Conventional and exotic states in the DSE/BSE framework

Christian S. Fischer

Justus Liebig Universität Gießen

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]; Sanchis-Alepuz, Williams, CPC [1710.04903]

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HIC for FAIR
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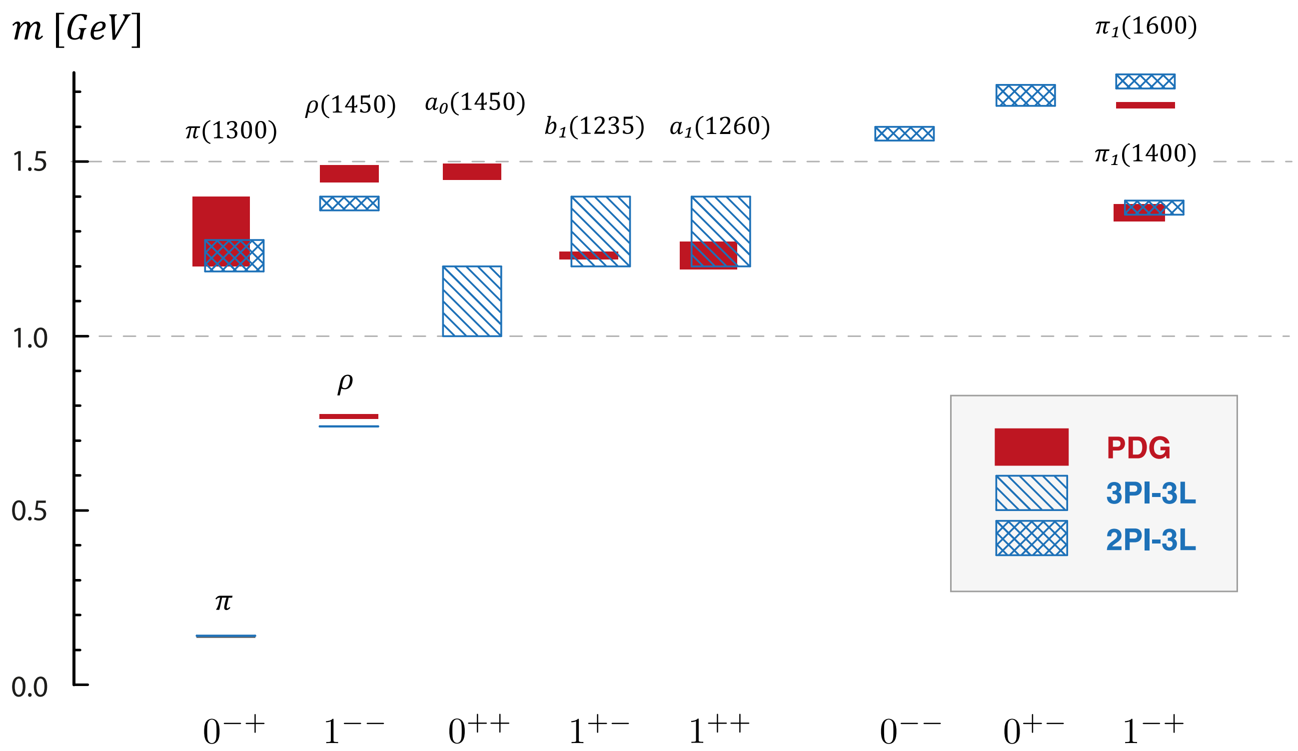
Overview - Take home messages

- Hybrids:

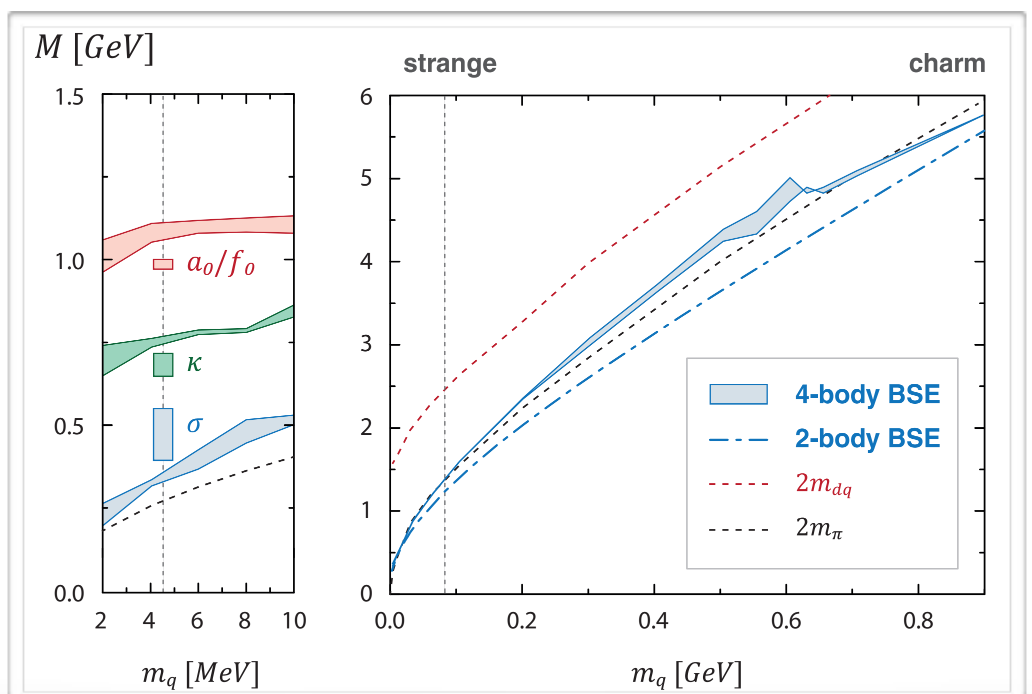


Williams, CF, Heupel, PRD93 (2016) 034026

- Glueballs: $$M(0^{++}) = 1.64\,\mathrm{GeV}$$

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

- Light tetraquarks:



Eichman, CF, Heupel, PLB 753 (2016) 282-287

Nonpert. QCD: Complementary approaches

Quarks and gluons

Quarks, gluons or… Hadrons

- Lattice simulations
  - Ab initio
  - Gauge invariant
- Effective theories (NRQCD, χPT, ...)
  - Dof integrated out
  - —>Physical dof
- Functional approaches (DSE, FRG,Hamilton):
  - Space-time continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible
  - Chemical potential: no sign problem
  - Access to structural information

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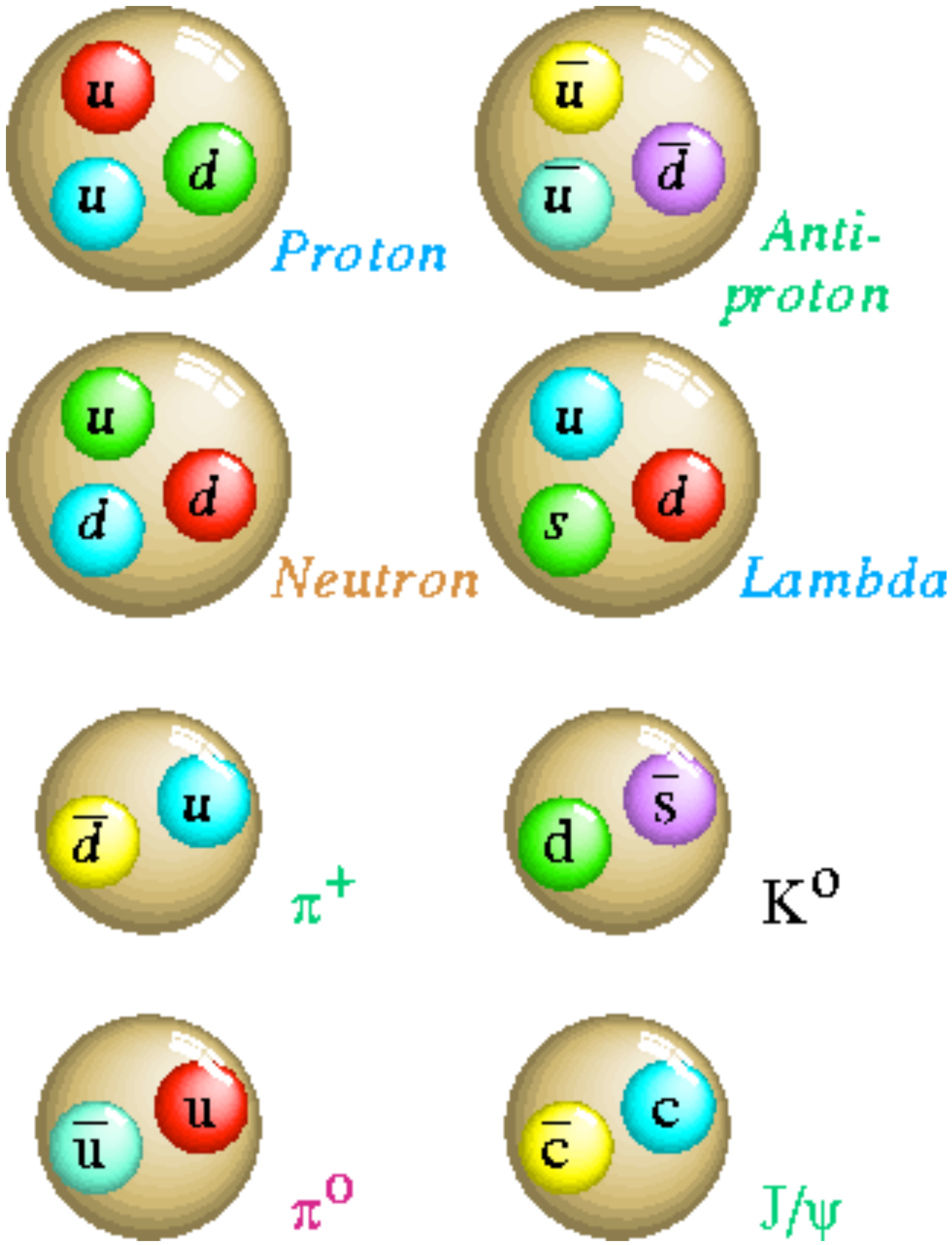
Phenomenological tool: Quark-model

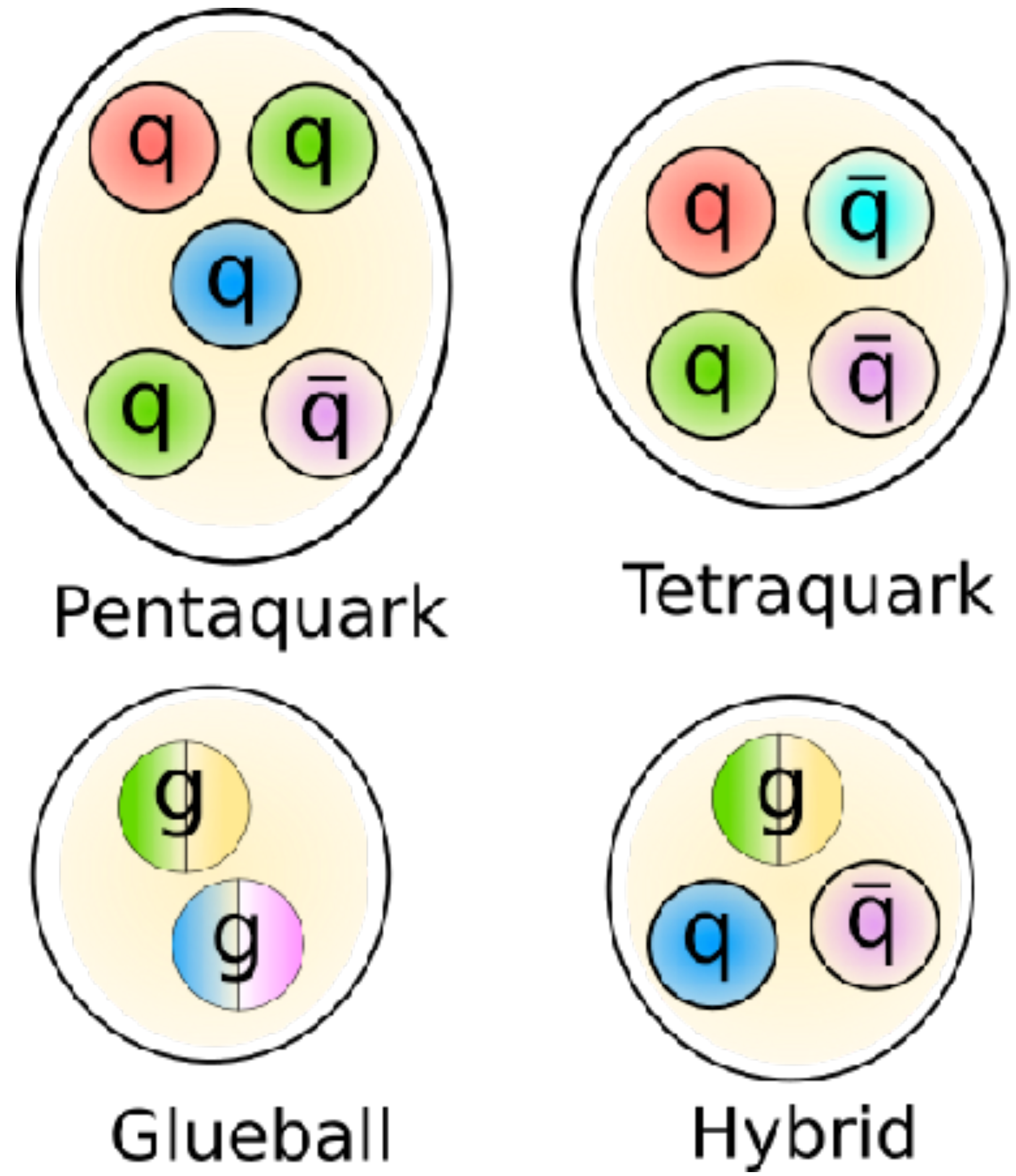
Hadrons: baryons, mesons and ... exotics !

-> Gernot Eichmann, Bruno El Bennich

Baryons

Mesons





Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

$$S : 1/2 \otimes 1/2 \to 0 \oplus 1$$

$$P : (-1)^{L+1}$$

S	L	J^{PC}	
0	0	0^{-+}	
1	0	1^{--}	
0	1	1^{+-}	**$^1P_1$**
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

$\boxed{J^{PC}}$ or $\boxed{^{2S+1}L_J}$

relativistic $q\bar{q}$

$$\begin{aligned}\Gamma_\pi(P,p) = \gamma_5 \, [F_1(P,p) & \quad \text{s-wave} \\ + F_2(P,p) i\not{P} & \\ + F_3(P,p) pP i\not{p} & \quad \text{p-wave} \\ + F_4(P,p) [\not{p}, \not{P}]] &\end{aligned}$$

(rest frame of π)

$$\cancel{P : (-1)^{L+1}}$$

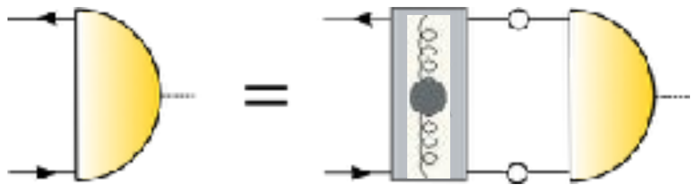
Bethe,Salpeter, Llewelyn-Smith 1950ies

- conventional states more complicated
- 'exotic' quantum numbers possible !

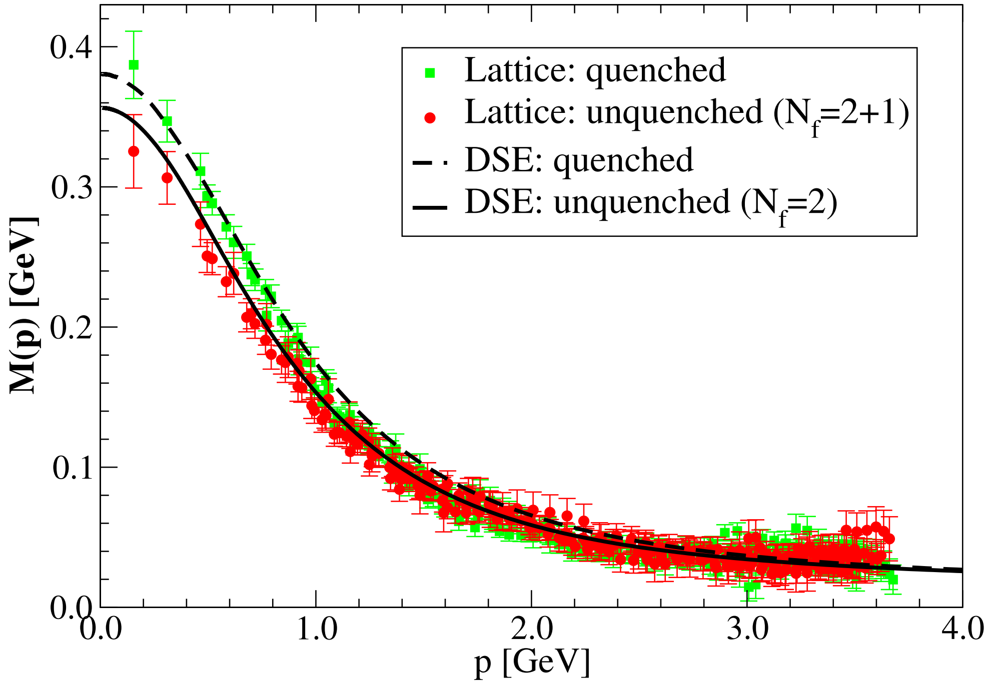
$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

Quark mass

Rainbow-Ladder (RL) vs beyond the rainbow (BRL)



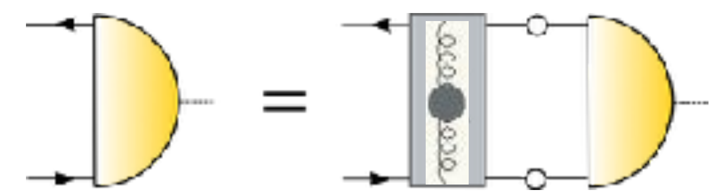
$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$



CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

Rainbow-Ladder (RL) vs beyond the rainbow (BRL)



$[S(p)]^{-1} =$

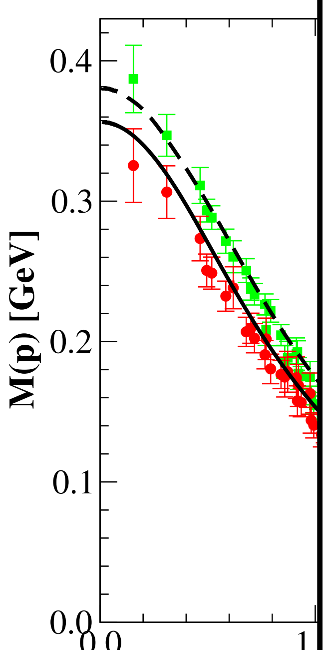
$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2)$$

"approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi}\tau(k^2)Z(k^2) \sim \alpha(k^2)$$



veak

$V)^3$

CF, Nickel, Williams, EPJ C 60 (2009) 47

Quark mass

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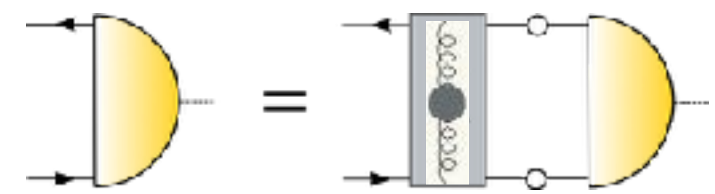

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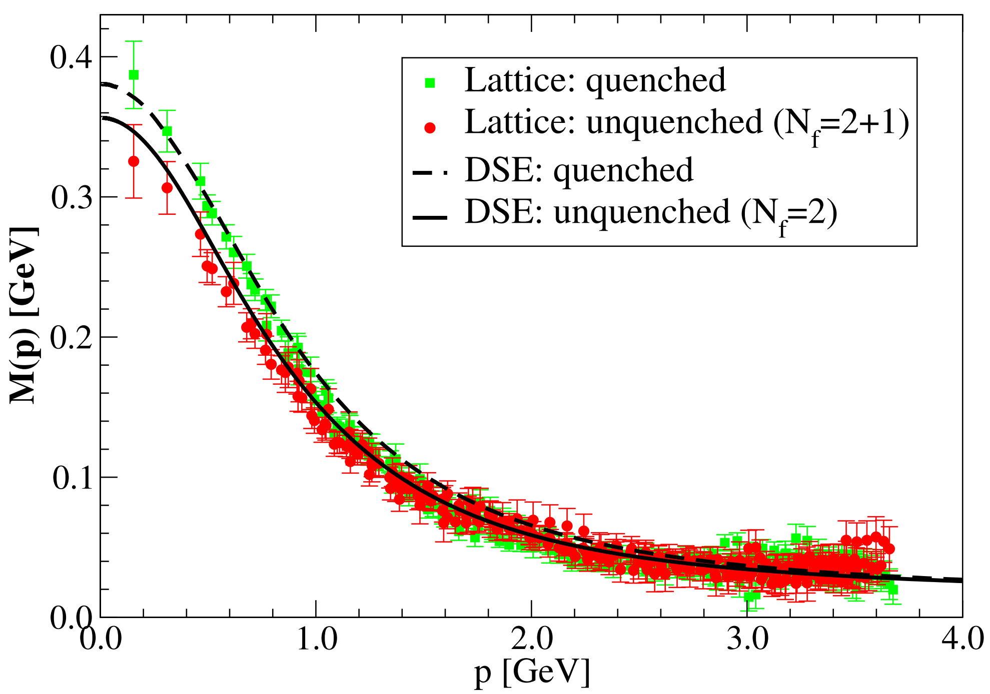

