TMD splitting functions in kT factorization and TMD parton showers

[EPJC 78 (2018) 174, 1711.04587]
[JHEP 01 (2016) 181, 1511.08439]
[PRD 94, 114013 (2016), 1607.01507]

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Introduction

- Parton distribution functions (PDFs) together with parton level matrix elements allow for a very accurate description of 'hard' events in hadron-hadron and hadron-electron collisions.

- The bulk of such analysis is carried out within the framework of collinear factorization.

- However, there exist classes of multi-scale processes where the use of more general schemes is of advantage
  - E.g. high-energy or low x limit of hard processes $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$ where $x = M^2/s$.
  - In such a scenario it is necessary to resum terms enhanced by logarithms $\ln 1/x$ to all orders in the α_s, which is achieved by BFKL evolution equation.
  - The resulting formalism called high energy (or k_T) factorization provides a factorization of such cross-sections into a TMD coefficient or 'impact factor' and an 'unintegrated' gluon density.

Introduction

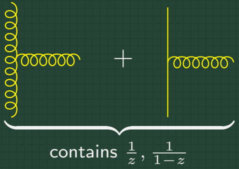
Limitations of high-energy factorization framework:

- valid only in low $x \lesssim 10^{-2}$ region
  - problems for observables involving fragmentation functions which involve integrals over the full x range of initial state PDFs
  - limited to exclusive observables which allow to fix x of both gluons
- limited to gluon-to-gluon splittings in the low x evolution, with quarks being absent.
  - omits a resummation of collinear logarithms associated with quark splittings which can provide sizable contributions at intermediate and large x
  - For hard processes initiated by quarks the appropriate unintegrated parton density functions are needed

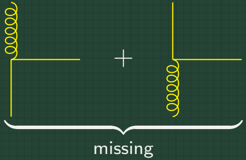
What can we do?

- Partial solution: use CCFM evolution equation instead of BFKL

  ✓ based on QCD coherence
  → includes also resummation
  of soft logarithms



  ✗ still limited to low x
  ✗ evolution equation for gluons only
  ✗ missing collinear logarithms



What do we want?

- Resum low x logarithms.
- Smooth continuation to the large x region.
- Reproduce the correct collinear limit (DGLAP).
- Ultimately: a coupled system of evolution equations for unintegrated PDFs
  - need: k_T-dependent splitting functions

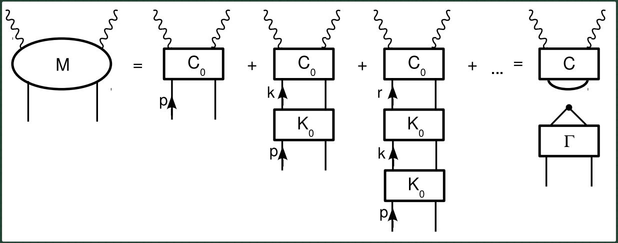
We will try to achieve this goal by extending Curci-Furmanski-Petronzio (CFP) and Catani-Hautmann (CH) formalisms.

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.
2. Generalization to the high-energy case and kernel calculation.
3. Results: new TMD splitting functions
4. Some results obtained within the k_T-factorization framework $(\mathrm{KaTie} + \mathrm{CASCADE})$.

Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

- Factorization based on generalized ladder expansion (in terms of 2PI kernels) [Ellis et al. NPB152 (1979), 285]



- Axial gauge instrumental
  - integration over outgoing legs leads to collinear singularities
  - incoming propagators amputated
- 2PI kernels connected only by convolution in x
  - this is achieved by introducing appropriate projector operators.

