# Subleading colour corrections in Herwig

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Section 1

# Motivation

# Why do subleading $N_c$ showers?

- $1/N_c^2$ is not that small and $1/N_c$ suppression possible if there are two quark-lines.
- More energy
  - many more coloured partons.
  - many more colour suppressed terms.
- For a leading $N_c$ shower, the number of colour connected pairs grow roughly as $N_{\text{partons}}$.
- The number of pairs of coloured partons grows as $N_{\text{partons}}^2$.
- Useful for exact NLO matching.

Section 2

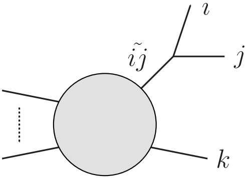
# Dipole showers

# Dipole Factorization

Dipole factorization gives, whenever $i$ and $j$ become collinear or one of them soft:

$$|\mathcal{M}_{n+1}(...,p_i,...,p_j,...,p_k,...)|^2 =$$
$$\sum_{k\neq i,j} \frac{1}{2p_i\cdot p_j}\langle\mathcal{M}_n(p_{\tilde{ij}},p_{\tilde{k}},...)\,|\mathbf{V}_{ij,k}(p_i,p_j,p_k)|\,\mathcal{M}_n(p_{\tilde{ij}},p_{\tilde{k}},...)\rangle$$

An emitter $\tilde{ij}$ splits into two partons $i$ and $j$, with the spectator $\tilde{k}$ absorbing the momentum to keep all partons (before and after) on-shell.
(Catani, Seymour arXiv:hep-ph/9605323)



# Dipole Factorization

The spin averaged dipole insertion operator is

$$\mathbf{V}_{ij,k}(p_i, p_j, p_k) = -8\pi\alpha_s V_{ij,k}(p_i, p_j, p_k)\frac{\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k}{\mathbf{T}_{\tilde{ij}}^2}$$

Where, for a final-final dipole configuration, we have for example

$$V_{q\to qg,k}(p_i, p_j, p_k) = C_F\left(\frac{2(1-z)}{(1-z)^2 + p_\perp^2/s_{ijk}} - (1+z)\right)$$

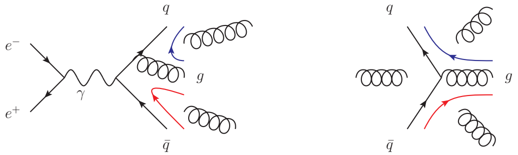
# Emission probability

For a leading $N_c$ shower, the emission probability would be

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{\delta(\tilde{ij}, \tilde{k}\text{ colour connected})}{1+\delta_{\tilde{ij}\,g}}$$

Including subleading emissions, instead gives

$$\mathrm{d}P_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{\mathrm{d}\phi_{n+1}(p_\perp^2, z)}{\mathrm{d}\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2}\frac{\langle\mathcal{M}_n|\mathbf{T}_{\tilde{ij}}\cdot\mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle}{|\mathcal{M}|^2}$$



Section 3

# Colour Matrix Element Corrections

# Overall picture

Using Herwigs dipole shower

- Instead of only colour connected emitter-spectator pairs radiating, all possible pairs can radiate.
- The emission probabilities are modified by a factor

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle}{|\mathcal{M}|^2}$$

  which is included using the reweighting in Herwig.
- We evolve the colour structure to be able to evaluate the factor above for the next emission. Color structure is calculated with ColorFull (arXiv:1412.3967).
- Continue for a set number of emissions and then do the rest with the standard shower.

# Density operator

We can write the amplitude as a vector in some basis (trace, multiplet, etc.),

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \leftrightarrow \mathcal{M}_n = (c_{n,1}, ..., c_{n,d_n})^T$$

And construct an "amplitude matrix" $M_n = \mathcal{M}_n\mathcal{M}_n^\dagger$, that we evolve by

$$M_{n+1} = -\sum_{i\neq j}\sum_{k\neq i,j} \frac{4\pi\alpha_s}{p_i\cdot p_j}\frac{V_{ij,k}(p_i,p_j,p_k)}{\mathbf{T}^2_{\tilde{ij}}} T_{\tilde{k},n} M_n T^\dagger_{\tilde{ij},n}$$

where

$$V_{ij,k} = \mathbf{T}^2_{\tilde{ij}}\frac{p_i\cdot p_k}{p_j\cdot p_k}.$$

This allows us to calculate the "colour matrix element corrections".

