Electroweak Logarithms in Inclusive Processes

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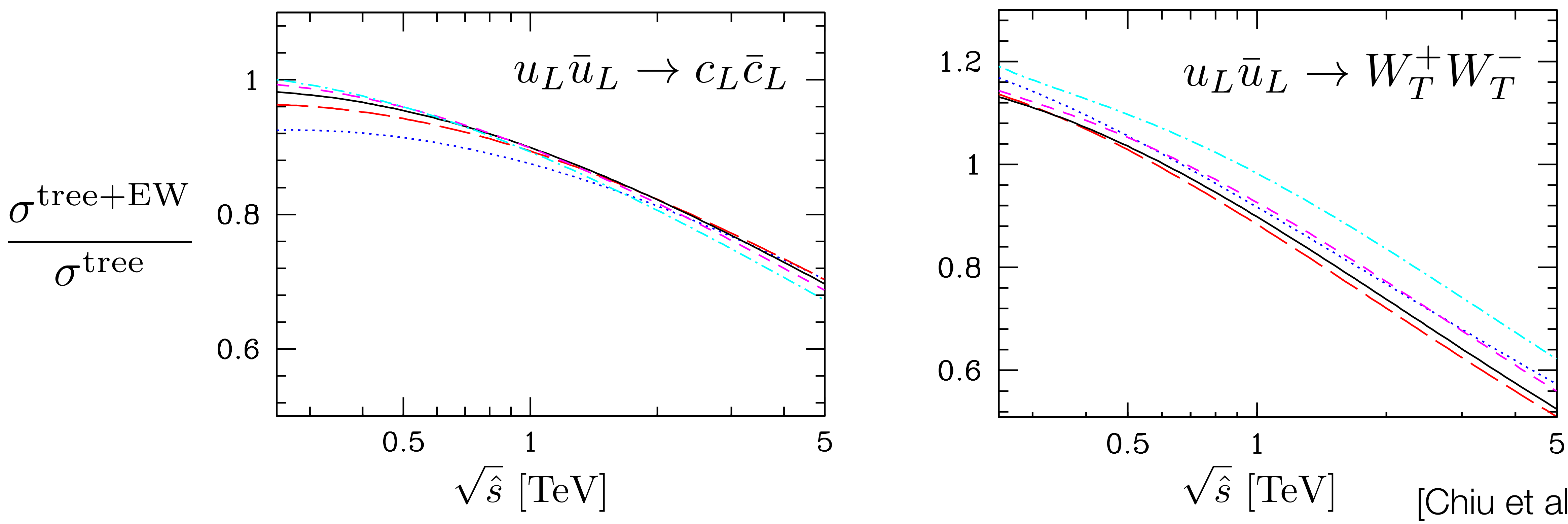
Outline

- Introduction
- Factorization
- Evolution
- Comparison and extensions
- EW gauge boson PDFs
- Conclusions

Based on arXiv:1802.08687, 1803.06347
with Aneesh Manohar and Bartosz Fornal

1. Introduction

Electroweak double logarithms



[Chiu et al.]

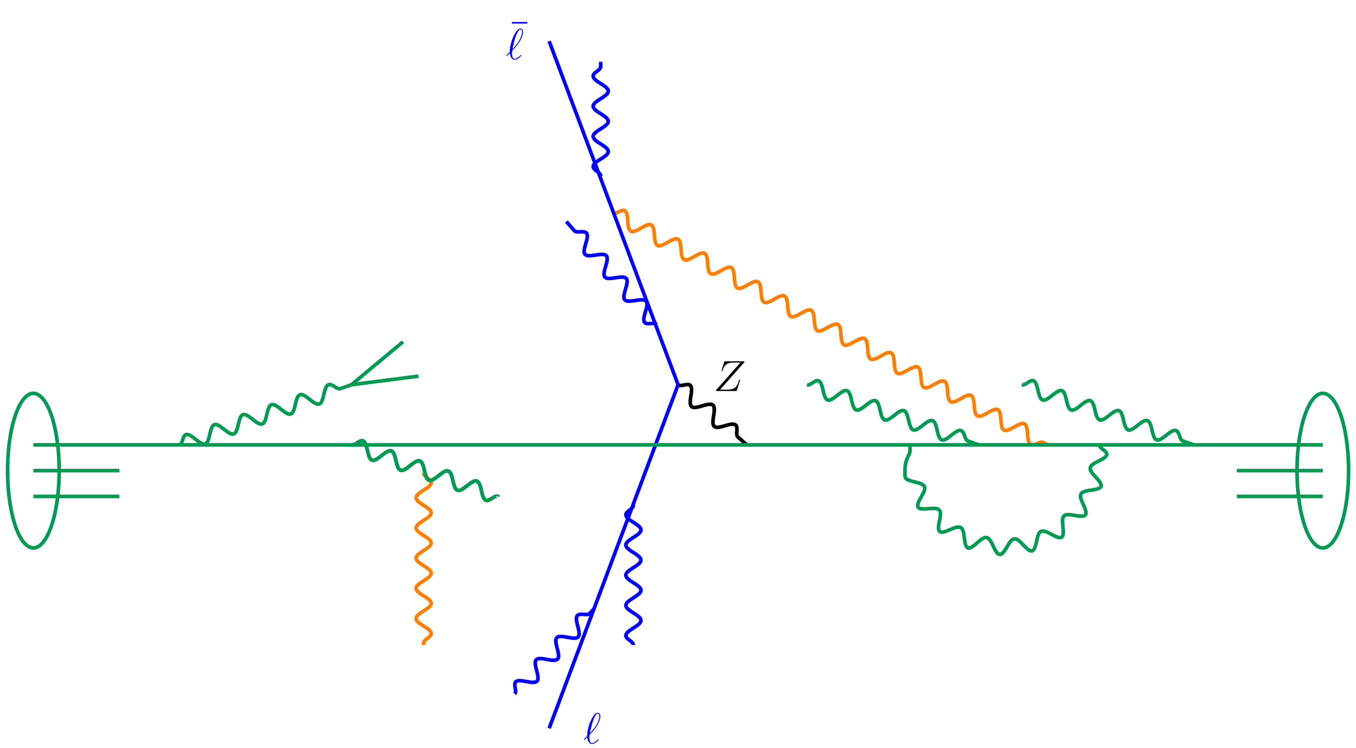
- At high energies Q, cross section contains $\alpha_W \ln^2(Q/M_W)$
  [Ciafaloni, Comelli; Kuhn, Penin; Fadin et al; Denner, Pozzorini; Chiu et al; …]
- $\mathcal{O}(10\%)$ effect at LHC, $\mathcal{O}(100\%)$ at FCC
- Problem for finding new physics in tails of distributions

Inclusive processes

- Exclusive production usually assumed: all W and Z resolved → only virtual corrections → EW double logs
- We consider inclusive processes, such as $pp \to \ell^+\ell^- X$, where the final state has invariant mass $Q^2 \gg M_W^2$
- Inclusive production also involves EW **double** logs [Ciafaloni et al] whereas QCD corrections only involve **single** logs

Electroweak resummation in inclusive processes

- We find that EW resummation is achieved by:
  - (Modified) DGLAP of PDFs **and** Fragmentation Functions
  - Soft function evolution
- Complications arise because initial/final-state particles are not electroweak singlets, e.g. $f_u \neq f_d$



