OVERVIEW OF RESUMMATION

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN

PARTON SHOWERS AND RESUMMATION

LUND, JUNE 4, 2018
WHERE IT ALL COMES FROM:
EIKONAL EMISSION...

EMISSION OF A SOFT \((q^\mu \to 0)\) GAUGE PARTICLE FROM EXTERNAL LINE

\[
\sigma(\alpha \to \beta) \to \sigma(\alpha \to \beta) \frac{e^{-p \cdot q \cdot i\epsilon}}{p \cdot q - i\epsilon}
\]

(Bloch, Nord Sec, 1937; Yenni, Frautschi, Suura, 1955; Weinberg, 1964)

- **SOFT EMISSION \Rightarrow EIKONAL FACTOR**
- **CROSS SECTION FOR SINGLE (DOUBLE . . .) EMISSION INFRARED DIVERGENT; DIVERGENCE CANCELLED BY VIRTUAL CORRECTIONS**
- **1,2, . . . ,N EMISSIONS EXPONENTIATE \Rightarrow \Gamma \sim \exp - \left[ \alpha \ln^2 \left( 1 - \frac{M^2}{s} \right) \right] \right (\text{Sudakov, 1956})**
- **AFTER CANCELLATION, LEFTOVER SOFT LOGS:**

\[
\sigma(\alpha \to \beta) \to \sigma(\alpha \to \beta) \ln^2 \left( 1 - \frac{M^2}{s} \right)
\]
...AND RESUMMATION

- **EXponentiation of LeftOver Logs** $\Rightarrow$
  **Threshold Resummation** of $\alpha_s \ln^2 (1 - x), \ x = \frac{M^2}{s}$

- **LogS come in Pairs:** **SOFT+Collinear** $\Rightarrow \ln p_t$ when integral over $p_t$ not performed $\Rightarrow$
  **Transverse Momentum Resummation** of $\alpha_s \ln^2 \frac{q_T}{M^2}$

- **In Gluon Channel** symmetry of the Triple Gluon Vertex $\Rightarrow$ Large Logs
  Also when the exchanged gluon is soft: no collinear contribution,
  **Single Logs** $\Rightarrow \ln \frac{Q^2}{Q^2} \Rightarrow$
  **High Energy Resummation** of $\alpha_s \ln \frac{1}{x}$

**Gluon Radiation**
\[
\sigma(\tau, M^2) = \int_y^1 \frac{dy}{y} \frac{P(x)}{y} \int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_T^2}{k_T^2} \hat{\sigma}(y, M^2)
\]

**The Gluon Splitting Function:**
\[
P_{gg}(x) = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \beta_0 \delta(1-x)
\]

**Logarithmically Enhanced Terms**

- **Infrared Logs:** $\int_\tau^1 dy \frac{1}{1-y_+} \sim \ln(1 - \tau)$

- **UV Logs:** $\int_\tau^1 dy \frac{1}{y} \sim \ln(\tau)$

- **Collinear Logs:** $\int_{\mu^2}^{(s-M^2)^2/s} \frac{dk_T^2}{k_T^2} \sim \ln \left[ \frac{Q^2}{\mu^2} \frac{(1-\tau)^2}{\tau} \right] = \ln \frac{Q^2}{\mu^2} + \ln(1 - \tau)^2 + \ln \tau$
RESUMMATION: A TIMELINE

EARLY

- Balitski, Fadin, Kuraev, Lipatov, 1976-1978 STRUCTURE OF HIGH-ENERGY LOGS
- Dokshitzer, Dyakonov, Troyan, 1979 STRUCTURE OF $p_T$ LOGS
- Parisi, Petronzio, 1979 LL TRANSVERSE MOMENTUM RESUMMATION
- Parisi, 1980; Curci, Greco, 1980 LL SOFT GLUON RESUMMATION
- Jaroszewicz, 1980 LL HIGH ENERGY RES. OF ANOMALOUS DIMENSIONS