# Colour matrix element correction

With a way to evolve the density operator we can calculate the colour matrix element corrections for any number of emissions

$$\omega_{ik}^{n} = \frac{-1}{\mathbf{T}_{\tilde{ij}}^{2}} \frac{\mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n} M_n T_{\tilde{ij},n}^{\dagger}\right)}{\mathrm{Tr}\left(S_n \times M_n\right)}$$

- $\omega_{ik}^{n}$ can be negative, this is included through the weighted Sudakov algorithm (Bellm, J. et. al. arXiv:1605.08256).
- This initially resulted in very large weights, modifications to the weighted Sudakov veto algorithm drastically reduced the weights.

Section 4

# Preliminary results

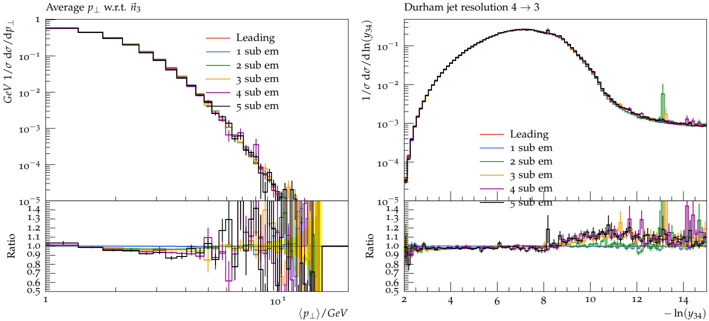
# $e^+e^-$ results

- Our results are in line with what S. Plätzer and M. Sjödahl found (Platzer, S., Sjodahl, M., arXiv:1206.0180).
- Differences are on the % level between leading and subleading shower ($\sim 10\%$ for tailored observables).

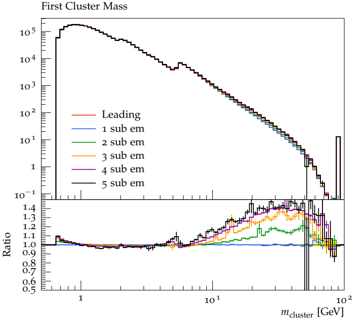
We have added

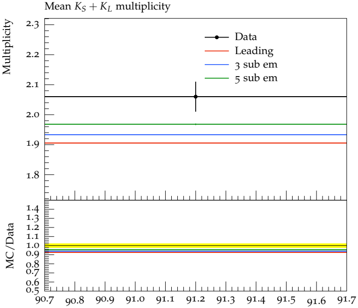
- Hadronic initial state, meaning initial state radiation (so we can do any process now, in particular LHC events).
- It is compatible with all of the additional functionality in Herwig $7.1$.
- After the subleading $N_c$ shower we continue with the standard Herwig dipole shower.
- $g \to q\bar{q}$ splittings.

# Preliminary $e^+e^- \to jj$ results



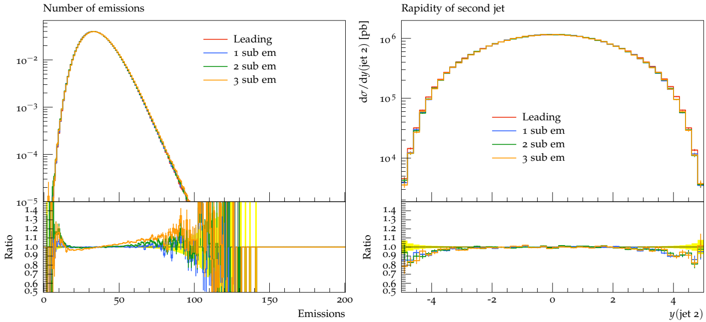
# Retune for hadronization model required?



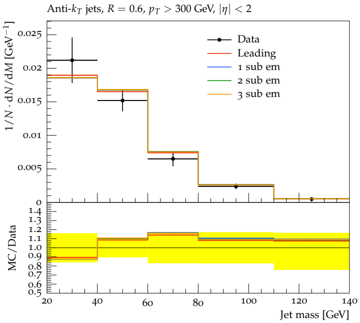

- First cluster mass seems to converges with more subleading emissions, kaon multiplicity does not.

# Preliminary $pp \to jj$ results, parton level




- Generation cut: $p_{\perp\,\text{cut}} = 40\,\text{GeV}$, analysis cut: $p_{\perp\,\text{cut}} = 50\,\text{GeV}$.
- Standard QCD observables converge, differences to the leading $N_c$ shower is a few percent for most observables.

# Preliminary $pp \to jj$ results, hadron level




- ATLAS_2012_I1119557 Rivet analysis.
- Full simulation can be run with the subleading colour shower.

Section 5

# Current status and future work

# Current status

- We can simulate full events with our subleading colour shower!
- Look at more processes (VBF, etc.)
- Look at the effect on analyses with data.
- Look for observables where subleading $N_c$ has a large effect (Simon Plätzer and Malin Sjödahl looked at some interesting ones).