2. Factorization

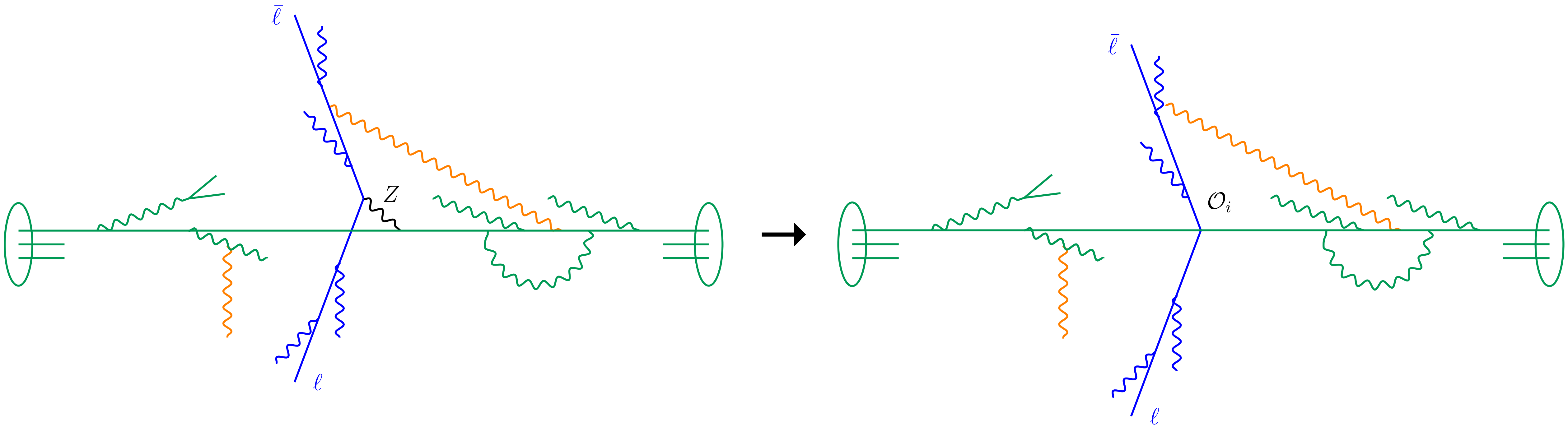
Hard matching

- Integrate out hard scattering at scale Q in **symmetric** phase

$$\mathcal{L}_{\text{hard}} = \sum_i \mathcal{H}_i O_i$$

$$\begin{aligned} O_{\ell q}^{(3)} &= (\bar{\ell}_1 \gamma^\mu t^a \ell_2)\,(\bar{q}_3 \gamma_\mu t^a q_4) \\ O_{\ell q} &= (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{q}_3 \gamma_\mu q_4) \\ O_{\ell u} &= (\bar{\ell}_1 \gamma^\mu \ell_2)\,(\bar{u}_3 \gamma_\mu u_4) \\ &\vdots \end{aligned}$$

- Remaining radiation is collinear or soft



Factorization of collinear and soft

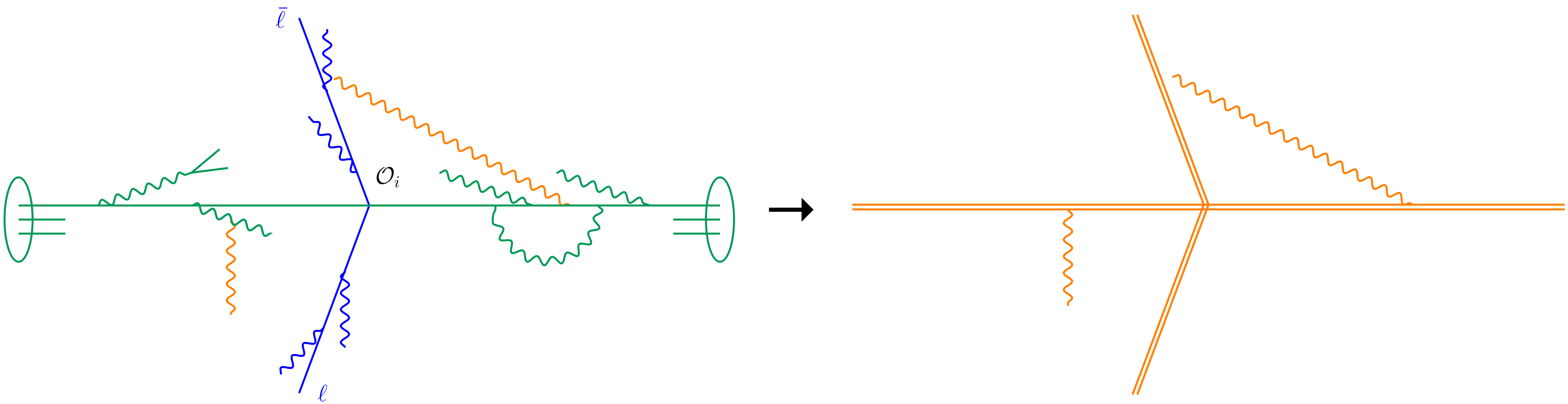
- Soft radiation is captured by Wilson lines

$$q \to \mathcal{S} q \quad \mathcal{S} = P \exp\left\{ \mathrm{i} \int_{-\infty}^{0} \mathrm{d}s\, n_4 \cdot \left[\underbrace{g_3 A_s(s\, n_4)}_{SU(3)} + \underbrace{g_2 W_s(s\, n_4)}_{SU(2)} + \underbrace{g_1 y_q B_s(s\, n_4)}_{U(1)} \right] \right\}$$

$$O_{\ell q}^{(3)} \to (\bar{\ell}_1 \mathcal{S}_1^\dagger \gamma^\mu t^a \mathcal{S}_2 \ell_2)(\bar{q}_3 \mathcal{S}_3^\dagger \gamma_\mu t^a \mathcal{S}_4 q_4)$$

$$\vdots$$

- There are also collinear Wilson lines (absorbed in PDFs/FFs)



Factorization of cross section

- Factorize cross section into PDFs, FFs and a soft function

$$\sigma \sim \sum_X \langle pp|\mathcal{L}_{\rm hard}|\mu^+\mu^- X\rangle\langle\mu^+\mu^- X|\mathcal{L}_{\rm hard}|pp\rangle$$

$$\sim |\mathcal{H}|^2 \underbrace{\langle p|\bar{q}_4 q_4|p\rangle}_{\rm PDF} \underbrace{\langle p|q_3\bar{q}_3|p\rangle}_{\rm PDF} \underbrace{\langle 0|\mathcal{S}_2^\dagger\mathcal{S}_1\mathcal{S}_4^\dagger\mathcal{S}_3\mathcal{S}_1^\dagger\mathcal{S}_2\mathcal{S}_3^\dagger\mathcal{S}_4|0\rangle}_{\rm soft}$$

$$\times \underbrace{\sum_{X_1}\langle 0|\ell_1|\mu^- X_1\rangle\langle\mu^- X_1|\bar{\ell}_1|p\rangle}_{\rm FF} \underbrace{\sum_{X_2}\langle 0|\bar{\ell}_2|\mu^+ X_2\rangle\langle\mu^+ X_2|\ell_2|p\rangle}_{\rm FF} + \ldots$$

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- **Nonsinglets** also contribute:

$$\langle p|\bar{q}_4 t^a q_4|p\rangle \quad \langle 0|\mathcal{S}_1 t^a \mathcal{S}_1^\dagger\, \mathcal{S}_2 t^b \mathcal{S}_2^\dagger|0\rangle \quad \cdots$$

- Can cancel soft Wilson lines **without** t^a in between: $S_i^\dagger S_i = 1$ (this is why for inclusive processes in QCD soft function is 1)

Matching onto broken phase

- Singlet and triplet fermion PDF are (essentially)

$$f_q^{(I=0)} \sim \langle p|\bar{q}q|p\rangle \qquad f_q^{(I=1)} \sim \langle p|\bar{q}t^a q|p\rangle$$

- Tree-level matching at the electroweak scale

$$\begin{aligned} f_u &= f_{u_R} \\ f_q^{(I=0)} &= f_{u_L} + f_{d_L} \\ f_q^{(I=1,I_3=0)} &= \frac{1}{2} f_{u_L} - \frac{1}{2} f_{d_L} \\ f_W^{(I=0)} &= f_{W^+} + f_{W^-} + \cos^2\theta_W f_Z \\ &\quad + \sin^2\theta_W f_\gamma + \sin\theta_W \cos\theta_W \left(f_{Z\gamma} + f_{\gamma Z}\right) \\ f_W^{(I=1,I_3=0)} &= f_{W^+} - f_{W^-} \\ &\vdots \end{aligned}$$

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&\quad + \sin^2\theta_W f_\gamma + \sin\theta_W \cos\theta_W (f_{Z\gamma} + f_{\gamma Z}) \\
f_W^{(I=1,I_3=0)} &= f_{W^+} - f_{W^-} \\
&\vdots
\end{aligned}$$

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3. Evolution

Rapidity divergences

- For transverse mom. factorization, rapidity divergences appear

$$S^{(1)}(p_T) \propto \alpha_s \, \frac{\mu^{2\epsilon}}{p_T^{1+2\epsilon}} \int \mathrm{d}y$$

