\bar{q}$ states?**
⇒ study with DSE/BSE!

Scattering amplitudes at quark-gluon level:
GE, Fischer, PRD 85 (2012), PRD 87 (2013)



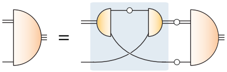
Diquarks?

- Suggested to resolve **'missing resonances'** in quark model:
  fewer degrees of freedom ⇒ fewer excitations



Anselmino et al., Rev. Mod. Phys. 65 (1993),
Klempt, Richard, Rev. Mod. Phys. 82 (2010)

- QCD version: assume qq scattering matrix as sum of diquark correlations
  ⇒ three-body equation simplifies to **quark-diquark BSE**



Oettel, Alkofer, Hellstern Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klähn, Roberts, FBS 46 (2009)

Quark exchange binds nucleon, gluons absorbed in building blocks.
Scalar diquark ~ 800 MeV, axialvector diquark ~ 1 GeV

Maris, FBS 32 (2002), GE, Krassnigg, Schwinzerl, Alkofer, Ann. Phys. 323 (2008), GE, FBS 57 (2016)

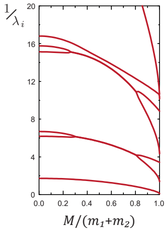
- N and Δ properties similar in quark-diquark and three-quark approach:
  quark-diquark approximation is good!

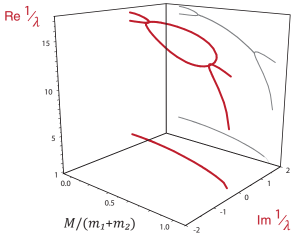
Complex eigenvalues?

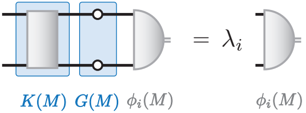
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\,L^\dagger L\,\phi_i = \lambda_i\,\phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

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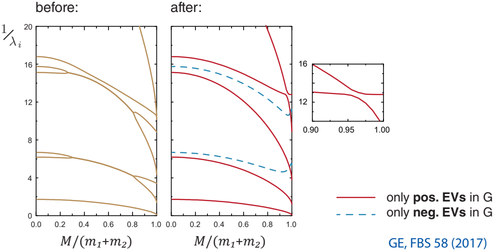
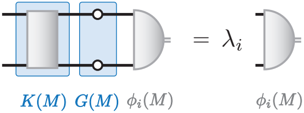


GE, FBS 58 (2017)



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$\Rightarrow$ Hermitian problem with same EVs!

$\Rightarrow$ all EVs strictly **real**
$\Rightarrow$ level repulsion
$\Rightarrow$ “anomalous states” removed?

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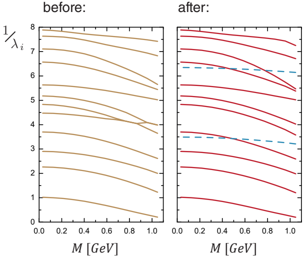
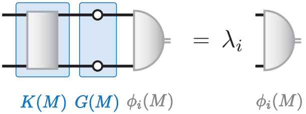


Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)

only **pos. EVs** in G
only **neg. EVs** in G



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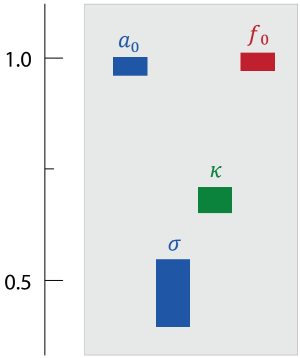
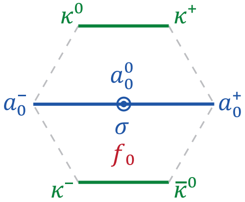
$\Rightarrow$ Hermitian problem with same EVs!

$\Rightarrow$ all EVs strictly **real**
$\Rightarrow$ level repulsion
$\Rightarrow$ "anomalous states" removed?

Tetraquarks?

