Soft gluon evolution & non-global logarithms

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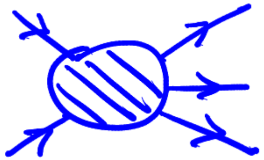
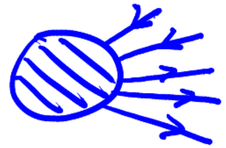
Soft gluon evolution

$|H\rangle =$  $=$ 

$|H\rangle\langle H| = H =$ 

$$D_i = \sum_j T_j \, E_i \, \frac{p_j}{p_j \cdot q_i}$$

$$V_{a,b} = \exp\left[\int_a^b \frac{dE}{E}\,\Gamma\right]$$

$$\Gamma = \frac{\alpha_s}{\pi} \sum_{i<j} \left(-T_i \cdot T_j\right) \left\{ \int \frac{d\Omega_k}{4\pi}\, w_{ij}(\hat{k}) - i\pi\, \tilde{\delta}_{ij} \right\}$$

$$w_{ij}(\hat{k}) = E_k^2 \, \frac{p_i \cdot p_j}{p_i \cdot k \;\, p_j \cdot k}$$

e.g. $A_1 =$

$$d\sigma_n = \mathrm{Tr}\, A_n(\mu)\, d\Pi_n$$

$\mu = 0$ always

& $\mu = Q_0$ if inclusive for $E < Q_0$

$$\Sigma = \sum_n \int d\sigma_n \; u_n(q_1, \ldots, q_n)$$

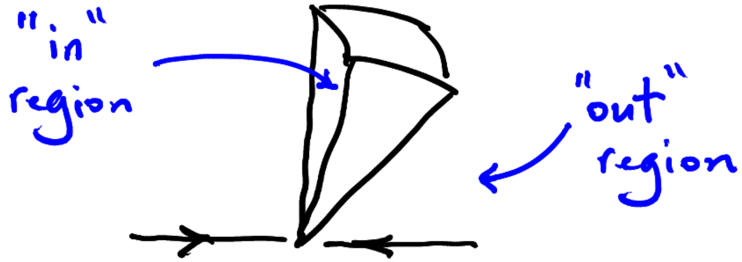
measurement function

$$A_n(E) = V_{E,E_n} D_n A_{n-1}(E_n) D_n^\dagger V_{E,E_n}^\dagger \, \Theta(E \leq E_n)$$

An IR finite formulation

e.g. $u_m = \prod_{i=1}^{m} \left[\Theta_{out}(q_i) + \Theta_{in}(q_i)\, \Theta(E_i < Q_0) \right]$

$= u(q_i)$



$\mu = Q_0$

Put $\Gamma = \Gamma_u + \bar{\Gamma}_u$

$\bar{\Gamma}_u \sim \int d\Omega_k \, (1 - u(k))\, W_{ij}(\hat{k}) \sim \int_{in} d\Omega_k$

$\Gamma_u \sim \int d\Omega_k \, u(k)\, W_{ij}(\hat{k}) \sim \int_{out} d\Omega_k$

Sum $\bar{\Gamma}_u$ and expand in Γ_u

$\Rightarrow$ IR poles cancel between real & virtual order-by-order

$$\Sigma_0 \sim \bar{V}\; H\; \bar{V} \qquad = \text{"global" part}$$

$$\Sigma_1 \sim \int_{\text{out}} \Big[\; \bar{V}\; D\; \bar{V}\; H\; \bar{V}\; D\; \bar{V} \qquad = \text{"non-global" part}$$

$$\qquad - \;\bar{V}\; H\; \bar{V}\; \tfrac{1}{2}D^2\; \bar{V}$$

$$\qquad - \;\bar{V}\; \tfrac{1}{2}D^2\; \bar{V}\; H\; \bar{V} \;\Big]$$

etc.

$$\Sigma_1 = -\frac{C_A C_F}{2}\,\zeta(2)\left(\frac{\alpha_s}{\pi}\right)^2 \log^2\left(Q/Q_0\right) \qquad (\text{"out"} = \text{hemisphere})$$

$$+\; 2\, C_A^2 C_F\, \zeta(3)\left(\frac{\alpha_s}{\pi}\right)^3 \frac{1}{3!}\log^3\left(Q/Q_0\right) \;+\; \cdots\cdots$$