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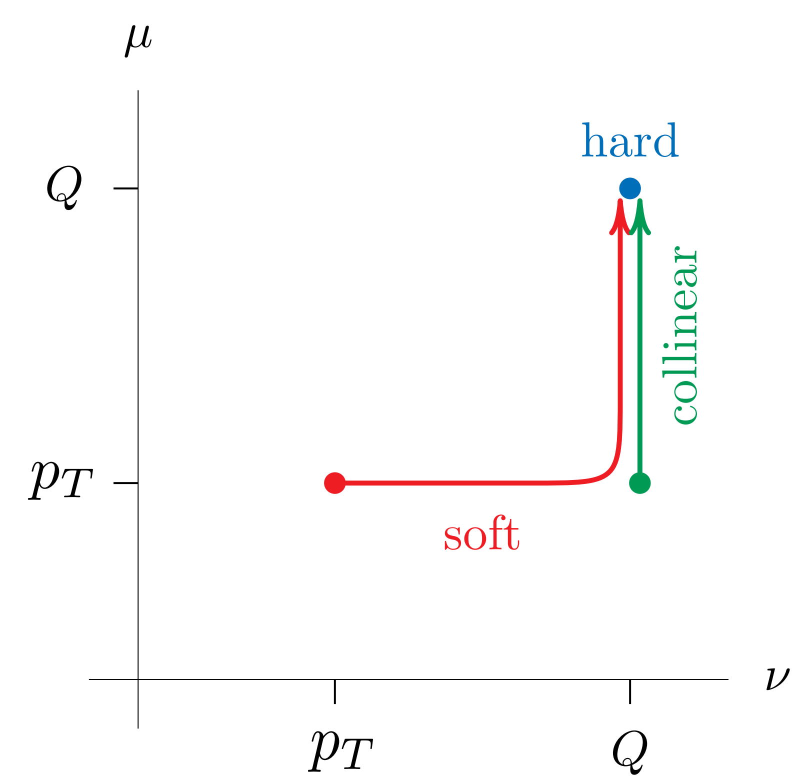
$$S^{(1)}(p_T) \propto \alpha_s \frac{\mu^{2\epsilon}\,\nu^{\eta}}{p_T^{1+2\epsilon+\eta}} \int \mathrm{d}y\, |2\sinh y|^{-\eta}$$

- We use the η-regulator, which acts very similar to dim. reg.
  [Chiu, Jain, Neill, Rothstein]

- Soft function contains $\ln \frac{\nu}{p_T}$
  Collinear function has $\ln \frac{\nu}{\bar{n}\cdot r} \sim \ln \frac{\nu}{Q}$

- ν-evolution sums single logs of Q/p_T



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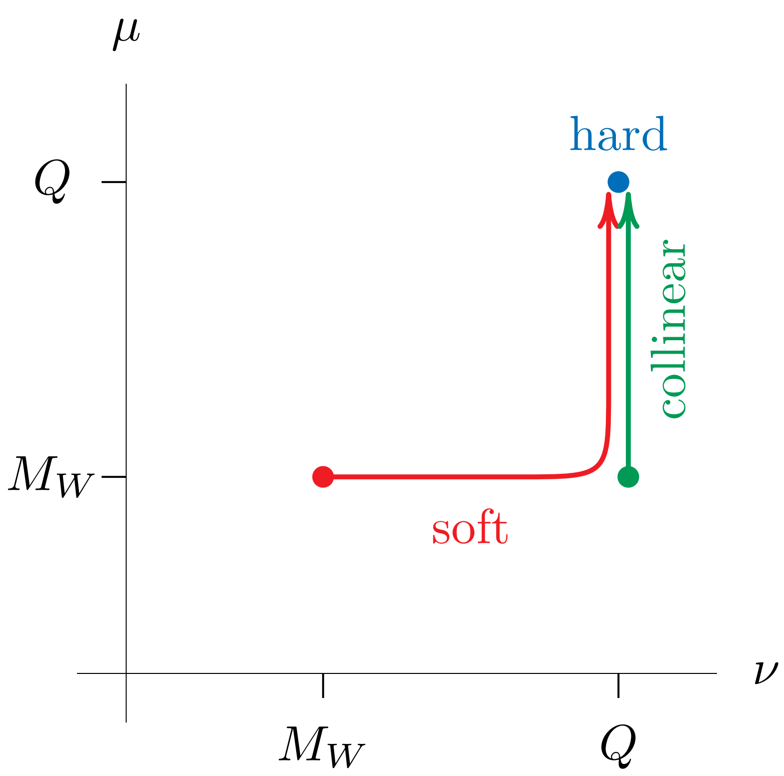
- Electroweak correction to nonsinglets have rapidity divergences ($p_T \to M_W$)



RG equations

- PDFs (and FFs):

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_i(x,\mu,\nu) = \sum_j \int_0^1 \frac{\mathrm{d}z}{z}\,\frac{\alpha}{\pi}\,\hat{\gamma}_{\mu,ij}(z,\mu,\nu)\, f_j\!\left(\frac{x}{z},\mu,\nu\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} f_i(x,\mu,\nu) = \frac{\alpha}{\pi}\,\hat{\gamma}_{\nu,i}(\mu,\nu)\, f_i(x,\mu,\nu)$$

- Soft function:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \mathcal{S}(\mu,\nu) = \frac{\alpha}{\pi}\,\hat{\gamma}_{\mu,\mathcal{S}}(\mu,\nu)\,\mathcal{S}(\mu,\nu)$$

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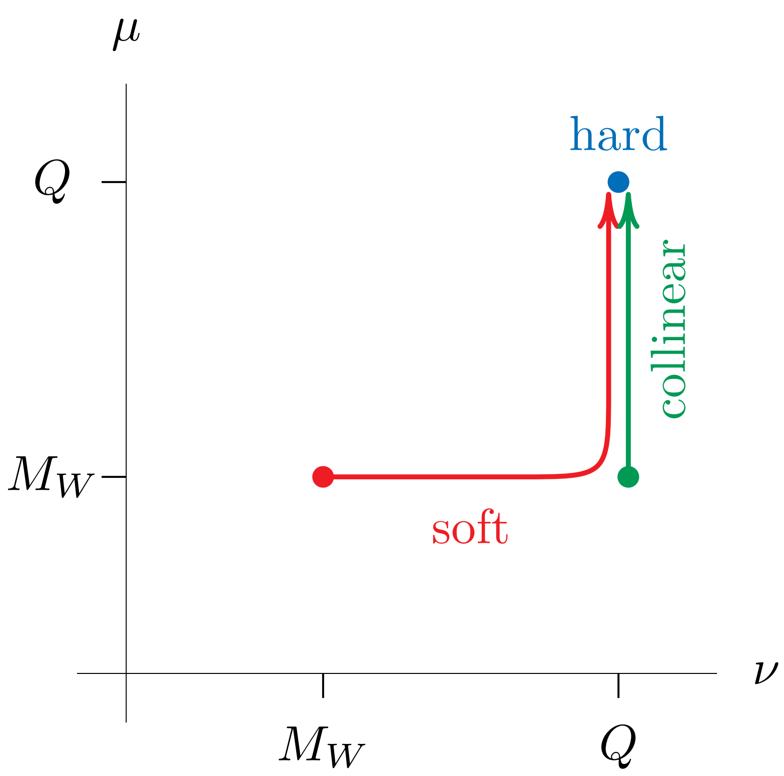


Fermion PDF anomalous dimension

- Singlet and triplet:

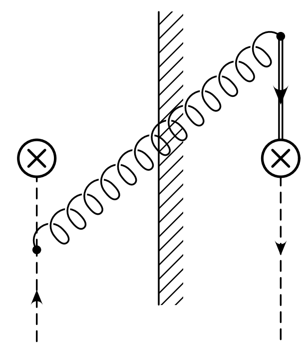
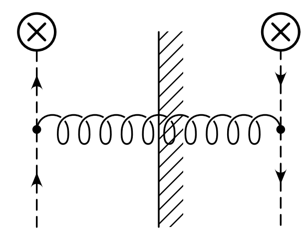
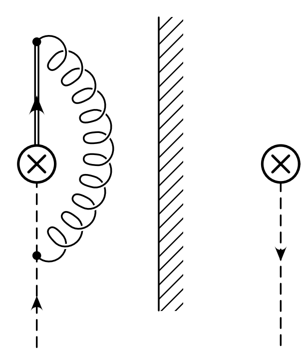
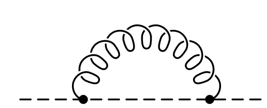
$$f_q^{(I=0)} \sim \langle p|\bar{q}q|p\rangle$$

$$f_q^{(I=1)} \sim \langle p|\bar{q}t^a q|p\rangle$$

- Virtual diagrams have c_F
- Real diagrams have

$$t^b t^b = c_F$$

$$t^b t^a t^b = (c_F - \tfrac{1}{2}c_A)t^a$$

Graph	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\frac{2}{(1-z)_+} - z - 2 - 2\delta(1-z)\ln\frac{\nu}{\bar{n}\cdot r}$	$-\ln\frac{\mu^2}{M^2}$
	z	0
Total_1	$\frac{2}{(1-z)_+} - 2 - 2\delta(1-z)\ln\frac{\nu}{\bar{n}\cdot r}$	$-\ln\frac{\mu^2}{M^2}$
	$\left(2\ln\frac{\nu}{\bar{n}\cdot r} + 1\right)\delta(1-z)$	$\ln\frac{\mu^2}{M^2}$
	$\delta(1-z)$	0
Total_2	$\left(2\ln\frac{\nu}{\bar{n}\cdot r} + 2\right)\delta(1-z)$	$\ln\frac{\mu^2}{M^2}$

Fermion PDF and FF

$$\hat{\gamma}^{(R)}_{\mu,qq} = c_{qq}(R) P_{qq}(z) + [c_F - c_{qq}(R)] \left(2 \ln \frac{\nu}{\bar{n} \cdot r} + \frac{3}{2}\right) \delta(1-z)$$

$$\hat{\gamma}^{(R)}_{\nu,q} = [c_F - c_{qq}(R)] \ln \frac{\mu^2}{M^2}$$

- Group theory: $c_{qq}(1) = c_F \,, \quad c_{qq}(\text{adj}) = c_F - \frac{1}{2} c_A \,.$
- Adjoint representation has **double** logs and rapidity logs

Fermion PDF and FF

$$\hat{\gamma}^{(R)}_{\mu,qq} = c_{qq}(R)P_{qq}(z) + [c_F - c_{qq}(R)]\left(2\,\ln\frac{\nu}{\bar{n}\cdot r} + \frac{3}{2}\right)\delta(1-z)$$

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- Group theory: $c_{qq}(1) = c_F\,, \quad c_{qq}(\text{adj}) = c_F - \frac{1}{2}c_A\,.$
- Adjoint representation has **double** logs and rapidity logs
- At one loop, anomalous dimensions of FF related to PDF
- If final-state particle not observed, completeness gives

$$\sum_h \int_0^1 \mathrm{d}x\, x\, D^{(I=0)}_{q\to h}(x,\mu,\nu) = 1 \qquad \sum_h \int_0^1 \mathrm{d}x\, x\, D^{(I=1)}_{q\to h}(x,\mu,\nu) = 0$$

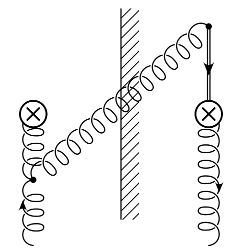
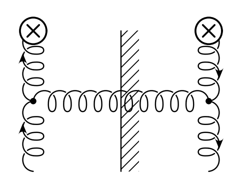
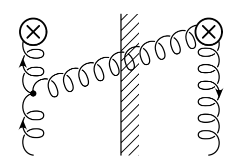
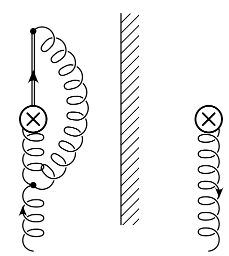
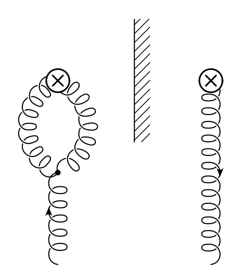
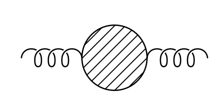
Gauge boson PDF anomalous dimension

- Virtual diagrams have c_A
- Real diagrams have

$$f_W^{(I=0)} : \quad c_A$$

$$f_W^{(I=1)} : \quad \frac{1}{2} c_A$$

$$f_W^{(I=2)} : \quad -1$$

Graph	$P_{G_+G_+}$		$P_{G_+G_-}$	
	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\frac{2}{(1-z)_+} - 1 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$	$-\ln\frac{\mu^2}{M^2}$	0	0
	$\frac{1}{z} + 1 - z^2$	0	$\frac{(1-z)^3}{z}$	0
	$-1 - z$	0	0	0
Total$_1$	$\frac{2}{(1-z)_+} + \frac{1}{z} - 1 - z - z^2 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$	$-\ln\frac{\mu^2}{M^2}$	$\frac{(1-z)^3}{z}$	0
	$c_A\left(2\ln\frac{\nu}{\bar{n}\cdot r} + \frac{5}{2}\right)\delta(1-z)$	$c_A\ln\frac{\mu^2}{M^2}$	0	0
	$-\frac{3}{2}c_A\delta(1-z)$	0	0	0
	$\left(\frac{b_0}{2} - c_A\right)\delta(1-z)$	0	0	0
Total$_2$	$\left(\frac{b_0}{2} + 2c_A\ln\frac{\nu}{\bar{n}\cdot r}\right)\delta(1-z)$	$c_A\ln\frac{\mu^2}{M^2}$	0	0

Gauge boson PDF anomalous dimension

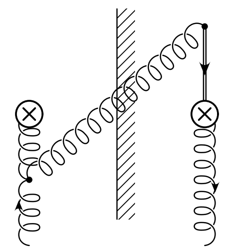
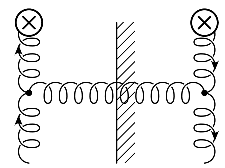
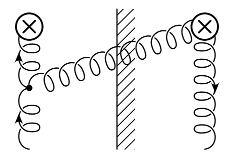
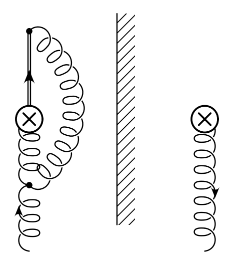
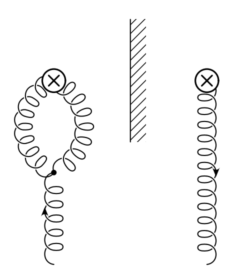
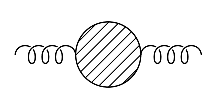
- Virtual diagrams have c_A
- Real diagrams have

$$f_W^{(I=0)} : \; c_A$$

$$f_W^{(I=1)} : \; \frac{1}{2} c_A$$

$$f_W^{(I=2)} : \; -1$$

- Fermions and gauge bosons of **same** rep mix
- Evolution polarizes gauge bosons, due to mixing and $f_{u_L} \neq f_{u_R}$

Graph	$P_{G_+G_+}$		$P_{G_+G_-}$	
	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\frac{2}{(1-z)_+} - 1 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$	$-\ln\frac{\mu^2}{M^2}$	0	0
	$\frac{1}{z} + 1 - z^2$	0	$\frac{(1-z)^3}{z}$	0
	$-1-z$	0	0	0
Total$_1$	$\frac{2}{(1-z)_+} + \frac{1}{z} - 1 - z - z^2 - 2\ln\frac{\nu}{\bar{n}\cdot r}\,\delta(1-z)$	$-\ln\frac{\mu^2}{M^2}$	$\frac{(1-z)^3}{z}$	0
	$c_A\left(2\ln\frac{\nu}{\bar{n}\cdot r} + \frac{5}{2}\right)\delta(1-z)$	$c_A\ln\frac{\mu^2}{M^2}$	0	0
	$-\frac{3}{2}c_A\delta(1-z)$	0	0	0
	$\left(\frac{b_0}{2} - c_A\right)\delta(1-z)$	0	0	0
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PDF evolution in Standard Model

Includes:

- $SU(3)$x$SU(2)$x$U(1)$
- Yukawa's
- Spin dependence
- Higgs
- Longitudinal W, Z
- γZ interference