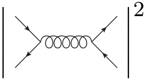
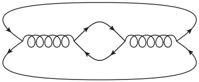
# Future work

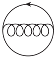
- Tuning with the subleading $N_c$ shower.
- Virtual corrections, which rearrange the colour structure without any real emissions.
- Updated hadronization model.

Section 6

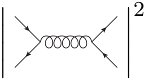
# Extra slides

Leading colour structure:

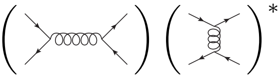
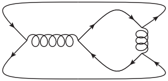
 $=$ 

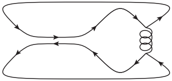
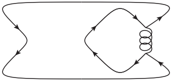
$= T_R$  $= T_R^2(N_c^2 - 1) \propto N_c^2.$

Leading colour structure:

 $\propto N_c^2.$

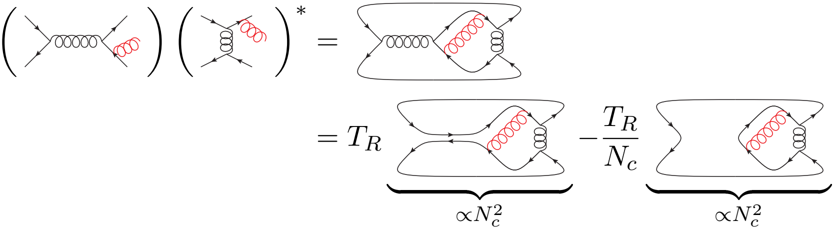
Interference term:

 $=$ 

$= T_R$  $- \frac{T_R}{N_c}$ 

$$= 0 - T_R^2 \frac{N_c^2 - 1}{N_c} \propto N_c.$$

# Example of $1/N_c$ suppressed terms



# Standard veto algorithm

Standard veto algorithm: we want to generate a scale $q$ and additional splitting variables $x$ (e.g. $z$ and $\phi$) according to a distribution $\mathrm{d}S_P$.

$$
\begin{aligned}
&\mathrm{d}S_P(\mu, x_\mu|q, x|Q) \\
&\qquad = \mathrm{d}q\mathrm{d}^d x \left(\Delta_P(\mu|Q)\delta(q-\mu)\delta(x-x_\mu)\right. \\
&\qquad\qquad \left. + P(q,x)\theta(Q-q)\theta(q-\mu)\Delta_P(q|Q)\right)
\end{aligned}
$$

Where $\Delta_P$ is the Sudakov form factor,

$$
\Delta_P(q|Q) = \exp\left(-\int_q^Q \mathrm{d}k \int \mathrm{d}^d z P(k,z)\right)
$$

To do this we use an overestimate of the distribution (with nicer analytical properties) $dS_R$ (change $P \to R$ in the above eqs.). Where we require $R(q,x) \geq P(q,x)$ for all $q, x$.

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$$\begin{aligned}\mathrm{d}S_P(\mu, x_\mu | q, x | Q) \\ &= \mathrm{d}q\mathrm{d}^d x \left(\Delta_P(\mu|Q)\delta(q-\mu)\delta(x-x_\mu)\right. \\ &\qquad \left. + P(q,x)\theta(Q-q)\theta(q-\mu)\Delta_P(q|Q)\right)\end{aligned}$$

Where $\Delta_P$ is the Sudakov form factor,

$$\Delta_P(q|Q) = \exp\left(-\int_q^Q \mathrm{d}k \int \mathrm{d}^d z P(k,z)\right)$$

To do this we use an overestimate of the distribution (with nicer analytical properties) $dS_R$ (change $P \to R$ in the above eqs.). Where we require $R(q,x) \geq P(q,x)$ for all $q, x$.

# Standard veto algorithm

$P(q,x) > 0$ and $R(q,x) \geq P(q,x)$. Set $k = Q$

1. Generate $q$ and $x$ according to $S_R(\mu, x_\mu | q, x | k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Else, accept the emission with the probability

$$\frac{P(q,x)}{R(q,x)}.$$

4. If the emission was vetoed, set $k = q$ and go back to 1.

# Weighted veto algorithm

Introduce an acceptance probability $0 \leq \epsilon(q, x|k, y) < 1$ and a weight $\omega$. Set $k = Q$, $\omega = 1$.

1. Generate $q$ and $x$ according to $S_R(\mu, x_\mu|q, x|k)$.
2. If $q = \mu$, there is no emission above the cutoff scale.
3. Accept the emission with the probability $\epsilon(q, x|k, y)$, update the weight
$$\omega \to \omega \times \frac{1}{\epsilon} \times \frac{P}{R}$$
4. Otherwise update the weight to
$$\omega \to \omega \times \frac{1}{1-\epsilon} \times \left(1 - \frac{P}{R}\right)$$
and start over at 1 with $k = q$.