Leading N_c : Σ_n is iterative solution to BMS

$$\frac{\partial g_{ab}(t)}{\partial t} = \int_{\text{out}} \frac{d\Omega_k}{4\pi}\, W_{ab}(\hat{k}) \left[\frac{V_{ak}\, V_{kb}}{V_{ab}}\, g_{ak}(t)\, g_{kb}(t) - g_{ab}(t) \right]$$

$t = \frac{C_A \alpha_s}{\pi} \log\left(E/Q_0\right)$

$V_{ij} = \exp\left[-t \int_{\text{in}} \frac{d\Omega_k}{4\pi}\, W_{ij}(\hat{k}) \right]$

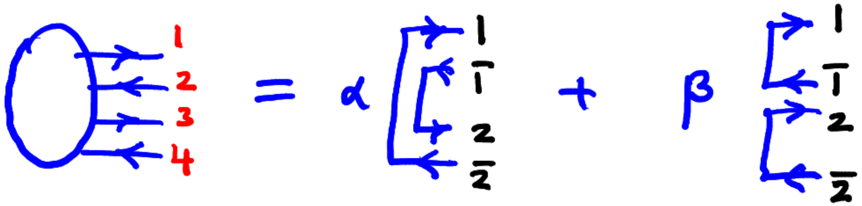
i.e. $\Sigma_n(E) = V_{ab}(t)\, g_{ab}^{(n)}(t)$

$g_{ab}^{(0)}(t) = 1$

Case of single $q\bar{q}$ pair

Colour evolution

Work in colour flow basis



i	c_i	$\bar{c}_i$
1	1	0
2	0	$\bar{1}$
3	2	0
4	0	$\bar{2}$

$$|M\rangle = \alpha \left|\begin{matrix}1 & 2\\ \bar{2} & \bar{1}\end{matrix}\right\rangle + \beta \left|\begin{matrix}1 & 2\\ \bar{1} & \bar{2}\end{matrix}\right\rangle$$

$$= \alpha\,|2\;1\rangle + \beta\,|1\;2\rangle$$

length of σ, τ

$$\langle\sigma|\tau\rangle = N_c^{\,n-\#(\sigma,\tau)}$$

transpositions by which σ & τ differ

e.g. $\langle 1\,2|1\,2\rangle = N_c^2$

$\langle 2\,1|1\,2\rangle = N_c$

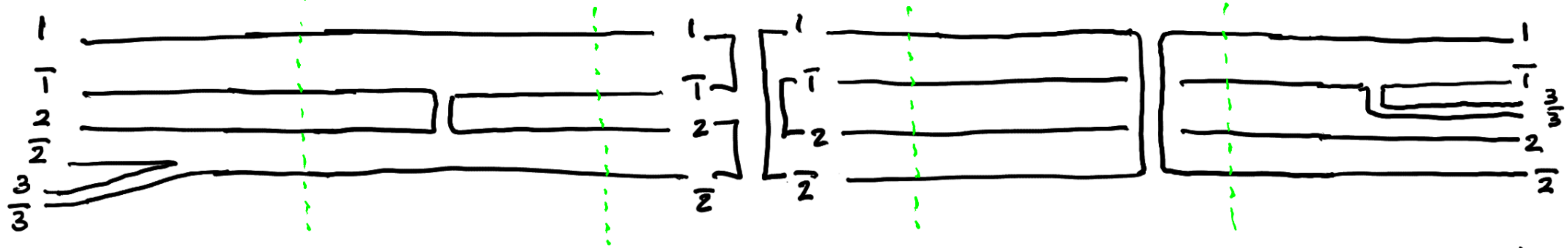
$$\sum_{\sigma} |\sigma\rangle[\sigma| = \mathbb{1} \qquad [\sigma|\tau\rangle = \langle\sigma|\tau] = \delta_{\sigma,\tau}$$

not orthonormal

"scalar product matrix"

$$\mathrm{Tr}\, A = \sum_{\tau,\sigma} [\tau|A|\sigma]\langle\sigma|\tau\rangle$$

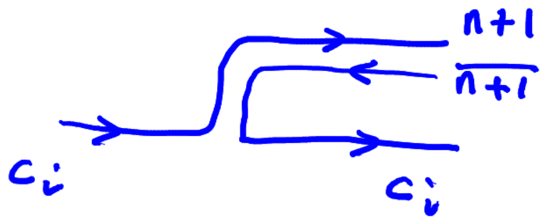
$$|\sigma_3\rangle[\sigma_3|D|\sigma_2\rangle[\sigma_2|V|\sigma_1\rangle[\sigma_1|H|\tau_1]\langle\tau_1|V^{\dagger}|\tau_2]\langle\tau_2|D^{\dagger}|\tau_3]\langle\tau_3|$$