Curci-Furmanski-Petronzio (CFP) methodology [NPB175 (1980) 2792]

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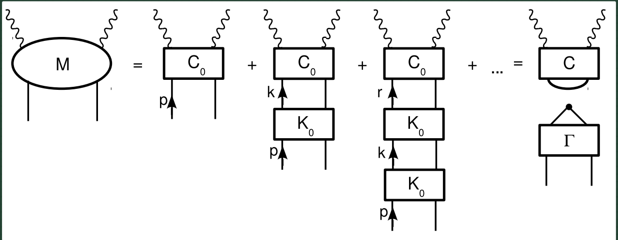


$$M = C^{(0)}\,\mathcal{G}^{(0)} \quad \text{with} \quad \mathcal{G}^{(0)} = \left(1 + K^{(0)} + K^{(0)}\,K^{(0)} + \ldots\right) = \frac{1}{1 - K^{(0)}}$$

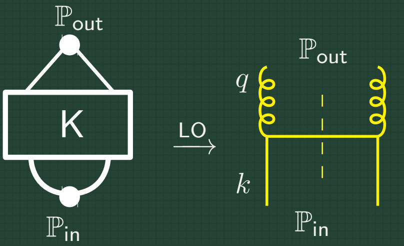
introduce projectros: $K^{(0)} = (1-\mathbb{P})\;K^{(0)} + \mathbb{P}\,K^{(0)}$

$$M = \underbrace{\left(C_0\,\frac{1}{1-(1-\mathbb{P})K^{(0)}}\right)}_{C}\underbrace{\left(\frac{1}{1-\mathbb{P}K}\right)}_{\Gamma} \quad \text{with} \quad K = K^{(0)}\left(\frac{1}{1-(1-\mathbb{P})K^{(0)}}\right)$$

Curci-Furmanski-Petronzio (CFP) methodology

[Nucl. Phys. B175 (1980) 2792]

- CFP projector operators: $\mathbb{P} = \mathbb{P}^{\epsilon} \otimes \mathbb{P}^{s} = \mathbb{P}^{\epsilon} \otimes \mathbb{P}^{s}_{\text{in}} \otimes \mathbb{P}^{s}_{\text{out}}$



$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{out}} = -g^{\mu\nu} \\ \mathbb{P}_{q,\,\text{out}}(q) = \frac{\not{n}}{2q\cdot n} \end{cases}$$

$$\begin{cases} \mathbb{P}^{\mu\nu}_{g,\,\text{in}}(k) = \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^{\mu}n^{\nu}+n^{\mu}k^{\nu}}{k\cdot n}\right) \\ \mathbb{P}_{q,\,\text{in}} = \frac{\not{k}}{2} \\ \text{incoming legs put on-shell } k^2 = 0 \end{cases}$$

- performs integration over $dq^2 d^{m-2}\mathbf{q}$
- takes pole part

- Splitting function definition
  - Extracted from PDF

$$\Gamma = \left(\frac{1}{1-\mathbb{P}K}\right) = 1 + \mathbb{P}\,K^{(0)} + \mathcal{O}(\alpha_S)$$

$$= x\,\mathrm{PP}\left\{\int \frac{d^{4+2\epsilon}k}{(2\pi)^{4+2\epsilon}}\delta\left(x - \frac{qn}{kn}\right)\mathbb{P}_{j,\,\mathsf{in}} \otimes K^{(0)}_{ij} \otimes \mathbb{P}_{i,\,\mathsf{out}}\right\}$$

  - splitting function is given as residuum of Γ

$$P(x) = 2\,\mathrm{Res}_0\left\{\int \frac{d^{4+2\epsilon}k}{(2\pi)^{4+2\epsilon}}\delta\left(x - \frac{qn}{kn}\right)\mathbb{P}_{j,\,\mathsf{in}} \otimes K^{(0)}_{ij} \otimes \mathbb{P}_{i,\,\mathsf{out}}\right\}$$

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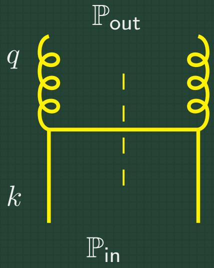
Generalization to high energy kinematics

- High energy kinematics



$$k^\mu = yp^\mu + k_\perp^\mu$$

$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2xp \cdot n} n^\mu$$

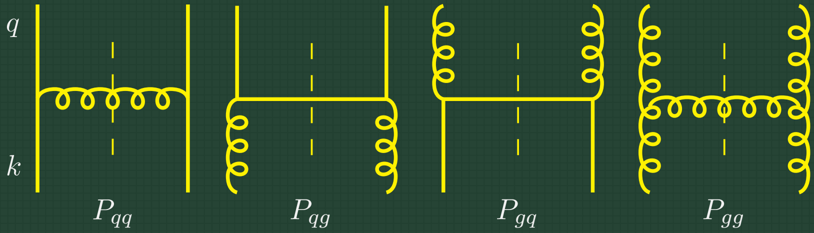
We will define/constrain splitting functions by requiring:

