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PARIS-SACLA

# Covariant extension of the GPD overlap representation



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QCD Workshop Hervé MOUTARDE

May 4<sup>th</sup>, 2018



# Context and key questions.



### Covariant extension

#### Phenomenology

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1 What do we need for high precision phenomenology?

2 How can we implement all theoretical constraints in flexible GPD parameterizations?

3 How do we relate all this to actual measurements?

# What do we need for high precision phenomenology?

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### Anatomy of hadrons. GPDs, 3D hadron imaging, and beyond (1/3).



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Probabilistic interpretation of Fourier transform of  $GPD(x, \xi = 0, t)$  in transverse plane.

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 $\rho(\mathbf{x}, \mathbf{b}_{\perp}, \lambda, \lambda_{N}) = \frac{1}{2} \left| \mathbf{H}(\mathbf{x}, 0, \mathbf{b}_{\perp}^{2}) + \frac{\mathbf{b}_{\perp}^{\prime} \epsilon_{ji} \mathbf{S}_{\perp}^{i}}{M} \frac{\partial \mathbf{E}}{\partial \mathbf{b}^{2}}(\mathbf{x}, 0, \mathbf{b}_{\perp}^{2}) \right|$  $+\lambda\lambda_N\tilde{H}(x,0,b_\perp^2)$  .

Notations : quark helicity  $\lambda$ , nucleon longitudinal polarization  $\lambda_N$  and nucleon transverse spin  $S_{\perp}$ .

Burkardt, Phys. Rev. **D62**, 071503 (2000)

## Can we obtain this picture from exclusive measurements?



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Proc. 1149.

150 (2009)



### Anatomy of hadrons. GPDs, 3D hadron imaging, and beyond (2/3).



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Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\left\langle N, P + \frac{\Delta}{2} \right| T^{\mu\nu} \left| N, P - \frac{\Delta}{2} \right\rangle = \bar{u} \left( P + \frac{\Delta}{2} \right) \left[ \mathbf{A}(t) \gamma^{(\mu} P^{\nu)} + \mathbf{B}(t) P^{(\mu} i \sigma^{\nu)\lambda} \frac{\Delta_{\lambda}}{2M} + \frac{\mathbf{C}(t)}{M} (\Delta^{\mu} \Delta^{\nu} - \Delta^{2} \eta^{\mu\nu}) \right] u \left( P - \frac{\Delta}{2} \right)$$

with 
$$t = \Delta^2$$
.

Key observation: link between GPDs and gravitational form factors

 $\int \mathrm{d}x \, \mathbf{x} \mathbf{H}^q(\mathbf{x}, \xi, t) = \mathbf{A}^q(t) + 4\xi^2 \mathbf{C}^q(t) ,$  $\int \mathrm{d}x \, \mathbf{x} \mathbf{E}^q(\mathbf{x}, \xi, t) = \mathbf{B}^q(t) - 4\xi^2 \mathbf{C}^q(t) .$ 

Ji, Phys. Rev. Lett. **78**, 610 (1997)



### Anatomy of hadrons. GPDs, 3D hadron imaging, and beyond (3/3).



### Spin sum rule:

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Ji, Phys. Rev. Lett. 78, 610 (1997)

**Shear** and **pressure** distributions:

$$\left\langle N \left| T^{ij}(\vec{r}) \right| N \right\rangle = s(r) \left( \frac{r^{i} r^{j}}{\vec{r}^{2}} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij} .$$

Polyakov and Shuvaev, hep-ph/0207153 Polyakov, Phys. Lett. **B555**, 57 (2003)

 Energy, radial pressure and transverse pressure distributions (u<sup>μ</sup> the 4-velocity at spacetime location χ<sup>ν</sup>):

$$\langle N | T^{\mu\nu} | N \rangle = (\epsilon + p_t) u^{\mu} u^{\nu} - p_t \eta^{\mu\nu} + (p_r - p_t) \chi^{\mu} \chi^{\nu} .$$

Trawinski *et al.*, *in preparation* H. Moutarde | QCD Workshop 2018 - Camburi | 6 / 46





### Exclusive processes of current interest (1/2). Factorization and universality.



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### Exclusive processes of present interest (2/2). Factorization and universality.



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### Bjorken regime : large $Q^2$ and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale  $\mu_F$ .
- **Consistency** requires the study of **different channels**.
- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^1 dx \, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F) ,$$

for a given GPD F.

- CFF  $\mathcal{F}$  is a **complex function**.
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### Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!