$\sigma_3 = (3\,1\,2)$ $\quad \sigma_2 = (2\,1)$ $\quad \sigma_1 = (1\,2)$ $\quad \tau_1 = (2\,1)$ $\quad \tau_2 = (2\,1)$ $\quad \tau_3 = (2\,3\,1)$

Real emissions:

$$T_i = \lambda_i \, t_{c_i} - \bar{\lambda}_i \, \bar{t}_{\bar{c}_i} - \frac{1}{N_c} \left(\lambda_i - \bar{\lambda}_i \right) s$$



$$s \left| \begin{matrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{matrix} \right\rangle = \left| \begin{matrix} 1 & 2 & \cdots & n & n+1 \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) & n+1 \end{matrix} \right\rangle$$

$$t_\alpha \left| \begin{matrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{matrix} \right\rangle$$

$$= \left| \begin{matrix} 1 & \cdots & \alpha & \cdots & n & n+1 \\ \sigma(1) & \cdots & n+1 & \cdots & \sigma(n) & \sigma(\alpha) \end{matrix} \right\rangle$$

$\lambda_i = 1/\sqrt{2}$ out q
in $\bar{q}$
$\bar{\lambda}_i = 0$

$\lambda_i = 0$ in q
$\bar{\lambda}_i = 1/\sqrt{2}$ out $\bar{q}$

$\lambda_i = \bar{\lambda}_i = 1/\sqrt{2}$ g

$$T\,|\sigma_n\rangle \cdots\cdots\cdots \langle\tau_n|\,T \;=\; |\sigma_{n+1}\rangle \cdots \langle\tau_{n+1}|$$

if σ_n & τ_n differ by n transpositions

then σ_{n+1} & τ_{n+1} differ by n or $n+2$ transpositions

or $n+1$ transpositions

$t \ldots s$ $\quad 1/N_c$

$t \ldots t$

$s \ldots s$ $\quad 1/N_c^2$

$t \ldots t$

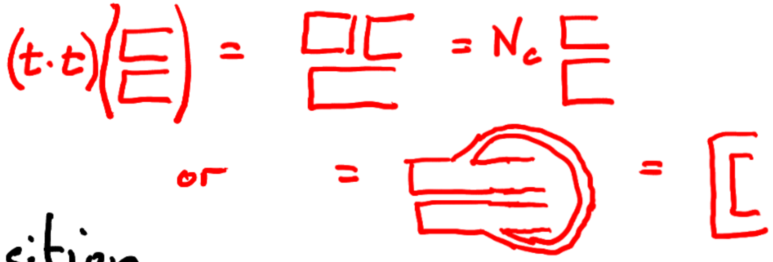
Note: cannot reduce # transpositions via real emissions

Virtual corrections

$$s \cdot t = t \cdot s = \mathbb{1}$$

$$s \cdot s = N_c \, \mathbb{1}$$

$$t \cdot t = N_c \, \mathbb{1} \quad \text{or} \quad 1 \text{ transposition}$$



$$\Gamma_\tau \left(\Gamma | \sigma \right\rangle = N_c \, \delta_{\tau\sigma} \, \Gamma_\sigma + \Sigma_{\sigma\tau} + \frac{1}{N_c} \delta_{\tau\sigma} \, \rho$$

$T_i \cdot T_j$

$\#(\sigma, \tau) = 1$

$$[\tau|e^{\Gamma}|\sigma\rangle = \sum_{l=0}^{\infty \to d} \frac{(-1)^l}{N_c^l} \sum_{k=0}^{l} \frac{(-\rho)^k}{k!} \sum_{\sigma_0,\sigma_1,\dots\sigma_{l-k}} \delta_{\tau\sigma_0}\,\delta_{\sigma_{l-k}\sigma} \prod_{\alpha=0}^{l-k-1} \Sigma_{\sigma_\alpha\sigma_{\alpha+1}}$$

$$\times\, R\left(\{\sigma_0,\sigma_1,\dots,\sigma_{l-k}\}\right)$$

{ Plätzer
Eur. Phys. J C (2014) 74
arXiv: 1312.2448 }

e.g. $d=1$

$$[\tau|e^{\Gamma}|\sigma\rangle = \delta_{\tau\sigma}\, e^{-N_c\Gamma_\sigma}\left(1+\rho/N_c\right) - \frac{1}{N_c}\Sigma_{\tau\sigma}\underbrace{\frac{e^{-N_c\Gamma_\tau}-e^{-N_c\Gamma_\sigma}}{\Gamma_\tau-\Gamma_\sigma}}$$