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{q,r,s}^{(I=1)} &= \frac{\alpha_3}{\pi}\frac{4}{3}\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=1)} \\
&+\frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=1)}+\Gamma_1 f_{q,r,s}^{(I=1)}(z)+\frac{1}{4}N_c\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{W_+}^{(I=1)}+\frac{1}{4}N_c\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{W_-}^{(I=1)}\right] \\
&+\frac{\alpha_1}{\pi}\mathrm{y}_q^2\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=1)}+\frac{g_1g_2}{4\pi^2}\mathrm{y}_qN_c\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes\left(f_{W_+B_+}^{(I=1)}+f_{B_+W_+}^{(I=1)}\right) \\
&+\frac{g_1g_2}{4\pi^2}\mathrm{y}_qN_c\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes\left(f_{W_-B_-}^{(I=1)}+f_{B_-W_-}^{(I=1)}\right) \\
&+\frac{Y_t^2}{4\pi^2}\left[-\frac{1}{8}\delta_{r3}f_{q,3,s}^{(I=1)}(z)-\frac{1}{8}\delta_{s3}f_{q,r,3}^{(I=1)}(z)+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f_{\bar{H}}^{(I=1)}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{\ell,r,s}^{(I=1)} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{Q_-Q_-}\otimes f_{\ell,r,s}^{(I=1)}+\Gamma_1 f_{\ell,r,s}^{(I=1)}(z)+\frac{1}{4}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{W_+}^{(I=1)}+\frac{1}{4}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{W_-}^{(I=1)}\right] \\
&+\frac{\alpha_1}{\pi}\mathrm{y}_\ell^2\widetilde{P}_{Q_-Q_-}\otimes f_{\ell,r,s}^{(I=1)}+\frac{g_1g_2}{4\pi^2}\mathrm{y}_\ell\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes\left(f_{W_+B_+}^{(I=1)}+f_{B_+W_+}^{(I=1)}\right) \\
&+\frac{g_1g_2}{4\pi^2}\mathrm{y}_\ell\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes\left(f_{W_-B_-}^{(I=1)}+f_{B_-W_-}^{(I=1)}\right),
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{W_\pm}^{(I=1)} &= \frac{\alpha_2}{\pi}\Bigg[\widetilde{P}_{G_\pm G_+}\otimes f_{W_+}^{(I=1)}+\widetilde{P}_{G_\pm G_-}\otimes f_{W_-}^{(I=1)}+\Gamma_2 f_{W_\pm}^{(I=1)}(z)+P_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},\bar{\ell}\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=1)} \\
&+P_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\ell,\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=1)}+P_{G_\pm H}(z)\otimes\sum_{i=H,\bar{H}}f_i^{(I=1)}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{W_\pm B_\pm}^{(I=1)} &= \left[\frac{\alpha_2}{\pi}\Gamma_3+\frac{\alpha_1}{\pi}\frac{1}{4}b_{0,1}\right]f_{W_\pm B_\pm}^{(I=1)}(z)+\frac{g_1g_2}{4\pi^2}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{i=q,\ell,r=1,\ldots,n_g}\mathrm{y}_if_{i,r,r}^{(I=1)} \\
&-\frac{g_1g_2}{4\pi^2}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{i=\bar{q},\bar{\ell},r=1,\ldots,n_g}f_{i,r,r}^{(I=1)},
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{H}^{(I=1)} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{HH}\otimes f_{H}^{(I=1)}+\Gamma_4 f_{H}^{(I=1)}(z)+\frac{1}{4}\widetilde{P}_{HG_+}\otimes f_{W_+}^{(I=1)}+\frac{1}{4}\widetilde{P}_{HG_-}\otimes f_{W_-}^{(I=1)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_H^2\widetilde{P}_{HH}\otimes f_{H}^{(I=1)}\right]+\frac{Y_t^2}{8\pi^2}\left[z\otimes f_{\bar{q},3,3}^{(I=1)}-N_cf_H^{(I=1)}(z)\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{\widetilde{H}H}^{(I=1)} &= \frac{\alpha_2}{\pi}\left[-\frac{1}{4}\widetilde{P}_{HH}\otimes f_{\widetilde{H}H}^{(I=1)}+\Gamma_4 f_{\widetilde{H}H}^{(I=1)}(z)\right] \\
&+\frac{\alpha_1}{\pi}\left[-\mathrm{y}_H^2\widetilde{P}_{HH}\otimes f_{\widetilde{H}H}^{(I=1)}+2\mathrm{y}_H^2\Gamma_4 f_{\widetilde{H}H}^{(I=1)}(z)\right]-\frac{Y_t^2}{8\pi^2}N_cf_{\widetilde{H}H}^{(I=1)}(z).
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{B_\pm}^{(I=0)} &= \frac{\alpha_1}{\pi}\Bigg[\frac{1}{2}b_{0,1}f_{B_\pm}^{(I=0)}(z)+\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},u,d,\bar{\ell},e\\ r=1,\ldots,n_g}}\mathrm{y}_i^2f_{i,r,r}^{(I=0)} \\
&+\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\bar{u},\bar{d},\ell,\bar{e}\\ r=1,\ldots,n_g}}\mathrm{y}_i^2f_{i,r,r}^{(I=0)}+\mathrm{y}_H^2\widetilde{P}_{G_\pm H}\otimes\sum_{i=H,\bar{H}}f_i^{(I=0)}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{H}^{(I=0)} &= \frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{HH}(z)\otimes f_{H}^{(I=0)}+\frac{1}{2}\widetilde{P}_{HG_+}\otimes f_{W_+}^{(I=0)}+\frac{1}{2}\widetilde{P}_{HG_-}\otimes f_{W_-}^{(I=0)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_H^2\widetilde{P}_{HH}(z)\otimes f_{H}^{(I=0)}+\mathrm{y}_H^2\widetilde{P}_{HG_+}\otimes f_{B_+}^{(I=0)}+\mathrm{y}_H^2\widetilde{P}_{HG_-}\otimes f_{B_-}^{(I=0)}\right] \\
&+\frac{Y_t^2}{8\pi^2}\left[z\otimes\left(f_{\bar{q},3,3}^{(I=0)}+2f_{\bar{u},3,3}^{(I=0)}\right)-N_cf_H^{(I=0)}(z)\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{q,r,s}^{(I=0)}(z) &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=0)}+\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{g_+}^{(I=0)}+\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{g_-}^{(I=0)}\right] \\
&+\frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=0)}+\frac{N_c}{2}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{W_+}^{(I=0)}+\frac{N_c}{2}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{W_-}^{(I=0)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_q^2\widetilde{P}_{Q_-Q_-}\otimes f_{q,r,s}^{(I=0)}+2N_c\mathrm{y}_q^2\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{B_+}^{(I=0)}+2N_c\mathrm{y}_q^2\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{B_-}^{(I=0)}\right] \\
&+\frac{Y_t^2}{4\pi^2}\Bigg[\delta_{r3}\delta_{s3}(1-z)\otimes f_{u,3,3}^{(I=0)}-\frac{1}{8}\delta_{r3}f_{q,3,s}^{(I=0)}(z)-\frac{1}{8}\delta_{s3}f_{q,r,3}^{(I=0)}(z) \\
&+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f_{\bar{H}}^{(I=0)}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{u,r,s}^{(I=0)} &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_+Q_+}\otimes f_{u,r,s}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f_{g_+}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f_{g_-}^{(I=0)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_u^2\widetilde{P}_{Q_+Q_+}\otimes f_{u,r,s}^{(I=0)}+N_c\mathrm{y}_u^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f_{B_+}^{(I=0)}+N_c\mathrm{y}_u^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f_{B_-}^{(I=0)}\right] \\
&+\frac{Y_t^2}{4\pi^2}\Bigg[\frac{1}{2}(1-z)\delta_{r3}\delta_{s3}\otimes f_{q,3,3}^{(I=0)}-\frac{1}{4}\delta_{r3}f_{u,3,s}^{(I=0)}(z)-\frac{1}{4}\delta_{s3}f_{u,r,3}^{(I=0)}(z) \\
&+\frac{N_c}{2}\delta_{r3}\delta_{s3}\,1\otimes f_{H}^{(I=0)}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{d,r,s}^{(I=0)} &= \frac{\alpha_3}{\pi}\left[\frac{4}{3}\widetilde{P}_{Q_+Q_+}\otimes f_{d,r,s}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f_{g_+}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f_{g_-}^{(I=0)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_d^2\widetilde{P}_{Q_+Q_+}\otimes f_{d,r,s}^{(I=0)}+N_c\mathrm{y}_d^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f_{B_+}^{(I=0)}+N_c\mathrm{y}_d^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f_{B_-}^{(I=0)}\right],
\end{aligned}
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{\ell,r,s}^{(I=0)} &= \frac{\alpha_2}{\pi}\left[\frac{3}{4}\widetilde{P}_{Q_-Q_-}\otimes f_{\ell,r,s}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{W_+}^{(I=0)}+\frac{1}{2}\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{W_-}^{(I=0)}\right] \\
&+\frac{\alpha_1}{\pi}\left[\mathrm{y}_\ell^2\widetilde{P}_{Q_-Q_-}\otimes f_{\ell,r,s}^{(I=0)}+\mathrm{y}_\ell^2\delta_{rs}\widetilde{P}_{Q_-G_+}\otimes f_{B_+}^{(I=0)}+\mathrm{y}_\ell^2\delta_{rs}\widetilde{P}_{Q_-G_-}\otimes f_{B_-}^{(I=0)}\right],
\end{aligned}
$$