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#### 1. Experimental data fits 2. GPD extraction Covariant extension $H^{+}(x, t; \Xi=0.2, O^{2}=4)$ $\Delta \sigma \, [\text{pb.GeV}^{-4}]$ 15. Phenomenology 0.1 Content of GPDs Experimental access = 0.5-10 Tomography $\langle x_R \rangle$ $= 6.3 \text{ GeV}^2$ Dispersion relations -1.08,05,0,4,02 0,02,0,4,08,08,1 0 $0.735 \text{ GeV}^2$ Modeling 0.2 $\phi$ [deg] Definition Polynomiality 3. Nucleon imaging Radon transform Positivity Inverse Radon Images from Guidal et al., Examples Rept. Prog. Phys. 76 (2013) 066202 The 2015 Long Range Plan for Nuclear Science PARTONS Design Sidebar 2.2: The First 3D Pictures of the Nucleon Fits Releases 2 A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken 1 Conclusion bones, and spot the early signs of osteoporosis. 0,[fm] Now physicists are using the principles behind the 0 procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new -1 concept in nuclear physics called generalized parton distributions. -2 -1 0 1 Ó -2 -1 b, [fm] b, [fm] An intense beam of high-energy electrons can be used

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### Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!



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**1** Extract  $H(x, \xi, t, \mu_F^{ref})$  from experimental data.

- **Extrapolate** to vanishing skewness  $H(x, 0, t, \mu_F^{ref})$ .
- **3** Extrapolate  $H(x, 0, t, \mu_F^{\text{ref}})$  up to infinite *t* and down to vanishing *t*.
  - **Compute** 2D Fourier transform in transverse plane:

$$\mathcal{H}(x,b_{\perp}) = \int_0^{+\infty} \frac{\mathrm{d}|\Delta_{\perp}|}{2\pi} \left|\Delta_{\perp}\right| J_0(|b_{\perp}||\Delta_{\perp}|) \mathcal{H}(x,0,-\Delta_{\perp}^2) \ .$$

- 5 Propagate uncertainties.
- 6 **Control** extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

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### GPD H at t = -0.23 GeV<sup>2</sup> and $Q^2 = 2.3$ GeV<sup>2</sup>.







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### Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.







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0-0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1

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### GPD model: see Kroll et al., Eur. Phys. J. C73, 2278 (2013)

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1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8



Fits

### From principles to actual data. Direct experimental access to a restricted kinematic domain.



#### $\xi_{\min}$ from finite beam energy. Covariant extension Phenomenology Content of GPDs Experimental access Tomography 4.5 Dispersion relations 3.5 Modeling 3 Definition 25 Polynomiality Radon transform 2 Positivity 1.5 Inverse Radon 1 Examples 0.5 PARTONS 0-0.9<sub>0.8</sub>0.7<sub>0.6</sub>0.5<sub>0.4</sub>0.3<sub>0.2</sub>0.1 Design ξ -0.8 -0.4 -0.2 0 0.2 0.4 0.6 0.8 Releases Conclusion х GPD model: see Kroll et al., Eur. Phys. J. C73, 2278 (2013)





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### $\xi_{\rm max}$ from kinematic constraint on 4-momentum transfer.







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### The black curve is what is needed for transverse plane imaging!







### Covariant extension

### Density plot of H at t = -0.23 GeV<sup>2</sup> and $Q^2 = 2.3$ GeV<sup>2</sup>

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Need for GPD modeling and flexible parameterizations.



### Dispersion relations and the lines $x = \pm \xi$ . Relation between $\operatorname{Re}\mathcal{H}(\xi)$ and $H(x = \pm \xi, \xi)$ at leading order.



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• Write dispersion relation **at fixed** *t* **and**  $Q^2$ :  $\operatorname{Re}\mathcal{H}(\xi) = \int_{1}^{\infty} \frac{\mathrm{d}\omega}{\pi} \operatorname{Im} \mathcal{C}(\omega) \left\{ \int_{-1}^{+1} \mathrm{d}x \left[ \frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] \mathcal{H}\left(x, \frac{x}{\omega}\right) + \mathcal{I}(\omega) \right\}.$ 

Diehl and Ivanov, Eur. Phys. J. C52, 919 (2007)
 At leading order in α<sub>s</sub> (no kinematic corrections):

$$\mathrm{m}\mathcal{C}(\omega) \propto \pi \Big[\delta(\omega-1) - \delta(\omega+1)\Big].$$

Dispersion relation simplifies to:

T

$$\begin{split} &\operatorname{Re}\mathcal{H}(\xi) \quad \propto \quad \int_{-1}^{+1} \mathrm{d}x \, \left[ \frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] H(x, x) + \mathcal{I} \ , \\ &\operatorname{Im}\mathcal{H}(\xi) \quad \propto \quad H(\xi, \xi) - H(-\xi, \xi) \ . \end{split}$$

■ In principle tomography not possible at leading, order ... , o ... H. Moutarde | QCD Workshop 2018 - Camburí | 11 / 46



### DVCS analysis and fits. No global GPD fit has been obtained so far.



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- GPD fits **only in the small** *x*<sub>*B*</sub> **region** with a **flexible** parameterization (kinematic simplifications).
- Global fits of CFFs in the sea and valence regions.
- Some GPD models with non-flexible parameterizations adjusted to experimental DVCS or DVMP data.