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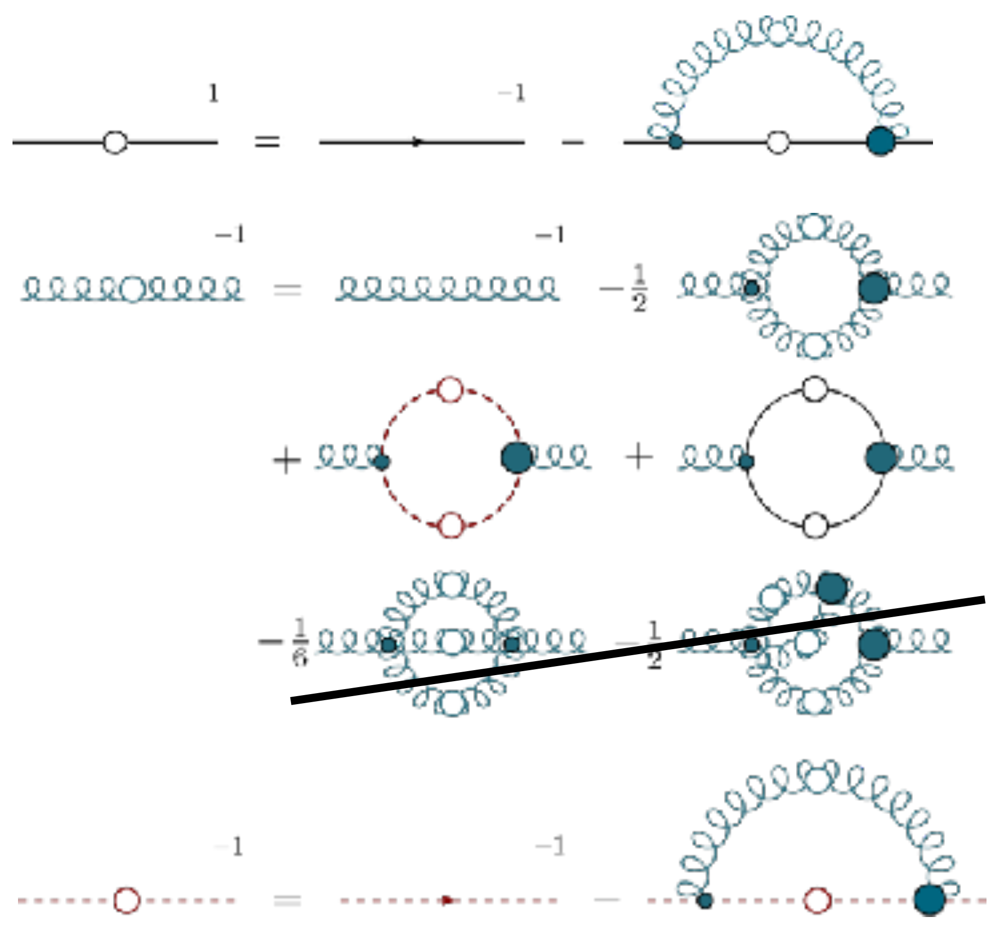
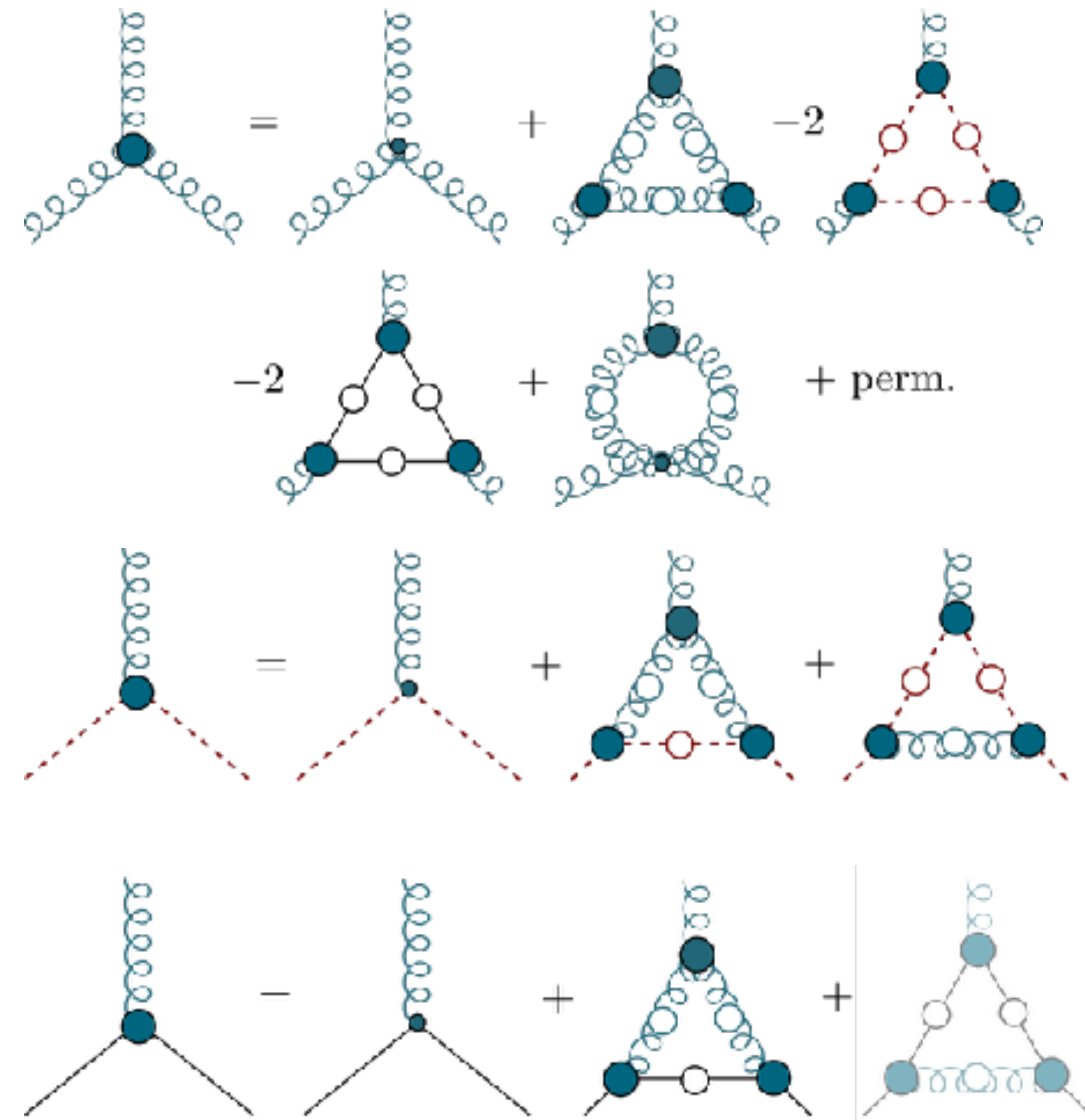
-> Richard Williams

propagators



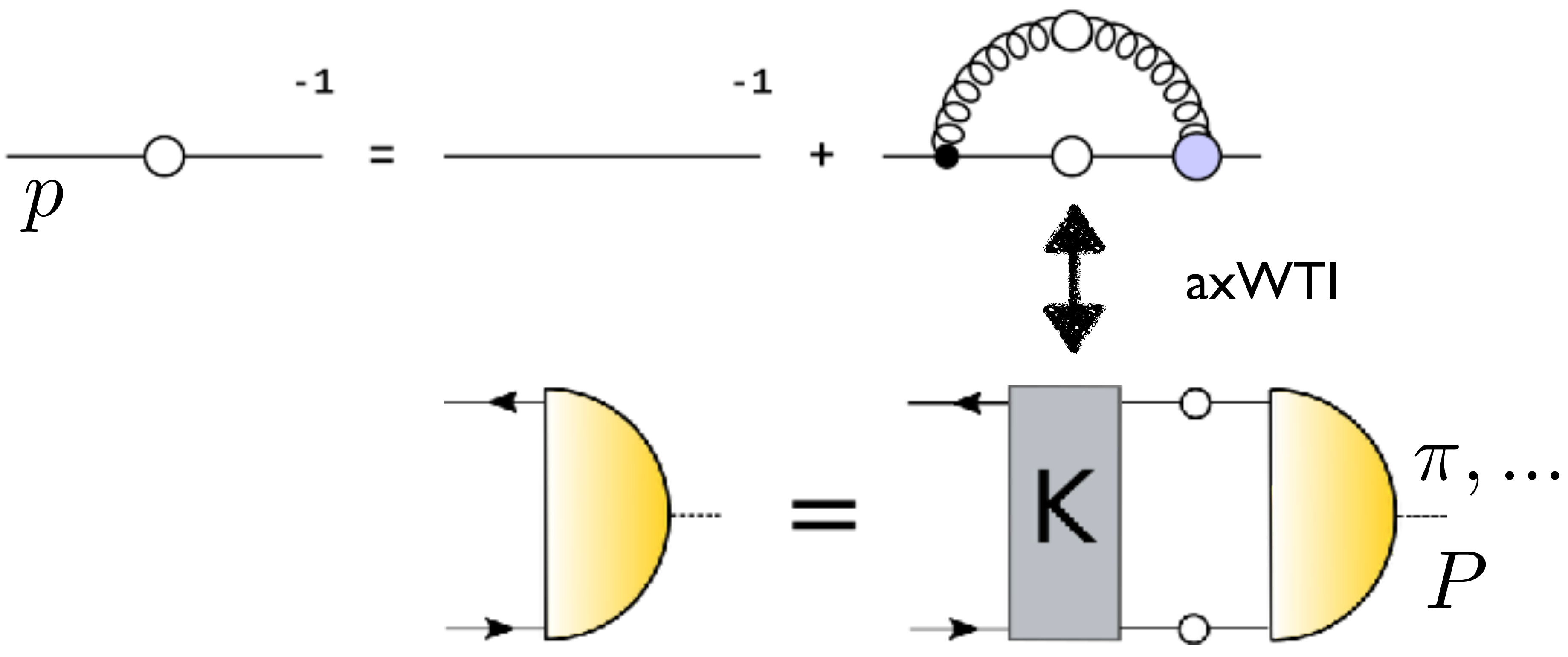
for different BRL approaches see work of Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices



Williams, CF, Heupel, PRD 93 (2016) 034026
CF, Williams, PRL 103 (2009) 122001

DSEs and Bethe-Salpeter equation



spacelike: $p^2 > 0$

$P^2 = -m_\pi^2$

timelike !

Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i \int (K\gamma_5 S_- + K S_+ \gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu} \Gamma_\nu \gamma_5 + \int \gamma_5 \gamma_\mu S_- D_{\mu\nu} \Gamma_\nu$$

$\boxed{\rightarrow \text{Pion is bound state } \textbf{and} \text{ Goldstone boson}}$

Maris, Roberts, Tandy, PLB 420 (1998) 267

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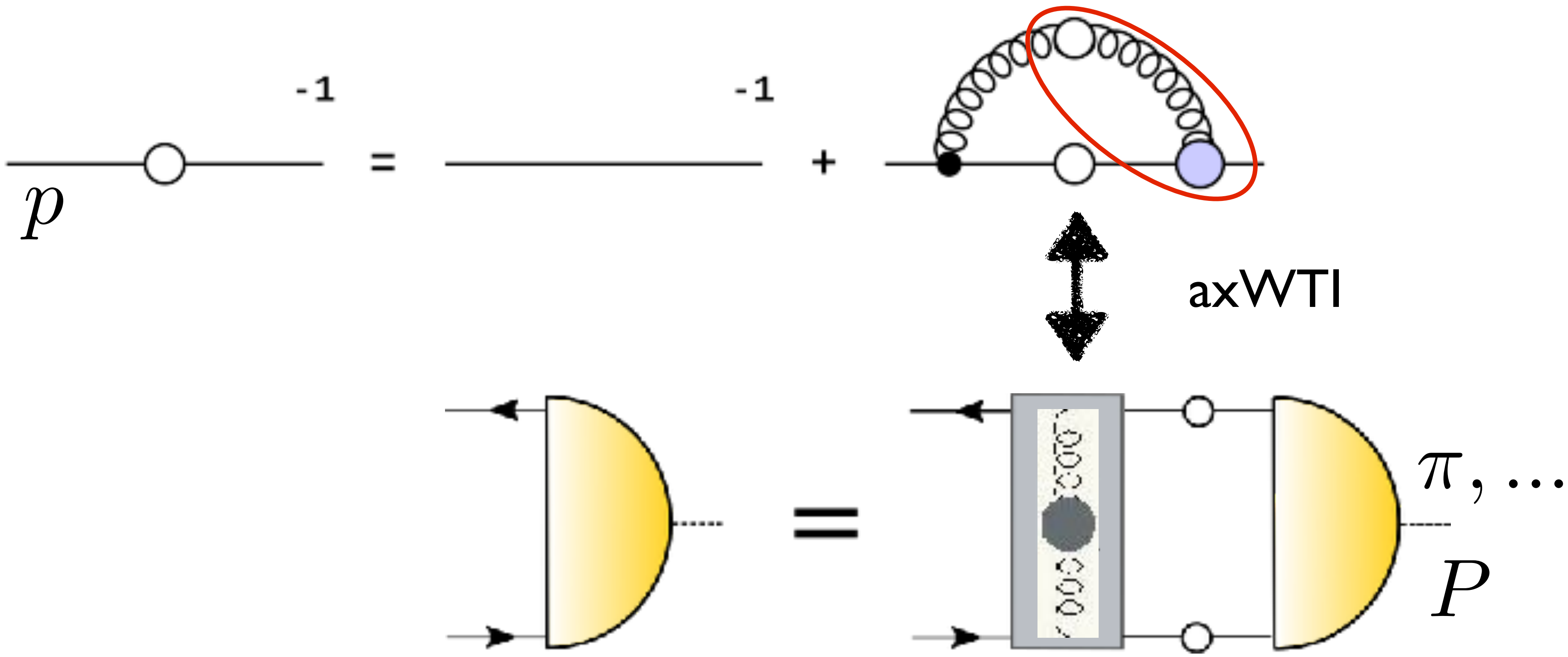

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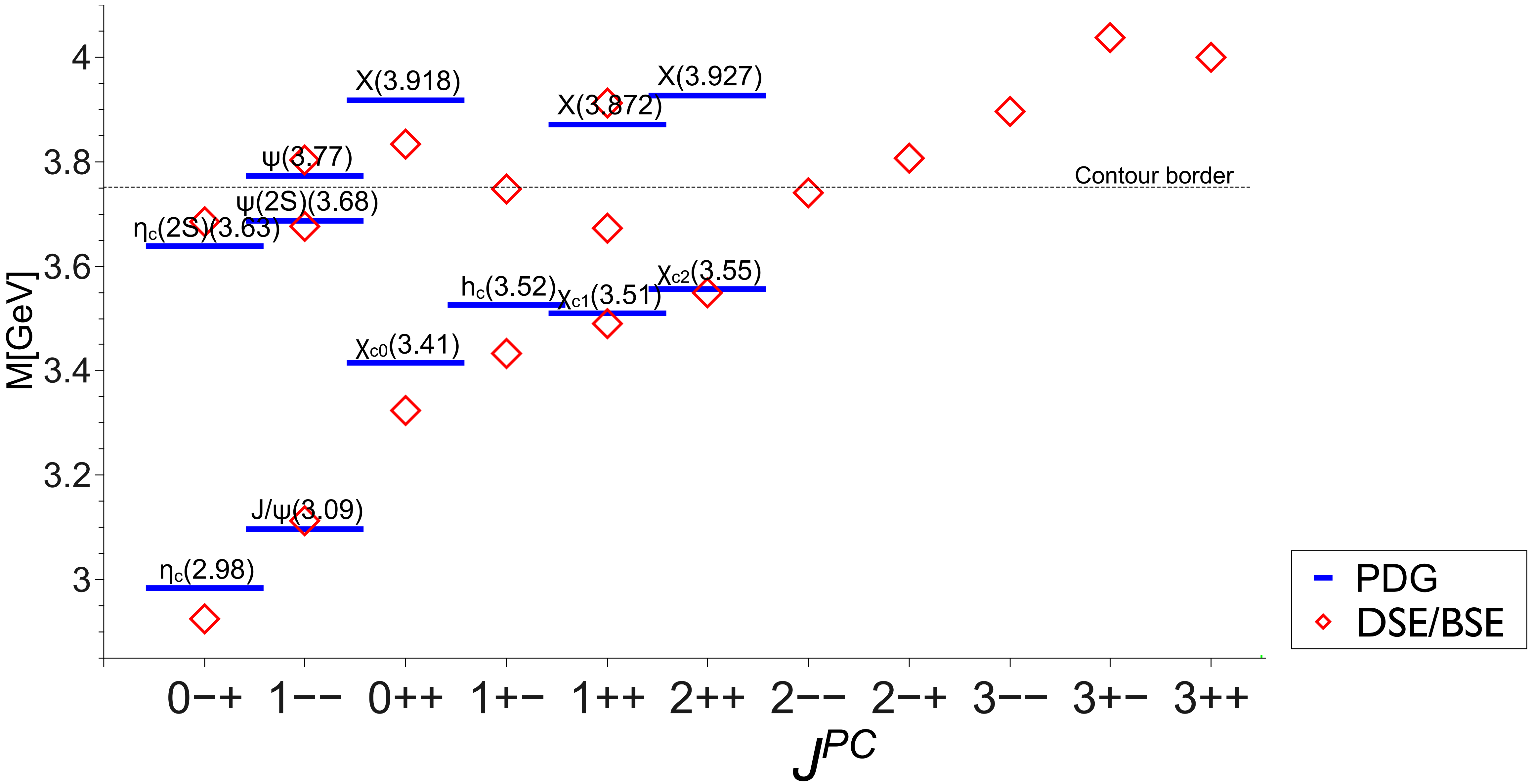
Charmonium spectrum



- good channels: 1^{--}, 2^{++}, 3^{--},...
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure
- excited states fine ! (in good channels)

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

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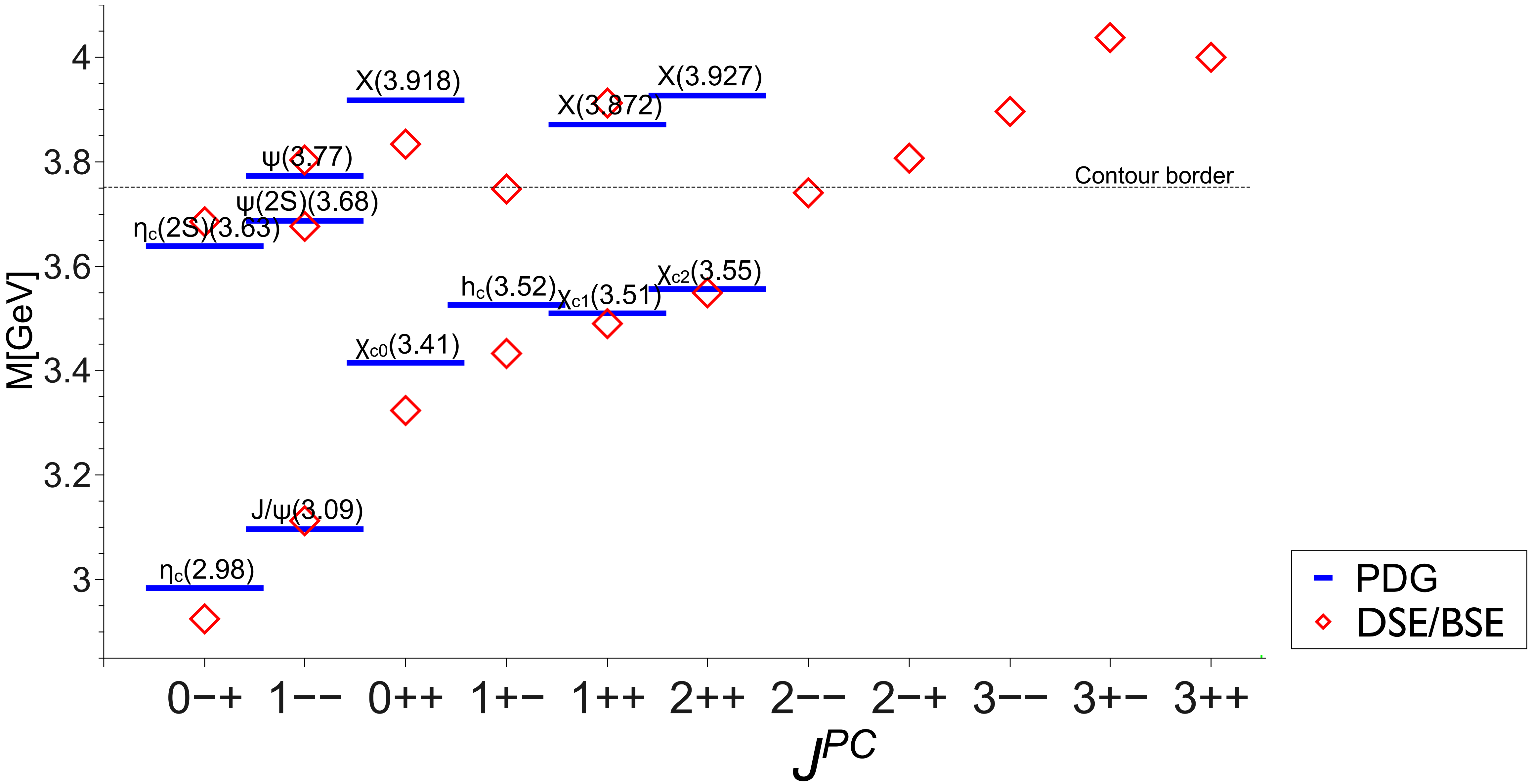

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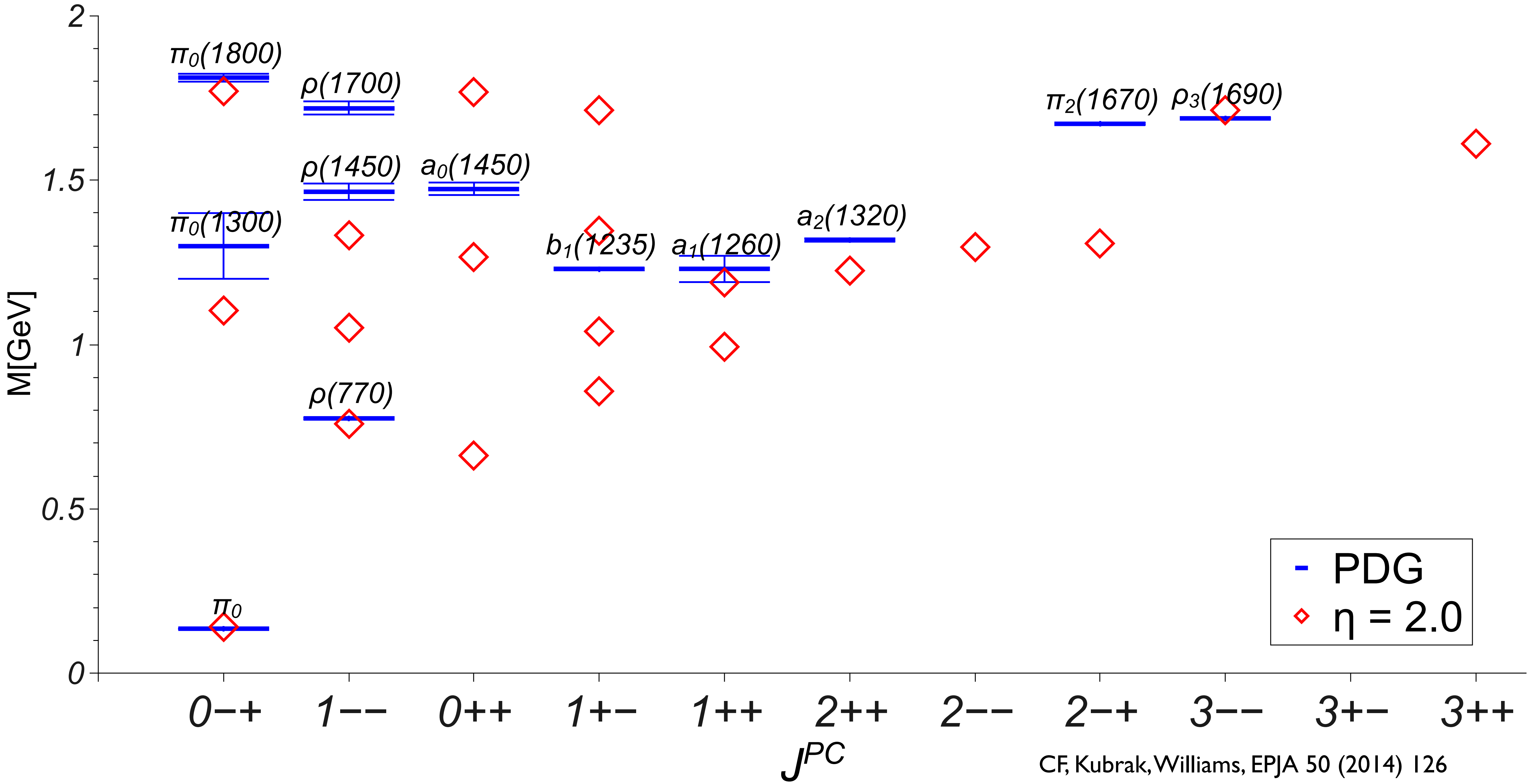
Light meson spectrum



CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states

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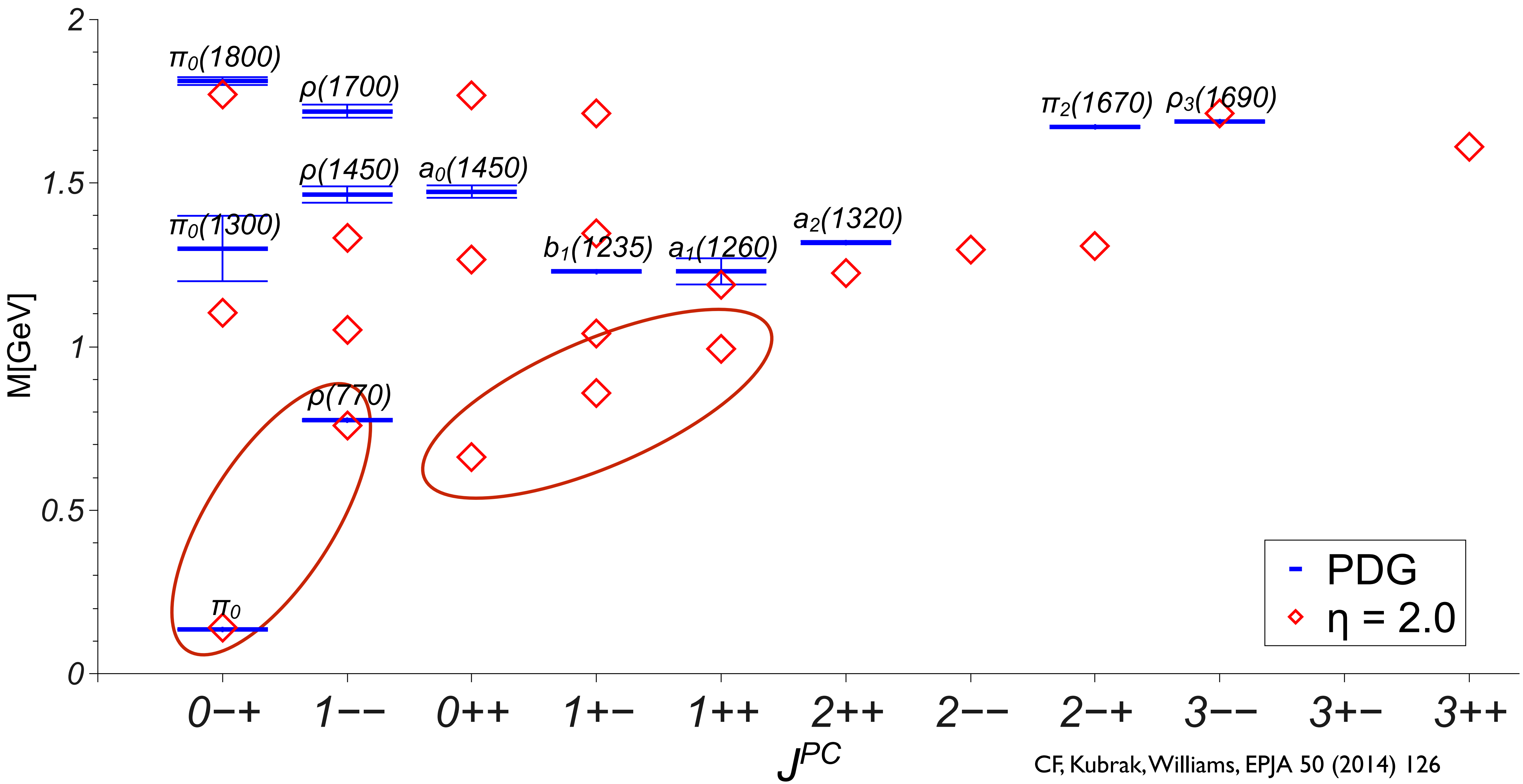

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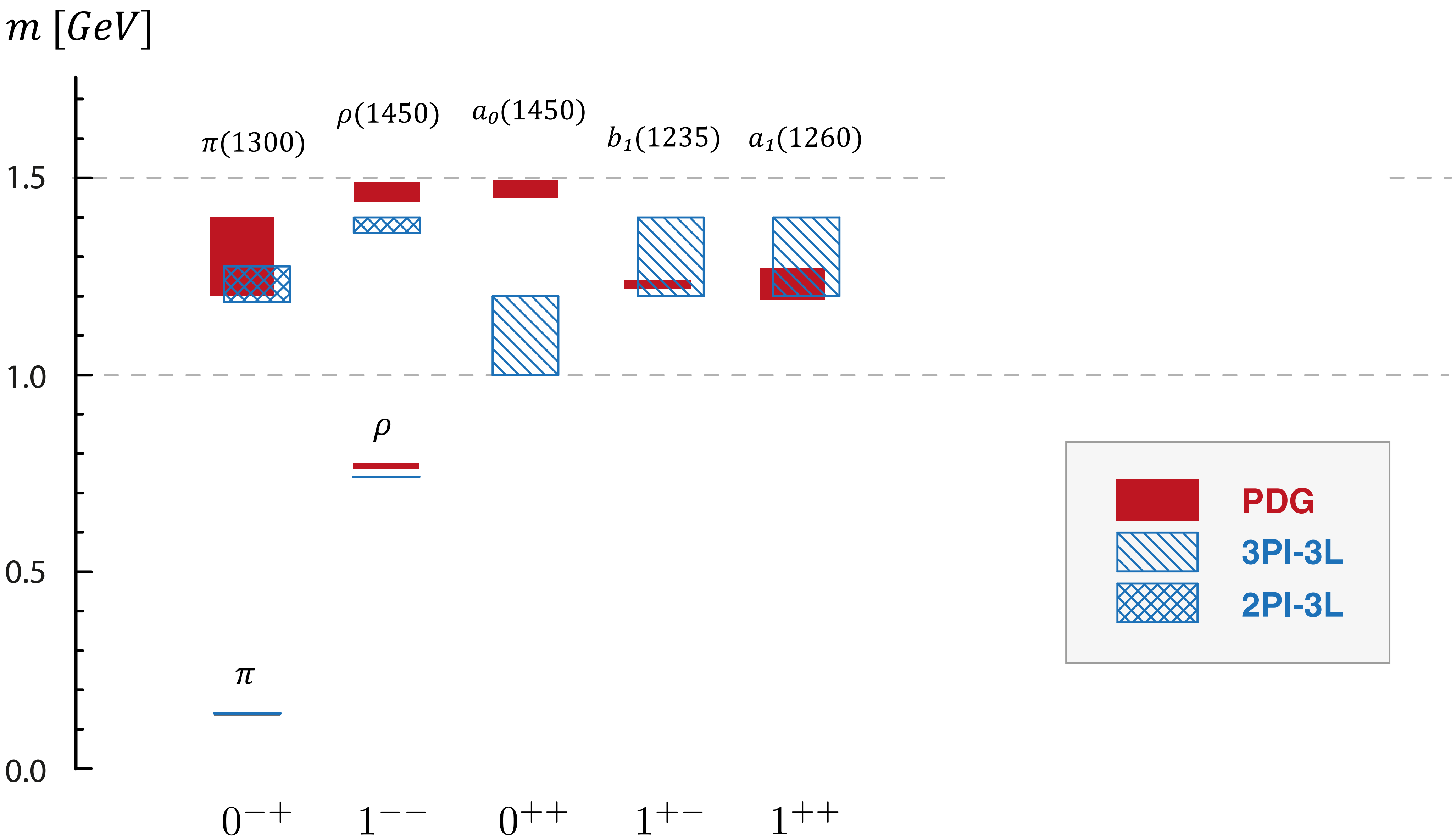

CF, Kubrak, Williams, EPJA 50 (2014) 126

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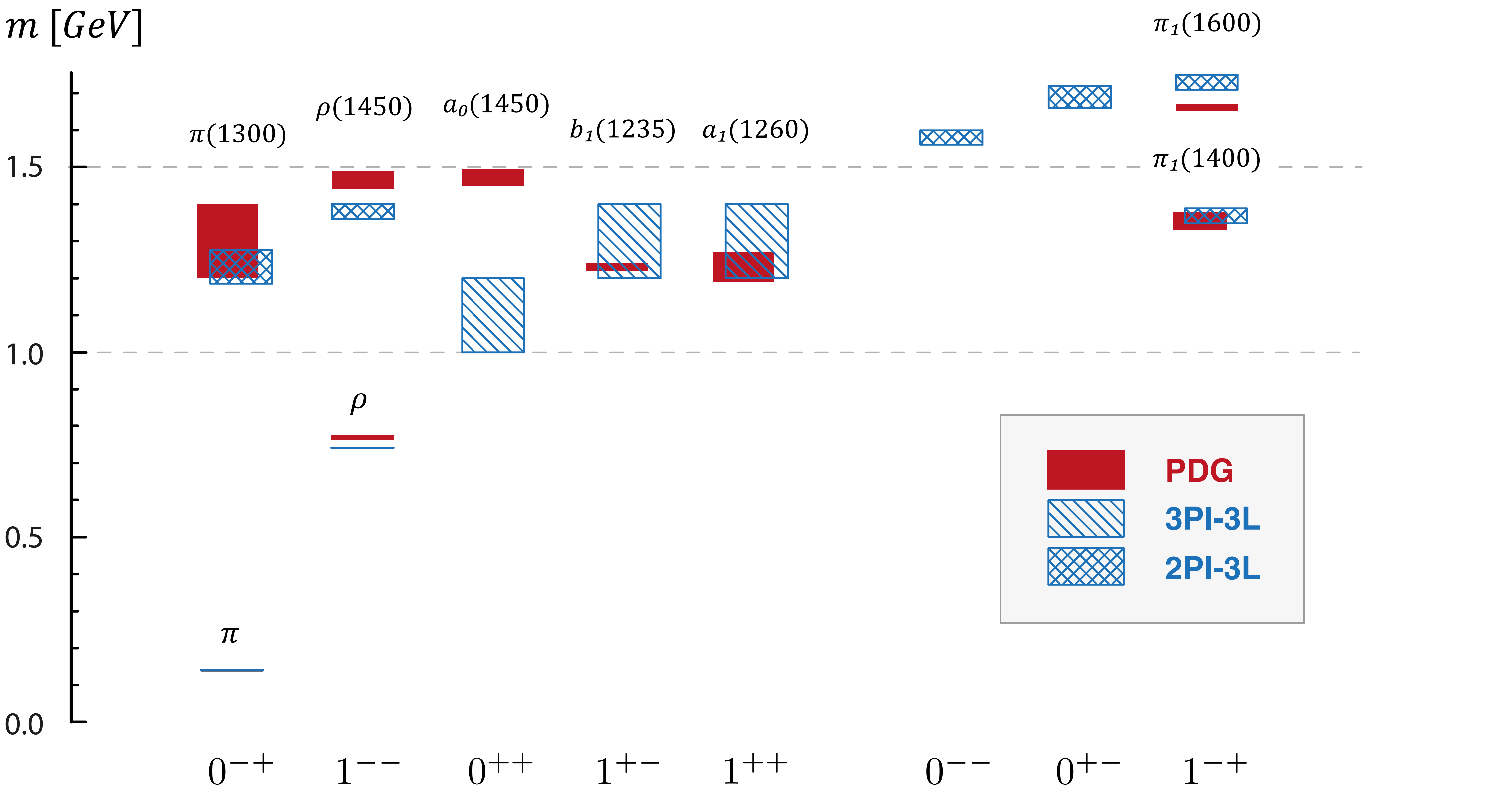

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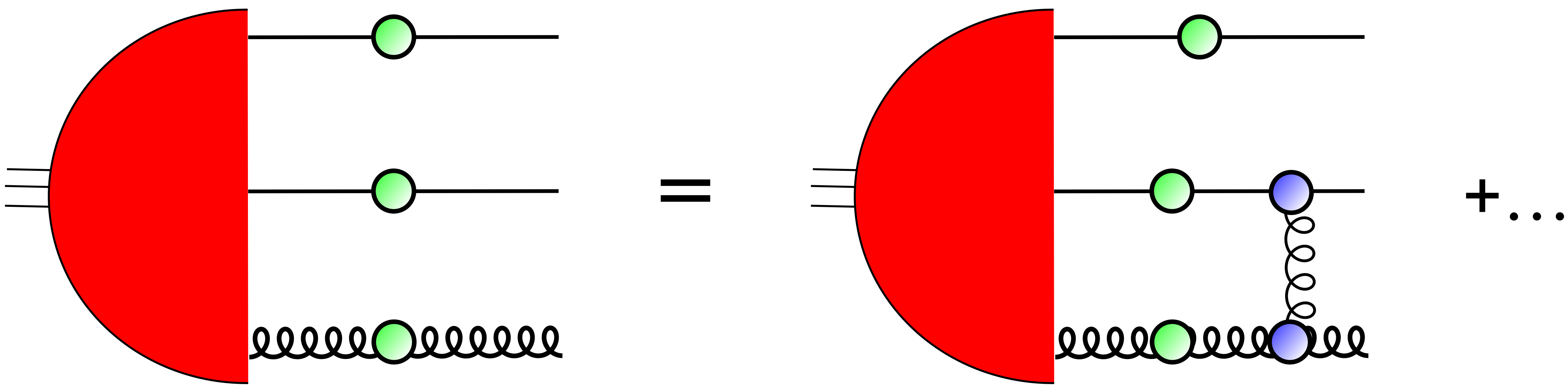
Hybrids as three-body states



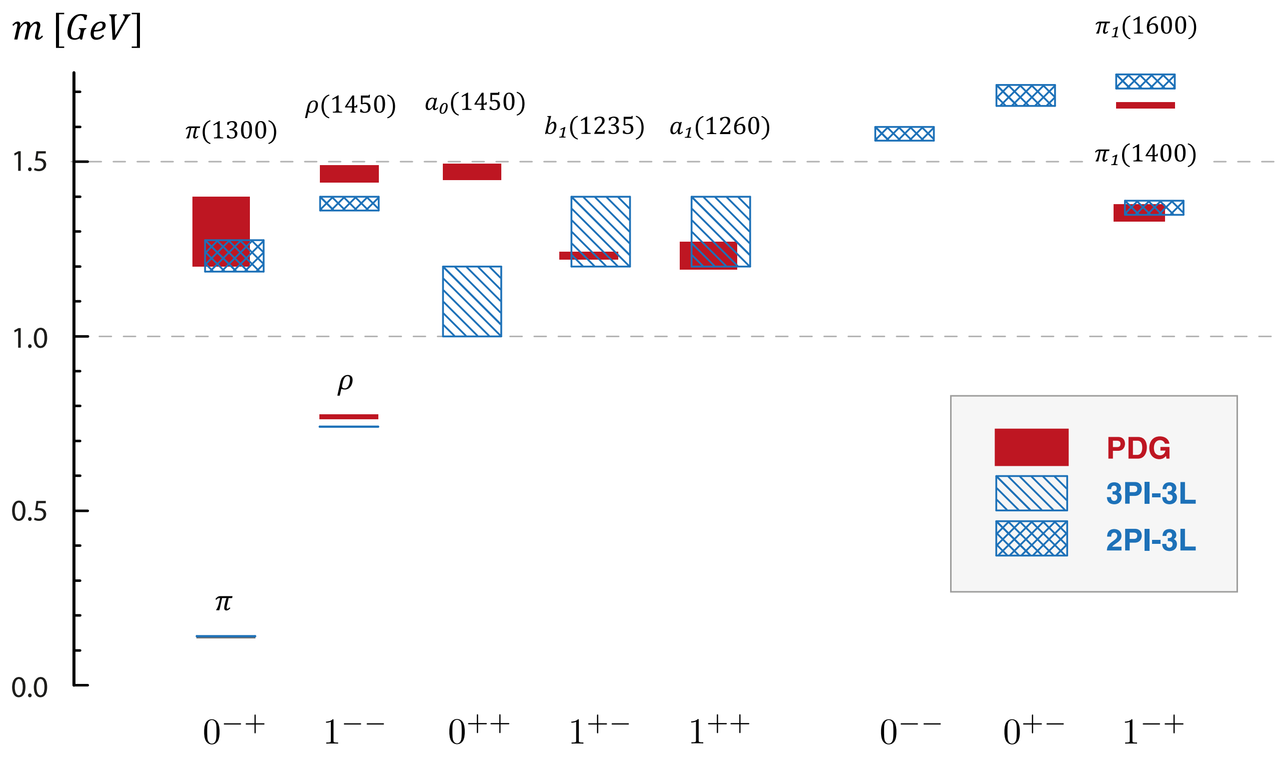
- Similar to Faddeev-eq. for baryons except for glue
- Expectation: bound states around 800 MeV higher than $q\bar{q}$ with same quantum numbers

Liu et al. (HSC), JHEP 1207 (2012) 126

> Working hypothesis:
> two-body BSE with lots of glue in kernel =
> three-body-BSE with glue in valence part

Overview - Take home messages

- **Hybrids:**

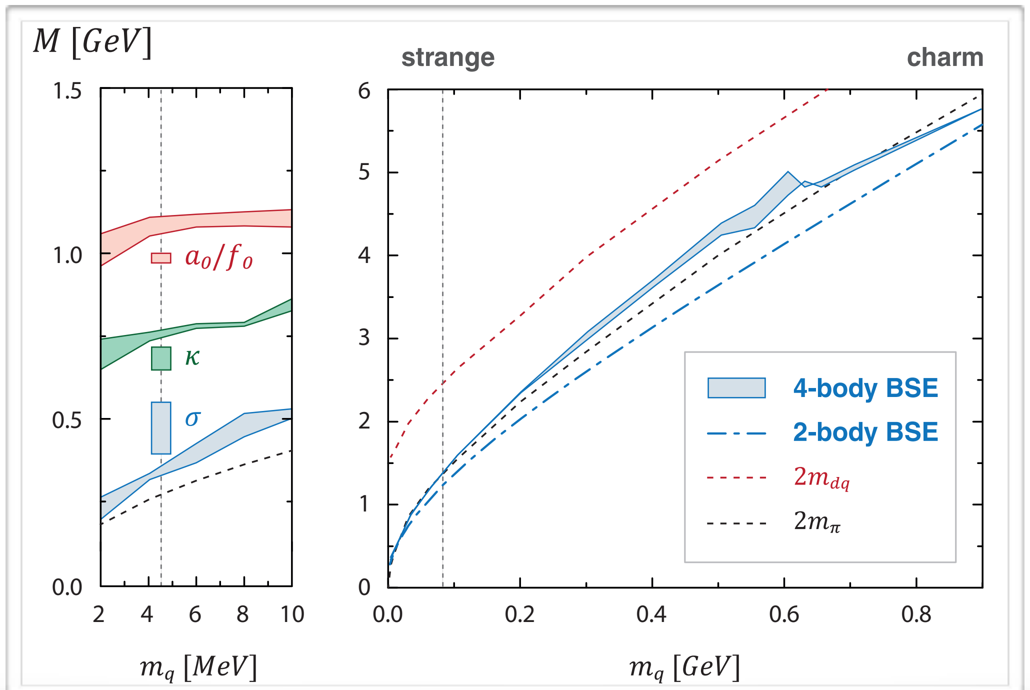


Williams, CF, Heupel, PRD93 (2016) 034026

- **Glueballs:** $M(0^{++}) = 1.64\,\text{GeV}$

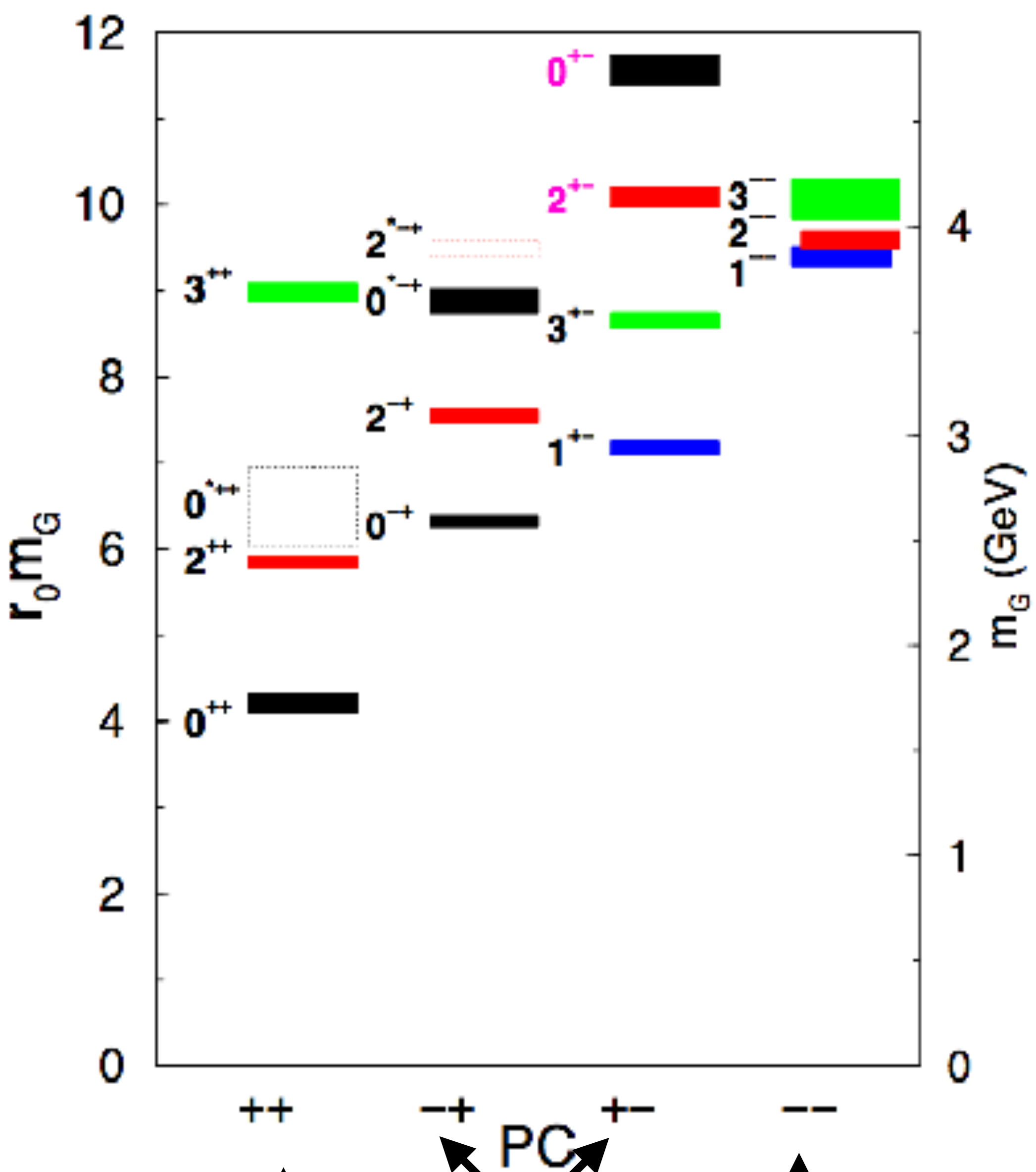
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

- **Light tetraquarks:**



Eichman, CF, Heupel, PLB 753 (2016) 282-287

Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

two gluons

three gluons

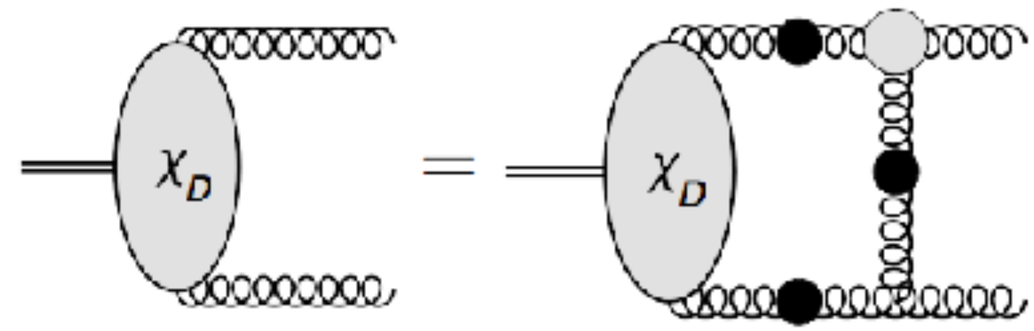
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Unquenched calculations very involved

Gregory et al., JHEP 1210 (2012) 170

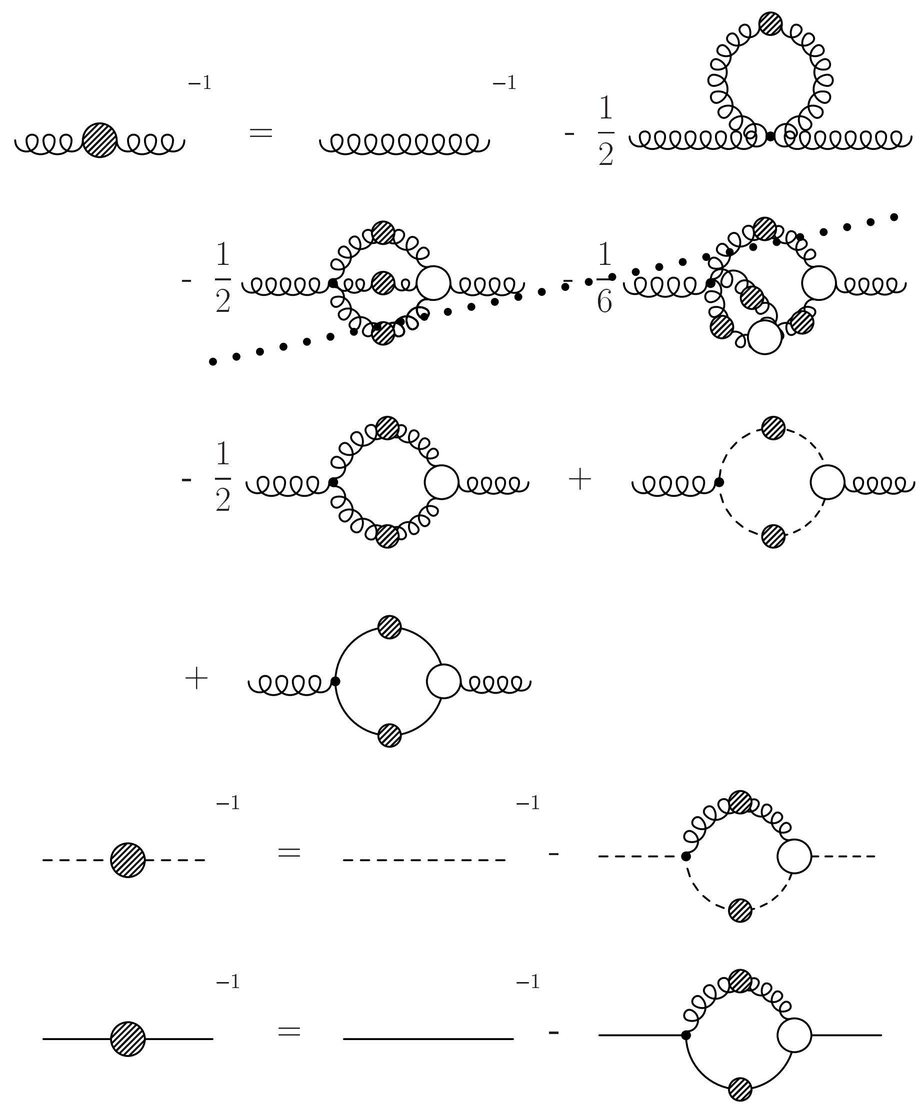
DSE:

- structural information

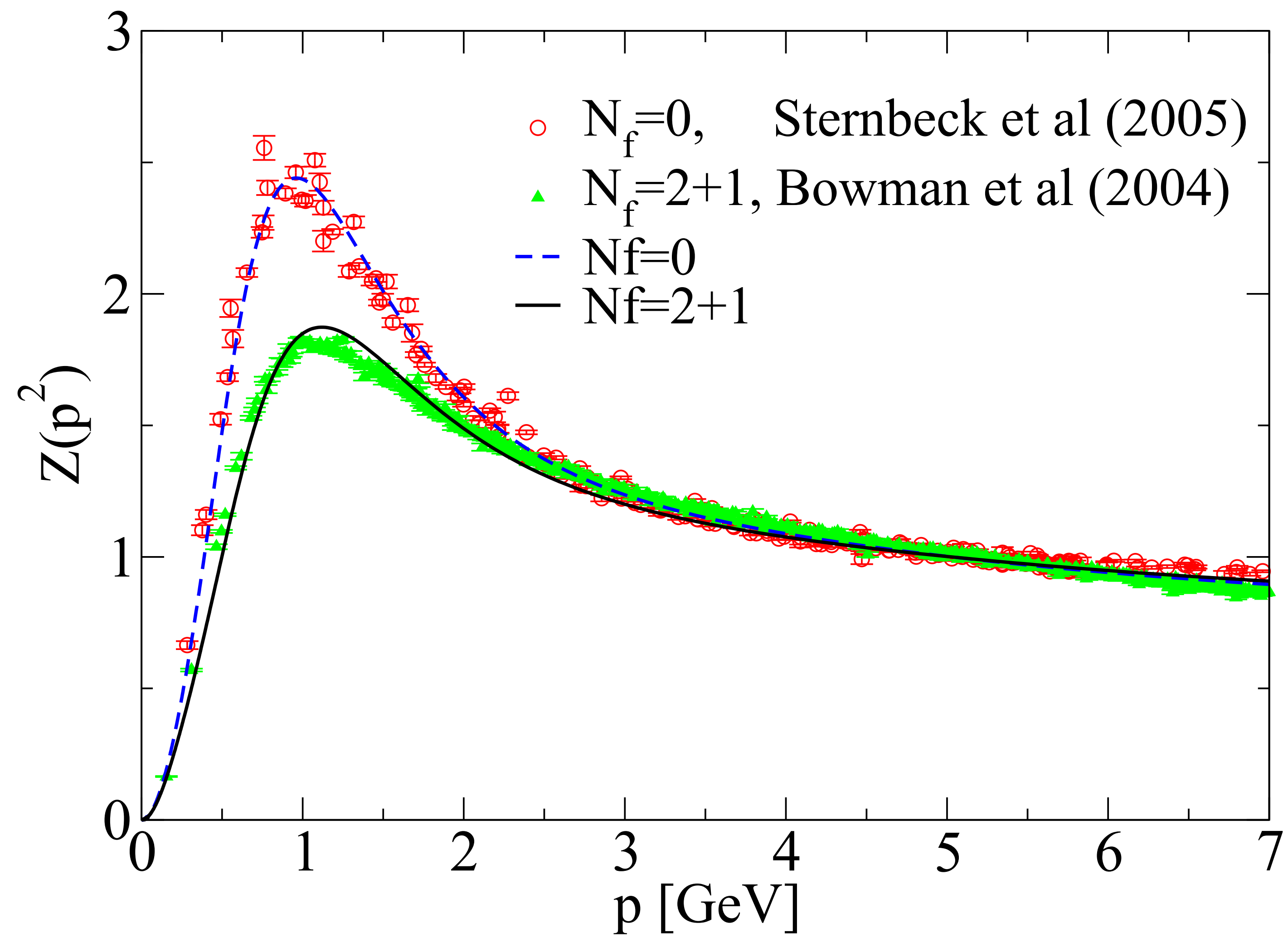


Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$



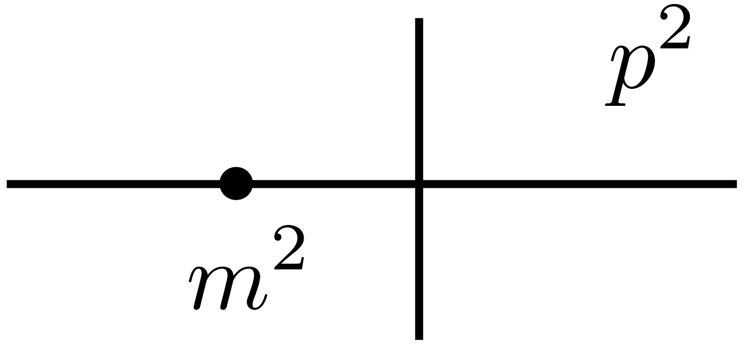
- spacelike momenta:
  good agreement with lattice
- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

Huber and von Smekal, JHEP 1304 (2013)149
Hopfer, CF and Alkofer, JHEP 1411 (2014) 035
Huber, EPJC 77 (2017)

Analytic structure of gluon I

massive physical particle $\dfrac{1}{p^2 + m^2}$



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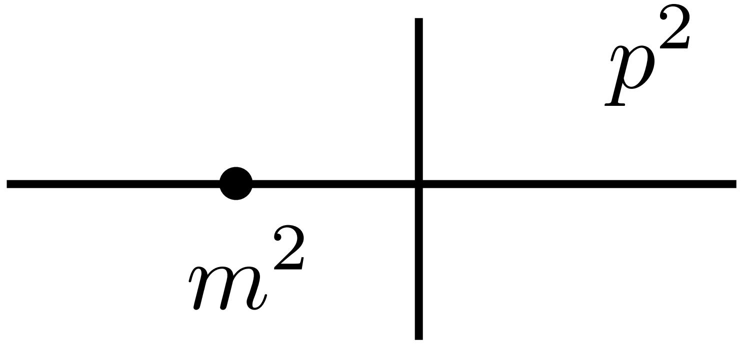
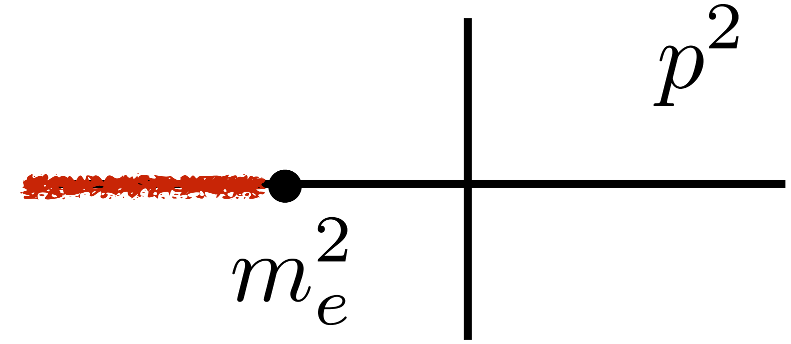
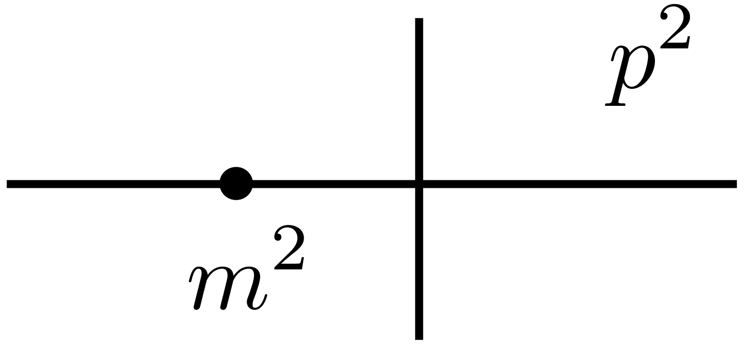


U(1) gauge theory: electron with photon cloud



Alkofer, Detmold, CF, Maris PRD70 (2004) 014014 massive physical particle

$$\frac{1}{p^2+m^2}$$



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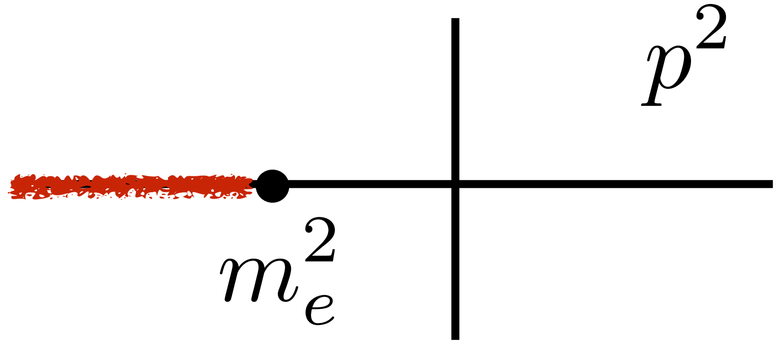
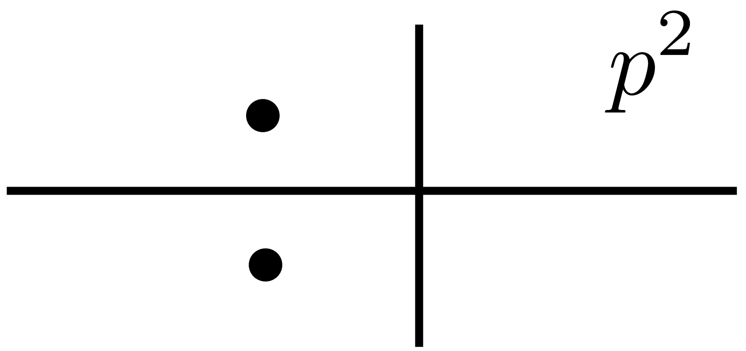


Alkofer, Detmold, CF, Maris PRD70 (2004) 014014

quark ? (complex conjugate singularities)



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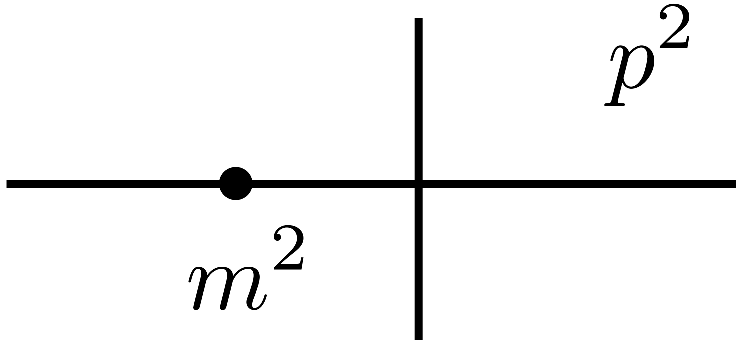

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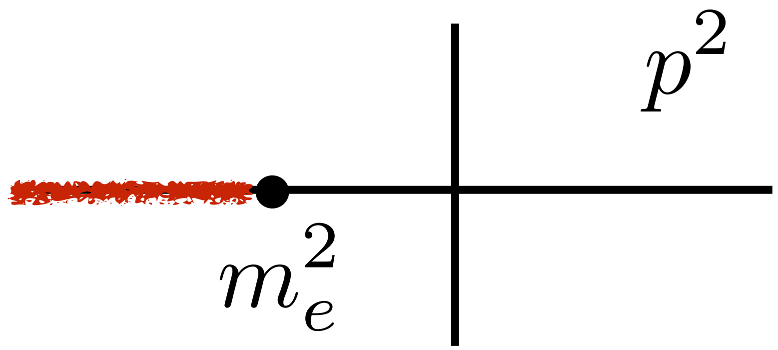

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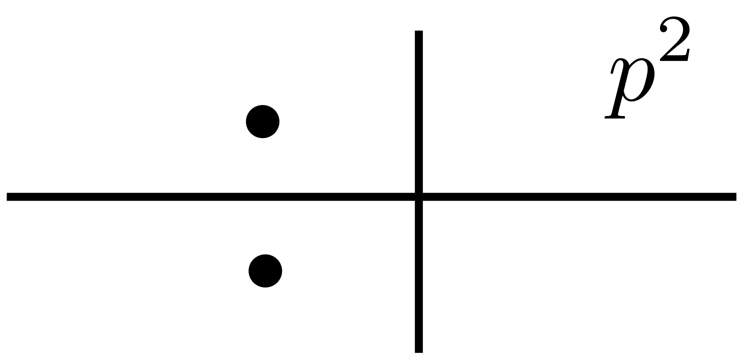
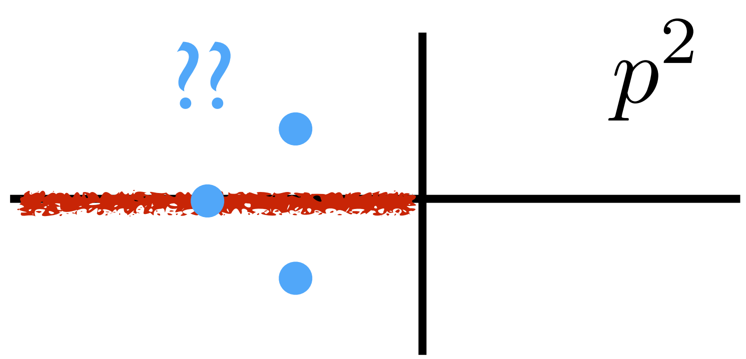


gluon ?



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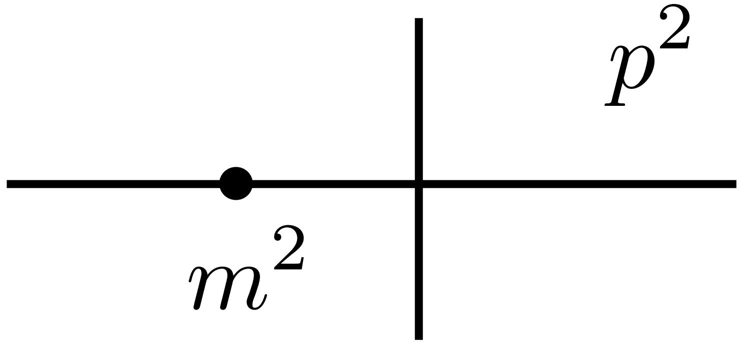

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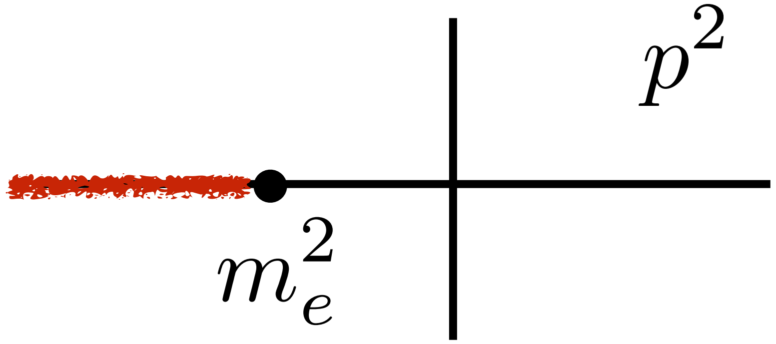

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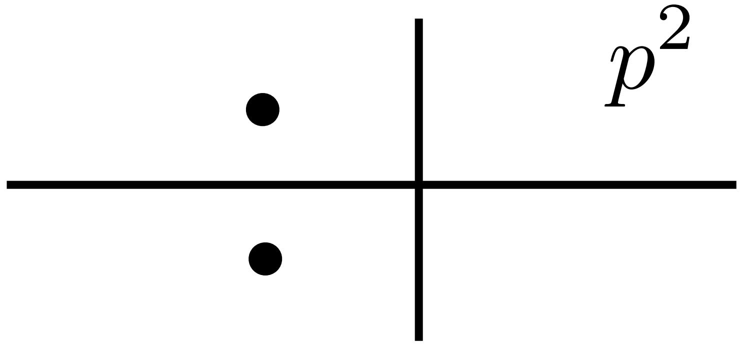

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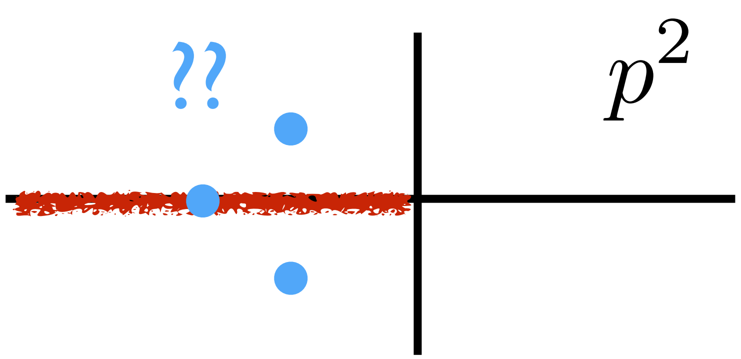
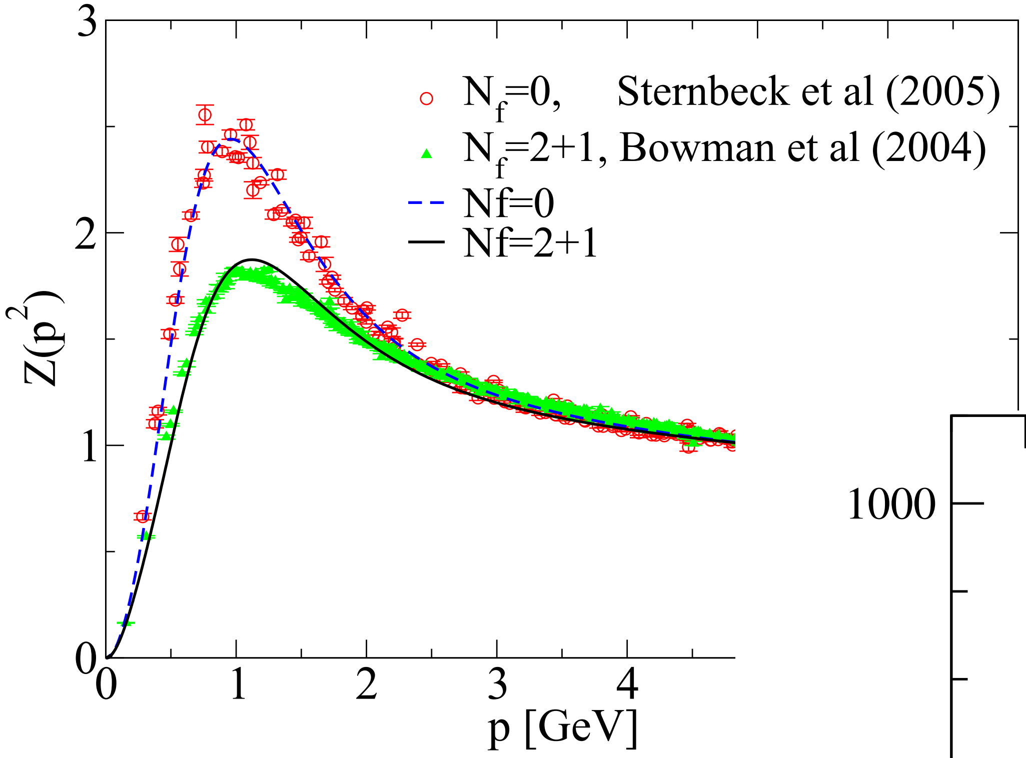
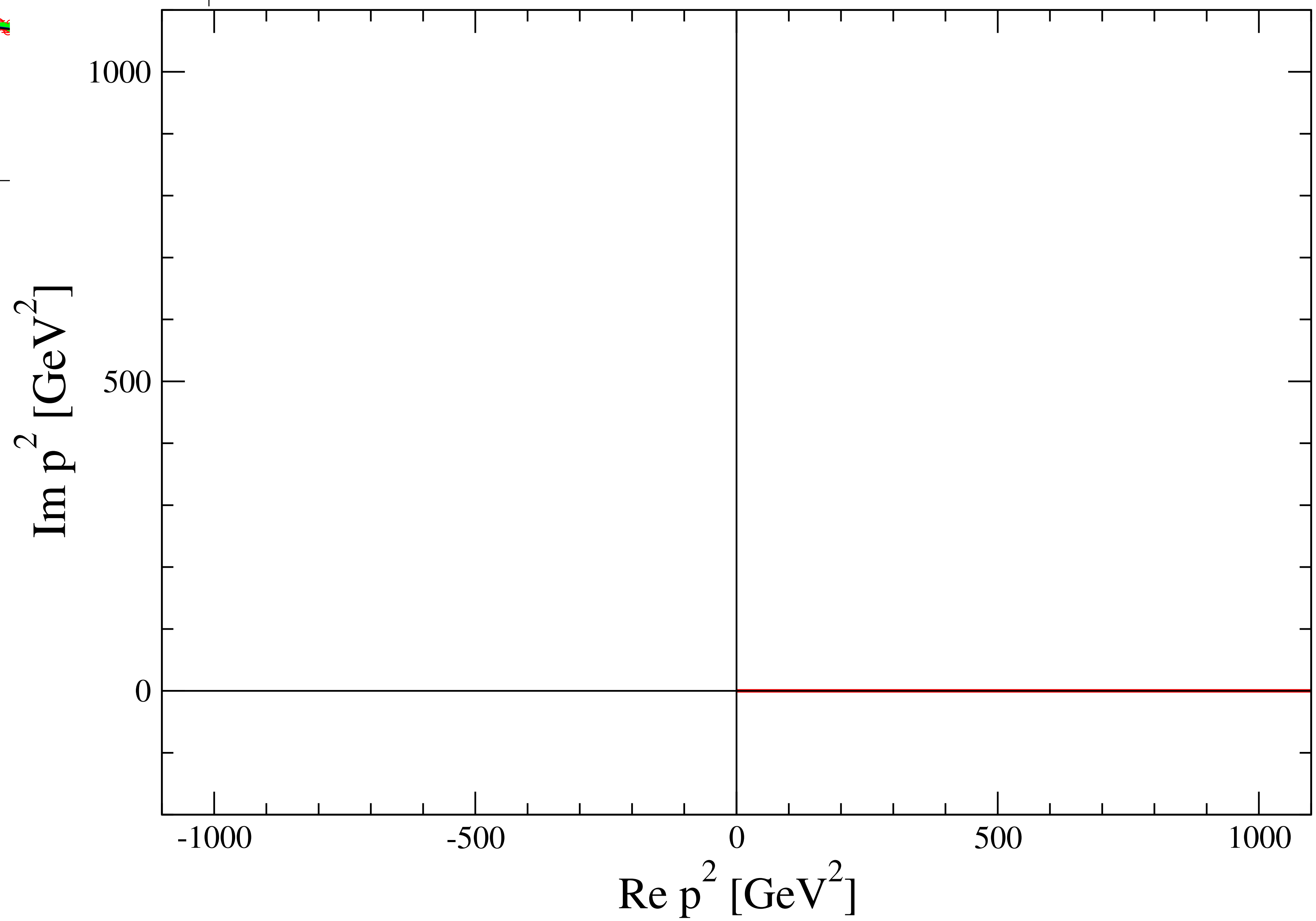


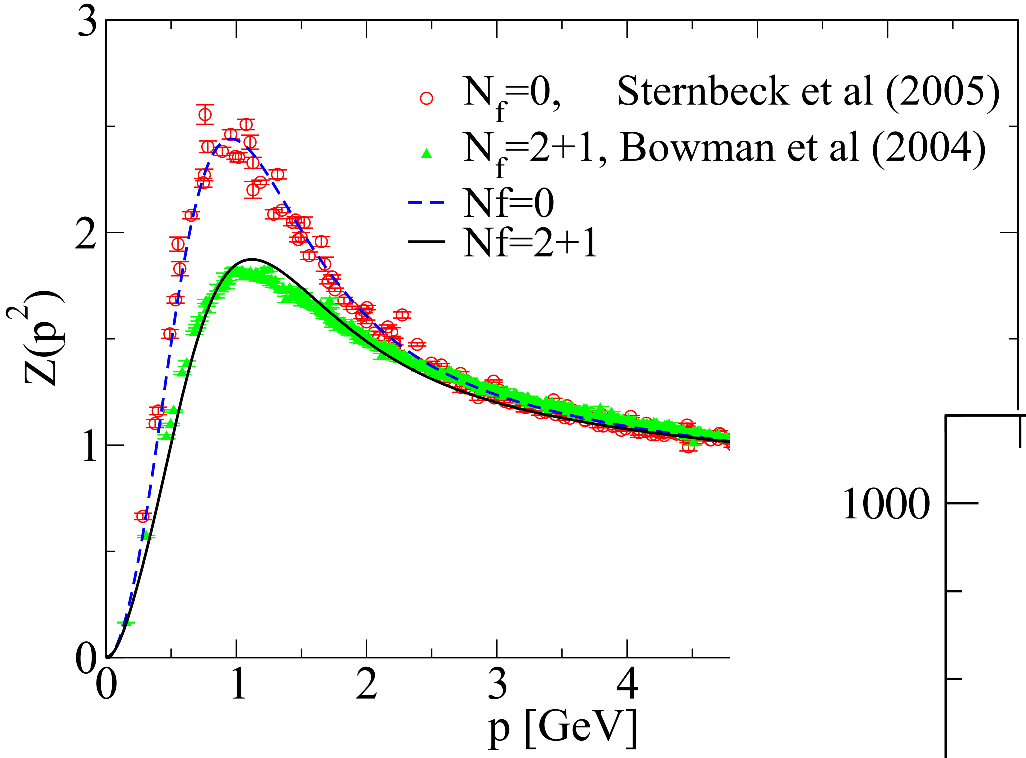
Fit ? Extrapolation ? Direct calculation !