CLASSIC

- Collins, Soper, Sterman, 1984 TRANSVERSE MOMENTUM: GENERAL STRUCTURE & ARGUMENT AND NLL RESULT (DRELL-YAN)
- Sterman, 1987: THRESHOLD: GENERAL STRUCTURE & ARGUMENT
- Catani, Trentadue, 1989 SOFT GLUON: NLL RESULT (DY, DIS)
- Catani, Ciafaloni, Hautmann, 1990 HIGH ENERGY: GENERAL FORMALISM FOR PURE GLUE
- Catani, Hautmann, 1993 HIGH ENERGY: GENERAL FORMALISM INCLUDING QUARKS
THE STRUCTURE OF RESUMMED EXPRESSIONS
THRESHOLD RESUMMATION

LOG COUNTING

\[ C_{\text{res}}(N, \alpha_s) = g_0(\alpha_s) \exp \left[ \frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots \right]; \]
\[ g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3); \]
\[ g_1(\lambda) = \sum_{k=2}^{\infty} g_{1,k} \lambda^k, \quad g_i(\lambda) = \sum_{k=1}^{\infty} g_{i,k} \lambda^k \quad \text{FOR} \quad i \geq 2. \]

LOG APPROX.  | XSECT ACCURACY | EXP. ACCURACY: $\alpha_s^n L^k$ | $g_0$ ACCURACY: $\alpha_s^i$
---|---|---|---
LL | $k = 2n$ | $k = n + 1$ | 0
NLL | $2n - 2 \leq k \leq 2n$ | $k = n$ | 1
NNLL | $2n - 4 \leq k \leq 2n$ | $k = n - 1$ | 2

THE RESUMMED EXPONENT

\[ S \left( \frac{M^2}{N^2}, \frac{M^2}{N^2} \right) = \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \tilde{\gamma} \left( \alpha_s(\mu^2), \frac{M^2}{N^2} \right) \]
\[ = \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} \left[ -A(\alpha_s(\mu^2)) \ln \left( \frac{M^2}{N^2} \right) + B[\alpha_s(\mu^2)] \right]. \]

- $A, B$ are power series in $\alpha_s$
- $A$ is universal, coefficient of $\ln N$ in (diagonal quark or gluon) anomalous dimension at each order
- $B$ contains process-dependent terms, starts at NLL
THE STRUCTURE OF RESUMMED EXPRESSIONS

$P_t$ RESUMMATION

THE CROSS SECTION

- **PARTONIC DIFFERENTIAL $p_T$ DISTRIBUTION:** $\Sigma = \sum_n \alpha_s^n(M^2) \Sigma^{(n)}(p_T, M^2)$;
  $\Sigma^{(n)}(p_T, M^2) = \left[ \frac{P_n(\ln(p_T^2/M^2))}{p_T^2/M^2} \right]_+ + Q_n(p_T^2/M^2) + D_n\delta(p_T^2/M^2)$

- **DIVERGES AS $p_T \to 0$ TO ANY FINITE ORDER**

RESUMMATION

- **PHASE-SPACE FACTORIZATION:** LONGITUDINAL $\leftrightarrow$ MELLIN; TRANSVERSE $\leftrightarrow$ FOURIER

- $\Sigma(\alpha_s, p_T^2/M^2) = \frac{M^2}{2\pi} \int d^2b e^{-i\vec{p}_T \cdot \vec{b}} \Sigma(\alpha_s, \alpha_s L) = \int_0^{+\infty} db J_0(bq_T) \Sigma(\alpha_s, \alpha_s L)$;
  $\alpha_s = \alpha_s(M^2)$; $L \equiv -\ln b^2 M^2$

- **EXPONENTIATION:** $\Sigma(\alpha_s, \alpha_s L) = \exp S(\alpha_s, \alpha_s L)$

THE RESUMMED EXPONENT

$$S(\alpha_s, \alpha_s L) \equiv - \int_{1/g_s}^\infty \frac{d\mu^2}{\mu^2} \left[ \ln \frac{Q^2}{\mu^2} A(\alpha_s(\mu^2)) + B(\alpha_s(\mu^2)) \right],$$

$$\Rightarrow \Sigma(\alpha_s, \alpha_s L) = H(\alpha_s) \exp \left[ Lg^{(1)}(\alpha_s L) + g^{(2)}(\alpha_s L) + \ldots \right]$$