- gauge invariance/current conservation of vertices
- correct collinear limit
- correct high energy limit

Generalization to high energy kinematics

- Partly obtained by Catani and Hautmann for the case of P_{qg}
  [Catani, Hautmann NPB427 (1994) 475524, hep-ph/9405388]

- We want to extend it to general case including all splittings
  - P_{gq} and P_{qq} case done in [JHEP 01 (2016) 181, 1511.08439]
  - P_{gg} done in [EPJC 78 (2018) 174, 1711.04587]



To achieve this goal we need to provide:

- Appropriate projector operators
- Generalize QCD vertices

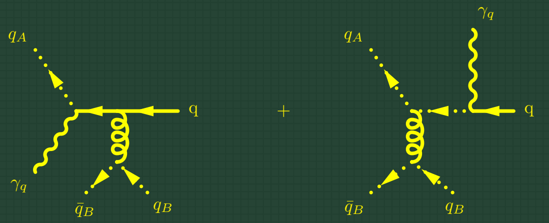
Generalization of QCD vertices

We need to ensure gauge invariance of vertices in the presence of off-shell k, q momenta.

- We use spinor helicity formalism to construct appropriate gauge invariant amplitudes and extracted vertices from them [Kutak, van Hameren, Serino, JHEP 02, 009 (2017)]
  - Off-shell particles are introduced as pairs of auxiliary on-shell particles $\rightarrow$ increased no. of Feynman diagrams.
  - Sum relevant diagrams to obtain gauge invariant amplitudes.
  - "Deconstruct" amplitude by removing polarisation vectors ($\epsilon^\mu(p)$ for an on-shell particle vs. p^μ for an off-shell particle)

- Alternatively (but with some ambiguity for P_{gg}) this can be obtained using the reggeized quark formalism (Lipatov high-energy action) [Lipatov, Vyazovsky, NPB 597 (2001) 399]

Generalization of QCD vertices: $\Gamma^{\mu}_{g^* q^* q}$ from $\mathcal{A}(1^*, \bar{q}^{*-}, q^+)$



$$\mathcal{A}(q,k,p') = \frac{\langle p|\gamma^\mu|p]}{\sqrt{2}} \frac{d_{\mu\nu}(q)}{q^2} \langle p| \frac{\not{\epsilon}_{p+}}{\sqrt{2}} \frac{\not{k}}{k^2} \frac{\gamma^\nu}{\sqrt{2}} - \frac{\gamma^\nu}{\sqrt{2}} \frac{\not{p}}{2p\cdot p'} \frac{\not{\epsilon}_{p+}}{\sqrt{2}} |n]$$

$$= n^\mu \frac{d_{\mu\nu}(q)}{q^2} [n| \left\{ \gamma^\nu - \frac{p^\nu}{p\cdot p'} \not{k} \right\} \frac{\not{k}}{k^2} |p]$$

$$\equiv n^\mu \frac{d_{\mu\nu}(q)}{q^2} [n| \Gamma^{\nu}_{g^* q^* q}(q,k,p') |p]$$

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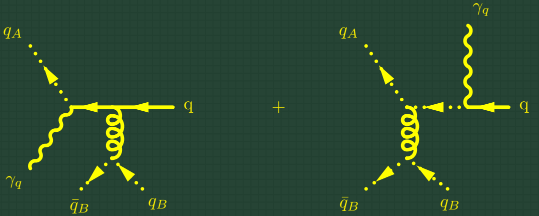
Generalization of QCD vertices: $\Gamma^{\mu}_{g^*q^*q}$ from $\mathcal{A}(1^*, \bar{q}^{*-}, q^+)$



