Much Excitement About Nothing?

adapted freely from William Shakespeare



Elastic and transition form factors

Probe the excited nucleon structures at perturbative and nonperturbative QCD scales

Distinctive information on the roles played by DCSB and confinement in QCD

Quark-Gap Equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.



$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_{\mu} S(q) \Gamma^a_{\nu}(q,p)$$

with the running mass function $M(p^2) = B(p^2)/A(p^2)$.



Each satisfies it's own DSE !

 $S^{-1}(p)|_{p^2 = \zeta^2} = i\gamma \cdot p + m(\zeta)$ where ζ is the renormalization point.

 $\Gamma^a_{\nu}(q,p)$: dressed quark-gluon vertex

dressed-gluon propagator \checkmark

 $D_{\mu\nu}$

- Z_2 : quark wave function renormalization constant
- Z_1 : quark-gluon vertex renormalization constant



For light quarks the Higgs mechanism is almost irrelevant!

Motivation: Connection with Real World



Motivation: Connection with Real World

- How does one incorporate the dressed-quark mass function M(p²) in study of mesons and baryons? The behavior of M(p²) is a quantum field theoretical effect.
- In quantum field theory a meson (nucleon) appears as a pole in the four (six)-point quark Green functions amplitude.
- Residue is proportional to meson's Bethe-Salpeter or nucleon's Faddeev amplitude.
- Poincaré covariant Bethe-Salpeter/Faddeev equation sum all possible exchanges and interactions that can take place between dressed-quarks ($Q^2 \gg M^2$).





MESON BOUND STATES



Bethe-Salpeter Equations for QCD Bound States



Nonperturbative QCD based ansatz for interaction kernel

$$\Gamma(P,p) = \int \frac{d^4k}{(2\pi)^4} K(P,p,k) S(k-\frac{P}{2}) \Gamma(P,k) S(k+\frac{P}{2})$$

Rainbow-Ladder truncation:

$$K(P, p, k) = -\frac{Z_2^2 \mathcal{G}(q^2)}{q^2} \left(\frac{\lambda^a}{2}\gamma_\mu\right) T_{\mu\nu}(q) \left(\frac{\lambda^a}{2}\gamma_\nu\right)$$

Rainbow-ladder truncation (leading symmetry-preserving approximation)



Model gluon propagator, solve quark propagator and 4-point Green function.

Bethe-Salpeter Equations for QCD Bound States



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General solution for Poincaré invariant ground- and excited-state amplitudes $\Gamma_{P_n}(p,P) = \gamma_5 \left[i \mathbb{I}_D E_{P_n}(p,P) + \gamma \cdot P F_{P_n}(p,P) + \gamma \cdot p \left(p \cdot P \right) G_{P_n}(p,P) + \sigma_{\mu\nu} p_{\mu} P_{\nu} H_{P_n}(p,P) \right]$



Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: *infrared massive fixed point; ultraviolet massless propagator.*

Dressed-gluon propagator



Qin, Chang, Liu, Roberts and Wilson 2011

$$\Delta(k^2) \approx \frac{4\pi\alpha(k^2)}{k^2 + m_g^2(k^2)}, \quad m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2} \qquad \longrightarrow \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4} De^{-s/\omega^2} + \frac{8\pi^2\gamma_m}{\ln\left[\tau + (1 + s/\Lambda_{QCD}^2)\right]} \mathcal{F}(s)$$

Pseudoscalar- and Vector-Meson Spectroscopy



E. Rojas, B. E. & J. P. B. C. de Melo, PRD (2014) F. Mojica, C. Vera, E. Rojas & B. E., PRD (2017)

Bethe-Salpeter Equations as an Eigenvalue Problem

$$\lambda(P^2) \,\Gamma_{P_n}(P,p) = \int \frac{d^4k}{(2\pi)^4} \, K(P,p,k) \,\chi_{P_n}(k,P)$$

 $\chi_{P_n}(k,P) = S(k-\frac{P}{2})\Gamma(P,k)S(k+\frac{P}{2})$: Bethe-Salpeter wave function

