

Effect of depolarizing noise on entangled photons

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Abstract. Entangled photons are important resources for quantum information. Especially, in quantum cryptography, the polarization entangled photons are highly secured. However, the entanglement of photon can hold in isolated system. In realistic system, the entangled photons inevitably interact with environment. In this work, we study the effect of depolarizing noise on the two general entangled qubits. The entangled photon are prepared with different entangled states and send to the depolarizing channel. We measure the entanglement quality by using concurrence. When the entangled photons interact to the depolarizing noise, the entanglement quality of photons will be degraded, and the concurrence of entangled photons reduces to 0. Moreover, we also apply Hadamard gate to the entangled qubits and send to the depolarizing channel. The Hadamard gate can filter the maximally entangled qubits in the depolarizing channel.

1. Introduction

In classical computer, the information is represented by a string of bits, which is 0 or 1. On the other hand, the information in quantum computer is represented by the quantum state, called qubit. An important feature of qubit is quantum entanglement. The entanglement of qubits is a promising scheme for carrying information in quantum communication [1]. However, the entangled qubits can preserve the entanglement in isolated system, where the influence of environment is ignored. For actual quantum communication, the entangled qubits inevitably interact to environment.

The interactions between the entangled qubits and environment cause the entanglement sudden death (EDS) [2], and the entanglement of qubits will be degraded. One of the noisy channels is the depolarizing channel. The depolarizing channel induce the sudden death of the maximally entangled qubits [3, 4]. This results to the information loss in quantum communication.

Another important issue in quantum computing is the information encryption. An important tool to manage the qubit is Hadamard gate. In quantum computing, the Hadamard gate can control the qubit and encrypt the information in qubit [5, 6]. Therefore, we apply the Hadamard gate to encrypt the information in entangled qubits, and the encrypted information can be decrypted by re-applying the Hadamard gate.

In this work, we study the influence of depolarizing noise on the general entangled state of photon. Moreover, we study the influence of the Hadamard gate on the general entangled photon in depolarizing noise. This paper is organized as follows. We analyze the entangled state of polarization in depolarizing noise and measure the quality of entanglement by using concurrence in Section 2. In Section 3, the time dependence of concurrence is presented for both cases. The conclusions are shown in Section 4.

2. Entanglement in Depolarizing noise

2.1. Depolarizing noise

The polarization states of photon are employed to represent the qubit $|0\rangle$ and $|1\rangle$. In this work, we use $|H\rangle$ for 1 and $|V\rangle$ for 0. A conventional protocol for sending quantum information is entangled states of photons [1]. We initially prepare the entangled states for the sender A to the receiver B in two combinations, as given by

$$|\psi_1\rangle = a_1|H\rangle_A|V\rangle_B + b_1|V\rangle_A|H\rangle_B, \quad |\psi_2\rangle = a_2|H\rangle_A|H\rangle_B + b_2|V\rangle_A|V\rangle_B, \quad (1)$$

where the coefficients satisfy the normalization conditions $|a_1|^2 + |b_1|^2 = 1$ and $|a_2|^2 + |b_2|^2 = 1$. These states correspond to the Bell states [7].

The entangled photons are prepared as an ensemble of entangled photons. We can describe the ensemble of entangled photons by using density matrix ρ . Thus, the initial states of the entangled photons are

$$\rho_1 = |\psi_1\rangle\langle\psi_1| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_1^2 & a_1b_1 & 0 \\ 0 & a_1b_1 & b_1^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)$$

$$\rho_2 = |\psi_2\rangle\langle\psi_2| = \begin{pmatrix} a_2^2 & 0 & 0 & a_2b_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_2b_2 & 0 & 0 & b_2^2 \end{pmatrix}. \quad (3)$$

When the entangled photons are sent to the receiver, they inevitably interact to the environments. One of the environment effects is depolarizing noise. The depolarizing noise is a non-dissipative channel that affects the entanglement feature of photon [8, 9]. The evolution of density states in depolarizing channel is given by

$$\rho' = \sum_{i=1}^4 E_i \rho E_i^\dagger, \quad (4)$$

where ρ is initial density matrix. The single qubit Kraus operators E_i for depolarizing channel are given by,

$$E_1 = \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad E_3 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad E_4 = \sqrt{\frac{p}{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (5)$$

where $p = 1 - \exp(-\gamma t)$ and γ is the decay factor of the depolarizing channel. Since only the qubit sending to the receiver B evolves under depolarizing noise, the Kraus operators become

$$D_1 = E_1^A \otimes E_1^B, \quad D_2 = E_1^A \otimes E_2^B, \quad D_3 = E_1^A \otimes E_3^B, \quad D_4 = E_1^A \otimes E_4^B. \quad (6)$$