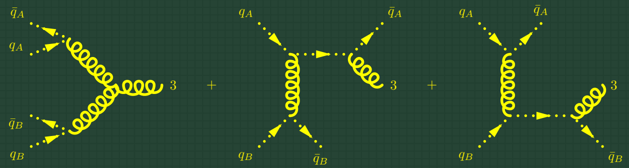
$$\mathcal{A}(q,k,p') = \frac{\langle p|\gamma^\mu|p]}{\sqrt{2}} \, \frac{d_{\mu\nu}(q)}{q^2} \, \langle p| \frac{\not\epsilon_{p+}}{\sqrt{2}} \frac{\not k}{k^2} \frac{\gamma^\nu}{\sqrt{2}} - \frac{\gamma^\nu}{\sqrt{2}} \frac{\not p}{2p\cdot p'} \frac{\not\epsilon_{p+}}{\sqrt{2}} |n]$$

$$= \frac{n^\mu}{q^2} \, [n| \left\{ d_{\mu\nu}(q) \left(\gamma^\nu - \frac{p^\nu}{p\cdot p'} \not k \right) \right\} \frac{\not k}{k^2} |p]$$

$$\equiv \frac{n^\mu}{q^2} \, [n| \, \Gamma^{\nu}_{g^*q^*q}(q,k,p') \, |p]$$

[EPJC 78 (2018) 174, 1711.04587]

Generalization of QCD vertices: $\Gamma^{\mu}_{g^* g^* g}$ from $\mathcal{A}(1^*, 2^*, 3)$



$$\mathcal{A}(q,k,p') = (\sqrt{2})\,\frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2} \Big\{ \mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'} \Big\}$$

$$\equiv (\sqrt{2})\,\frac{p_{\mu_1}\, n_{\mu_2}\, \epsilon_{\mu_3}(p')}{q^2\, k^2}\,\Gamma^{\mu_1\mu_2\mu_3}(q,k,p')$$

- $d_{\mu\nu}(q) = -g_{\mu\nu} + \frac{n^\mu q^\nu + n^\nu q^\mu}{q^2}$ not invertable in light-cone gauge $(n^2 = 0) \Rightarrow$ it has to be kept everywhere!

Generalization of QCD vertices

The full set of gauge invariant off-shell vertices are:

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(k) \left(\gamma^{\nu} - \frac{n^{\nu}}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(q) \left(\gamma^{\nu} - \frac{p^{\nu}}{p\cdot q}\not{k}\right)$$

$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = igt^a \left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot p'}\not{k} + \frac{n^{\mu}}{n\cdot p'}\not{q}\right)$$

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = i\,g\,f^{abc} \Big\{ \mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\, \frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\, \frac{k^2 p^{\mu_3}}{p\cdot p'} \Big\}$$

Generalization of projector operators 1

Since the incoming momentum is no longer collinear the corresponding projector operators need to be modified.

- Gluon case [Catani, Hautmann NPB427 (1994) 475524]:

$$\mathbb{P}^{\mu\nu}_{g,\,\mathrm{in}} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2}$$

- Quark case [JHEP 01 (2016) 181]:

$$\mathbb{P}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}$$

✓ Both operators reduce to the CFP projectors in the collinear limit

$$< \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2} >_\phi \overset{k_\perp \to 0}{=} \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n}\right)$$

$$k \overset{k_\perp \to 0}{=} yp$$

Generalization of projector operators 2

- The form of the CH projectors were derived based on heavy quark production in which case numerators of the gluon propagators factorize [Catani, Ciafaloni, NPB 366 (1991) 135-188]

$$\mathcal{M}^{g^* g^* \to q\bar{q}}(k_1, k_2; p_3, p_4) = \frac{2\, x_1\, x_2\, p_1^{\mu_1}\, p_2^{\mu_2}}{\sqrt{k_{1\,\perp}^2\, k_{2\,\perp}^2}}\, d_{\mu_1 \nu_1}(k_1)\, d_{\mu_2 \nu_2}(k_2)\, \hat{\mathcal{M}}^{g^* g^* \to q\bar{q}}_{\mu_1, \mu_2}(k_1, k_2; p_3, p_4)$$