$= -N_c\, e^{-N_c\Gamma_\sigma}$ if $\sigma=\tau$

"N^dLC"

= much more than N^dLC for observables

Monte Carlo implementation

{ de Angelis, JF, Plätzer
based heavily on CVolver }

for each real choose next pair of colour states, σ_m & τ_m, so that they differ by $n, n+1$ or $n+2$ transpositions
($n = \#$ transpositions by which σ_{m-1} & τ_{m-1} differ $= \#_{\sigma_{m-1},\tau_{m-1}}$)

Do this by adding a colour line & making 0 or 1 flips making sure to exclude emission off same particle in amplitude & conjugate.

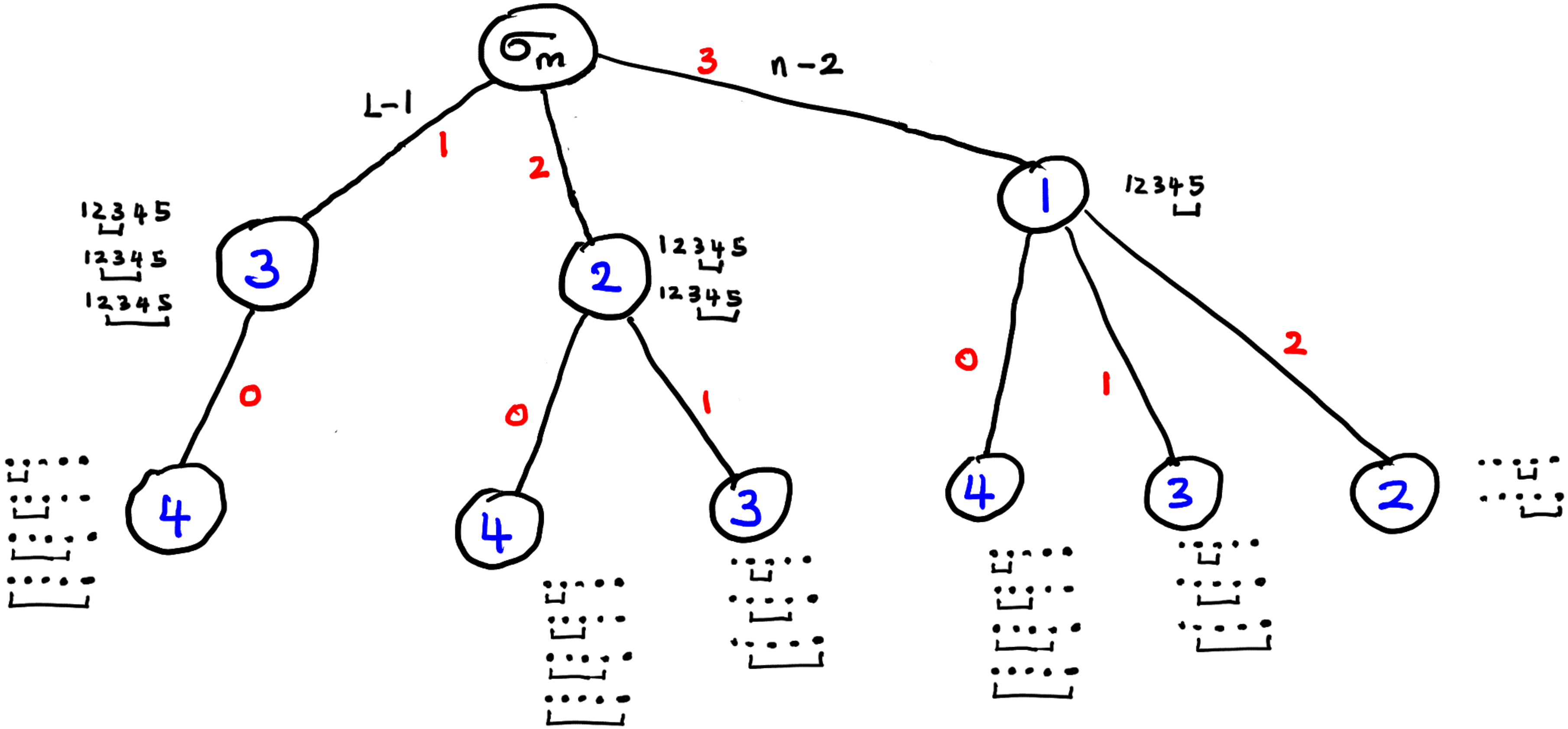
for every e^{Γ} virtual choose # flips to make, p, from a $(1/N_c)^p$ distribution up to $p_{max} = d$