$$
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{e,r,s}^{(I=0)} = \frac{\alpha_1}{\pi}\left[\mathrm{y}_e^2\widetilde{P}_{Q_+Q_+}\otimes f_{e,r,s}^{(I=0)}+\mathrm{y}_e^2\delta_{rs}\widetilde{P}_{Q_+G_+}\otimes f_{B_+}^{(I=0)}+\mathrm{y}_e^2\delta_{rs}\widetilde{P}_{Q_+G_-}\otimes f_{B_-}^{(I=0)}\right],
$$

$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{g_\pm}^{(I=0)} &= \frac{\alpha_3}{\pi}\Bigg[3\widetilde{P}_{G_\pm G_+}\otimes f_{g_+}^{(I=0)}+3\widetilde{P}_{G_\pm G_-}\otimes f_{g_-}^{(I=0)}+\frac{1}{2}b_{0,3}f_{g_\pm}^{(I=0)}(z) \\
&+\frac{4}{3}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},u,d,\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=0)}+\frac{4}{3}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\bar{u},\bar{d}\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=0)}\Bigg],
\end{aligned}
$$

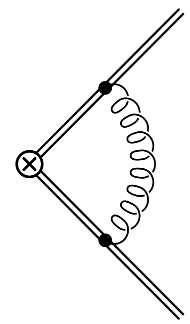
$$
\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{W_\pm}^{(I=0)} &= \frac{\alpha_2}{\pi}\Bigg[2\widetilde{P}_{G_\pm G_+}(z)\otimes f_{W_+}^{(I=0)}+2\widetilde{P}_{G_\pm G_-}(z)\otimes f_{W_-}^{(I=0)}+\frac{1}{2}b_{0,2}f_{W_\pm}^{(I=0)}(z) \\
&+\frac{3}{4}\widetilde{P}_{G_\pm Q_+}\otimes\sum_{\substack{i=\bar{q},\bar{\ell}\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=0)}+\frac{3}{4}\widetilde{P}_{G_\pm Q_-}\otimes\sum_{\substack{i=q,\ell,\\ r=1,\ldots,n_g}}f_{i,r,r}^{(I=0)}+\frac{3}{4}\widetilde{P}_{G_\pm H}\otimes\sum_{i=H,\bar{H}}f_i^{(I=0)}\Bigg],
\end{aligned}
$$

$$
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}f_{W_\pm}^{(I=2)} = \frac{\alpha_2}{\pi}\left[-\widetilde{P}_{G_\pm G_+}(z)\otimes f_{W_+}^{(I=2)}-\widetilde{P}_{G_\pm G_-}(z)\otimes f_{W_-}^{(I=2)}+\left(\frac{b_{0,2}}{2}+6\ln\frac{\nu}{\bar{n}\cdot r}\right)f_{W_\pm}^{(I=2)}(z)\right]
$$

$$
\begin{aligned}
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f_i^{(I=0)} &= 0, \\
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f_i^{(I=1,I_3=0)} &= \frac{\alpha_2}{\pi}\ln\frac{\mu^2}{M_W^2}f_i^{(I=1,I_3=0)}, \\
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f_i^{(I=2,I_3=0)} &= \frac{3\alpha_2}{\pi}\ln\frac{\mu^2}{M_W^2}f_i^{(I=2,I_3=0)},
\end{aligned}
$$

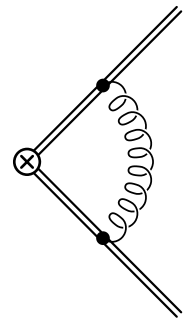
$$
\begin{aligned}
\nu\frac{\mathrm{d}}{\mathrm{d}\nu}f_{\widetilde{H}H}^{(I=1,I_3=1)} &= \left[\frac{\alpha_2}{2\pi}\ln\frac{\mu^2}{M_W^2}+\frac{(\alpha_2+4\mathrm{y}_H^2\alpha_1)}{2\pi}\ln\frac{\mu^2}{M_Z^2}\right]f_{\widetilde{H}H}^{(I=1,I_3=1)} \\
&= \left[\frac{\alpha_2}{2\pi}\ln\frac{\mu^2}{M_W^2}+\frac{\alpha_{\mathrm{em}}}{2\pi\sin^2\theta_W\cos^2\theta_W}\ln\frac{\mu^2}{M_Z^2}\right]f_{\widetilde{H}H}^{(I=1,I_3=1)}.
\end{aligned}
$$

Soft function anomalous dimension

Graph	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\ln \frac{(-n_i \cdot n_j - \mathrm{i}0)\nu^2}{2\mu^2}$	$\ln \frac{\mu^2}{M^2}$

- $S_{12\ldots m}^{a_1 a_2 \ldots a_m} = \langle 0 | \mathrm{tr}[(\mathcal{S}_1 t^{a_1} S_1^\dagger)(\mathcal{S}_2 t^{a_2} S_2^\dagger) \ldots (\mathcal{S}_m t^{a_m} S_m^\dagger)] | 0 \rangle$
- Wilson line direction of $\mathcal{S}_i$ denoted by $n_i = \pm(1, \hat{n}_i)$
- ν-evolution cancels against collinear: $\hat{\gamma}_\nu = -\frac{1}{2} m \, c_A \ln \frac{\mu^2}{M^2}$

Soft function anomalous dimension

Graph	$\hat{\gamma}_\mu$	$\hat{\gamma}_\nu$
	$\ln \frac{(-n_i \cdot n_j - \mathrm{i}0)\nu^2}{2\mu^2}$	$\ln \frac{\mu^2}{M^2}$

- $S_{12\ldots m}^{a_1 a_2 \ldots a_m} = \langle 0 | \mathrm{tr}[(\mathcal{S}_1 t^{a_1} S_1^\dagger)(\mathcal{S}_2 t^{a_2} S_2^\dagger) \ldots (\mathcal{S}_m t^{a_m} S_m^\dagger)] | 0 \rangle$
- Wilson line direction of $\mathcal{S}_i$ denoted by $n_i = \pm(1, \hat{n}_i)$
- ν-evolution cancels against collinear: $\hat{\gamma}_\nu = -\frac{1}{2} m\, c_A \ln \frac{\mu^2}{M^2}$
- For two Wilson line directions

$$\langle 0 | \mathcal{S}_1 t^a \mathcal{S}_1^\dagger\, \mathcal{S}_2 t^b \mathcal{S}_2^\dagger | 0 \rangle : \quad \hat{\gamma}_\mu = c_A \Big[\ln \frac{\mu^2}{\nu^2} - \ln \Big| \frac{n_1 \cdot n_2}{2} \Big| \Big]$$

- In- vs. outgoing Wilson line does not matter

Mixing and angular dependence

- For four Wilson line directions, there are multiple *SU*(2) reps.:

$$\langle 0|\mathrm{tr}[\mathcal{S}_1 t^a \mathcal{S}_1^\dagger\, \mathcal{S}_2 t^b \mathcal{S}_2^\dagger]\,\mathrm{tr}[\mathcal{S}_3 t^c \mathcal{S}_3^\dagger\, \mathcal{S}_4 t^d \mathcal{S}_4^\dagger]|0\rangle$$
$$\langle 0|\mathrm{tr}[\mathcal{S}_1 t^a \mathcal{S}_1^\dagger\, \mathcal{S}_3 t^c \mathcal{S}_3^\dagger]\,\mathrm{tr}[\mathcal{S}_2 t^b \mathcal{S}_2^\dagger\, \mathcal{S}_4 t^d \mathcal{S}_4^\dagger]|0\rangle$$
$$\langle 0|\mathrm{tr}[\mathcal{S}_1 t^a \mathcal{S}_1^\dagger\, \mathcal{S}_4 t^d \mathcal{S}_4^\dagger]\,\mathrm{tr}[\mathcal{S}_2 t^b \mathcal{S}_2^\dagger\, \mathcal{S}_3 t^c \mathcal{S}_3^\dagger]|0\rangle$$