The kernel $\mathcal{K}(P^2)$ has a complete set of real eigenvectors ϕ_i with eigenvalues $\lambda_i(P^2)$ which are ordered as $\lambda_0(P^2) > \lambda_1(P^2) > \lambda_2(P^2) > \dots > \lambda_i(P^2)$.

$$|\phi_n\rangle \stackrel{n \to \infty}{=} \lambda_0^n a_0 |\phi_0\rangle \simeq \lambda_0 \mathcal{K}^{n-1}(P^2) |\Phi\rangle$$

B. El-Bennich & E. Rojas arXiv:1509.02919 (2015)

- Eigenvalue spectrum is not limited to the ground state.
- Excited states with smaller eigenvalues can be determined with the same iterative methods.
- Usage of Gram-Schmidt orthogonalization process:

$$|\tilde{\Phi}\rangle = |\Phi\rangle - \frac{\langle \phi_0 \,|\, \Phi\rangle}{\langle \phi_0 \,|\, \phi_0\rangle} \,|\phi_0\rangle$$

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- Modern and more efficient approach is the implicitly restarted Arnoldi method (IRAM).
- Based on the stabilized Gram-Schmidt orthogonalization in the Krylov subspace obtained by iteration:

$$\mathcal{S}_r := \left\{ \Phi, K\Phi, K^2\Phi, K^3\Phi, \dots, K^{r-1}\Phi \right\}$$

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 The Arnoldi method generalizes the Gram-Schmidt process by computing the eigenvalues of the orthogonal projection of K onto the Krylov subspace ⇒ yields smaller eigenvalues.

Examples of eigenvalue spectrum: pion ground and radially excited states



Chebyshev expansion of 1st excited state: $E_{P_1}(p, P) = \sum_{m=0}^{\infty} E_{P_1}^m(p, P) U_m(\cos \theta)$

 $\Gamma_{P_n}(p,P) = \gamma_5 \left[i \mathbb{I}_D E_{P_n}(p,P) + \gamma \cdot P F_{P_n}(p,P) + \gamma \cdot p (p \cdot P) G_{P_n}(p,P) + \sigma_{\mu\nu} p_{\mu} P_{\nu} H_{P_n}(p,P) \right]$

Lowest Chebyshev moment : $E_{P_1}^0(p^2, P^2)$



Pseudoscalar- and Vector-Meson Spectroscopy

$J^{PC} = 0^{-+}$	DSE-BSE	PDG
m_{π}	0.136	0.139
f_{π}	0.139	0.1304
$m_{\pi(1300)}$	1.414	1.30 ± 0.10
$f_{\pi(1300)}$	8.3×10^{-4}	—
m_K	0.493	0.493
f_K	0.164	0.156
$m_{K(1460)}$	1.580	1.460
$f_{K(1460)}$	0.017	—
$m_{\eta_c(1S)}$	3.065	2.984
$f_{\eta_c(1S)}$	0.389	0.395
$m_{\eta_c(2S)}$	3.784	3.639
$f_{\eta_c(2S)}$	0.105	

E. Rojas, **B.** E. & J. P. B. C. de Melo, PRD (2014) F. Mojica, C. Vera, E. Rojas & **B.** E., PRD (2017)

Weak decay constant for radially excited states vanish — only strong decays possible:

$$f_{P_n}^0 \equiv 0 \,, \ n \ge 1$$

$J^{PC} = 1^{}$	DSE-BSE	PDG
$m_{ ho^0(770)}$	0.742	0.775
$f_{ ho^0(770)}$	0.231	0.221
$m_{ ho^0(1450)}$	1.284	1.465
$f_{\rho^0(1450)}$	0.150	—
$m_{K^*(892)}$	0.951	0.896
$f_{K^{*}(892)}$	0.287	0.217
$m_{K^*(1410)}$	1.217	1.414
$f_{K^{*}(1410)}$	0.127	
$m_{\phi(1020)}$	1.087	1.019
$f_{\phi(1020)}$	0.305	0.322
$m_{\phi(1680)}$	1.650	1.659
$f_{\phi(1680)}$	0.138	
$m_{J/\psi}$	3.114	3.097
$f_{J/\psi}$	0.433	0.416
$m_{\psi(2S)}$	3.760	3.689
$f_{\psi(2S)}$	0.176	0.295
$m_{\Upsilon(1S)}$	9.634	9.460
$f_{\Upsilon(1S)}$		0.715
$m_{\Upsilon(2S)}$	10.140	10.023
$f_{\Upsilon(2S)}$	0.564	0.497



BARYON SPECTRUM



Thanks to G. Eichmann.