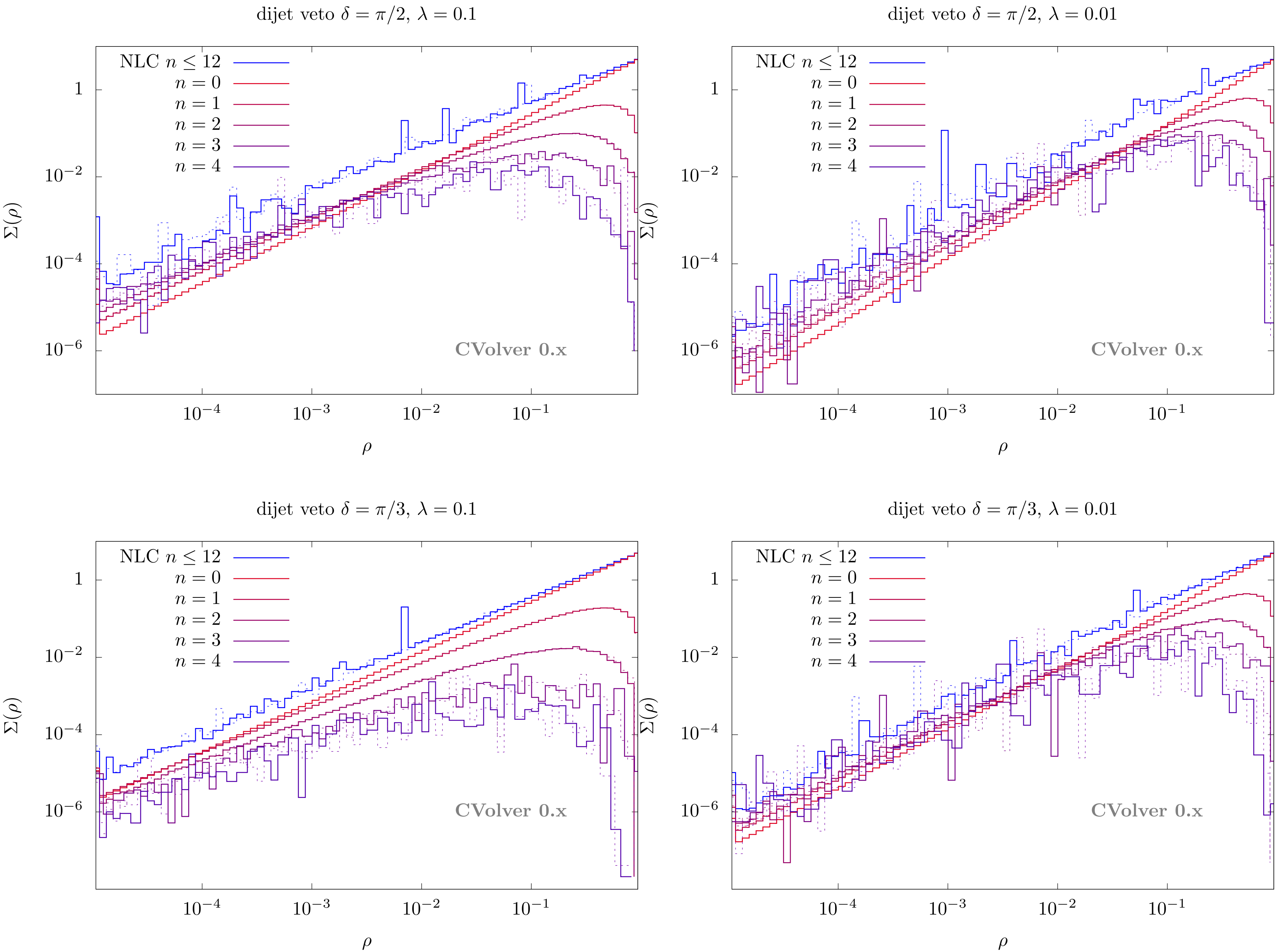
nb this can reduce $\#(\sigma_n,\tau_n)$