- **$A, B$ ARE POWER SERIES IN $\alpha_s$**

- **$A$ CONTAINS SOFT CONTRIBUTIONS, $B$ PURELY COLLINEAR (FLAVOR-CONSERVING) CONTRIBUTIONS**

- **RESUMMATION SET UP SO THAT $A$ AND $B$ ARE PROCESS-INDEPENDENT (ONLY DEPEND ON QUARK VS GLUON)** (Catani, de Florian, Grazzini, 2000)
RESUMMATION TIMELINE II

MODERN

- Catani, Mangano, Nason, Trentadue, 1996 SOFT RESUMMATION FORMALISM FOR HADRONIC PROCESSES
- Contopanagos, Laenen, Sterman, 1997 GENERAL RG ARGUMENT FOR SUDAKOV RESUMMATION
- Altarelli, Ball, SF, 2001 HIGH ENERGY RESUMMATION IN DGLAP EQUATION

CONTEMPORARY

- Catani, de Florian, Grazzini, 2001 TRANSVERSE MOMENTUM RESUMMATION UNIVERSALITY & FORMALISM FOR HADRONIC PROCESSES
- Manohar, 2003 SCET APPROACH TO THRESHOLD RESUMMATION
- Ciafaloni, Colferai, Salam, Stasto, 2004 FULL HIGH ENERGY RESUMMATION OF SPLITTING FUNCTIONS (PURE GLUE)
- Altarelli, Ball, SF, 2008 HIGH ENERGY RESUMMATION FOR PHYSICAL PROCESSES
RESUMMATION NOW

- **SCET vs. $dQCD$ NOT AN ISSUE**
- **THRESHOLD RESUMMATION ⇒ IMPROVE FIXED ORDER**
- **MATCHING OF $p_t$ RESUMMATION TO FIXED ORDER STANDARD FOR $p_T$ SPECTRA**
- **IMPLEMENTED IN SEVERAL PUBLIC CODES:** E.G. xFitter/APFEL (Bertone, 2016-2017), TROLL, HELL (Bonvini, 2016-2018), TOP++ (Czakon, Mitov, 2016-2018), iHixs (Anastasiou, Bühler, Dulat, Herzog, Lazopoulos, Mistlberger 2012-2018),...
- **PROGRESS**
  - HIGHER ORDERS, MORE PROCESSES
  - COMBINED, CONSISTENT & MORE EXCLUSIVE
SIX GREAT IDEAS ABOUT RESUMMATION

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN

PARTON SHOWERS AND RESUMMATION

LUND, JUNE 4, 2018
MOMENTUM-SPACE RESUMMATION
MELLIN/FOURIER
THE PROBLEM:

- **RG argument for resummation naturally formulated in Fourier-Mellin:**
  - **Mellin** ⇒ Longitudinal momentum conservation (undoes convolution)
  - **Fourier** ⇒ Transverse momentum conservation (factorizes $k_t$ delta)

- **Eikonal emission factorizes** ⇒ can resum (both soft & $p_t$) in momentum space

- **Schematic behaviour (fixed coupling)**
  \[
  K^{res} \sim \frac{d}{dp_t} \exp - \left[ \alpha_s \ln \frac{Q^2}{p_t^2} F\left( \alpha, \ln \frac{Q^2}{p_t^2} \right) \right]
  \]

  ⇒ **zero radius of convergence**! (diverges faster than any power of log)

- **Expansion at LL, NLL... leads to convergent result,**

  \[
  K^{res} \sim \frac{d}{dp_t} \exp - \left[ \alpha_s \ln \frac{Q^2}{p_t^2} \right] \frac{\Gamma\left(1-2\alpha_s \ln \frac{Q^2}{p_t^2}\right)}{\Gamma\left(1+2\alpha_s \ln \frac{Q^2}{p_t^2}\right)}
  \]

  ⇒ **bad perturbative expansion**
MOMENTUM SPACE
A SOLUTION
(Monni, Re, Torrielli, 2016; also Ebert, Tackmann, 2017)