- Since the numerators of gluon propagators do not factorize in case of $\Gamma^{\mu_1 \mu_2 \mu_3}_{g^* g^* g}$ we need further modifications.
- The form of the projectors is determined by
  - condition: $\mathbb{P}^2 = \mathbb{P}$
  - and a proper collinear limit

Final set of projectors:

$$\mathbb{P}^{\mu\nu}_{g,\,\text{in}} = -y^2\, \frac{p^\mu p^\nu}{k_\perp^2} \qquad \mathbb{P}^{\mu\nu}_{g,\,\text{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} - k^2\, \frac{n_\mu n_\nu}{(k \cdot n)^2}$$

$$\mathbb{P}_{q,\,\text{in}} = \frac{y\, \not{p}}{2} \qquad \mathbb{P}_{q,\,\text{out}} = \frac{\not{n}}{2\, n \cdot l}$$

Rest of the talk

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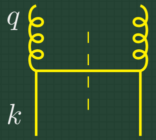
TMD splitting function definition

- Angular-dependent splitting function

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon\right) = z \int \frac{d^{2+2\epsilon}\mathbf{q}}{2(2\pi)^{4+2\epsilon}} \underbrace{\int dq^2\, \mathbb{P}_{j,\,\text{in}} \otimes \hat{K}^{(0)}_{ij}(q,k) \otimes \mathbb{P}_{i,\,\text{out}}}_{\tilde{P}^{(0)}_{ij}(z,\mathbf{k},\tilde{\mathbf{q}},\epsilon)} \Theta(\mu_F^2 + q^2)$$

$$\hat{K}^{(0)}_{ij}(q,k) =$$



$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2xp\cdot n} n^\mu$$

$$k^\mu = yp^\mu + k_\perp^\mu$$

$$\tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}, \qquad z = x/y$$

- Angular-average splitting function

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \epsilon\right) = \frac{\alpha_s}{2\pi} z \int_0^{(1-z)(\mu_F^2 - z\mathbf{k}^2)} \frac{d\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^\epsilon \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} \bar{P}^{(0)}_{ij}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right)$$

Results for splitting functions

With the new projection operators we reproduce our earlier results [JHEP 01 (2016) 181, 1511.08439]

$$\tilde{P}_{qg}^{(0)} = T_R \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\,\mathbf{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right]$$

$$\tilde{P}_{gq}^{(0)} = C_F \left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} - \frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z) + \mathbf{k}^2 z(1-z^2))}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} \right. \\ \left. + \frac{\epsilon z \tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2 + (1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2} \right]$$

$$\tilde{P}_{qq}^{(0)} = C_F \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2} \right) \left[\frac{\tilde{\mathbf{q}}^2 + (1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} \right. \\ \left. + \frac{z^2\tilde{\mathbf{q}}^2 - z(1-z)(1-3z+z^2)\mathbf{k}^2 + (1-z)^2\epsilon(\tilde{\mathbf{q}}^2 + z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)} \right]$$

✓ In the collinear limit, $\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \to 0$, standard DGLAP results are reproduced.

Results for splitting functions

The new result is [EPJC 78 (2018) 174, 1711.04587]

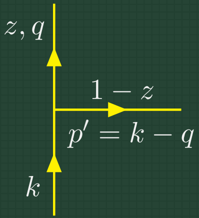
$$\tilde{P}_{gg}^{(0)}(z,\tilde{\mathbf{q}},\mathbf{k}) = C_A \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\right)^2 \frac{\tilde{\mathbf{q}}^2}{\left(\tilde{\mathbf{q}}-(1-z)\mathbf{k}\right)^2}$$
$$\times\Bigg[-\frac{4z^2-4z+2}{z(1-z)} - z(1-z)(4z^4-12z^3+9z^2+1)\frac{\mathbf{k}^4}{\tilde{\mathbf{q}}^4}$$
$$-4z(1-z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^2}{\mathbf{k}^2\tilde{\mathbf{q}}^2} + 2(4z^3-6z^2+6z-3)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2}$$
$$-4z(1-z)^2(3-5z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^4} - (4z^4-8z^3+5z^2-3z-2)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}$$
$$+8z(1-z)^2\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}^3}{\mathbf{k}^2\tilde{\mathbf{q}}^4} - 2z^2(1-z)(3-4z)(3-2z)\frac{\mathbf{k}^2\,\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^4}\Bigg]$$
$$-\epsilon\, C_A\, z(1-z)\frac{\tilde{\mathbf{q}}^2}{\mathbf{k}^2}\left(\frac{(2z-1)\mathbf{k}^2+2\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\right)^2$$