Currently no attempt to prevent $\#(\sigma_n,\tau_n)$ from becoming too large key component is how to select σ_{m+1} from σ_m

s.t. $\#(\sigma_{m+1}, \sigma_m) = L$

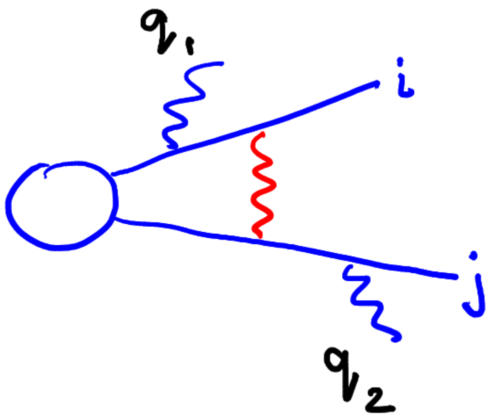
e.g. $n = 5$, $L = 2$ $\qquad \sigma_m = (1\ 2\ 3\ 4\ 5)$





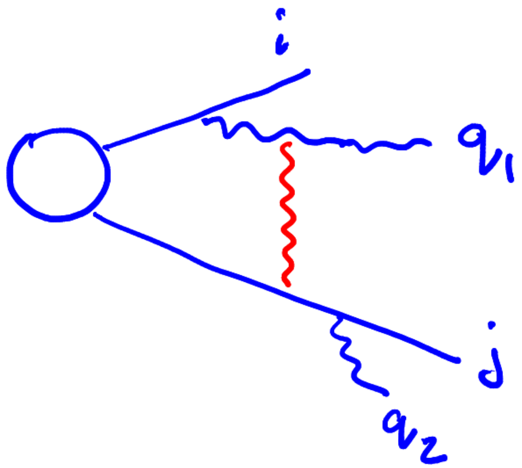
dijet veto $\delta = \pi/2$, $\lambda = 0.1$
dijet veto $\delta = \pi/2$, $\lambda = 0.01$
dijet veto $\delta = \pi/3$, $\lambda = 0.1$
dijet veto $\delta = \pi/3$, $\lambda = 0.01$
NLC $n \leq 12$
$n = 0$
$n = 1$
$n = 2$
$n = 3$
$n = 4$
$\Sigma(\rho)$
ρ
1
10^{-2}
10^{-4}
10^{-6}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
CVolver 0.x

The ordering variable



$$q_{1\perp}^{ij} = \left[\frac{2\, p_i\cdot q_1 \; p_j\cdot q_1}{p_i\cdot p_j}\right]^{1/2}$$

$$T_j \frac{p_j}{p_j\cdot k} \int_{\tilde{q}_2}^{\tilde{q}_1} \frac{dk_\perp}{k_\perp} \{\ldots\} \; T_i \frac{p_i}{p_i\cdot k}$$



$$T_j \frac{p_j}{p_j\cdot k} \int_{\tilde{q}_2}^{q_{1\perp}^{ij}} \frac{dk_\perp}{k_\perp} \{\ldots\} \; T_i \left(\frac{p_i}{p_i\cdot k} - \frac{p_j}{p_j\cdot k} \right)$$

$\sum_i$ 

$$= \sum_{i\neq j} \frac{i f^{abc}\, T_i^b\, T_j^c}{C_A} \left(\frac{p_i}{p_i\cdot k} - \frac{p_j}{p_j\cdot k} \right)$$

Bierenbaum, Czakon & Mitov
arXiv: 1107.4384

Thanks to Mike Seymour & René Ángeles Martínez

Highly non-trivial

- full 3 & 4 gluon vertices
- exact Θ functions

R. Angeles Martinez
JF & Seymour

arXiv: 1510.07998
1602.00623

$$“ \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1+\alpha\cos\phi}{1+2\alpha\cos\phi+\alpha^2} = \Theta\left(1-|\alpha|\right) ”$$

$$\alpha = k_\perp / (q_\perp^{ij})$$

Conclusions

* CVolver-based event generator seems to be feasible

  soft gluons in e^+e^- at present but framework is ready to accommodate a full-fledged parton shower

* Loop integrals simplify dramatically due to remarkable cancellations (Eikonal couplings <u>only</u> to external hard partons).

  $\rightsquigarrow$ Amplitude-level ordering in "dipole $k_\perp$"