DIVERGENCE & SINGULARITIES WHEN EXPONENTIATING INDEPENDENT SMALL $p_t$ EMISSION DUE TO THE NEGLECT OF CONFIGURATIONS WHERE $p_t$ SMALL DUE TO CANCELLATIONS
$\Rightarrow$ DOMINANT AS $p_t \to 0$ (Parisi, Petronzio, 1979)

- CONSTRUCT CUMULATIVE CROSS-SECTION
  \[ \Sigma(p_t) = \int_0^{p_t} dp_t' \frac{d\sigma(p_t')}{dp_t'} \]

- DIVERGENT $\Leftrightarrow$ EXPANDING EMISSIONS $k_t^i$ ABOUT $p_t$

- ORDER EMISSIONS IN $k_t$, EXPAND ABOUT LARGEST $k_T^1$

- CAN DEFINE ITERATIVELY $n$-EMISSION CUMULANT IN TERMS OF \( n - 1 \)

- CAN PROVE EQUIVALENCE TO STANDARD FORMALISM UP TO SUBLEADING CORRECTIONS; SINGULAR TERMS SHIFTED TO HIGHER ORDERS

(Monni, Re, Torrielli, 2016)
MASSIVE EVOLUTION
AN OLD PROBLEM: MASSIVE QUARK SCHEMES

EXAMPLE: $b\bar{b} \rightarrow H$

- **4FS** ⇒ MASSIVE $b$, NO $b$ IN DGLAP EVOLUTION AND $\beta$ FUNCTION
- **5FS** ⇒ $mb$ IN DGLAP EVOLUTION AND $\beta$ FUNCTION BUT $b$ MASS NEGLECTED

CLASSIC SOLUTION: MATCHED SCHEMES

- COLLINEAR FACTORIZATION ALSO HOLDS FOR MASSIVE QUARKS (Collins, 1998)
- RE-EXPRESS 4FS IN TERMS OF 5FS (OR VICE-VERSA)
- COMBINE & SUBTRACT DOUBLE COUNTING
- $b\bar{b}H$: PERTURBATIVE INSTABILITY OF 4FS REMOVED BY RESUM- MATION
- MASS CORRECTIONS NEEDED FOR PERCENTAGE ACCURACY, EVEN FOR TOTAL XSECTS

4FS, 5FS VS MATCHED (FONLL) RENORM. SCALE DEP.
WHAT'S THE PROBLEM?

- CUMBERSOME, ESPECIALLY FOR DIFFERENTIAL OBSERVABLES
- NOT EASY TO IMPLEMENT/INTERFACE IN MONTECARLOS

SOLUTION: MASSIVE 5FS

((Krauss, Napoletano, 2018))

- MASSIVE SPLITTING FUNCTION
- SUBSET OF UNIVERSAL INITIAL-STATE MASS CORRECTIONS RESUMMED
- SUBSET OF MATCHED SCHEME CAPTURED, BUT RESUMMED TO ALL ORDERS

5FS vs MFS

\[ b\bar{b} \rightarrow H \]

(Figueroa, Honeywell, Quackenbusch, Reina, Reuschle, Wackeroth, 2018)
MIXED QCD-EW EVOLUTION
THE PROBLEM OF EW SUDAKOVs

- **KLN** cancellation of IR singularities holds in QED
- In nonabelian case, cancellation only for gauge group singlet
- **Proton is not EW singlet** $\Rightarrow$ IR mass singularities do not cancel between real and virtual
- Explicit mass dependence in virtual correction, real contribution has no $+ \text{prescription}$ $\Rightarrow$ singular splitting functions (M. Ciafaloni, P. Ciafaloni, Comelli, 2000-2005):
  \[ P_V \sim \delta(1-x) \ln \frac{Q^2}{k_t^{2\text{max}}}, \]
  \[ P_R \sim \frac{1}{1-x}; \]
  \[ \frac{d}{d \ln Q^2} F(x, Q^2) = \int_x^{y^{\text{max}}} P f(\frac{y}{x}, Q^2); \quad P = P^V + P^R \]
- **Double logs** $\alpha_{EW} \ln^2 \frac{Q^2}{M_W^2}$ to all orders
A PRACTICAL SOLUTION
SOLVING SM EVOLUTION EQUATIONS

(Bauer, Ferland, Webber, 2017)