Kumerički et al., Eur. Phys. J. A52, 157 (2016)

 Unclear model-dependence on tomographic images obtained from CFF fits relying on leading order and leading twist analysis.

### The situation can be improved!

- GPD parameterizations satisfying a priori all theoretical constraints on GPDs.
- Computing framework to go beyond leading order and leading twist analysis.

How can we implement all theoretical constraints in flexible GPD parameterizations?

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 $H^q_{\pi}(x,\xi,t) =$  $\frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{i\mathbf{x}P^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+}q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{z^{+}=0}$ with  $t = \Delta^2$  and  $\xi = -\Delta^+/(2P^+)$ . References Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)  $z^3$ Ji, Phys. Rev. Lett. 78, 610 (1997) Radyushkin, Phys. Lett. B380, 417 (1996) PDF forward limit

Conclusion

$$H^q(x,0,0) = q(x)$$

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- PDF forward limit
- Form factor sum rule

$$\int_{-1}^{r+1} dx H^q(x,\xi,t) = F_1^q(t)$$

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Covariant extension  $H^q_{\pi}($ 

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- PDF forward limit
  - Form factor sum rule
  - $H^q$  is an even function of  $\xi$  from time-reversal invariance.

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$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+}q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}}$$
with  $t = \Delta^{2}$  and  $\xi = -\Delta^{+}/(2P^{+})$ .
  
References
  
Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)  
Ji, Phys. Rev. Lett. **78**, 610 (1997)  
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
  - Form factor sum rule
  - $H^q$  is an even function of  $\xi$  from time-reversal invariance.
- H<sup>q</sup> is real from hermiticity and time-reversal invariance.





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# $\int_{-1}^{+1} dx x^n H^q(x,\xi,t) = \text{polynomial in } \xi$

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### Lorentz covariance

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Lorentz covariance

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### Positivity

$$H^{q}(x,\xi,t) \leq \sqrt{q\left(rac{x+\xi}{1+\xi}
ight)q\left(rac{x-\xi}{1-\xi}
ight)}$$





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### Positivity of Hilbert space norm

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•  $H^q$  has support  $x \in [-1, +1]$ .

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### Spin-0 Generalized Parton Distribution. Not so simple properties.



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### • $H^q$ has support $x \in [-1, +1]$ .

### Relativistic quantum mechanics

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Lorentz covariance

• 
$$H^q$$
 has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

**Soft pion theorem** (pion target)

$$H^{q}(x,\xi=1,t=0) = \frac{1}{2}\phi_{\pi}^{q}\left(\frac{1+x}{2}\right)$$

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•  $H^q$  has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

**Soft pion theorem** (pion target)

Dynamical chiral symmetry breaking

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### Polynomiality. Mixed constraint from Lorentz invariance and discrete symmetries.

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• Express Mellin moments of GPDs as **matrix elements**:

$$\int_{-1}^{+1} \mathrm{d}x \, x^m H^q(x,\xi,t)$$
  
=  $\frac{1}{2(P^+)^{m+1}} \left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| P - \frac{\Delta}{2} \right\rangle$ 

- Identify the Lorentz structure of the matrix element: linear combination of (P<sup>+</sup>)<sup>m+1-k</sup>(∆<sup>+</sup>)<sup>k</sup> for 0 ≤ k ≤ m+1
- Remember definition of **skewness**  $\Delta^+ = -2\xi P^+$ .
- Select even powers to implement time reversal.
- Obtain polynomiality condition:

$$\int_{-1}^{1} \mathrm{d}x \, x^{m} H^{q}(x,\xi,t) = \sum_{i=0 \text{ even}}^{m} (2\xi)^{i} C^{q}_{mi}(t) + (2\xi)^{m+1} C^{q}_{mm+1}(t) \; .$$

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# Polynomiality.

Abstract formulation: the range of the Radon transform.