- These mix under renormalization and depend on angles

$$\hat{\gamma}_\mu = c_A \left[2\ln\frac{\mu^2}{\nu^2} - \begin{pmatrix} L_{12}+L_{34} & 0 & 0 \\ 0 & L_{13}+L_{24} & 0 \\ 0 & 0 & L_{14}+L_{23} \end{pmatrix} \right] + \begin{pmatrix} 0 & w & -w \\ v & 0 & -v \\ u & -u & 0 \end{pmatrix}$$

where $L_{ij} = \ln|n_i \cdot n_j/2|$, and *u*, *v*, *w*, are conformal ratios

EW resummation

- ν-evolution vanishes for $\mu = M_W$ (at NLL)

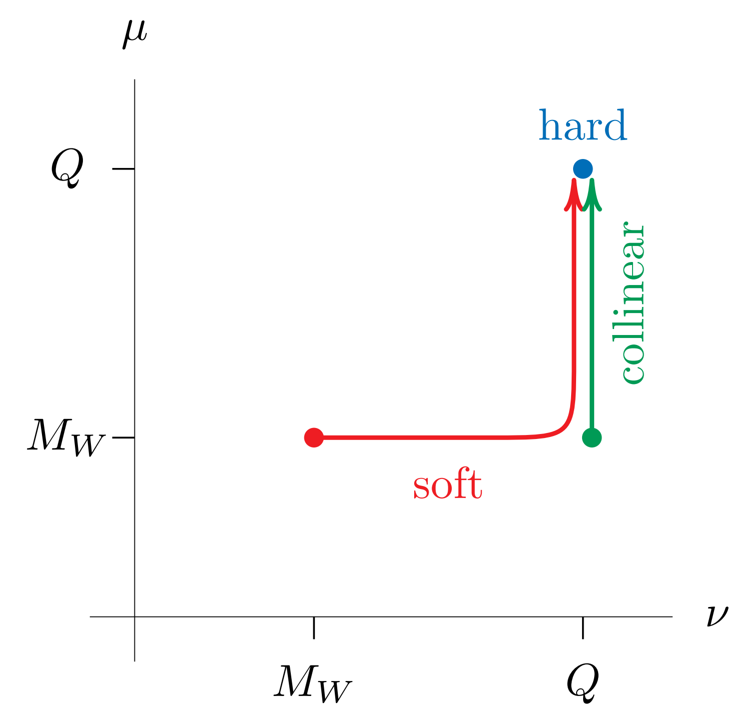
$$U_\nu = \exp\left[\int_{M_W}^{Q} \frac{\mathrm{d}\nu}{\nu}\gamma_{\nu,S}\right] = \exp\left[-m\frac{\alpha_2(\mu)}{\pi}\ln\frac{Q}{M_W}\ln\frac{\mu^2}{M_W^2}\right]$$

number of triplets

- μ-evolution gives rise to double logarithms [See Ciafaloni et al]

$$U_\mu^{\mathrm{DL}} = \exp\left[\int_{M_W}^{Q}\frac{\mathrm{d}\mu}{\mu}\,m\,\frac{2\alpha_2}{\pi}\ln\frac{\mu}{\bar{n}\cdot r}\right] \approx \exp\left[-m\frac{\alpha_2}{\pi}\ln^2\frac{Q}{M_W}\right]$$

- Single logarithms for nonsinglets:
  - Different coefficient splitting function
  - Angular dependence through soft



4. Comparison and extensions

Comparison with Bauer, Ferland, Webber

- They cut off soft singularity in PDF evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_q^{(I=1,I_3=0)}(x,\mu) = \frac{\alpha_2}{\pi}\int_0^{1-M/\mu} \mathrm{d}z \left[-\frac{1}{4}\widetilde{P}_{QQ}(z)\, f_q^{(I=1,I_3=0)}\left(\frac{x}{z},\mu\right) + \frac{1}{4}N_c\widetilde{P}_{QG}(z)\, f_W^{(I=1,I_3=0)}\left(\frac{x}{z},\mu\right) + \ldots \right]$$

- Fix $\delta(1-z)$ contribution from momentum sum rule

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_q^{(I=1,I_3=0)}(x,\mu) = \frac{\alpha_2}{\pi}\left(\frac{3}{2}\ln\frac{M}{\mu} + \frac{9}{8}\right) f_q^{(I=1,I_3=0)}(x,\mu) + \ldots$$

- Agrees with our result for $z < 1$ and at LL. Differences at NLL
- They do not account for polarization of gauge bosons

Some extensions

- Higher orders:

	Matching	Non-cusp	Cusp
LL	tree	-	1-loop
NLL	tree	1-loop	2-loop
NLL′	1-loop	1-loop	2-loop
NNLL	1-loop	2-loop	3-loop

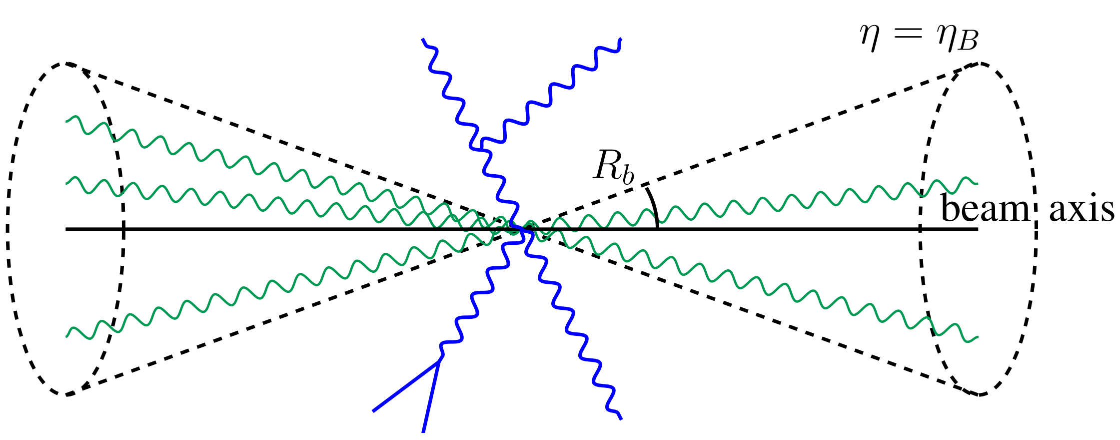
- Jets: fragmentation function for jets, match at scale $\mu = p_T R$
  [Kang, Ringer, Vitev; Dai, Kim, Leibovich]

$$D^{(I=0)}_{W_\pm\to\mathrm{jet}}(x,\mu,\nu) = \delta(1-x)\,, \qquad D^{(I=1,I_3=0)}_{W_\pm\to\mathrm{jet}}(x,\mu,\nu) = 0\,, \qquad D^{(I=2,I_3=0)}_{W_\pm\to\mathrm{jet}}(x,\mu,\nu) = 0\,,$$

$$D^{(I=0)}_{q\to\mathrm{jet}}(x,\mu,\nu) = \delta(1-x)\,, \qquad D^{(I=1,I_3=0)}_{q\to\mathrm{jet}}(x,\mu,\nu) = 0\,,$$