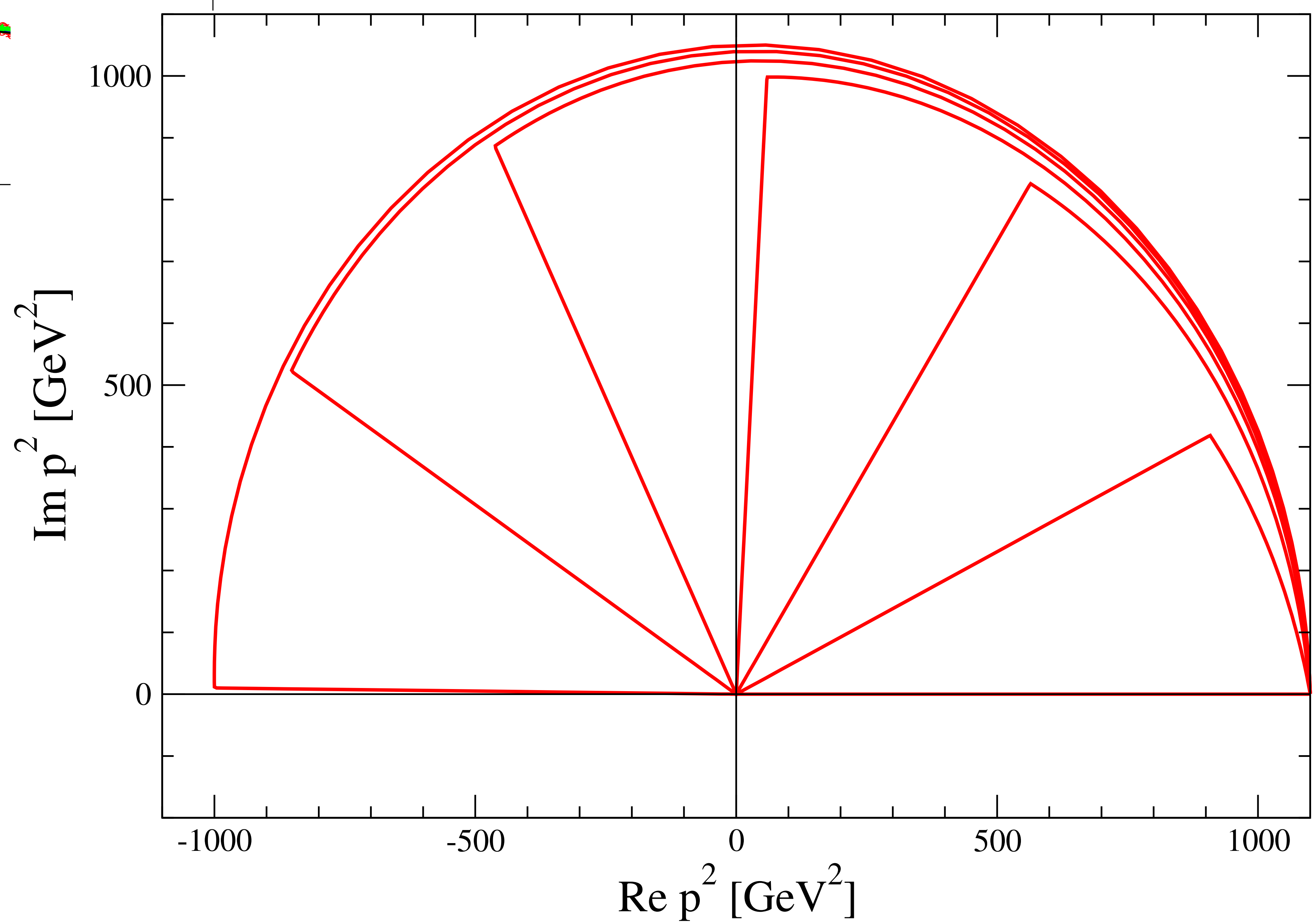
Analytic structure II





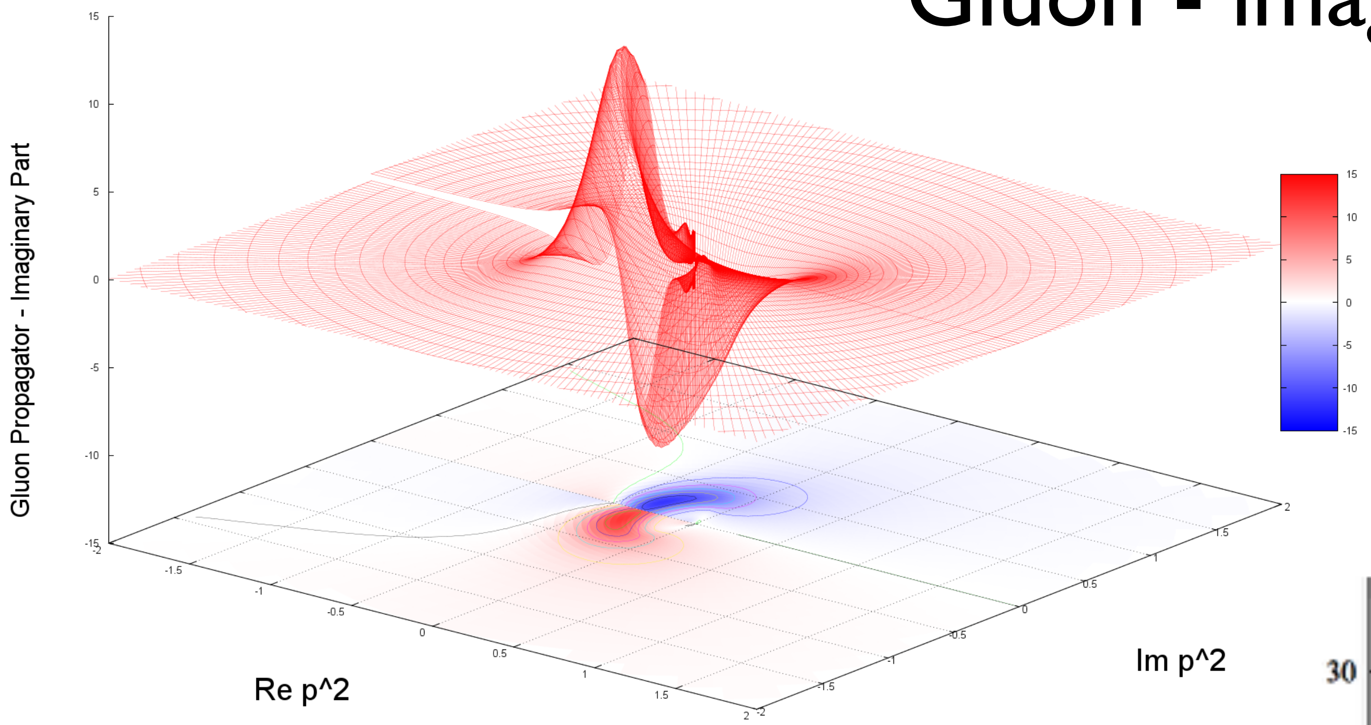
Analytic structure II

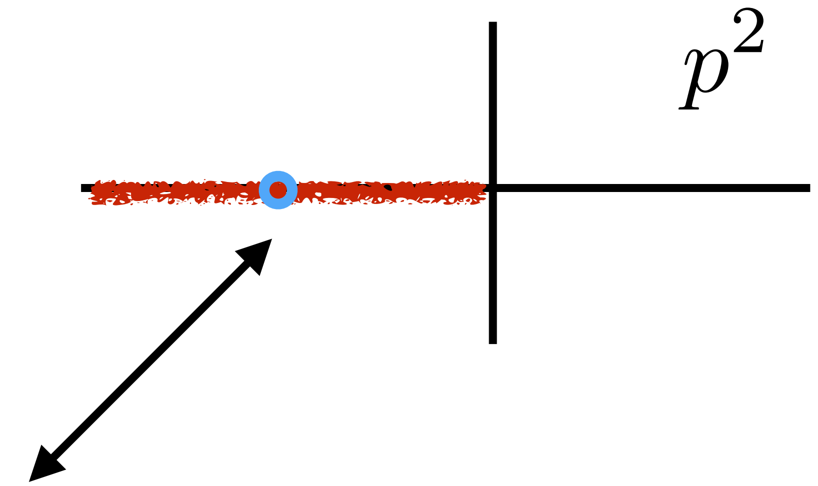


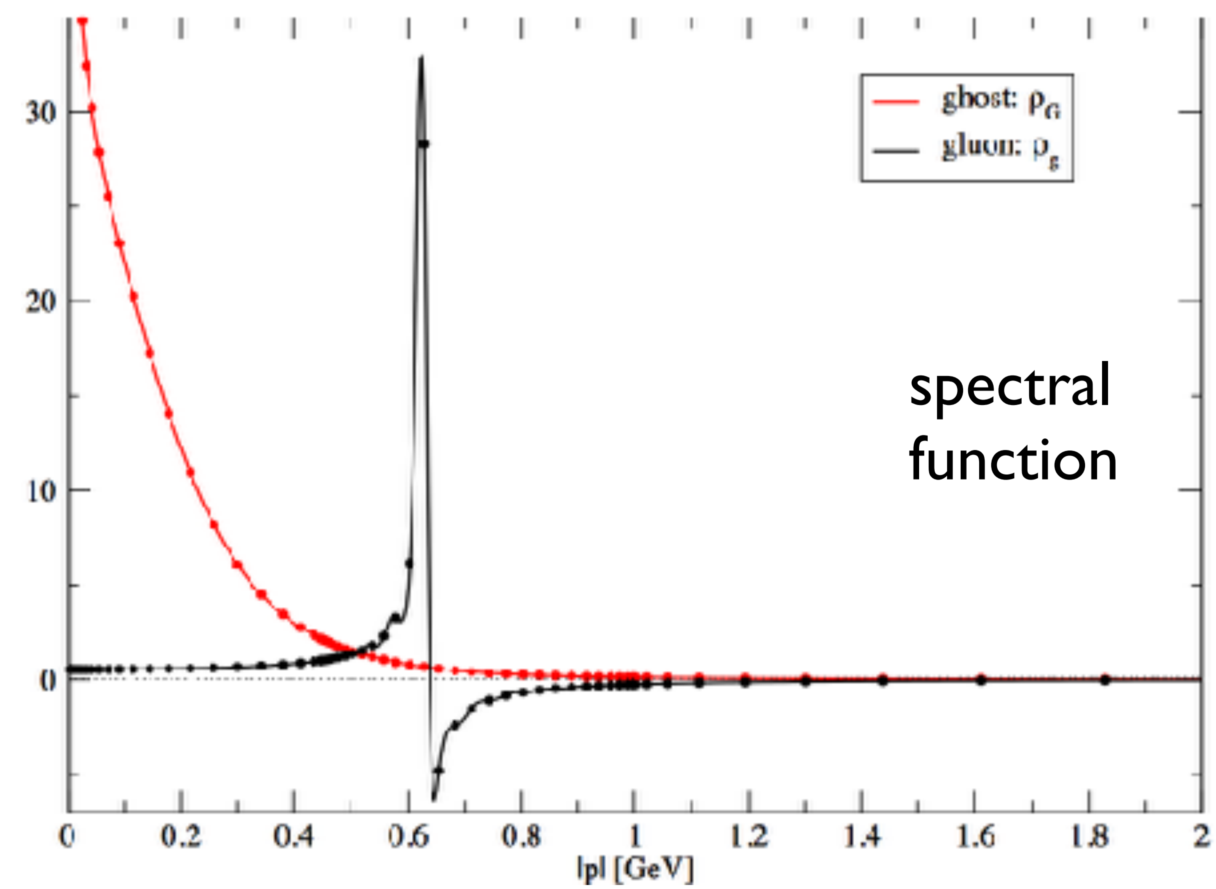


Analytic structure III

Gluon - imaginary part







- spectral function: positivity violations
  Gluon cannot appear in detector!

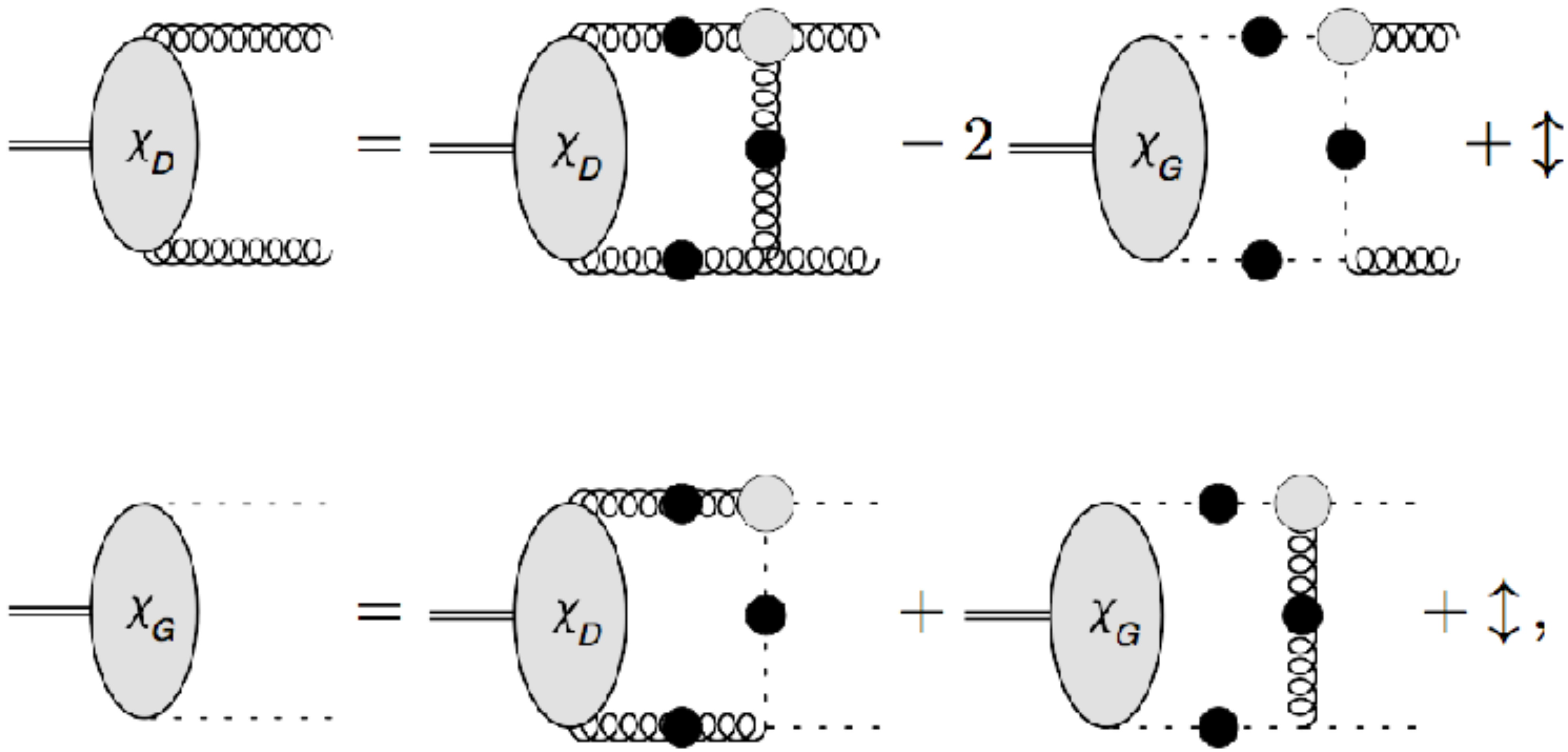
- $600\,\mathrm{MeV} < m_g < 700\,\mathrm{MeV}$

  Cornwall PRD 26 (1982); Cucchieri, Mendes PoS Lat2007 297
  Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
  Boucaud et al. JHEP 0806 (2008) 099

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

see also: Cyrol, Pawlowski, Rothkopf and Wink, 1804.00945

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

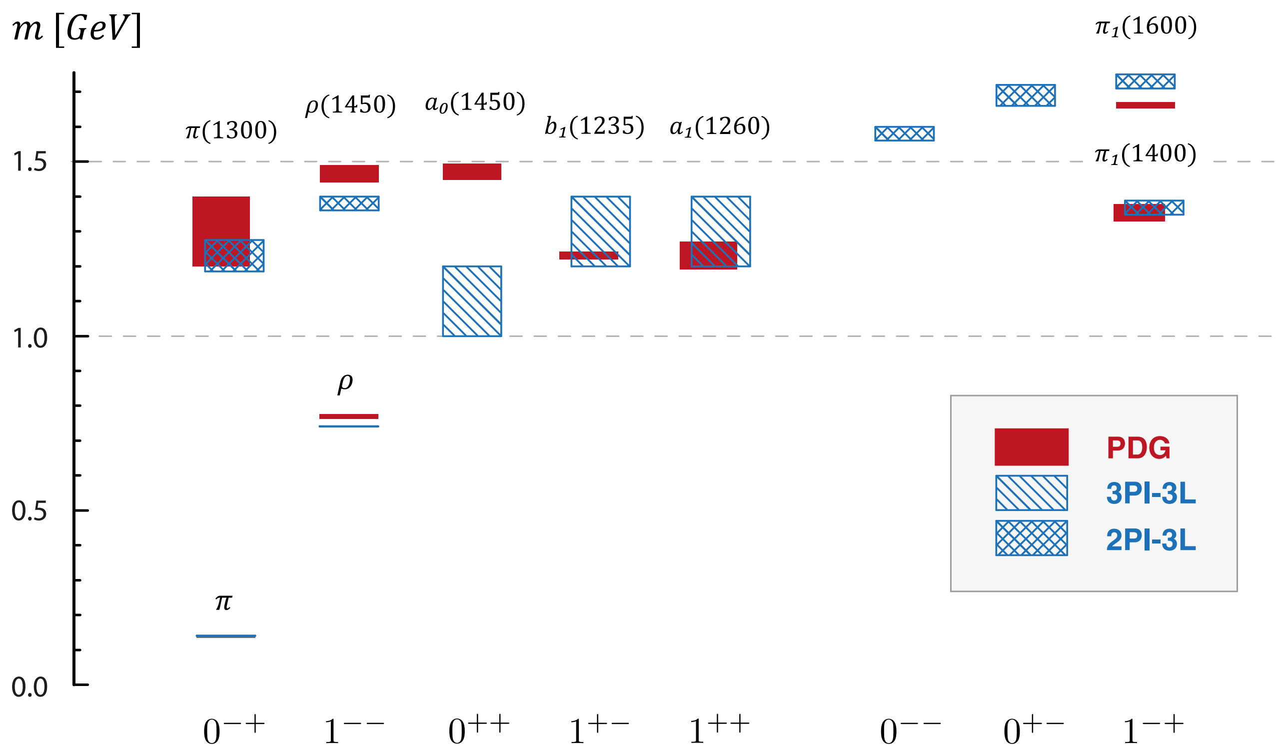
Results: $M(0^{++}) = 1.64\,\mathrm{GeV}$

$M(0^{-+}) = 4.53\,\mathrm{GeV}$ $\leftarrow$ ghost do not contribute !