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• Assume the existence of  $D^q(z, t)$  such that:

$$\int_{-1}^{+1} \mathrm{d}z \, z^m D(z,t) = C^q_{mm+1}(t) \; .$$

 $= H^q(x,\xi,t) - D(x/\xi,t) \text{ satisfies polynomiality at order } m: \\ \int_{-1}^1 \mathrm{d}x \, x^m \Big( H^q(x,\xi,t) - D(x/\xi,t) \Big) = \sum_{i=1}^m (2\xi)^i C^q_{mi}(t) \; .$ 

• Thus, there exists a function  $F_D$  such that:

$$H(x,\xi,t) = D(x/\xi,t) + \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \, F_D(\beta,\alpha,t) \, \delta(x-\beta-\alpha\xi) \; .$$

even

• The support  $\Omega_{DD} = \{ |\alpha| + |\beta| \le 1 \}$  is related to the GPD physical domain  $|x|, |\xi| \le 1$ . H. Moutarde | QCD Workshop 2018 - Camburi | 17 / 46



### Double Distributions. Relation to Generalized Parton Distributions.



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Most general representation of GPD:

 $H^{q}(x,\xi,t) = \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \,\delta(x-\beta-\alpha\xi) \big(F^{q}(\beta,\alpha,t) + \xi \,G^{q}(\beta,\alpha,t)\big)$ 

- Support property:  $x \in [-1, +1]$ .
- Discrete symmetries:  $F^q$  is  $\alpha$ -even and  $G^q$  is  $\alpha$ -odd.
  - **Gauge**: any representation  $(F^q, G^q)$  can be recast in one representation with a single DD  $f^q$ :

$$H^{q}(x,\xi,t) = x \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \, f^{q}_{\rm BMKS}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$$

Belitsky et al., Phys. Rev. **D64**, 116002 (2001)  $H^{q}(x,\xi,t) = (1-x) \int_{\Omega_{\rm DD}} d\beta d\alpha f_{\rm P}^{q}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$ 

Pobylitsa, Phys. Rev. **D67**, 034009 (2003)

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### Double Distributions. Lorentz covariance by example.



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• Choose 
$$F^q(\beta, \alpha) = 3\beta\theta(\beta)$$
 ad  $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$ :

$$H^{q}(x,\xi) = 3x \int_{\Omega} d\beta d\alpha \,\delta(x - \beta - \alpha\xi)$$

Simple analytic expressions for the GPD:

$$\begin{aligned} &\mathcal{H}(x,\xi) &= \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1, \\ &\mathcal{H}(x,\xi) &= \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1. \end{aligned}$$

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### Double Distributions. Lorentz covariance by example.



Covariant	Compute first Mellin moments.			
extension	п	$\int_{-\xi}^{+\xi} \mathrm{d}x  x^n H(x,\xi)$	$\int_{+\xi}^{+1} \mathrm{d}x x^n H(x,\xi)$	$\int_{-\xi}^{+1} \mathrm{d}x  x^n H(x,\xi)$
Phenomenology Content of GPDs Experimental access Tomography Dispersion relations	0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
Modeling Definition Polynomiality	1	$\frac{1 + \xi + \xi^2 - 3\xi^3}{2(1 + \xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
Radon transform Positivity Inverse Radon Examples	2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
PARTONS Design Fits Releases	3	$\frac{1\!+\!\xi\!\!+\!\xi^2\!+\!\xi^3\!+\!\xi^4\!-\!5\xi^5}{5(1\!+\!\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$
Conclusion	4	$\tfrac{1\!+\!\xi\!\!+\!\xi^2\!+\!\xi^3\!+\!\xi^4\!+\!\xi^5\!-\!6\xi^6}{7(1\!+\!\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$
		<ul> <li>Expressions get mor they always yield po</li> </ul>	e complicated as <i>n</i> olynomials!	increases But
			H. Moutarde   QCD Works	shop 2018 - Camburí   19 / 46



# The Radon transform.

Definition and properties.





#### Conclusion

Relation between GPD and DD in Belistky et al. gauge

$$\frac{\sqrt{1+\xi^2}}{x}H(x,\xi) = \mathcal{R}f_{\rm BMKS}\left(s,\phi\right) \;,$$



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# The Radon transform.

### Definition and properties.





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Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x}H(x,\xi) = \mathcal{R}f_{\mathrm{P}}(s,\phi) \ ,$$

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### The range of the Radon transform. The polynomiality property a.k.a. the Ludwig-Helgason condition.



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 The Mellin moments of a Radon transform are homogeneous polynomials in ω = (sin φ, cos φ).

The converse is also true:

### Theorem (Hertle, 1983)

Let  $g(s, \omega)$  an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the **Ludwig-Helgason consistency** condition hold:

(i) g is 
$$C^{\infty}$$
 in  $\omega$ ,

(ii)  $\int ds \, s^m g(s, \omega)$  is a homogeneous polynomial of degree m for all integer  $m \ge 0$ .