Results for splitting functions

or in an angular integrated form (with $\epsilon = 0$)

$$\bar{P}_{gg}^{(0)} = \frac{1}{\pi}\int_0^\pi d\phi \, \sin^{2\epsilon}\phi \, \tilde{P}_{gg}^{(0)}$$
$$= C_A \frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}\left[\frac{(2-z)\tilde{\mathbf{q}}^2 + (z^3 - 4z^2 + 3z)\mathbf{k}^2}{z(1-z)\,|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\mathbf{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\mathbf{k}^2}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)}\right]$$

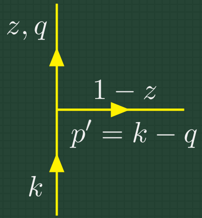
Kinematic limits of $\tilde{P}_{gg}^{(0)}$



- Collinear (DGLAP) limit

$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

Kinematic limits of $\tilde{P}_{gg}^{(0)}$



- Collinear (DGLAP) limit

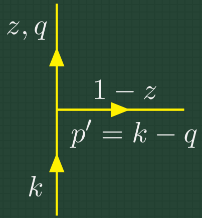
$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

- High-energy (BFKL) limit ($z \to 0$)

$$\lim_{z \to 0} \hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s \right) = \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^\epsilon} \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - (\mathbf{k} - \tilde{\mathbf{p}})^2\right) \frac{1}{\tilde{\mathbf{p}}^2}$$

$$= \int \frac{d^{2+2\epsilon}\mathbf{q}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - \mathbf{q}^2\right) \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^\epsilon} \frac{1}{(\mathbf{q} - \mathbf{k})^2},$$

Kinematic limits of $\tilde{P}_{gg}^{(0)}$



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$$\lim_{\mathbf{k}^2 \to 0} \bar{P}_{gg}^{(0)} = 2\, C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z) \right]$$

- High-energy (BFKL) limit ($z \to 0$)

$$\lim_{z \to 0} \hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - (\mathbf{k} - \tilde{\mathbf{p}})^2\right) \frac{1}{\tilde{\mathbf{p}}^2}$$

$$= \int \frac{d^{2+2\epsilon}\mathbf{q}}{\pi^{1+\epsilon}} \Theta\left(\mu_F^2 - \mathbf{q}^2\right) \frac{\alpha_s C_A}{\pi (e^{\gamma_E} \mu^2)^{\epsilon}} \frac{1}{(\mathbf{q} - \mathbf{k})^2},$$

- Soft (CCFM) limit ($\tilde{\mathbf{p}} = \frac{\mathbf{k} - \mathbf{q}}{1-z}$): $\tilde{\mathbf{p}}^2 \to 0$

$$\hat{K}_{gg}\left(z, \frac{\mathbf{k}^2}{\mu^2}, 0, \alpha_s\right) = z \int_0 \frac{d\tilde{\mathbf{p}}^2}{\tilde{\mathbf{p}}^2} \frac{\alpha_s C_a}{\pi} \left[\frac{1}{z} + \frac{1}{1-z} + \mathcal{O}\left(\frac{\tilde{\mathbf{p}}^2}{\mathbf{k}^2}\right) \right]$$

we obtain real/unresummed CFFM kernel "for free"

Rest of the talk

1. Basics of Curci-Furmanski-Petronzio method of splitting function calculation in collinear factorization.
2. Generalization to the high-energy case and kernel calculation.
3. Results: new TMD splitting functions.
4. Some results obtained within the k_T-factorization framework $(\mathrm{KaTie} + \mathrm{CASCADE})$.