Covariant Fadeev Equation

SU(3): 3⊗3=3⊕6

- The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for 3_c quark-quark correlations within a color-singlet baryon.
- *Diquark correlations* provide a tractable truncation of the Faddeev equation.
- We use non-pointlike color-antitriplet and fully interacting *diquarks* in the description of the Baryon Octet and Decuplet.
- Scalar and axialvector diquarks: dominant right parity.



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- We use non-pointlike color-antitriplet and fully interacting *diquarks* in the description of the Baryon Octet and Decuplet.
- Scalar and axialvector diquarks: dominant right parity.
- Typically, $r_{0+} \sim r_{\pi} \& r_{1+} \sim r_{\rho}$ (actually 10% larger).
- Pseudoscalar and vector diquarks: initially neglected, now included.
- Diquarks: have soft form factors.

Covariant Fadeev Equation



R.T. Cahill, C.D. Roberts, J. Praschifka (1989)

- M. Oettel, L. von Smekal, R. Alkofer (2001)
- I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts (2009)
- G. Eichmann, C. Fischer, H. Sanchis-Alepuz (2016)

Linear homogeneous matrix equation yields Poincaré covariant Faddeev amplitude (wave function) that describes relative motion of quark-diquark within nucleon.

Nucleon Electromagnetic Form Factors

- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Takahashi identities (EM gauge invariance).
- Coupling of the photon to the dressed quark.
- Coupling of the photon to the dressed diquark: Elastic & induced transitions
- Exchange and seagull terms.

$$\begin{array}{ll} J_{\mu}(P',P) &=& ie\, \bar{u}(P')\, \Lambda_{\mu}(q,P)\, u(P)\,,\\ &=& ie\, \bar{u}(P')\, \left(\gamma_{\mu}F_{1}(Q^{2})+\frac{1}{2M}\,\sigma_{\mu\nu}\,Q_{\nu}\,F_{2}(Q^{2})\right) u(P)\,,\\ &G_{E}(Q^{2})=F_{1}(Q^{2})-\frac{Q^{2}}{4M^{2}}F_{2}(Q^{2})\,,\ G_{M}(Q^{2})=F_{1}(Q^{2})+F_{2}(Q^{2})\,.\\ &\mu_{n}=\kappa_{n}=G_{M}^{n}(0)\,,\ \mu_{p}=1+\kappa_{p}=G_{M}^{p}(0) \end{array}$$



 $\frac{\mathbf{P}_{\mathrm{f}}}{\mathbf{I}}$

 $\nabla \Psi_i = \frac{P_i}{4}$









Dressed quark propagator solutions of QCD's Dyson-Schwinger equations.

⇒ momentum dependence !

$$S(p) \ = \ -i\gamma \cdot p \, \sigma_V(p^2,\zeta^2) + \sigma_S(p^2,\zeta^2) = rac{1}{i\gamma \cdot p \, A(p^2,\zeta^2) + B(p^2,\zeta^2)} = rac{Z(p^2,\zeta^2)}{i\gamma \cdot p + M(p^2)}$$

Proton's Sachs Electric and Magnetic Form Factors

$$G_E(Q^2) = F_1(Q^2) - rac{Q^2}{4M^2}F_2(Q^2)\,, \ G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts, Few Body Syst. 46 (2009)

Electric Sachs form factors: mass function dependence



▶ Both CI and QCD-based frameworks predict a zero crossing in µ_p G^p_E/G^p_M.
 ▶ The possible existence and location of the zero in µ_p G^p_E/G^p_M is an indirect measure of the nature of the quark-quark interaction.