Double Distributions and the Radon transform are the natural solution of the polynomiality condition.

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### Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.

# CEA - Saclay

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OGLAP and ERBL regions
$\begin{array}{ll} (x,\xi) \in \mbox{ DGLAP } \Leftrightarrow &  s  \ge  \sin \phi  \ , \\ (x,\xi) \in \mbox{ ERBL } \Leftrightarrow &  s  \le  \sin \phi  \ . \end{array}$
$\beta = (x - \xi)/(1 + \xi)$ $\alpha = \frac{1}{\xi}(x - \beta)$ $\beta = (x - \xi)/(1 - \xi)$ Each point $(\beta, \alpha)$ $\beta$ with $\beta \neq 0$ contributes $\beta = (x + \xi)/(1 + \xi)$ to <b>both</b> DGLAP and ERBL regions. $\Omega_{\text{DD}}( \alpha  +  \beta  = 1)$
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# Positivity.

A consequence of the positivity of the nom in a Hilbert space.



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- Identify the matrix element defining a GPD as an inner product of two different states.
- Apply Cauchy-Schwartz inequality, and identify PDFs at specific kinematic points, *e.g.*:

$$|H^{q}(x,\xi,t)| \leq \sqrt{\frac{1}{1-\xi^{2}}q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}$$

 This procedures yields infinitely many inequalities stable under LO evolution.

Pobylitsa, Phys. Rev. D66, 094002 (2002)

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The overlap representation guarantees a priori the fulfillment of positivity constraints.

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### Overlap representation. A first-principle connection with Light Front Wave Functions.



Covariant extension

• Decompose an hadronic state  $|H; P, \lambda\rangle$  in a Fock basis:

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• Derive an expression for the pion GPD in the DGLAP region  $\xi \le x \le 1$ :

$$f(\mathbf{x},\xi,t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{\mathbf{x}}\mathrm{d}\bar{\mathbf{k}}_{\perp}]_N \delta_{j,q} \delta(\mathbf{x}-\bar{\mathbf{x}}_j) \left(\psi_N^{(\beta,\lambda)}\right)^* (\hat{\mathbf{x}}',\hat{\mathbf{k}}'_{\perp}) \psi_N^{(\beta,\lambda)}(\tilde{\mathbf{x}},\tilde{\mathbf{k}}_{\perp})$$

with  $\tilde{x}, \tilde{\mathbf{k}}_{\perp}$  (resp.  $\hat{x}', \hat{\mathbf{k}}'_{\perp}$ ) generically denoting incoming (resp. outgoing) parton kinematics.

### Diehl et al., Nucl. Phys. B596, 33 (2001)

Similar expression in the ERBL region  $-\xi \le x \le \xi$ , but with overlap of *N*- and (N+2)-body LFWFs.

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### Overlap representation. Advantages and drawbacks.



### Covariant extension

- Physical picture.
- Positivity relations are fulfilled **by construction**.
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Implementation of symmetries of *N*-body problems.

### What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at**  $x = \pm \xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

### Diehl, Phys. Rept. 388, 41 (2003)



### Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.



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For **any model of LFWF**, one has to address the following three questions:

- 1 Does the extension exist?
- 2 If it exists, is it unique?
- 3 How can we compute this extension?

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### Implementing Lorentz covariance. Uniqueness of the extension.



#### Covariant extension

Consider a GPD H vanishing on the DGLAP region and write it as a Radon transform:

#### Phenomenology

Content of GPDs Experimental access Tomography Dispersion relations

#### Modeling

Definition

Polynomiality

Radon transform

Positivity

#### Inverse Radon

Examples

#### PARTONS

Design Fits

Releases

#### Conclusion

$$H(x,\xi) = \int_{\Omega_{\rm DD}} \mathrm{d}\beta \mathrm{d}\alpha \left[ F_D(\beta,\alpha) + \delta(\beta) D(\alpha) \right] \delta(x-\beta-\alpha\xi) \; .$$

•  $F_D(\beta, \alpha) = 0$  for all  $\alpha$  and  $\beta > 0$ .

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

- Up to D-term-like contributions, the DGLAP region completely characterizes a GPD.
- Modeling strategy:
  - Ensure positivity by modeling the DGLAP region as an overlap of LFWFs.
  - 2 Ensure polynomiality by inverting the Radon transform to identify an underlying DD.

### Chouika et al., Eur. Phys. J. C77, 906 (2017)

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Examples

Design

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### III-posedness in the sense of Hadamard. A first glimpse at the inverse Radon transform.