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





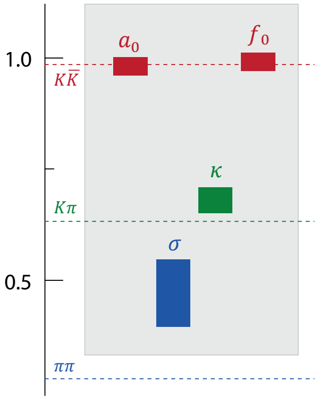
f_0 (980 MeV) $s\bar{s}$
κ (680 MeV) $u\bar{s}, d\bar{s}$
$\left.\begin{array}{l} a_0 \text{ (980 MeV)} \\ \sigma \text{ (500 MeV)} \end{array}\right\} u\bar{u}, d\bar{d}, u\bar{d}$

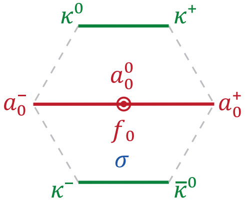
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV
- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have
  masses similar to axialvector & tensor mesons ~ 1.3 GeV

Tetraquarks?

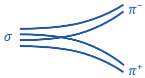
What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004





f_0 (980 MeV) } $us\overline{us}$, ...
a_0 (980 MeV) }
κ (800 MeV) $us\overline{ud}$, ...
σ (500 MeV) $ud\overline{ud}$

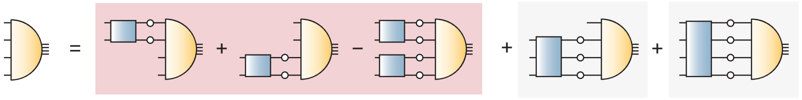
- Explains **mass ordering** & **decay widths**:
  f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ



- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-q$\bar{\text{q}}$ nature** of σ supported by dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models
  Pelaez, Phys. Rept. 658 (2016)

Four-body equation



Two-body interactions
... plus permutations:

$(qq)(\bar{q}\bar{q}),\ (q\bar{q})(q\bar{q}),\ (q\bar{q})(q\bar{q})$

(12) (34) (23) (14) (13) (24)

3-body
(+ permutations)

4-body

Bethe-Salpeter amplitude:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2$$

$$\begin{aligned} \omega_1 &= q\cdot k & \eta_1 &= p\cdot P \\ \omega_2 &= p\cdot k & \eta_2 &= q\cdot P \\ \omega_3 &= p\cdot q & \eta_3 &= k\cdot P \end{aligned}$$

$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3}, \quad 6\otimes\bar{6}$ or
$1\otimes 1, \quad 8\otimes 8$
(Fierz-equivalent)

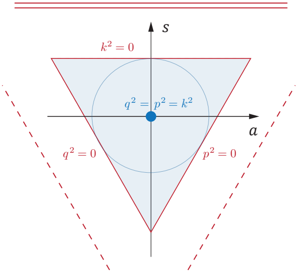
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{1}{4}(p^2+q^2+k^2)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}\sqrt{3}\,(q^2-p^2)\\ p^2+q^2-2k^2\end{bmatrix}$

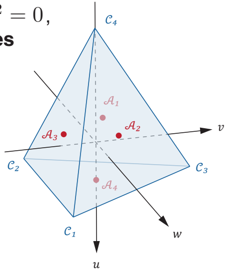
Mandelstam triangle,
outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2\,(\omega_1+\omega_2+\omega_3)\\ \sqrt{2}\,(\omega_1+\omega_2-2\omega_3)\\ \sqrt{6}\,(\omega_2-\omega_1)\end{bmatrix}$

tetrahedron bounded by $p_i^2=0$,
outside: **quark singularities**



- Second **triplet:** 3dim. sphere

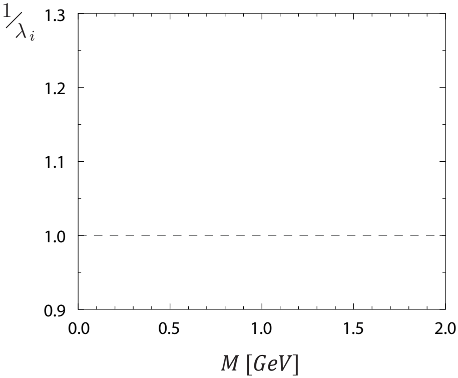
$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2\,(\eta_1+\eta_2+\eta_3)\\ \sqrt{2}\,(\eta_1+\eta_2-2\eta_3)\\ \sqrt{6}\,(\eta_2-\eta_1)\end{bmatrix}$$