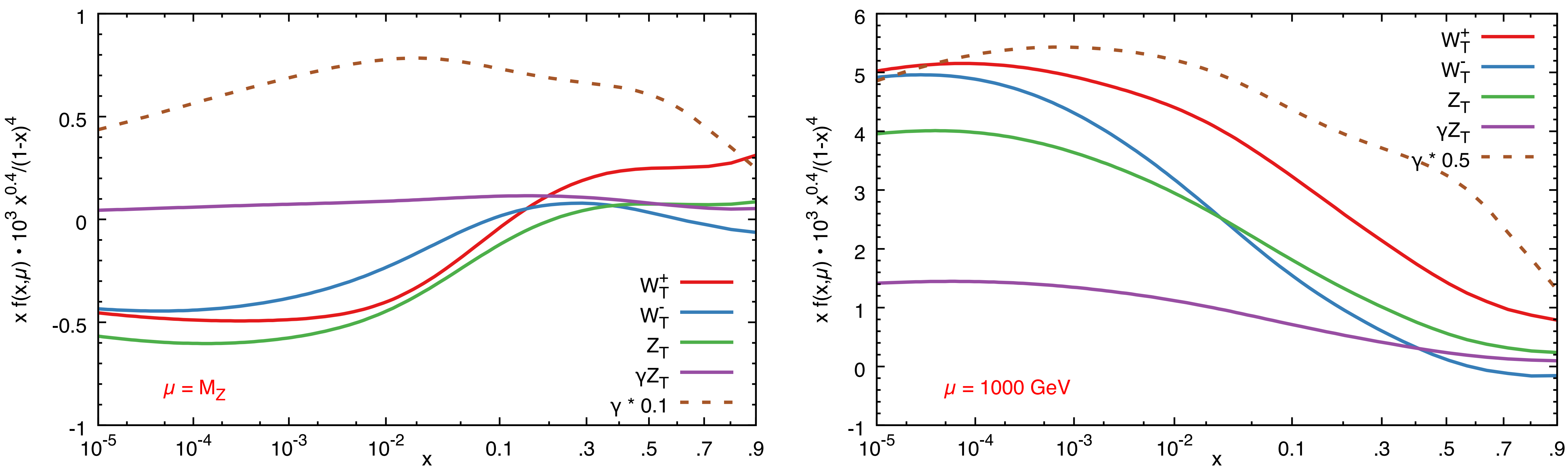
$$D_{u\to\mathrm{jet}}(x,\mu,\nu) = \delta(1-x)$$

- Inclusive beams and exclusive central (detector) region
  [Chien, Hornig, Lee; Becher, Neubert, Rothen, Shao]



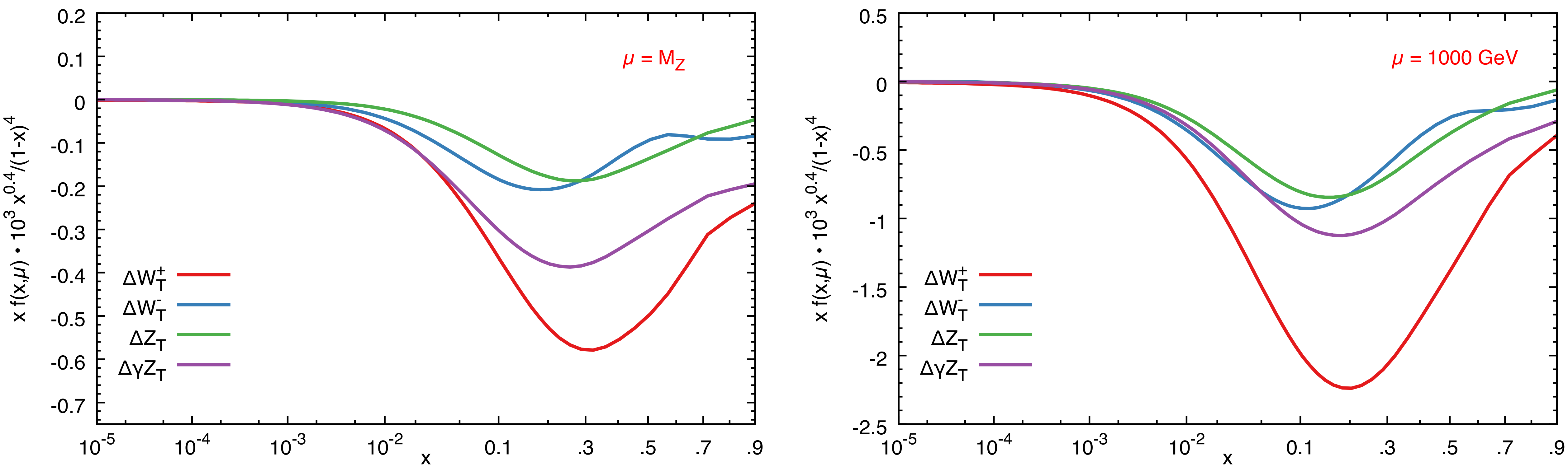
5. Electroweak gauge boson PDFs

Transverse gauge boson PDFs



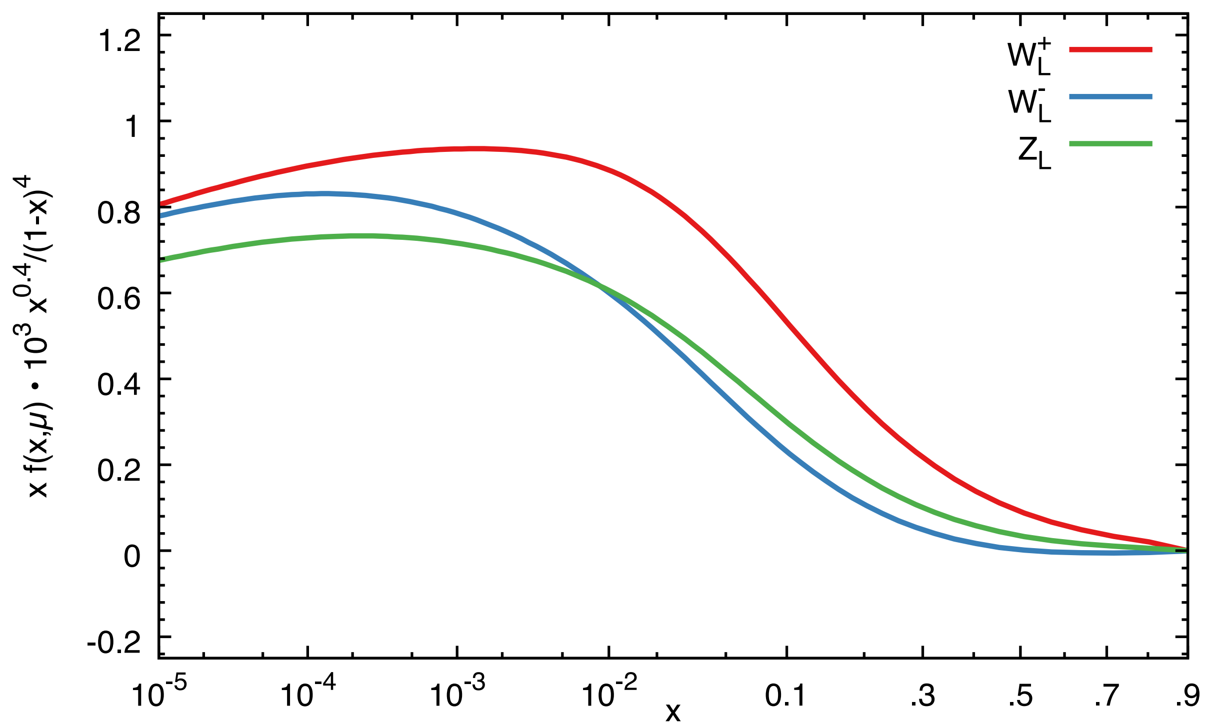
- Tree-level matching vanishes, first contribution at one-loop
- Does not have to be positive ($\overline{\text{MS}}$ subtraction)
- At higher energies comparable to photon PDF

Polarized gauge boson PDFs



- Polarization effects size-able, especially at largish *x*
- Proton contains more quarks than anti-quarks, and left-handed quarks preferably emit helicity -1 gauge bosons

Longitudinal gauge boson PDFs



- Similar in size to transverse PDFs at low scales
- μ-independent at this order

Summary

- Electroweak resummation for inclusive processes involves double logs because initial/final particles are not $SU(2)$ singlets
- Factorization in symm. phase but includes $SU(2)$ nonsinglets $\rightarrow$ soft functions, double logs, rapidity logs
- Beyond LL: modified DGLAP, angular dependence (through soft function), evolution polarizes gauge boson PDFs
- Can also consider mixed inclusive/exclusive setup and jets
- Matched the EW gauge boson PDFs at one-loop
- Coming soon: phenomenology

Tack!