A world with only scalar diquarks

The singly-represented d-quark in the proton $\equiv u[ud]_{0^+}$ is sequestered inside a soft scalar diquark correlation.



Contributions coming from u-quark



Contributions coming from d-quark



A world with scalar and axialvector diquarks

The singly-represented d-quark in the proton is not always (but often) sequestered inside a soft scalar diquark correlation.

Observation:

 $\mathcal{P}_{\mathrm{scalar}} \sim 0.62, \quad \mathcal{P}_{\mathrm{axial}} \sim 0.38$



Contributions coming from u-quark Contributions coming from d-quark 22 22 Ψ_{f} $P_f =$ Ψ_i Ψ_f $rac{1}{r}P_i$ P_f Ψ_{i} $\mathbf{F}_{\mathbf{i}}$ Ψ_{f} Ψ_f $\underline{-} P_i$ Ψ_i $\Psi_i = P_i$

A world with scalar and axialvector diquarks



Observations:

- F_{1p}^d is suppressed with respect F_{1p}^u in the whole range of momentum transfer.
- The location of the zero in F_{1p}^d depends on the relative probability of finding 1⁺ and 0⁺ diquarks in the proton.
- F_{2p}^d is suppressed with respect F_{2p}^u but only at large momentum transfer.
- There are contributions playing an important role in F₂, like the anomalous magnetic moment of dressed-quarks or meson-baryon final-state interactions.

Scalar and axialvector diquark contributions

J. Segovia & C. D. Roberts Phys. Rev. C94 (2016)



Axialvector diquark contribution is not enough in order to explain the proton's electromagnetic ratios.

Scalar diquark contribution is dominant and responsible of the Q^2 -behavior of the the proton's e.m. ratios.

▶ Higher quark-diquark angular momentum components of the nucleon are critical in explaining the data.

The presence of higher orbital angular momentum components in the nucleon is an inescapable consequence of solving a realistic Poincaré-covariant Faddeev equation





THE ROPER

PHYSICAL REVIEW LETTERS





- EBAC examined dynamical origins of two poles associated with the Roper resonance.
- Both of them, together with the next higher resonance in the P_{11} partial wave have the same originating bare state.
- The meson cloud shields quark-core state and diminishes its mass considerably.

Ground and Radially Excited States of the Nucleon



Roper Quark-Core Mass



DSE: Faddeev quark-diquark amplitude of 1st excited state with dressed quark propagators.

- J. Segovia, B. El-Bennich, E. Rojas, I.C. Cloët, C.D. Roberts, S.-S. Xu, H.-S. Zhong, Phys. Rev. Lett. 115 (2015)
- G. Eichmann, C. Fischer, H. Sanchis-Alepuz, Phys.Rev. D94 (2016)
- DSE: Faddeev three-quark interaction amplitude of 1st excited state with dressed propagators. G. Eichmann, C. Fischer, H. Sanchis-Alepuz, Phys.Rev. D94 (2016)

Contact : Faddeev amplitude of 1st excited state with contact interaction gap equation.

D.J. Wilson, I. C. Cloët, L. Chang, C.D. Roberts, Phys. Rev. C85 (2012)

DCCM : Dynamical Coupled Channel Model.

N. Suzuki, B. Julio-Díaz, H. Kamano, T.-S. H. Lee, A. Matsuyama, T. Sato, Phys. Rev. Lett. 104 (2010)

Chebyshev Moments

First three Chebyshev moments of leading S₁ component of 1st excited state's Faddeev amplitude



Chebyshev Moments



Zeroth Chebyshev moments of all S-wave components in the Faddeev wave function. S_1 is associated with the baryon's scalar diquark; A_2 , A_3 , A_5 associated with axialvector correlation.

$\gamma p \rightarrow R^+$ Dirac and Pauli Transition Form Factors



- Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on x > 2.
- The mismatch between our prediction and the data on x = 2 is due to meson cloud contribution.
- The dotted-green curve is an inferred form of meson cloud contribution from the fit to the data.
- The contact-interaction prediction disagrees both quantitatively and qualitatively with the data.