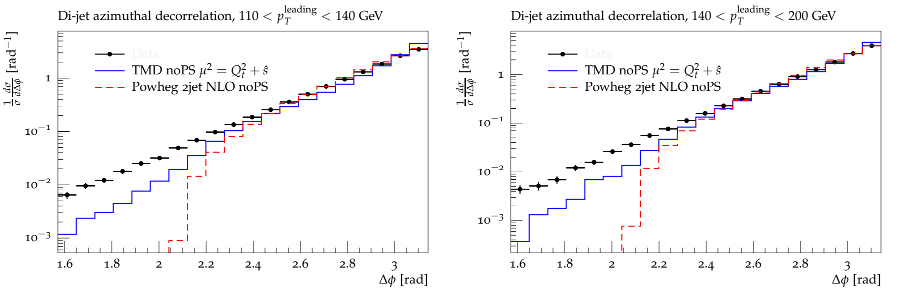
k_T-factorization framework: KaTie + CASCADE

- Idea behind new splitting functions:
  - construct a set of coupled evolution equations (including quarks, having correct collinear limit),
  - use it for evolution in Monte Carlo generator.
- Currently we already have a high-energy factorization toolbox:
  - off-shell matrix element generator: KaTie [CPC 224 (2018) 371],
  - MC generator: CASCADE [EPJC 70 (2010) 1237],
  - k_T-dependent PDFs:
    - KMRW [EPJC 31, 73 (2003)],
    - Parton Branching [JHEP 01 (2018) 070].
- In what follows I present some selected results obtained using this toolbox.

Results from k_T-factorization framework:

[EPJC 78 (2018) 137, 1712.05932]

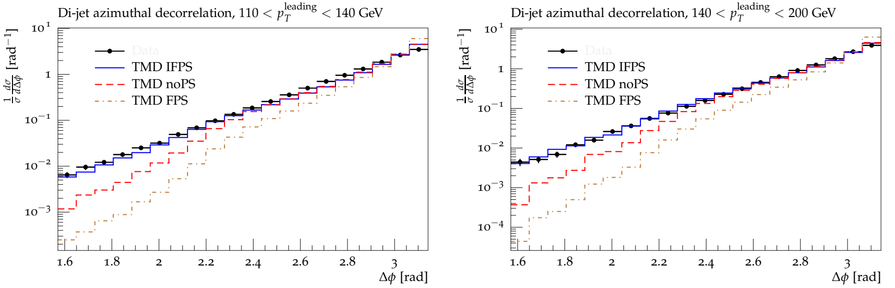
- Dijet Azimuthal Decorrelations measured by CMS [PRL 106 (2011) 122003, 1101.5029]
  - parton level



Results from k_T-factorization framework:

[EPJC 78 (2018) 137, 1712.05932]

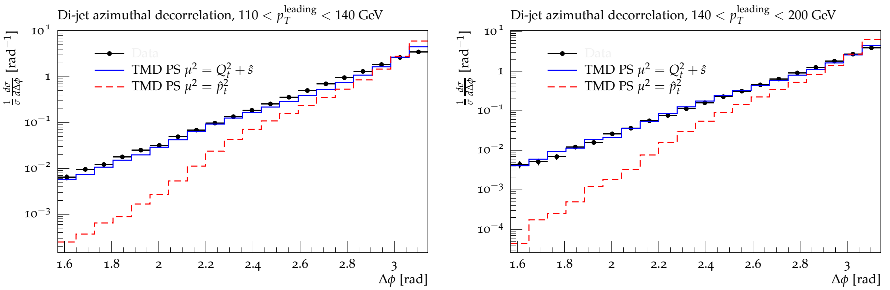
- Dijet Azimuthal Decorrelations measured by CMS [PRL 106 (2011) 122003, 1101.5029]
  - including showers
    - initial state PS – "TMD shower" (follows TMD PDFs)
    - final state PS – from PYTHIA



Results from k_T-factorization framework:

[EPJC 78 (2018) 137, 1712.05932]

- Dijet Azimuthal Decorrelations measured by CMS [PRL 106 (2011) 122003, 1101.5029]
  - scale choice for initial state PS
    - $\mu^2 = Q_t^2 + \hat{s}$ – angular ordering
    - $\mu^2 = \hat{p}_t{}^2$ – conventional ordering