Covariant extension	<ul> <li>Numerical evaluation <i>almost unavoidable</i> (polar vs cartesian coordinates).</li> </ul>
Phenomenology	<ul> <li>Ill-posedness by lack of continuity.</li> </ul>
Content of GPDs Experimental access Tomography Dispersion relations	The unlimited Radon inverse problem is mildly ill-posed while the limited one is severely ill-posed.
Modeling Definition Polynomiality Radon transform Positivity	Even if it existed, an analytic expression of the invert Radon transform would be of <b>limited practical use</b> .
Inverse Radon	





# Computation of the extension.



### How can we get a DD from a GPD in the DGLAP region?

#### Phenomenology

Covariant extension

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#### Modeling

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### Computation of the extension. Problem reduction.



#### Covariant extension

### How can we get a DD from a GPD in the DGLAP region? Restrict to quark GPDs ( $\beta > 0$ ).

Only ERBL region "sees" both  $\beta > 0$  and  $\beta < 0$ . 

#### Phenomenology

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### Computation of the extension. Problem reduction.







### Computation of the extension. Finite elements.

Covariant

extension

Phenomenology Content of GPDs Experimental access Tomography Dispersion relations Modeling





- Definition Polynomiality Radon transform Positivity
- Inverse Radon

Examples

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#### Conclusion

- Discretize the DD on a mesh with  $n \simeq 800$  triangular cells.
- Compute the Radon transform of a P1 basis function.
- Sample  $m \simeq 4n (x, \xi)$ -lines intersecting the DD support.
- Solve a linear system AX = B with A a sparse m × n matrix.
- Adopt an iterative regularization method: LSMR.

Fong and Saunders, arXiv:1006.0758

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### Examples - benchmarks (1/4). Algebraic Bethe-Salpeter model.



Covariant extension

#### Phenomenology

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#### Modeling



Fits Releases

#### Conclusion

-1 -0.75 -0.5 -0.25

0







# Examples - benchmarks (2/4). Algebraic spectator model.



### Covariant extension

$$\varphi(\mathbf{x}, \mathbf{k}_{\perp}) = \frac{g \mathcal{M}^{2p}}{\sqrt{1-x}} \mathbf{x}^{-p} \left( \mathcal{M}^2 - \frac{\mathbf{k}_{\perp}^2 + m^2}{x} - \frac{\mathbf{k}_{\perp}^2 + \lambda^2}{1-x} \right)^{-p-1}$$

#### Phenomenology

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#### Conclusion

Hwang and Müller, Phys. Lett. **B660**, 350 (2008)





### Examples - benchmarks (3/4). Regge-behaved Radyushkin DD Ansatz model.



### Covariant extension

### Radyushkin DD Ansatz with phenomenological PDF:

$$q_{\text{Regge}}(x) = rac{35 \ (1-x)^3}{32 \ \sqrt{x}} \ .$$

#### Phenomenology Content of GPDs

Content of GPDs Experimental access Tomography Dispersion relations

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### Chouika et al., Eur. Phys. J. C77, 906 (2017)

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### Examples - benchmarks (4/4). Gaussian wave function model.



#### Covariant extension

#### Phenomenology

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### Chouika et al., Eur. Phys. J. C77, 906 (2017)

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### Take home message.

What has been done, what remains to be done.



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# 1 Modeling: GPDs (not CFFs) have to be extracted from measurements to learn about hadron structure.

- 2 Generic procedure to build models satisfying all theoretical constraints.
- Remark: soft pion theorem can be fulfilled too!
   Chouika et al., Phys. Lett. B780, 287 (2018)
  - Extension to spin-1/2 hadron in progress.
- Integration in computing chain from GPDs to observables (PARTONS framework) in progress.
- 6 Still have to figure out how to input phenomenological parameterizations of PDFs and form factors for global GPD fits.

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### How do we relate all this to actual measurements?



PARtonic Tomography Of Nucleon Software

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Differential studies: physical models and numerical methods.



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Experimental data and phenomenology

Computation of amplitudes

principles and

fundamental parameters

First

Small distance contributions

Full processes

Large distance contributions

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### Computing chain design. Differential studies: physical models and numerical methods.



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Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters Small distance

Full processes

Large distance contributions

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Differential studies: physical models and numerical methods.



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Computation of amplitudes

Experimental

data and

First principles and fundamental parameters



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Differential studies: physical models and numerical methods.





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Differential studies: physical models and numerical methods.



### Covariant extension

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- Experimental data and phenomenology Need for modularity Computation
- of amplitudes

First principles and fundamental parameters



### Many observables.

Kinematic reach.