Tetraquark mass

Idea: use symmetries to figure out **relevant** momentum dependence

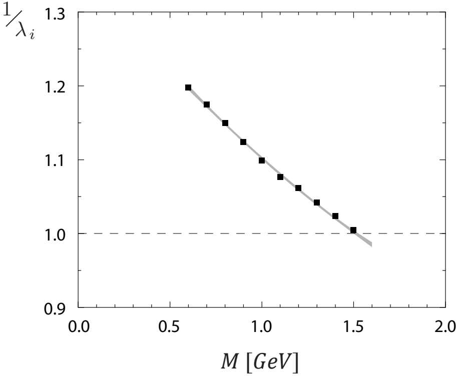
similar:

- **Three-gluon vertex**
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)
- HLbL scattering for **muon g-2**
  GE, Fischer, Heupel, PRD 92 (2015)



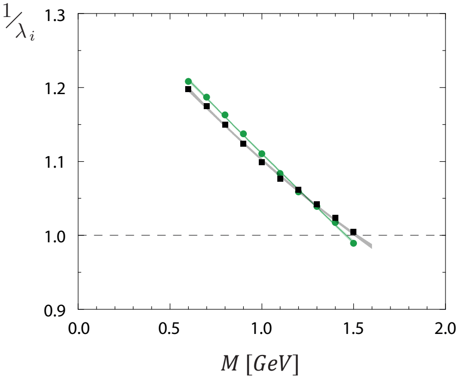
Tetraquark mass

$f_i(\mathcal{S}_0, \bigtriangledown, \triangle, \bigcirc)$



Tetraquark mass

$$f_i\,(\,\mathcal{S}_0\,, \triangledown\,, \triangle\,, \bigcirc\,)$$



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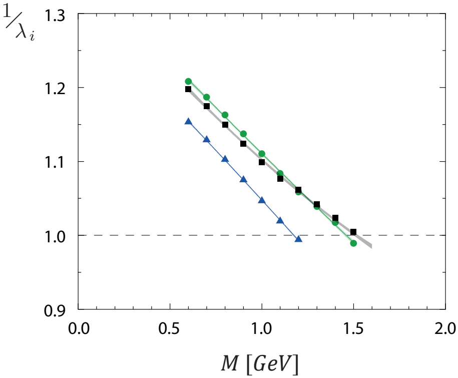
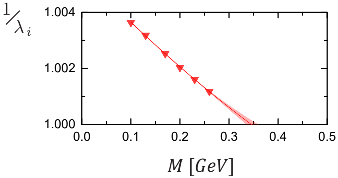
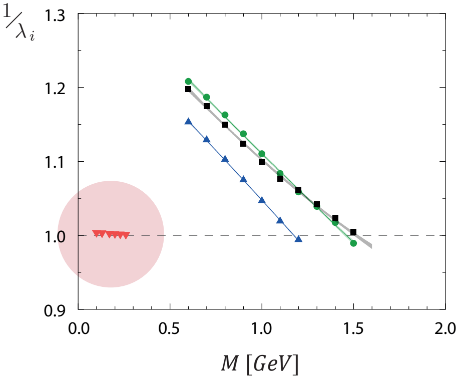


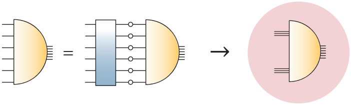
Tetraquark mass





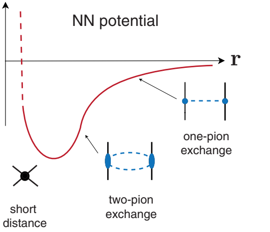
Towards multiquarks

Transition from **quark-gluon** to **nuclear degrees of freedom:**

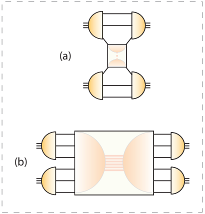


- 6 ground states, one of them **deuteron**
  Dyson, Xuong, PRL 13 (1964)
- Dibaryons vs. **hidden color?**
  Bashkanov, Brodsky, Clement, PLB 727 (2013)
- **Deuteron FFs** from quark level?

Microscopic origins of nuclear binding?



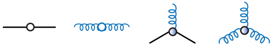
Weise, Nucl. Phys. A805 (2008)

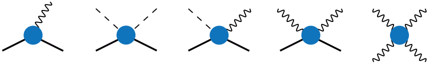


- only quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

Hadron physics with functional methods

Understand properties of **elementary n-point functions** ↔ Calculate hadronic **observables**: mass spectra, form factors, scattering amplitudes, . . .