J. Segovia, B. El-Bennich, E. Rojas, I.C. Cloët, C.D. Roberts, S.-S. Xu, H.-S. Zhong, Phys. Rev. Lett. (2015)

$\gamma p \rightarrow R^+$ Dirac Transition Form Factor



- The Dirac transition form factor is primarily driven by a photon striking a bystander dressed quark that is partnered by a scalar diquark.
- Lesser but non-negligible contributions from all other processes are found.
- In exhibiting these features, $F_{1p}^*(q^2)$ shows marked qualitative similarities to the proton's elastic Dirac form factor.

$\gamma p \rightarrow R^+$ Pauli Transition Form Factor



- A single contribution is overwhelmingly important: photon strikes a bystander dressed-quark in association with a scalar diquark.
- No other diagram makes a significant contribution.
- $F_{2p}^*(q^2)$ shows marked qualitative similarities to the proton's elastic Pauli form factor.

Nucleon & Parity Partner

Including pseudoscalar and vector diquarks ...

- The Nucleon and Roper remain dominated by scalar and axialvector diquark correlation.
- Both, the Nucleon and Roper are dominated by S-waves (~75% & 85%).
- However, while the N*(1535) and N*(1650) are still dominated by scalar and axialvector correlations, the Faddeev amplitude is dominated by P-waves (~70% & 85%).

	$N_{n=0}$	$N_{n=1}$	$N_{n=0}^{*}$	$N_{n=1}^{*}$
m^{DSE}	1.19	1.73	1.83	1.94
$m^{ ext{expt.}}$	0.94	1.44	1.54	1.65
$m^{\text{DSE}} - m^{\text{expt.}}$	0.25	0.29	0.29	0.29

 $m_N^*(1535) > m_N^*(1440)$

N*(1535) & N*(1650)



N(940) & N*(1535) — P-waves



Baryon Octet & Decuplet

	N	Λ	Σ	[1]	Δ	Σ^*	[I] *	Ω
m^{real}	1.14	1.28	1.35	1.46	1.39	1.51	1.64	1.76
$m^{ ext{CI}}$	1.14	1.26	1.35	1.43	1.39	1.51	1.63	1.76
$m^{ ext{expt.}}$	0.94	1.12	1.19	1.31	1.23	1.39	1.53	1.67
$m^{\text{real}} - m^{\text{expt.}}$	0.2	0.16	0.16	0.15	0.16	0.12	0.11	0.09
$m^{\mathrm{real}} - m^{\mathrm{CI}}$	0	0.02	0	0.03	0	0	0.01	0

CI: C. Chen, L. Chang, C. D. Roberts, S. Wan and D. J. Wilson, Few Body Syst. 53, 293 (2012)

- Contact model is good enough to calculate the spectrum.
- ➤ However, in the CI model the Faddeev amplitudes are constant ⇒ not suitable for quantitative predictions of large-q² form factors.

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Conclusive Remarks

- We extracted the ground states and first radial excitations of the nucleon, its parity partner and hyperons using a quark-diquark Faddeev kernel that described the quark core.
- Dynamical chiral symmetry breaking and its correct implementation produces pions as well as strong electromagnetically-active diquark correlations.
- Poincaré covariance demands the presence of dressed-quark orbital angular momentum in the baryon.
- The presence of strong diquark correlations within the nucleon is sufficient to understand empirical extractions of the flavor-separated form factors.
- * Scalar diquark dominance and the presence of higher orbital angular momentum components are responsible of the Q²-behavior of $\mu_p\,G^p_E/G^p_M$ and F^p_2/F^p_1 .

Conclusive Remarks about the Roper

- The Roper is the proton's first radial excitation. It's mass agrees with that of the bare unclothed Roper quark core.
- * Our calculation agrees quantitatively in magnitude and qualitatively in trend with the data on $x \ge 2$. The mismatch below $x \ge 2$ is due to meson cloud contribution.
- Flavor-separated versions of transition form factors reveal that, as in the case of the elastic form factors, the d-quark contributions are suppressed with respect the u-quark ones.