When the entangled qubits evolve under the depolarizing channel, the density states become

$$\rho'_1 = \begin{pmatrix} \frac{2a_1^2 p}{3} & 0 & 0 & 0 \\ 0 & a_1^2 \left(1 - \frac{2p}{3}\right) & a_1 b_1 \left(1 - \frac{4p}{3}\right) & 0 \\ 0 & a_1 b_1 \left(1 - \frac{4p}{3}\right) & b_1^2 \left(1 - \frac{2p}{3}\right) & 0 \\ 0 & 0 & 0 & \frac{2b_1^2 p}{3} \end{pmatrix} \quad (7)$$

$$\rho'_2 = \begin{pmatrix} a_2^2 \left(1 - \frac{2p}{3}\right) & 0 & 0 & a_2 b_2 \left(1 - \frac{4p}{3}\right) \\ 0 & \frac{2a_2^2 p}{3} & 0 & 0 \\ 0 & 0 & \frac{2b_2^2 p}{3} & 0 \\ a_2 b_2 \left(1 - \frac{4p}{3}\right) & 0 & 0 & b_2^2 \left(1 - \frac{2p}{3}\right) \end{pmatrix}. \quad (8)$$

2.2. Measurement of entanglement quality

The entanglement quality of photon can be measured by using concurrence [10, 11]. This measurement tool can measure entanglement for both pure state and mixed state. It is conventionally defined as

$$C = \text{Max}\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (9)$$

where λ_i ($i = 1, 2, 3, 4$) are the square root of the eigenvalues in decreasing order of the non-Hermitian matrix $\rho(\sigma_y^A \otimes \sigma_y^B)\rho^*(\sigma_y^A \otimes \sigma_y^B)$. The concurrence ranges from 0 to 1. If the qubits are entangled, the concurrence C will be 1. On the other hand, if the qubits are separable, the concurrence C becomes 0.

When the qubits are in the standard basis of polarization $|HH\rangle, |HV\rangle, |VH\rangle, |VV\rangle$, the concurrence can be simplified as

$$C = 2\text{Max}\{0, \sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}, \sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\}, \quad (10)$$

where ρ_{ij} ($i, j = 1, 2, 3, 4$) are the elements of density matrix ρ . According to the concurrence in the equation (10), when the entangled photons evolve under depolarizing noise, the concurrences of density matrix equations (7) and (8) are

$$C_\alpha = 2\text{Max}\left\{0, -\frac{1}{3}\sqrt{a_\alpha^2(1-a_\alpha^2)\exp(-2\gamma t)(\exp(\gamma t)+2)^2}, \frac{1}{3}\left(\sqrt{a_\alpha^2(1-a_\alpha^2)\exp(-2\gamma t)(\exp(\gamma t)-4)^2} - 2\sqrt{a_\alpha^2(1-a_\alpha^2)\exp(-2\gamma t)(\exp(\gamma t)-1)^2}\right)\right\}, \quad (11)$$

where $\alpha = 1, 2$. The concurrence in the equation (10) is evaluated by assigning $|b_\alpha|^2 = 1 - |a_\alpha|^2$ and $p = 1 - \exp(-\gamma t)$. We note that the concurrences of $|\psi_1\rangle$ and $|\psi_2\rangle$ in the equation (1) have the same expressions.

2.3. Hadamard gate

The common quantum gate in quantum information is Hadamard gate. It is defined by

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (12)$$

This operator transforms a single qubit to superposition state. Since the entangled states $|\psi_1\rangle$ and $|\psi_2\rangle$ is two-qubits system, the Hadamard gate for the two-qubits system can be defined by

$$H = H_A \otimes H_B, \quad (13)$$

where H_A and H_B are the Hadamard gate acting on the first and the second qubits, respectively.