- SOLVE EVOLUTION EQUATIONS IN THE FULL STANDARD MODEL
- MATCH RESULTS TO FIXED ORDER CALCULATIONS
- INITIAL GAUGE BOSON PDF CAN BE DETERMINED BY THE LUX METHOD (Fornal, Manohar, Waalewijn, 2019)

FULL SM PDF EVOLUTION: RATIO TO QCD

“LUX” TRANSVERSE W PDFS

$Q = M_Z$

$Q = 1$ TeV

$Q = M_Z$

$Q = 1$ TeV
NEXT-TO LEADING POWER
SOFT EXPANSION(S)

- BOTH IN THRESHOLD AND HE LIMIT: RESUMMATION ⇒
  - IMPROVEMENT OF FIXED ORDER
  - APPROXIMATE HIGHER ORDER (HQ MASS DEP, MULTISCALE)

- NEXT-TO-LEADING POWER LOGARITHMICALLY ENHANCED:
  \[ C^{(n)}(x) \approx \alpha_s \frac{\ln^{2n-1}(1-x)}{1-x} + \delta(1-x) + \ln^2(1-x) + \ldots \]

- LL, NLP RESUMMATION ⇔ DGLAP SPLITTING FUNCTION EXPANDED TO NLP

- NLP GOOD APPROX, ESPECIALLY IF RESCALED

NNLO HIGGS IN GLUON FUSION: SOFT EXPANSION

(Dulat, Mistlberger, Pelloni, 2018)
NLP PROGRESS

• **METHOD OF REGIONS** (Laenen et al, 2013....)
  - SEPARATE INTEGRATION REGIONS OVER MOMENTA THROUGH SCALING
  - COMBINE WITH FACTORIZATION ⇒ CLASSIFY EMISSION FROM INTERNAL BLOBS DRESSED WITH RADIATION FROM EXTERNAL LINES

• **RAPID RECENT PROGRESS** both using dQCD & SCET, both in HE & SOFT LIMIT

  **NLO NLP UNIVERSALITY**

  (Del Duca, Laenen, Magnea, Vernazza, White, 2018)

• **NEXT-TO-SOFT THEOREM** (E. Casali, 2014; Bonocore et al, 2015)
  ⇔ EIKONAL-LIKE NLP EMISSION FORMULA

• **EFFECTIVELY, NLP OBTAINED FROM SHIFT IN BORN KINEMATICS:**
  Born: \( p_1 + p_2 \rightarrow X \); NLP NLO \( p_1 \rightarrow p_1 - \frac{1}{2} \left( p_1 \frac{p_2 \cdot k}{p_1 \cdot p_2} - p_2 \frac{p_2 \cdot k}{p_1 \cdot p_2} + k \right) \), & similar for \( p_2 \)

• **NLP NLO:** \( C(x) = K(x, \varepsilon) C^{\text{Born}}(xs) \);
  \( K(x, \varepsilon) = \left( \frac{\mu^2}{s} \right)^\varepsilon \left[ \frac{2-2D_0(x)}{\varepsilon} + 4D_1(x) - 4 \ln(1-x) \right] \);
  \( D_n(x) = \left( \frac{\ln^n(1-x)}{1-x} \right)^+ \)

• **HIGGS IN GLUON FUSION:**
  - ALL NLO LOG DEPENDENCE COMES FROM UNIVERSAL NLP FACTOR \( K(x, \varepsilon) \)
  - FULL \( m_t \) DEPENDENCE INCLUDED THROUGH BORN FUNCTION \( F(x\tau, \varepsilon), \tau = \frac{m^2_H}{s} \),
  \( F(\tau, \varepsilon) = \frac{9}{4} \frac{1}{\tau^2} \left[ 1 + (1 - \frac{1}{\tau}) \arcsin^2(\sqrt{\tau}) \right]^2 + O(\varepsilon) \)
DIVERGENT OBSERVABLES
**RESUMMING DIVERGENCES**

- **DIVERGENCES AT KINEMATIC BOUNDARIES** routinely removed by resummation
- **IRC SAFE BUT DIVERGENT** observables made finite by resummation

**AZIMUTHAL VARIABLES**

(Catani, Grazzini, Sargsyan, 2017)