# Perturbative approximations.

- Physical models.
- Fits.

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- Numerical methods.
- Accuracy and speed.

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Differential studies: physical models and numerical methods.



### Covariant extension

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- Experimental data and phenomenology Need for modularity
- Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.
- Perturbative approximations.
- Physical models.
- Fits.

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- Numerical methods.
- Accuracy and speed.

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## Computing chain design.

Differential studies: physical models and numerical methods.



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### Covariant extension

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Computation of amplitudes





### Many observables.

- Kinematic reach.
- Perturbative approximations.
  - Physical models.

Fits.

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- Numerical methods.
- Accuracy and speed.

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# Computing chain design.

Differential studies: physical models and numerical methods.



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### Covariant extension

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First principles and fundamental parameters



- Many observables.
- Kinematic reach.
- Perturbative approximations.
- Physical models.

Fits.

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- Numerical methods.
- Accuracy and speed.

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## Computing chain design.

Differential studies: physical models and numerical methods.



### Covariant extension

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Computation of amplitudes





# Many observables. Kinematic reach.

- Perturbative approximations.
  - Physical models.

Fits.

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- Numerical methods.
- Accuracy and speed.

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# GPD or CFF fits (1/3). Local fit of CFFs.



### Covariant extension

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#### Conclusion

### First local fit of pseudo DVCS data, Sep. $26^{\mathrm{th}}$ , 2016

	Mattermost
PARTONS :	partons_fits → 7 🎍 Search @
@ partons_fits	Mon, Sep 26, 2016
	pawel 3:16 PM
	FCN=1.00128e-11 FROM MIGRAD STATUS=CONVERGED 44 CALLS 45 TOTAL
	EDM=2.00186e-11 STRATEGY= 1 ERROR MATRIX ACCURATE
	EXT PARAMETER STEP FIRST
	1 fit_CFF_H_Re 6.67247e-02 1.34241e+00 2.92531e-05 -7.02262e-07
	2 fit_CFF_H_Im 1.24231e+01 1.07342e+00 1.80608e-05 1.71071e-04 3 fit_CFF_F_Re3_94789e+00 fixed
	4 fit_CFF_E_Im -1.64116e-01 fixed
	5 fit_CFF_Ht_Re 1.54183e+00 fixed 6 fit_CFF_Ht_Im 2.59017e+00 fixed
	7 fit_CFF_Et_Re 5.41102e+01 fixed 8 fit_CFF_Et_Tm 3.79052e+01 fixed
	EXTERNAL ERROR MATRIX. NDIM= 25 NPAR= 2 ERR DEF=1
	1.804e+00 7.961e-03 7.961e-03 1.153e+00
	PARAMETER CORRELATION COEFFICIENTS NO. GLOBAL 1 2
	1 0.00552 1.000 0.006
	2 000.0 2000
	The first reasonable fit with PARTONS_Fits! 12 AUL and 12 ALU asymmetries fitted together.
	The true values of fit CFF H Re and fit CFF H Im are 0.06672466940113253 and
	12.423114181138908
	Write a message



PARTONS Design Fits Releases Conclusion

### GPD or CFF fits (2/3). Global fit of CFFs using a analytic parameterization.



#### Parametric global fit of JLab DVCS data, Apr. 5<sup>th</sup>, 2017 Covariant extension RESULTS Phenomenology Content of GPDs Kinematic cuts $O^2 > 1.5 \text{ GeV}^2$ (where we can rely on LO approximation) Experimental access Tomography $-t/Q^2 < 0.25$ (where we can rely on GPD factorization) Dispersion relations Modeling x<sup>2</sup> / ndf 3272.6 / (3433 - 7) ≈ 0.96 Definition Polynomiality Free parameters a<sub>Hsea</sub>, a<sub>Ĥval</sub>, a<sub>Ĥsea</sub>, C<sub>sub</sub>, a<sub>sub</sub>, N<sub>F</sub>, N<sub>F</sub> Radon transform Positivity x<sup>2</sup> / ndf per data set Inverse Radon Examples

[1] Phys. Rev. C 92, 055202 (2015) [2] Phys. Rev. Lett. 115, 212003 (2015) [3] Phys. Rev. D 91, 052014 (2015)

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Experiment	Reference	Observables	N points all	N points selected		chi2 / ndf
Hall A	[1] KINX2	σUU	120	120	135.0	1.19
Hall A	[1] KINX2	ΔσLU	120	120	98.9	0.88
Hall A	[1] KINX3	συυ	108	108	274.8	2.72
Hall A	[1] KINX3	ΔσLU	108	108	107.3	1.06
CLAS	[2]	συυ	1933	1333	1089.2	0.82
CLAS	[2]	ΔσLU	1933	1333	1171.9	0.88
CLAS	[3]	AUL, ALU, ALL	498	305	338.1	1.13
Paweł Sznajder		0	NS 2017			12

H. Moutarde

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Fits

### GPD or CFF fits (2/3). Global fit of CFFs using a analytic parameterization.





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# GPD or CFF fits (3/3). Global fit of CFFs using neural networks.