When we apply the Hadamard gate to the $|\psi_1\rangle$ and $|\psi_2\rangle$ and send to the depolarizing channel, The concurrences become

$$\begin{aligned} C_\alpha^H = & 2 \text{Max}\left\{0, \frac{1}{12} \left(\sqrt{\left(\sqrt{1-a_\alpha^2} + a_\alpha\right)^4 (4p-3)^2} \right. \right. \\ & \left. \left. - \sqrt{\left(8a_\alpha p \sqrt{1-a_\alpha^2} + 3a_\alpha^2 - 6\sqrt{1-a_\alpha^2}a_\alpha + 3(1-a_\alpha^2)\right)^2} \right), \right. \\ & \left. \frac{1}{12} \left(\sqrt{\left(a_\alpha - \sqrt{1-a_\alpha^2}\right)^4 (4p-3)^2} \right. \right. \\ & \left. \left. - \sqrt{\left(\sqrt{1-a_\alpha^2}a_\alpha(6-8p) + 3a_\alpha^2 + 3(1-a_\alpha^2)\right)^2} \right) \right\}, \quad (14) \end{aligned}$$

where $\alpha = 1, 2$. We assign $|b|^2 = 1 - |a|^2$ and $p = 1 - \exp(-\gamma t)$. The concurrences of both $|\psi_1\rangle$ and $|\psi_2\rangle$ with Hadamard operation also have the same expressions.

3. Results and Discussion

We study the characteristics of the concurrence in the equations (10) and (14) with parameter a and γt . The results are shown in figures 1 and 2. In figure 1, the concurrence of the

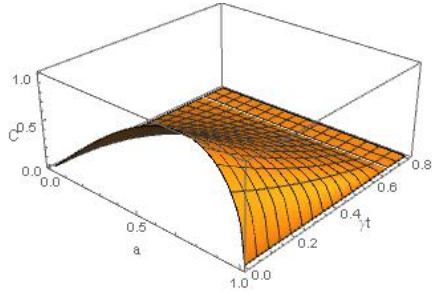


Figure 1. The behavior of concurrence depending on a and γt . The concurrence will be degraded in depolarizing channel. All entangled states will become separated states after $\gamma t = 0.7$.

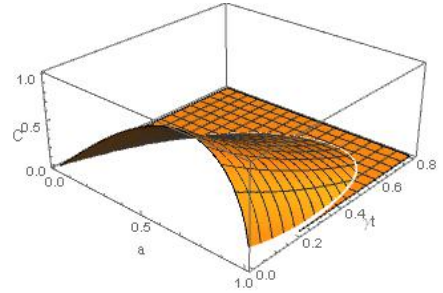


Figure 2. The behavior of concurrence depending on a and γt . The Hadamard gate affects the entangled states with different values of a . The concurrence with different a will decay with different rates.

entangled states $|\psi_1\rangle$ and $|\psi_2\rangle$ will be degraded by the influence of depolarizing noise. The initial concurrence depends on the value of a . When $a = 0.0$ and $a = 1.0$, the concurrence is 0. This means that the prepared state is initially separable. The maximally entangled state happens when $a \approx 0.707 = 1/\sqrt{2}$. All the combinations of $|HV\rangle$ and $|VH\rangle$ or $|HH\rangle$ and $|VV\rangle$ will decay to separable states after $\gamma t = 0.7$.

On the other hand, when we apply the Hadamard gate to the $|\psi_1\rangle$ and $|\psi_2\rangle$ and send to the depolarizing channel, the decay of concurrence depends on the value of a . In figure 2, the

maximally entangled state still happens when $a = 1/\sqrt{2}$, and it decays to 0 when $\gamma t = 0.7$. However, other combinations are degraded faster than $a = 1/\sqrt{2}$. For example, if $a = 0.5$, the concurrence will degrade from $C = 0.87$ to 0 when $\gamma t = 0.65$. This means that the Hadamard gate can be used to filter the maximally entangled states to carry the informations in the depolarizing channel.

4. Conclusions

The entangled states of photon are initially prepared in two types of combinations $|\psi_1\rangle$ and $|\psi_2\rangle$. The entangled photons are sent from sender A to the receiver B via depolarizing noise. The entanglement quality can be measured by using concurrence. The evolution of concurrence shows the degradation of entanglement quality in depolarizing channel.

We also apply the Hadamard gate to the qubits before sending them through the depolarizing noise. The result shows that the Hadamard gate can change the evolution of concurrence of the qubits in the depolarizing noise, and the maximally entangled state can keep the entanglement quality in depolarizing noise longer than the other. This suggests that we can use the Hadamard gate to filter the maximally entangled qubits in depolarizing noise. In future work, we plan to study the protection of entanglement quality in depolarizing noise in different methods.

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