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Overview - Take home messages

- **Hybrids:**

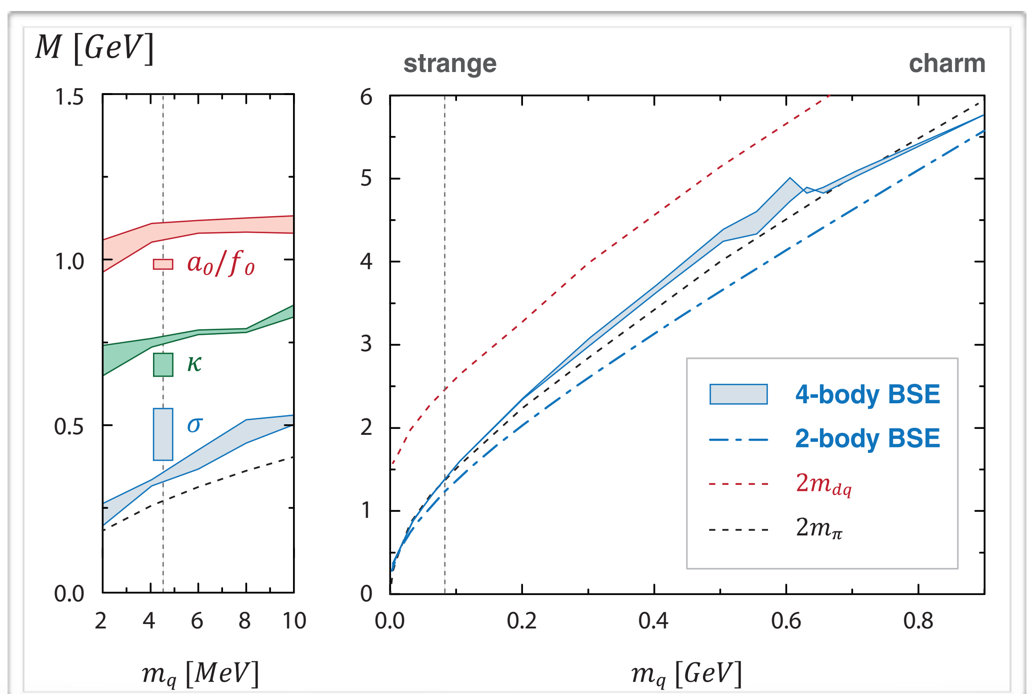


Williams, CF, Heupel, PRD93 (2016) 034026

- **Glueballs:** $M(0^{++}) = 1.64\,\mathrm{GeV}$

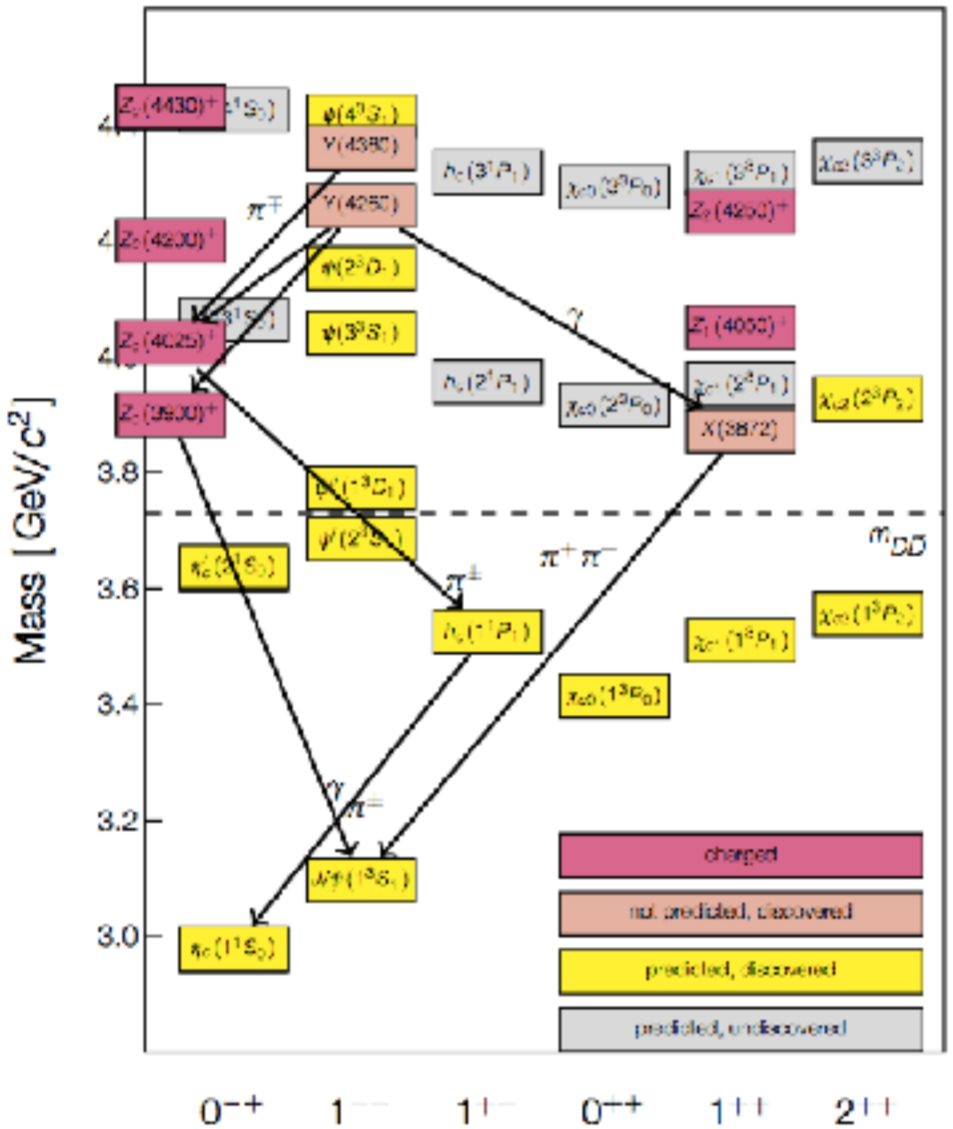
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

- **Light tetraquarks:**



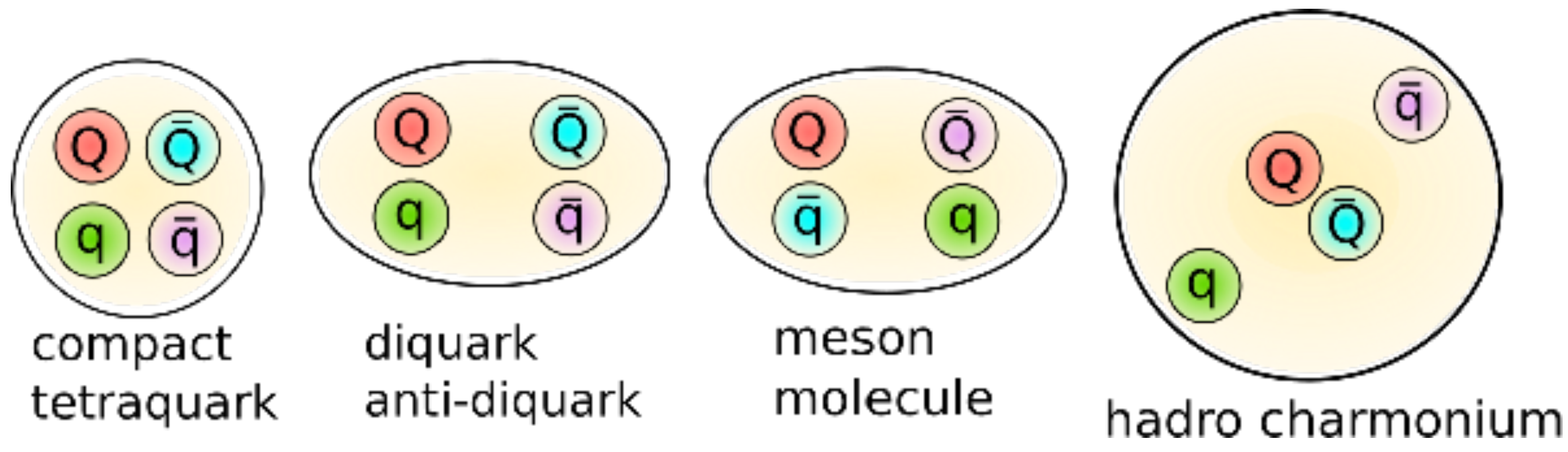
Eichman, CF, Heupel, PLB 753 (2016) 282-287

Heavy and light tetraquark



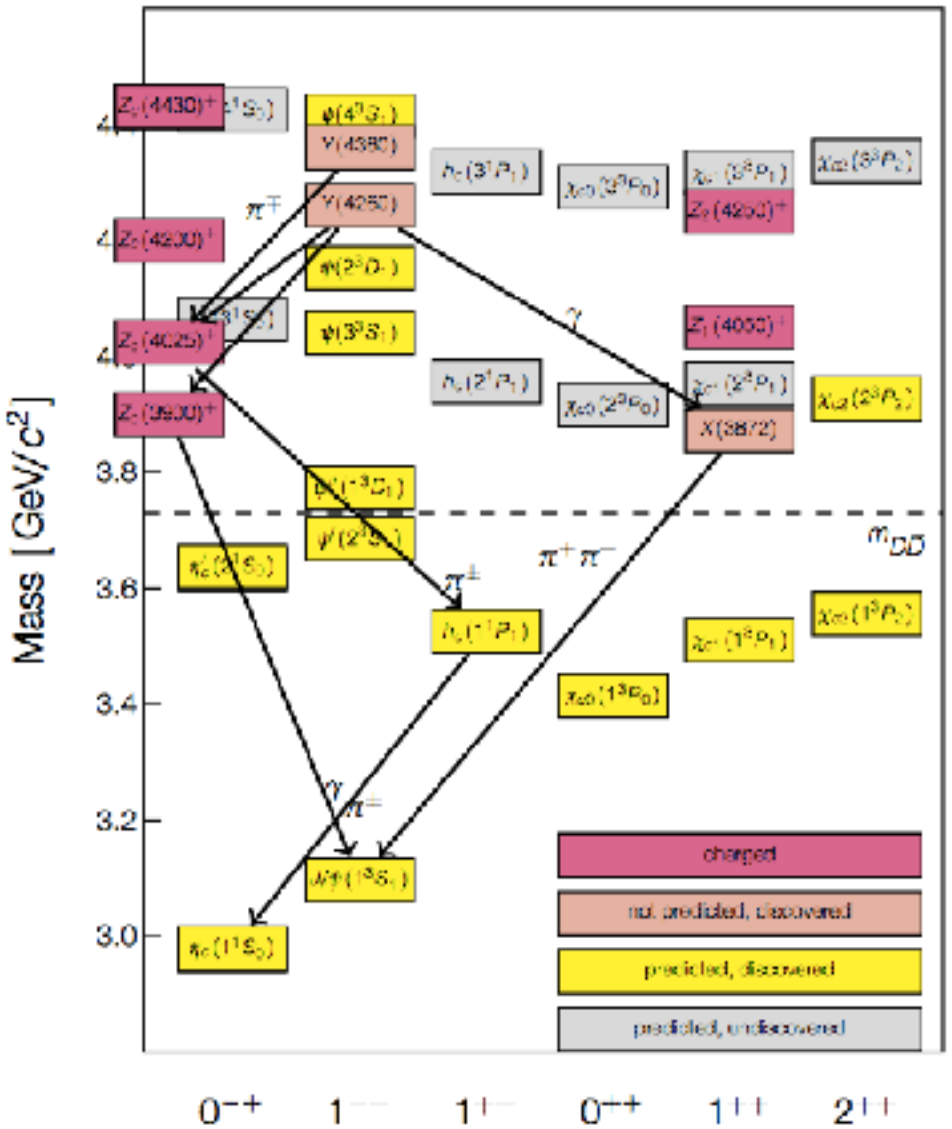
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks

Heavy and light tetraquark



Wolfgang Gradl, BESIII, St Goar 2015

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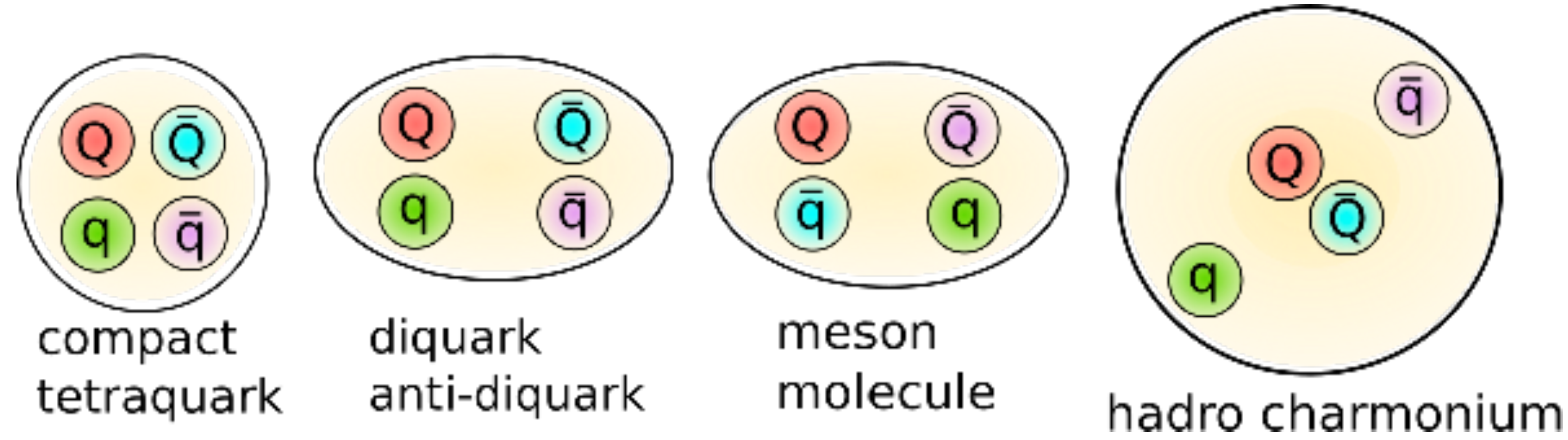
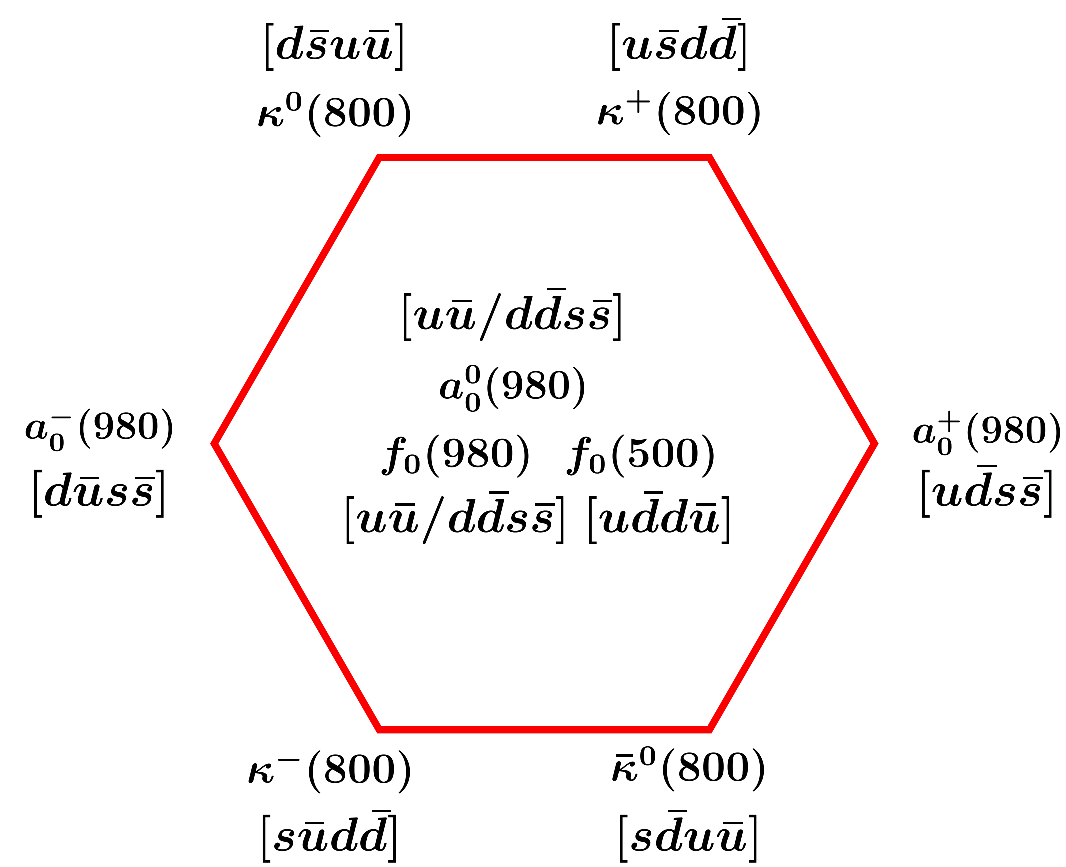
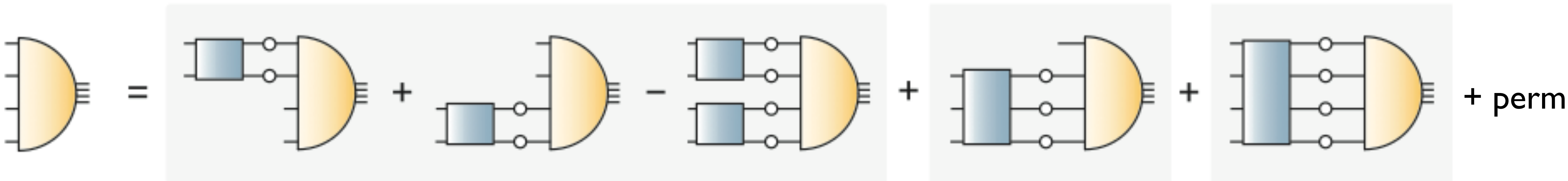




Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

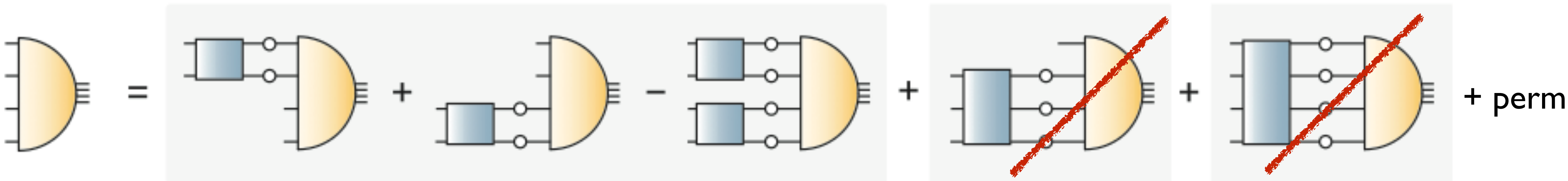
Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Tetraquarks from the four-body equation

Exact equation:



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Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
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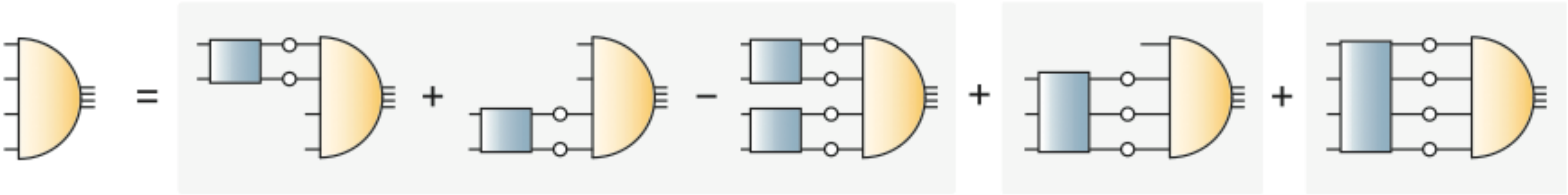
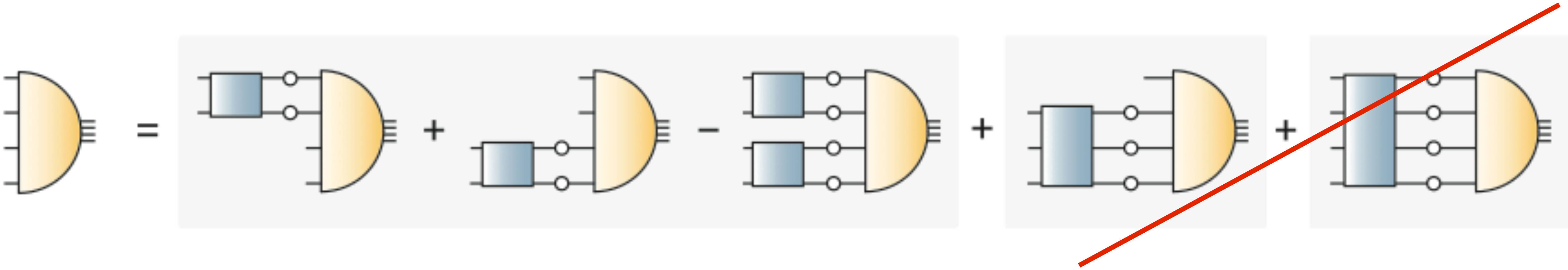
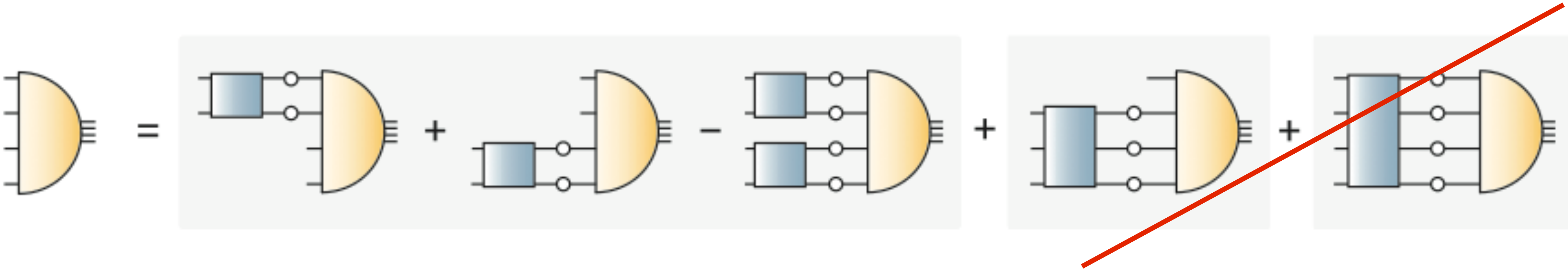
Two-body approximation



Two-body approximation

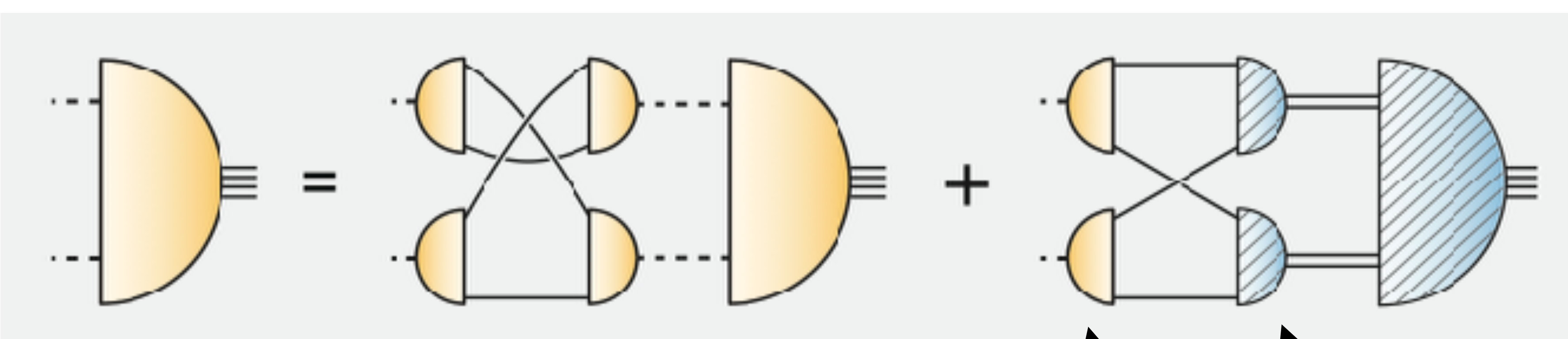


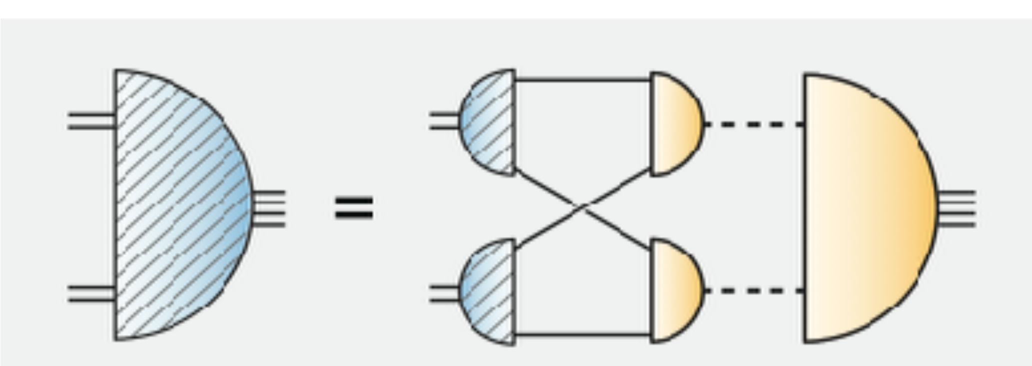
Two-body approximation



approximation: separable ansatz for interaction kernel

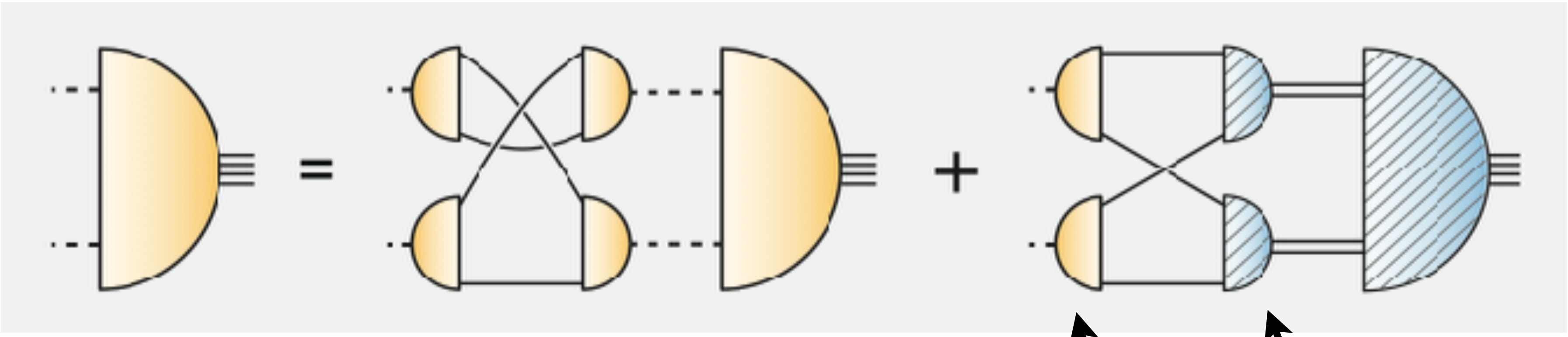
Heupel, Eichman, CF, PLB 718 (2012) 545-549