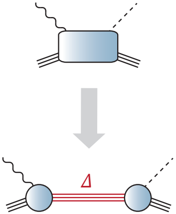


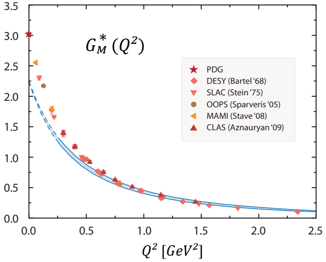
- **QCD**
- **symmetries** intact (Poincare invariance & chiral symmetry important)
- access to all momentum scales & all quark masses
- compute mesons, baryons, tetraquarks, . . . **from same dynamics**

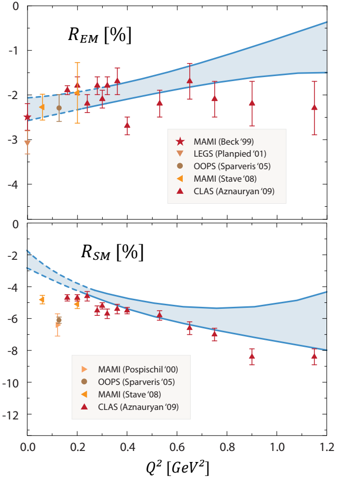
- **systematic** construction of truncations
- technical challenges: coupled integral equations, complex analysis, structure of 3-, 4-, ... point functions, **need lots of computational power!**

access to underlying nonperturbative dynamics!

Nucleon-Δ-γ transition





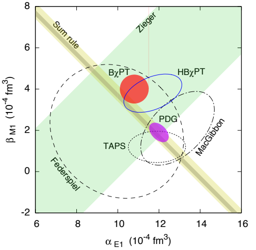


- **Magnetic dipole transition (**G_M^***)** dominant: quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole ratios** small & negative, encode deformation. Reproduced without pion cloud: **OAM** from **p waves!**

GE, Nicmorus, PRD 85 (2012)

Compton scattering

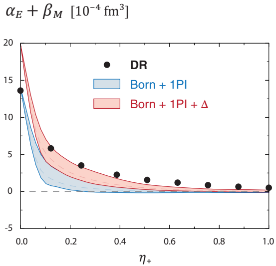
Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)

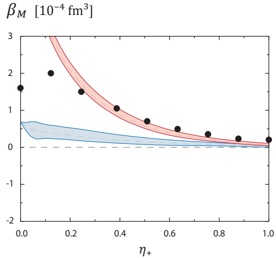


First DSE results:
GE, FBS 57 (2016)

- Quark Compton vertex (Born + 1PI) calculated, added Δ exchange
- compared to DRs
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E dominated by handbag, β_M by Δ contribution

$\Rightarrow$ **large "QCD background"!**



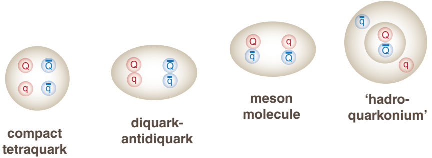


In total: polarizabilities ≈

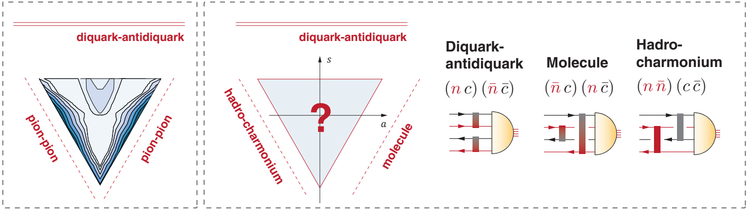
Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly Δ)
+ pion cloud (at low η_+)?

Tetraquarks in charm region?

- Can we **distinguish** different tetraquark configurations?

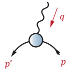


- **Four quarks** dynamically rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium; strengths determined by four-body BSE:



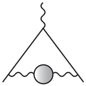
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

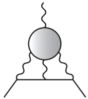


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



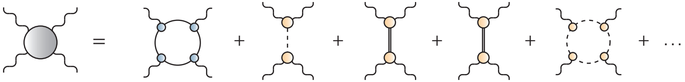
Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

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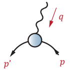

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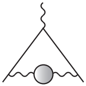
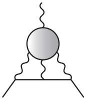


Hadronic vacuum polarization

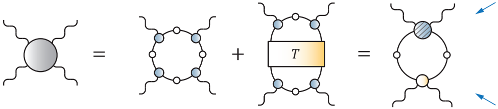


Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$ — Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



quark Compton vertex

Born terms

- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)