- \( h(p_1) h(p_2) \to F(p_3, p_4) + X \Rightarrow \text{DY into dileptons, top pairs, ...} \)
- **FINAL-STATE KINEMATICS** \( M, p_T, y \) of \( F; \theta, \phi \) of \( p_3 \) in Collins-Soper frame (rest frame of \( F \))

- \( \frac{d\sigma}{d\phi dp_t} \Rightarrow \text{NONINTEGRABLE SINGULARITY as } p_t \to 0 \Rightarrow \frac{d\sigma}{d\phi} \)

DIVERGENT IF EITHER

- INITIAL STATE GLUON (SOMETIMES)
- COLORED FINAL STATE (ALWAYS)

**AZIMUTHAL ANGULAR HARMONICS INTEGRATED DOWN TO** \( p_t = r_{\text{cut}} \)

---

**DRELL-YAN**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( f_{\text{Drell-Yan}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 0 )</td>
<td>0.001</td>
</tr>
<tr>
<td>( n = 1 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( n = 2 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**TOP PAIRS**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( f_{\text{Top Pairs}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 0 )</td>
<td>0.001</td>
</tr>
<tr>
<td>( n = 1 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( n = 2 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0.001</td>
</tr>
<tr>
<td>( n = 6 )</td>
<td>0.001</td>
</tr>
</tbody>
</table>
RESUMMING DIVERGENCES

(Catani, Grazzini, Sargsyan, 2017)

• DIVERGENCE DUE TO MISMATCH BETWEEN VIRTUAL & REAL AT FIXED ORDER

• VIRTUAL CANNOT LEAD TO AZIMUTHAL CORRELATION ⇒ CANNOT CANCEL
  – SINGULAR REAL CONTRIBUTIONS DUE TO COLLINEAR SPIN-CORRELATIONS (INITIAL-STATE GLUONS)
  – LARGE-ANGLE RADIATION FROM FINAL-STATE PARTICLES (FINAL-STATE COLORED STATES)

• RESUMMATION REMOVES THE DIVERGENCE THROUGH SUDAKOV SUPPRESSION

TOP PRODUCTION: FIRST HARMONIC

[Graphs showing fixed order and resummed vs. NLO & matched distributions]
RESUMMED PDFs
WHAT ABOUT PDFs?

- RATIO OF PHYSICAL PROCESSES $\Rightarrow$ RESUMMATION
- THE PDF IS THE DENOMINATOR
- BUT NOT RESUMMED IN AVAILABLE PDF SETS
- KEPT UNDER CONTROL WITH CUTS

$\Delta_N(Q^2) = \frac{w_N(Q^2)}{[F_N(Q^2)]^2}$.

(Catani, Trentadue, 1987)
**PDFs with threshold resummation**

**Effects are small**

- **First set:** NNPDF3.0 resum
- **Effect on PDFs comparable to effect on matrix element, anticorrelated to it**
- **Resummation included in fit (DIS, DY, Top data), effects visible at NLO, large x**
- **Perhaps relevant for new physics searches**

(Bonvini et al., 2015)
PDFs with high energy resummation

- **First set**: NNPDF3.0sx
- **High energy resummation included** in GLAP evolution & for DIS coefficient functions
- **Stabilizes** perturbative expansion
- **Visible effects** for light final states (b production at LHC) & for future colliders

(Ball et al., 2017)
AN OUTLOOK?
A RESUMMATION WISHLIST

- CONSISTENT, FULLY EXCLUSIVE SOFT RESUMMATION (ggHiggs, Drell-Yan, top, inclusive jets):
  differential in $p_T$, rapidity, always threshold-resummed upon integration

- NEXT-TO-LEADING LOG HIGH-ENERGY FACTORIZATION AND RESUMMATION FOR HADRONIC PROCESSES (GLUON-INDUCED: Higgs, heavy quarks)

- MULTISCALE RESUMMATION WITH HEAVY QUARKS (ggHiggs $p_t$ spectrum $\ln \frac{m_b}{p_t}$)

- THE LIMITS OF FACTORIZATION: WHEN DOES IT BREAK DOWN?

  - . . .