### Covariant extension

### Neural network global fit of CLAS asymmetries, May $31^{ m st}$ , 2017

Re CFF

Im CFF

### NEURAL NETWORK

Paweł Sznajder

#### Phenomenology

Content of GPDs Experimental access Tomography Dispersion relations

#### Modeling

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- Examples

#### PARTONS

- Design Fits
- Releases

#### Conclusion

- Our very first attempt to use NN technique → proof of feasibility
- Genetic algorithm (GA) to learn NN
- NN and GA libraries by PARTONS group
- Very simple design of NN
- CLAS asymmetry data only
- x<sup>2</sup> / ndf = 273.9 / (305 68) ≈ 1.16

#### Nucleon and Resonance Structure Workshop 2017

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### Open source release. Publicly available on CEA GitLab server.



### Covariant extension

#### Phenomenology

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#### Modeling

- Definition
- Polynomiality
- Radon transform
- Positivity
- Inverse Radon
- Examples

### PARTONS

- Design Fits
- Releases

#### Conclusion



The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron lon Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments.

### Get PARTONS

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### Open source release. Publicly available on CEA GitLab server.



### Covariant extension

Content of GPDs
Experimental access
Tomography
Dispersion relations

#### Modeling

Definition

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	partons partons project	★2 @
	elementary-utils Utility softwares (logger, parser, threads, string and file manipulation)	<b>*</b> 0 @
	numa     NumA++: numerical analysis C++ routines	<b>★</b> 0 @
	partons-example     Running version of PARTONS with examples (C++ code and XML computing scenarios)	★0 @
	Prev 1 Next	



### Open source release. Publicly available on CEA GitLab server.



### Covariant extension

#### Phenomenology

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Developm	ent team				
The list of development	ent team members can b	e found at our GitLab page.			
		1.0			
If you want to join the	development team of P	ARTONS, contact Hervé Mouta	rde.		

H. Moutarde | QCD Workshop 2018 - Camburí | 41 / 46



### First release content. DVCS channel only.

CEA - Saclay

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QCD Workshop 2018 - Camburí

### Covariant extension

	GPD modules	CFF modules
Phenomenology Content of GPDs	GK	LO
Experimental access Tomography	VGG	NLO
Dispersion relations Modeling	<ul> <li>Vinnikov (evolution)</li> </ul>	NLO Noritzsch
Definition Polynomiality	<ul> <li>MPSSW13 (NLO study)</li> </ul>	
Radon transform Positivity	<ul> <li>MMS13 (DD study)</li> </ul>	Evolution modules
Inverse Radon Examples		<ul> <li>Vinnikov (LO)</li> </ul>
PARTONS Design	DVCS modules	
Fits Releases	■ VGG	$\alpha_{s}$ modules
Conclusion	GV	■ 4-loop perturbation
	BMJ	constant value

H. Moutarde



### Future releases.

A lot remains to be integrated...Contributors welcome!



#### Covariant extension

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	Channel modules
Phenomenology	DVMP
Content of GPDs	
Experimental access	TCS
Tomography	
Dispersion relations	$\sim M$ production
Modeling	
Definition	777
Polynomiality	<b>—</b>
Radon transform	
Positivity	
Inverse Radon	Other modules
Examples	Other modules
PARTONS	Mellin moments (EM)
Design	
Fits	tensor, lattice QCD)
Releases	, ,
Conclusion	???

### Hadron structure modules

- DAs
- DDs
- Form factors
- PDFs
- LFWFs
- ????

### Nonperturbative QCD modules

- Gap equation solver?
- $\alpha_{\rm s}$  models?

```
????
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### Conclusion

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### Conclusion and prospects. Putting all the pieces together.



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Design Fits Releases

#### Conclusion

- We can now build generic GPD model satisfying a priori all theoretical constraints.
- We now have tools to systematically relate these models to experimental data. Open source release under GPLv3.0. of the PARTONS framework.
- We have an **operating fitting engine** for global CFF fits.

### New studies become possible!

- Global GPD fits.
- Energy-momentum structure of hadrons.
- Impact of nonperturbative QCD ingredients on 3D hadron structure studies.
  - ???

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