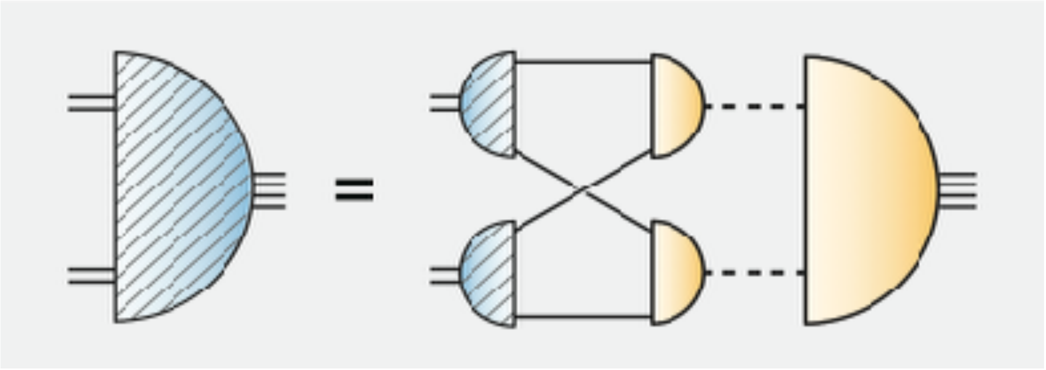


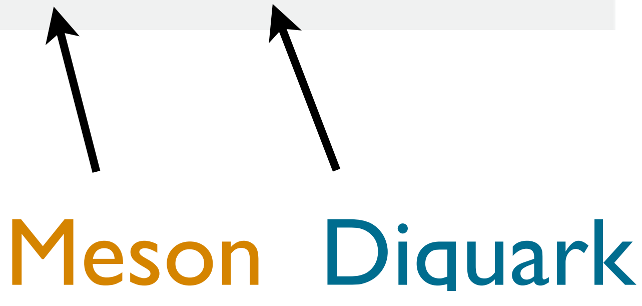


Meson Diquark

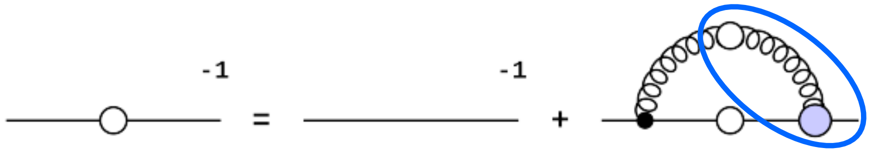
Tetraquark-BSEs - two-body equations





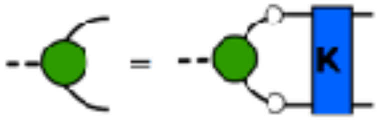


- Input: Covariant Quark-Gluon interaction - Maris-Tandy model



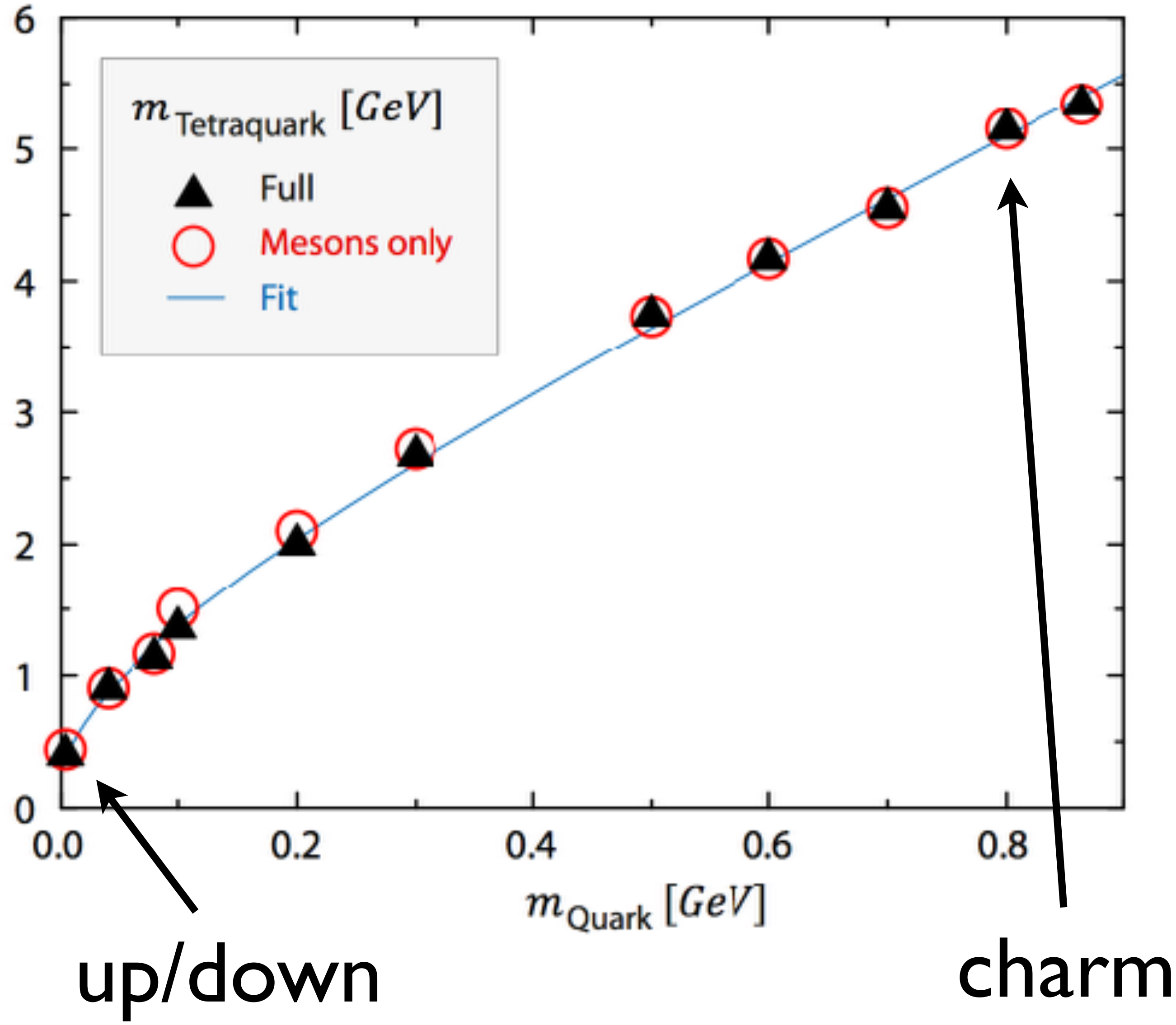
$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- Mesons and Diquarks via Bethe-Salpeter equation



Dynamical decision between Meson- and Diquark-configurations

Results: scalar tetraquarks



Heupel, Eichman, CF, PLB 718 (2012) 545-549

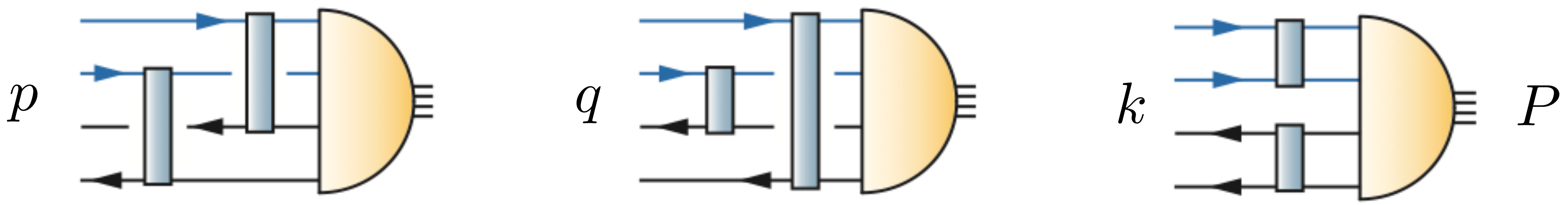
- Pion-Pion-contribution dominates !
- m(0^{++}) = 403 MeV

} $f_0(500)$

see also Caprini, Colangelo and Leutwyler, PRL. 96 (2006) 132001
Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011

- Narrow scalar $cc\overline{cc}$: m(0^{++}) = 5.3 ± (0.5) GeV

Scalar tetraquark:



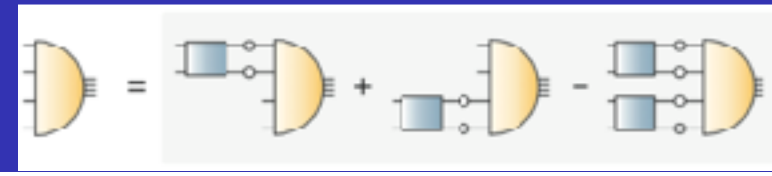
$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

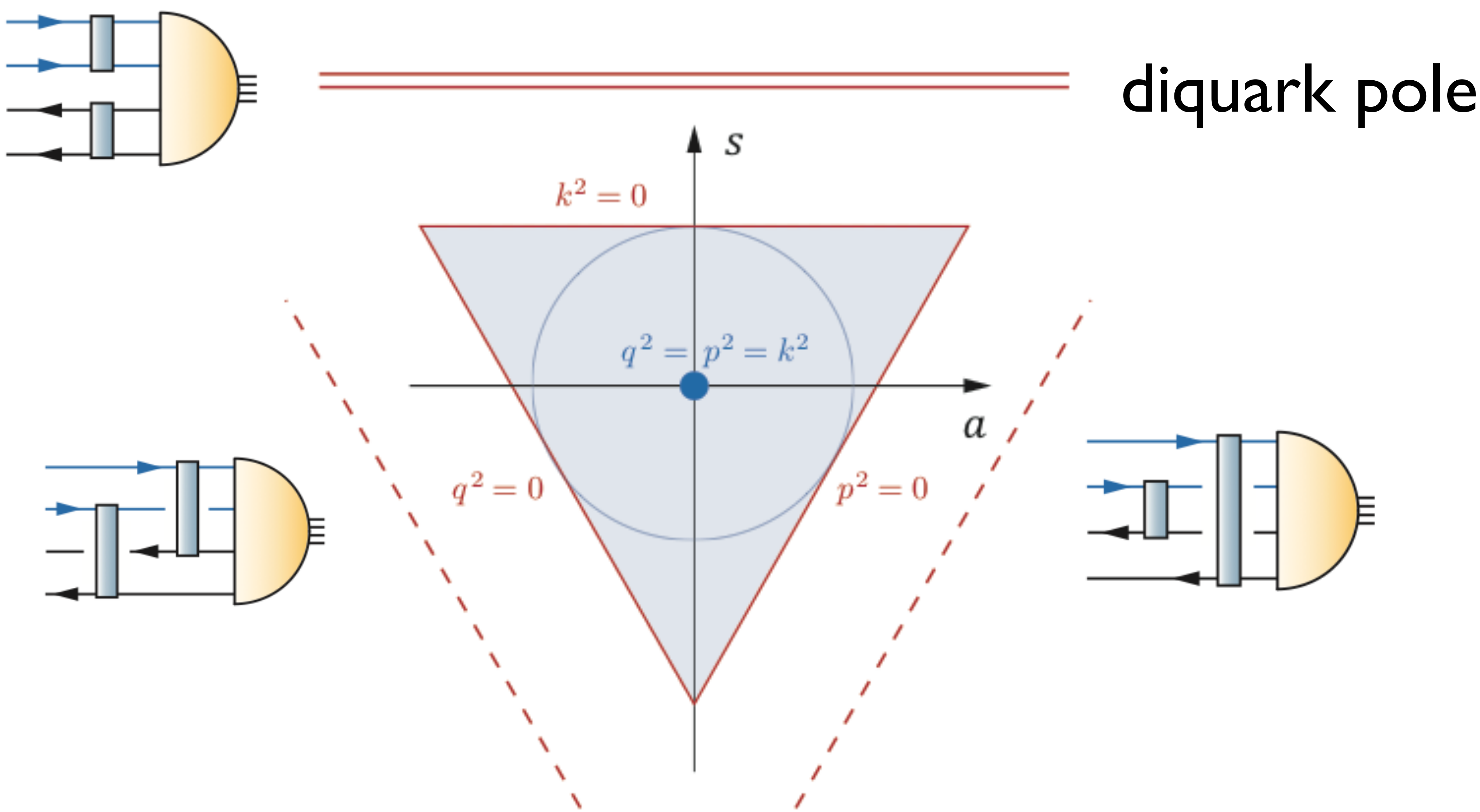
$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reasonable approximation: keep s-waves only;
  → 16 tensor structures

Four-body equation: 

Organise Dirac-Lorentz-tensors into multiplets of **S4**

- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$, carries overall scale
- Doublet: $a = \sqrt{3}(q^2 - p^2)/(4S_0); \;\; s = (p^2 + q^2 - 2k^2)/(4S_0)$



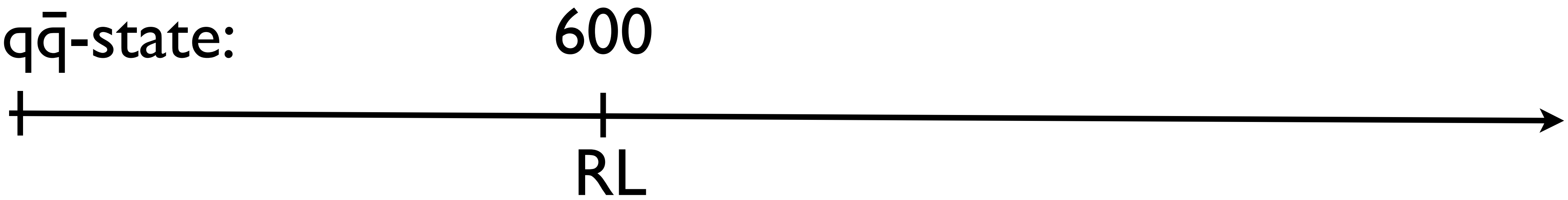
meson poles

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Two triplets

Bound state vs resonance: light scalars

$q\bar{q}$-state:



Bound state vs resonance: light scalars

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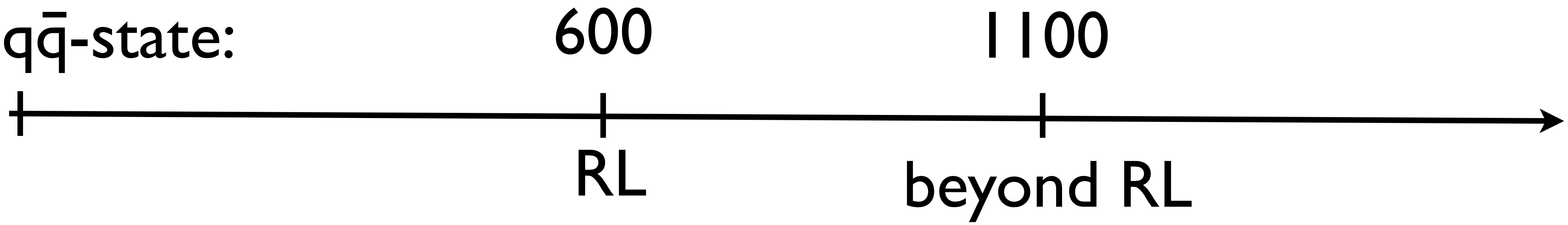
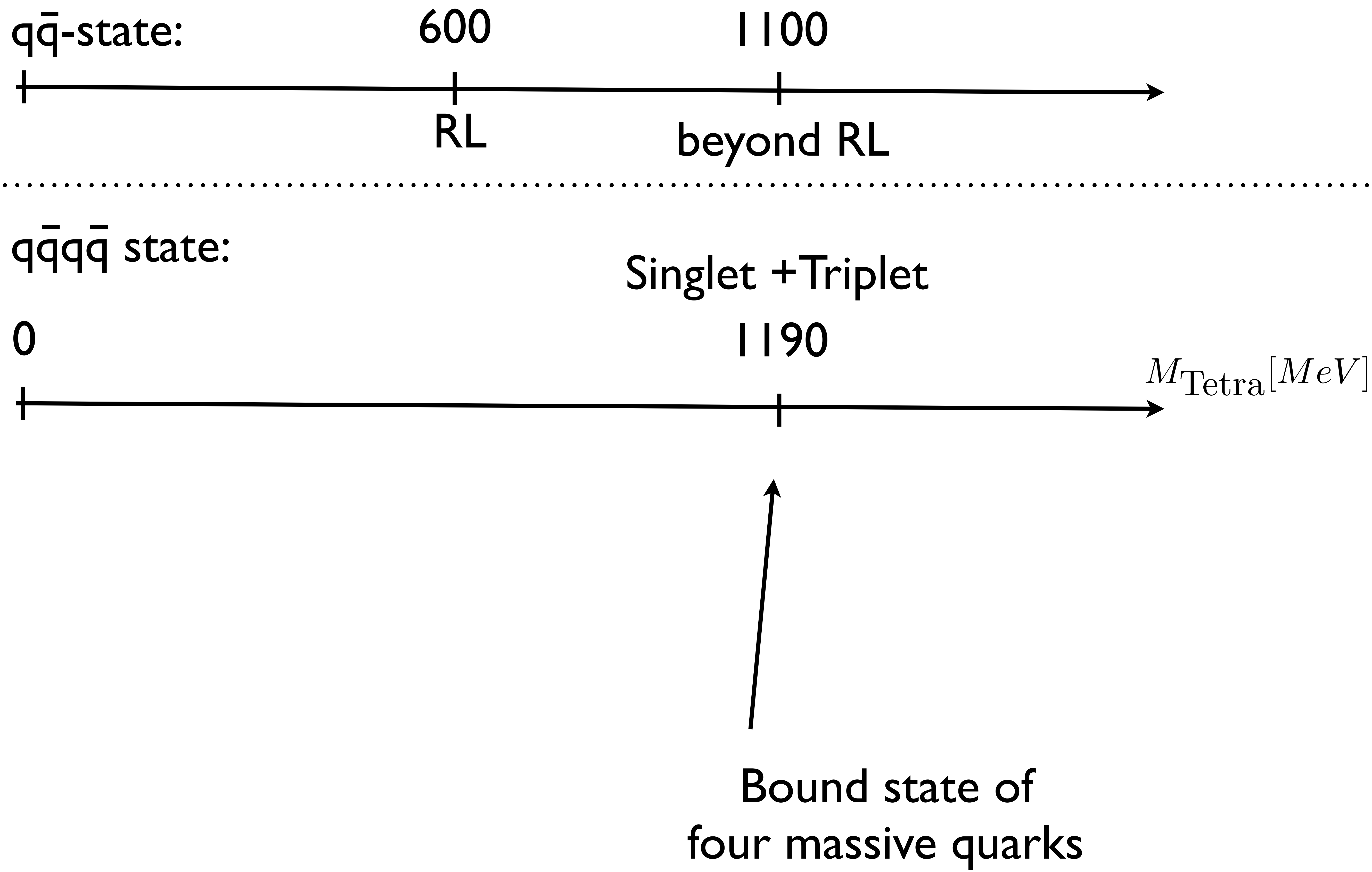


Bound state vs resonance: light scalars



Bound state vs resonance: light scalars

$q\bar{q}$-state: 600 1100

RL beyond RL

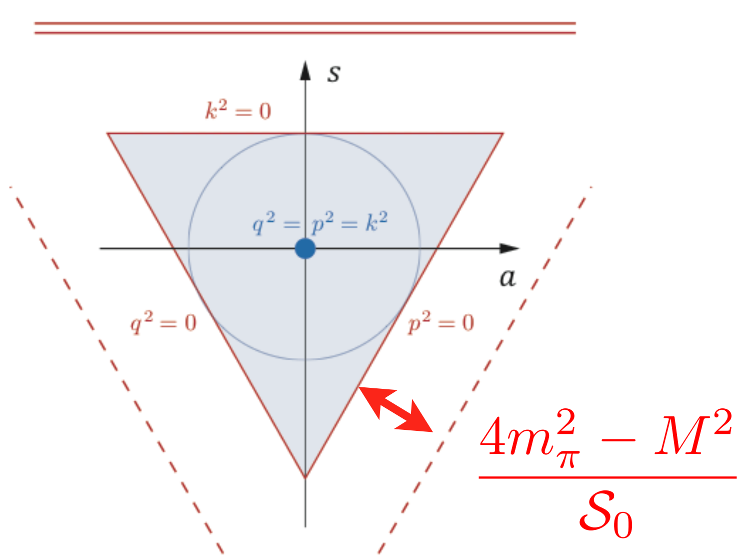
$q\bar{q}q\bar{q}$ state:

Singlet +Triplet

0 350 1190

$M_{\mathrm{Tetra}}[MeV]$

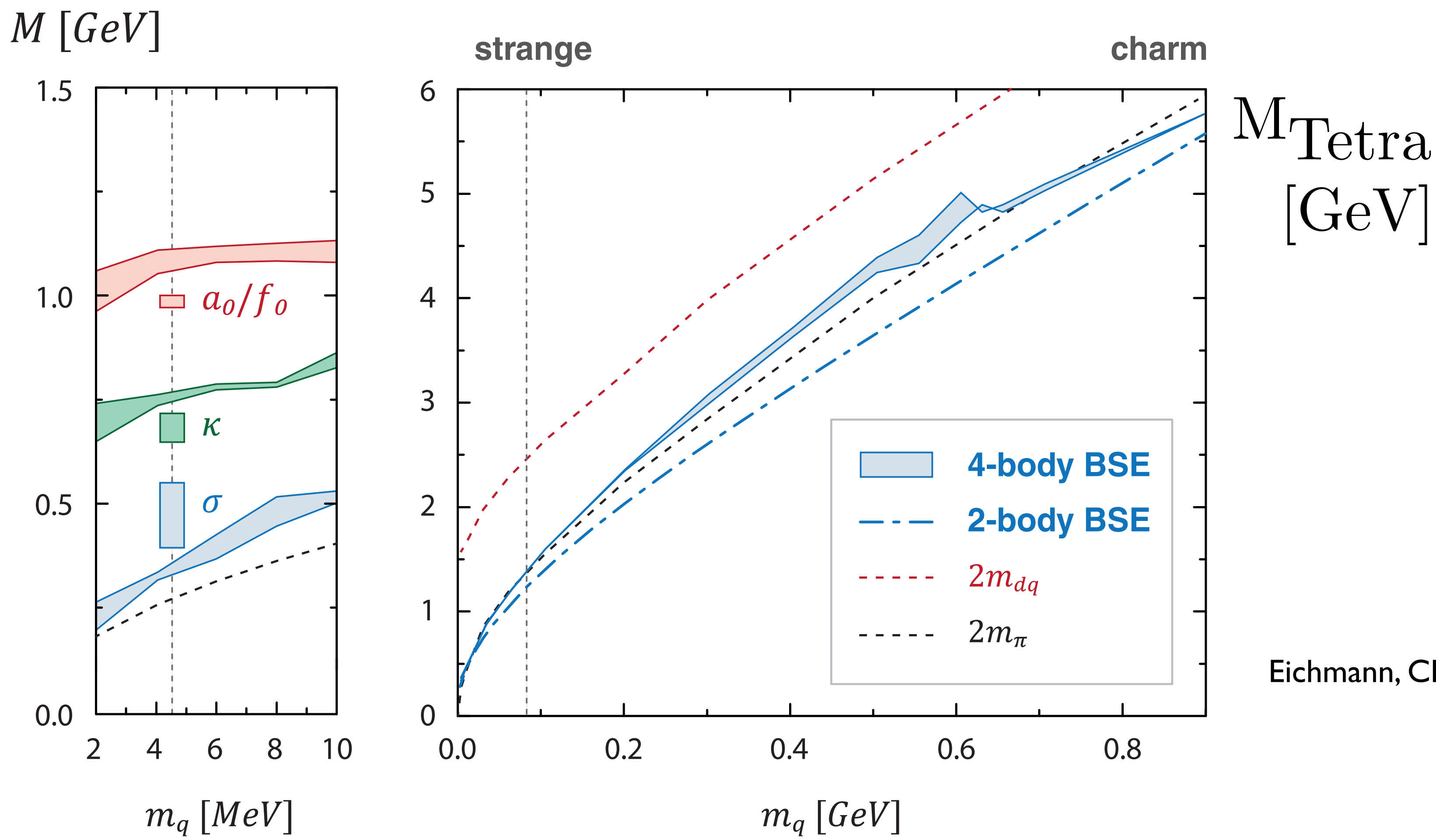
Singlet + Doublet



Two-pion resonance

Bound state of
four massive quarks

Mass evolution of tetraquark



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\text{MeV}$$

$$m_\kappa \sim 750\,\text{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\text{GeV}$$