Summary and Outlook

- We successfully extended method of Curci, Furmanski and Petronzio to the TMD case using gauge invariant vertices.
  - The essential subtleties which prevent the Catani-Hautmann generalisation from being directly extended to the P_{gg} case were uncovered and worked out.
- With the new projectors we have reproduced our earlier results for real emission $k_\perp$-dependent P_{qq}, P_{gq} and P_{qg} splitting functions confirming our formalism.
- We used the formalism to calculate P_{gg} TMD splitting function which feature correct
  - collinear limit (DGLAP kernels)
  - high-energy limit (BFKL kernel)
  - soft limit (CCFM kernel)

Summary and Outlook

- The next step is to calculate virtual corrections.
- In a longer perspective construct a complete set of evolution equations.
- I briefly showed results obtained using:
  $\mathrm{TMDs} \ + \mathrm{KaTie\ ME\ generator} + \mathrm{CASCADE\ PS}$
  - which provides a first complete toolbox for calculations in k_T-factorization.
- The presented splitting functions can form a basis for a new extraction of TMDs as well as a new shower.

Workshop on Resummation, Evolution, Factorization 2018

19-23 November 2018
Other Institutes
Europe/Warsaw timezone

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REF 2018 is the 5th workshop in the series of workshops on Resummation, Evolution, Factorization. The workshop wishes to bring together experts of different communities specialized in: nuclear structure; transverse momentum dependend distributions; small-x physics; effective field theories.

On Friday the 23rd of November there will be a tutorial on the use of existing software for the calculation of hadron scattering processes. The emphasis will be on programs that employ TMDs, for example those provided by TMDlib. The fixed-order program KaTie, and the parton shower program CASCADE will be addressed, as well as their merging.

Previous meetings

- 13-16 November 2017 Madrid (Spain)
- 7-10 November 2016 Antwerp (Belgium)
- 2-5 November 2015 DESY Hamburg (Germany)
- 8-11 December 2014 Antwerp (Belgium)

Scientiffic committee:

Elke Aschenauer Daniel Boer
Igor Cherednikov Markus Diehl
Didar Dobur David Dudal
Miguel García Echevarría
Laurent Favart Francesco Hautmann
Hannes Jung Fabio Maltoni
Piet Mulders Gunar Schnell
Andrea Signori Pierre Van Mechelen

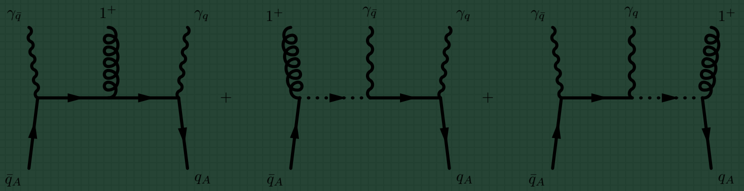
Starts 19 Nov 2018, 13:00
Ends 23 Nov 2018, 15:00
Europe/Warsaw

Other Institutes
Institute of Nuclear Physics
Polish Academy of Sciences
Kraków, Poland

Krzysztof Kutak (chairman)
Andreas van Hameren
Piotr Kotko
Sebastian Piotr Sapeta

BACKUP SLIDES

$\Gamma^{\mu}_{q^*,q^*,g}$ from $\mathcal{A}(g^+, \bar{q}^{*+}, q^{*-})$



$$\mathcal{A}(1^+, \bar{q}^{*+}, q^{*-}) \to \langle \bar{q}| \frac{\not{k}_{\bar{q}}}{k_{\bar{q}}^2} \left\{ \gamma^{\mu} + \frac{p_{\bar{q}}^{\mu}}{p_{\bar{q}} \cdot k_q} \not{k}_{\bar{q}} + \frac{p_q^{\mu}}{p_q \cdot k_{\bar{q}}} \not{k}_q \right\} \frac{\not{k}_q}{k_q^2} |q]$$

Dijet Azimuthal Decorrelations measured by CMS

[PRL 106 (2011) 122003, 1101.5029]