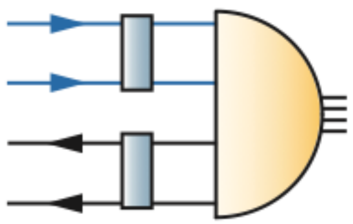
$$m_{a_0,f_0} \sim 1080\,\text{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\text{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

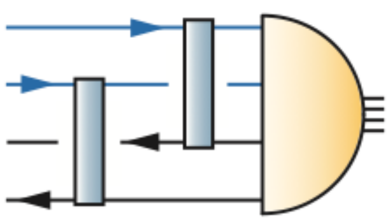
Outlook: heavy-light systems

Dynamical situation in **S4**-doublet:



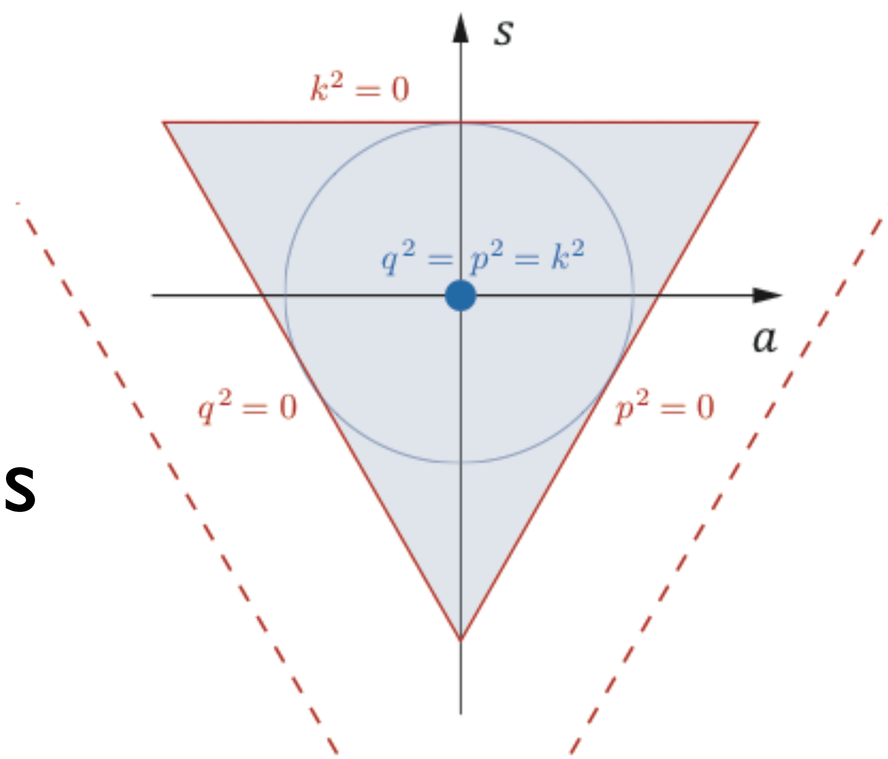
heavy-light diquarks

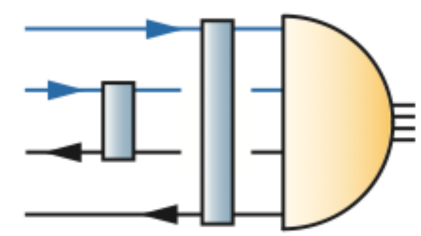
$$cq - \bar{c}\bar{q}$$



meson-molecules

$$c\bar{q} - \bar{c}q$$





hadro-charmonium

$$c\bar{c} - q\bar{q}$$

Dynamical decision of most important clustering!

Summary and outlook

One approach to find them all...

Summary

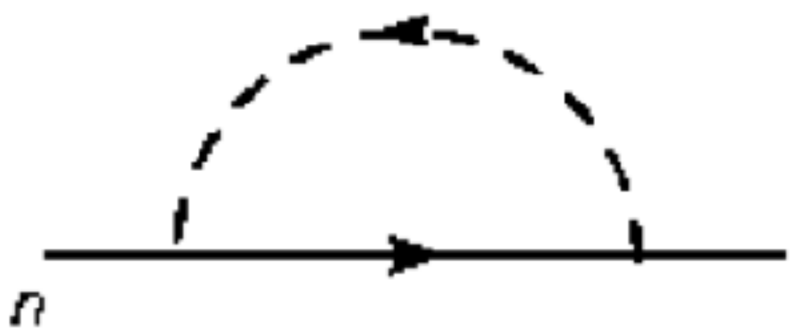
- Glueballs: $M(0^{++}) = 1.64\ \mathrm{GeV}$
- Hybrids in $q\bar{q}$-BRL
- Four-quarks states dominated by meson-meson configurations
- Dynamical description of σ as π-π resonance

Outlook

- Glueballs: refine, explore other quantum numbers, unquench
- Hybrids: $q\bar{q}g \longrightarrow$ work in progress
- Tetraquarks: explore heavy-light systems

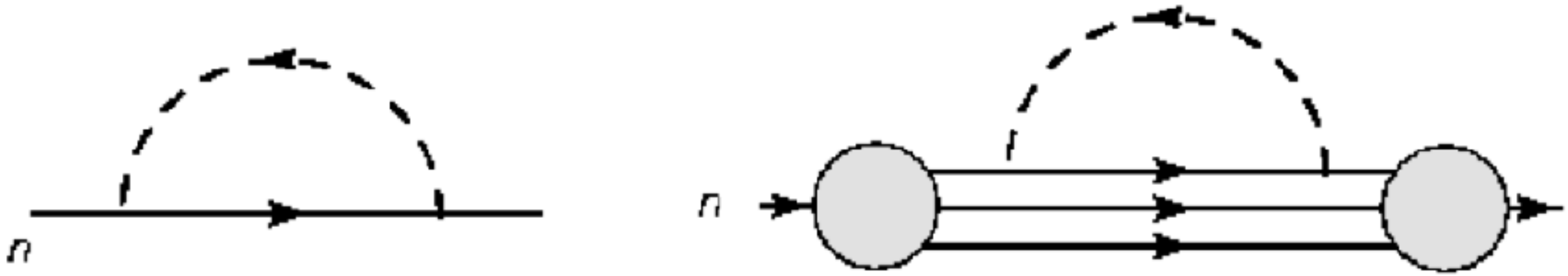
Backup

Pion cloud

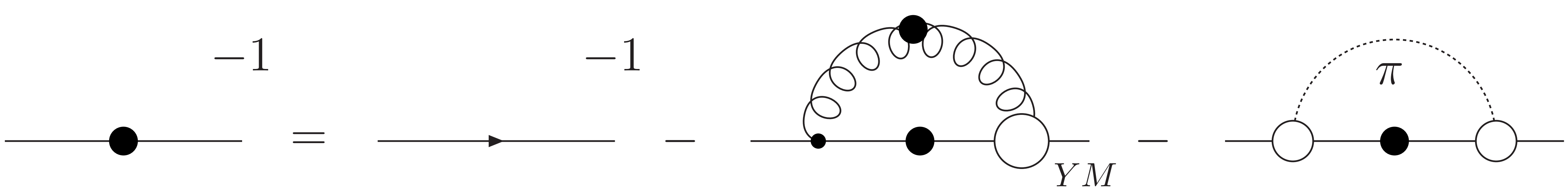



- Hadron level: πN-contributions to nucleon self-energy
- Quark-level: π-contributions to quark self-energy

Pion cloud

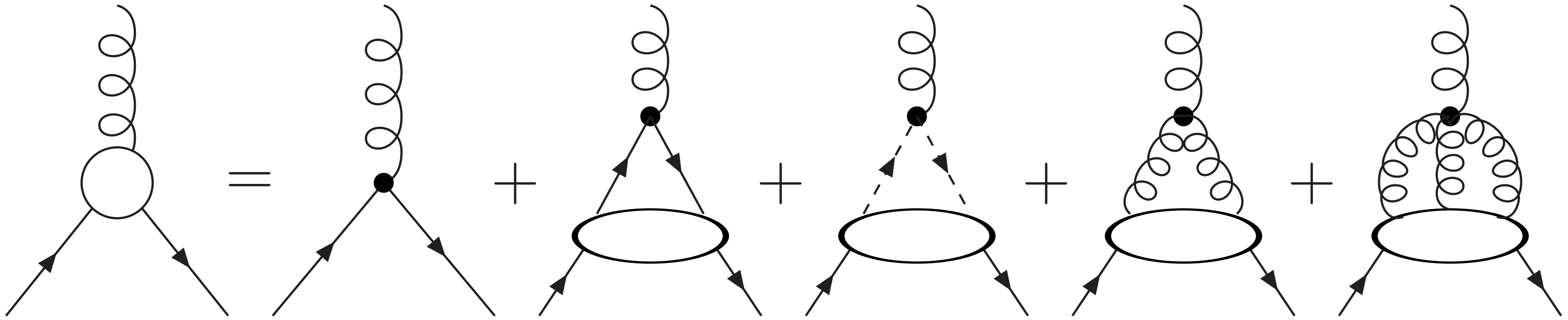


- Hadron level: πN-contributions to nucleon self-energy
- Quark-level: π-contributions to quark self-energy

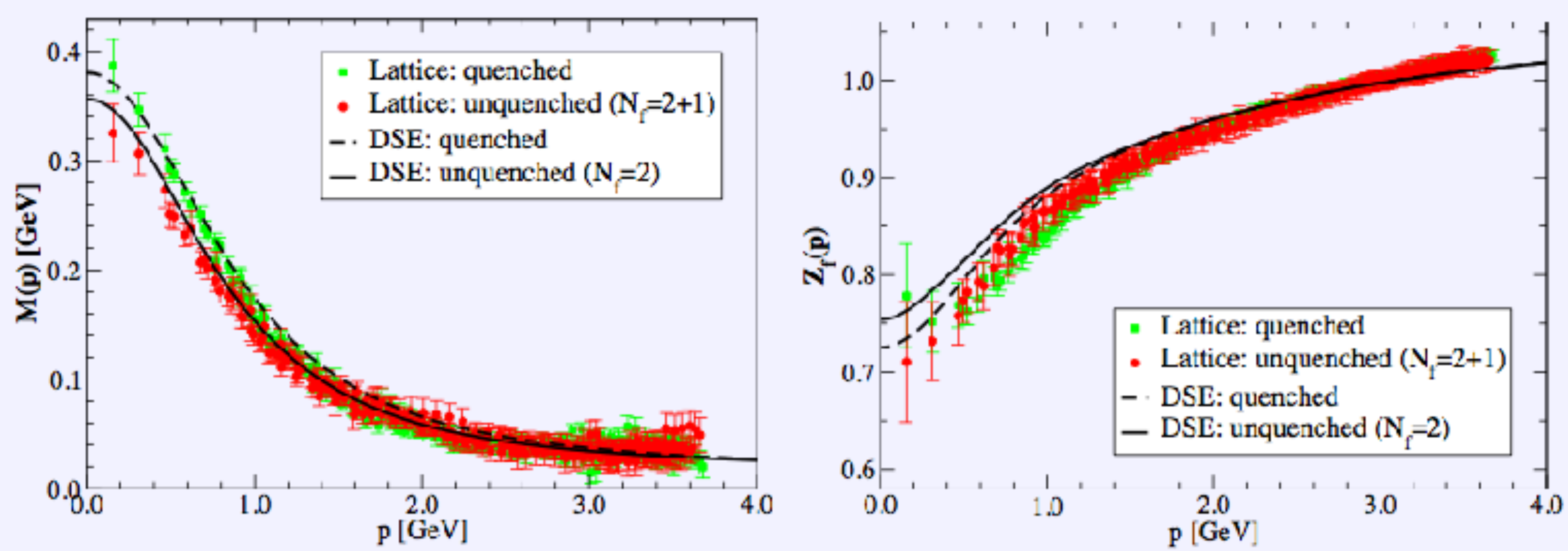


Pion effects in quark-gluon interaction

quark-gluon vertex:



quark:



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

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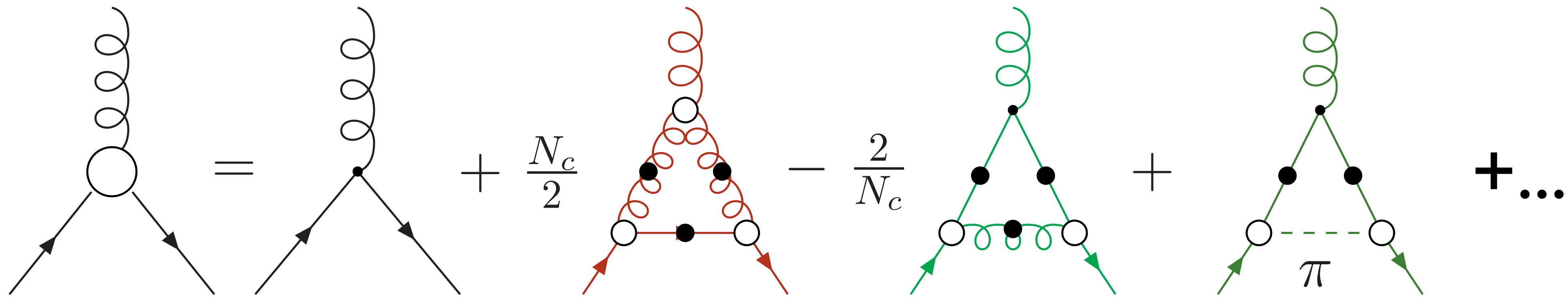
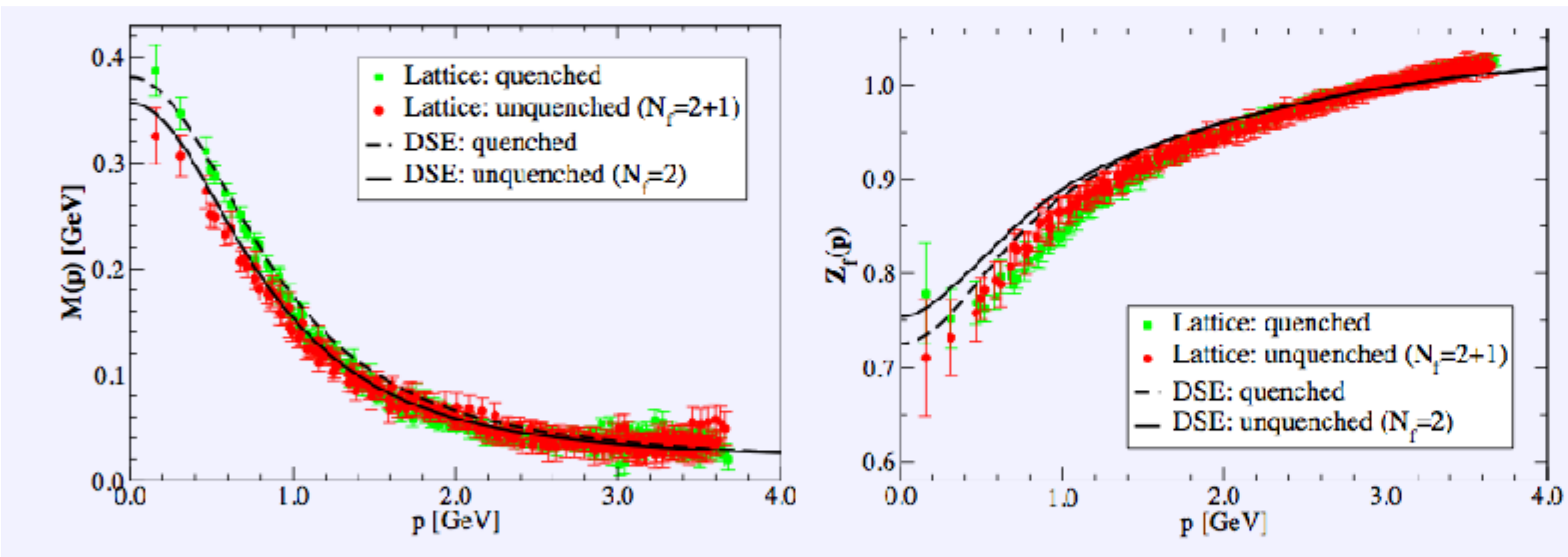


quark:



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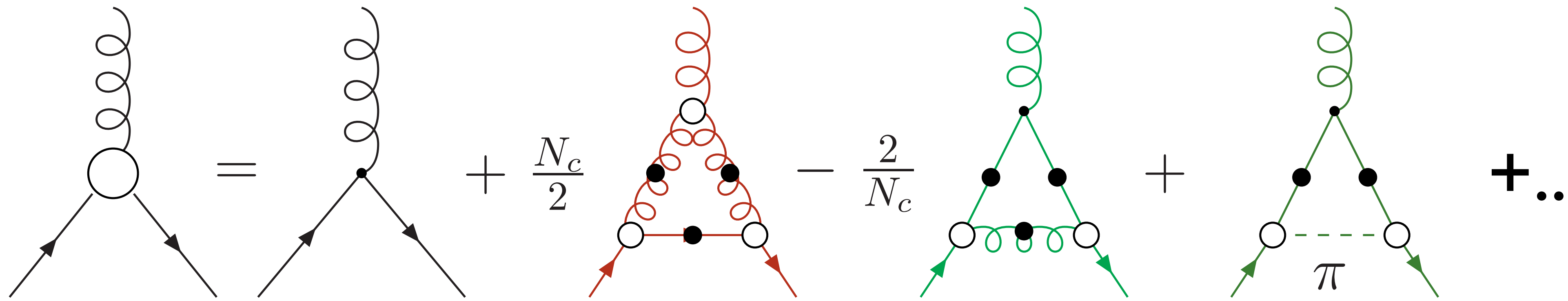
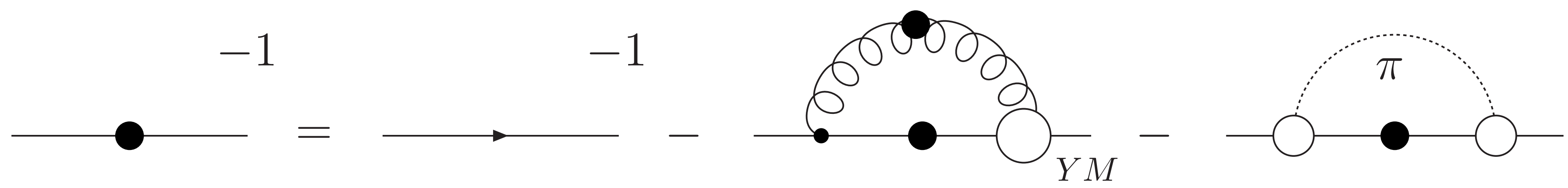
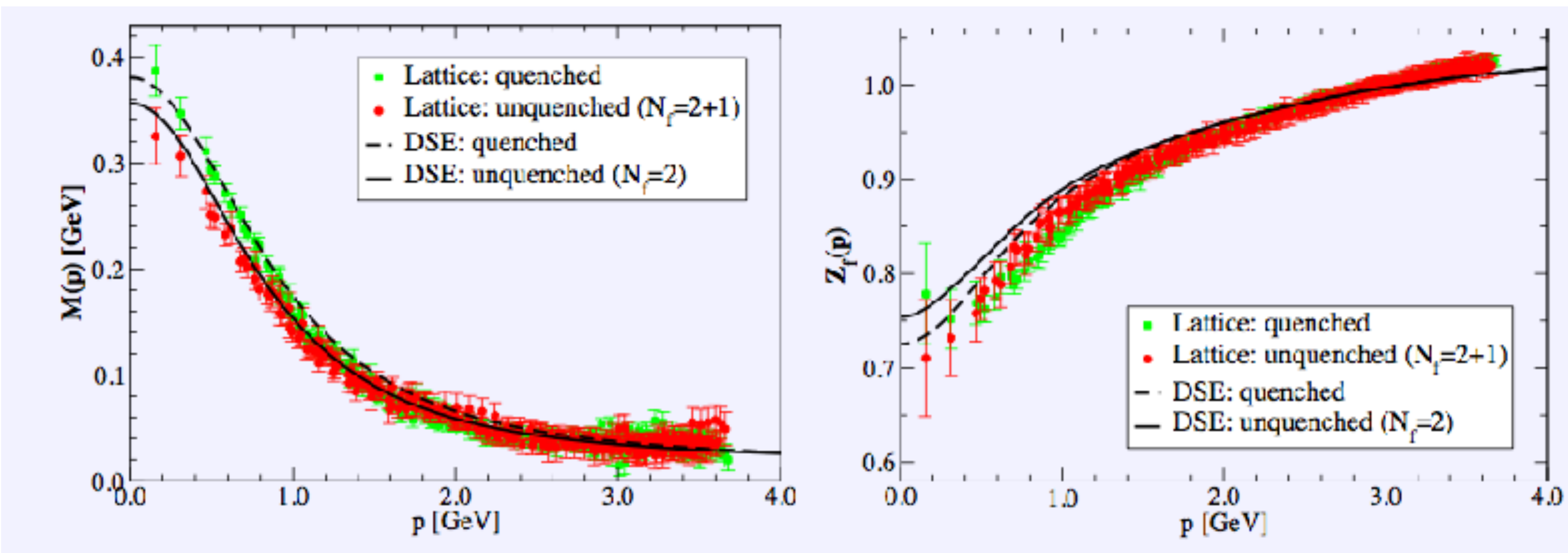




quark:



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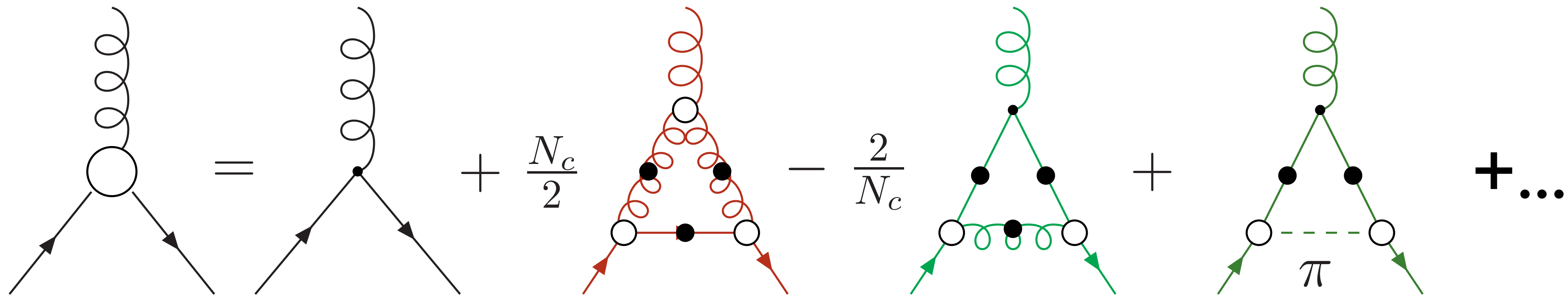

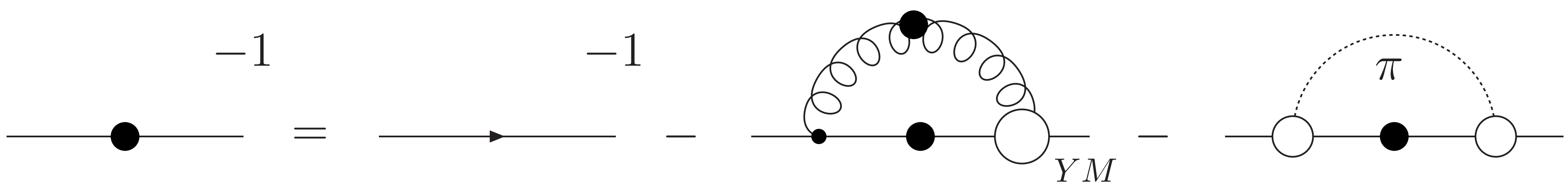

Bethe-Salpeter equation: