

Effects of global and local rewiring on SIS epidemic adaptive networks

Suwakan Piankoranee¹ and Surachate Limkumnerd^{1,2}

¹Department of Physics, Faculty of Science, Chulalongkorn University, Bangkok, Thailand

²Thailand Center of Excellence in Physics (ThEP Center), CHE, Bangkok, Thailand

E-mail: suwakan.pi@student.chula.ac.th

Abstract. Adaptive epidemic network is generally driven by two main processes: (1) infection-recovery process that changes the states of the nodes, and (2) rewiring process that modifies the topology of the network. We consider an adaptive susceptible-infected-susceptible (SIS) epidemic on a network. In this work, a link is rewired randomly to a chosen susceptible node according to a prescribed global or local rewiring method. In the global rewiring case, a susceptible node could break the link with its infected neighbour to form a new link with another susceptible in the network. In this rewiring mechanism, the node must know health status of every other node in the network in order to rewire. This is however impractical in real life because the knowledge is only limited to a certain neighbouring group of nodes surrounding a given point. We propose a more realistic rewiring method called the local rewiring method, where a new link is limited to join a susceptible within a neighbouring distance. We investigate the impact of local versus global rewiring on an epidemic network. A new disease prevention behaviour emerges as a result.

1. Introduction

Incorporating human behaviour in the dynamics of epidemic spread has received much attention recently [1–3]. One of the important strategies in controlling disease spreading can be modelled by adaptive networks. The basic idea is that individuals may change their behaviours under the threat of an emerging disease. For example, susceptible individuals may try to reduce their chance of catching the disease by avoiding contacts with infected individuals. The effects of this simple strategy have been explored by epidemic models on networks [1–4]. In these models, each member of a population such as an individual and an interaction between a pair of individuals, respectively, are represented by a node and a link of a network. The network is considered as bidirectional, with disease transmission possible in either direction along a link.

The mechanism of avoiding contact with infected individuals in current adaptive network models is to break the links between susceptible and infected individuals. According to Gross et al. [1], the number of links is conserved for the purpose of preserving the number of social contacts. Those broken links must be replaced by new connections which are established with any individuals in the network. In order to create the new connections, individuals need information about which other individuals they want to rewire to. Indeed, this knowledge is limited to their acquaintances. This allows us to modify the existing models to move closely simulate real-world network evolution by suppressing information accessible to a node about its neighbours.

2. SIS epidemic network model with rewiring mechanisms

We study a susceptible-infected-susceptible (SIS) model on an adaptive network. At a given time, each individual can be either susceptible (S) or infected (I). The infection is transmitted from infected individuals to their susceptible neighbours, who become immediately infected ($S \rightarrow I$) at the rate of b . These infected individuals may be recovered at the recovery rate of r , after which they become susceptible again ($I \rightarrow S$). Both the infection and recovery processes are independent. The disease dynamics is described by the continuous-time SIS epidemic model.

In an adaptive SIS model, another process is included, links joining susceptible individuals with infected individuals are broken at rewiring rate of w , and the corresponding susceptible individual is then connected to a randomly chosen susceptible individual in the network. Self-connections and multiple connections between pairs of individuals are prohibited. In the global rewiring mechanism proposed in [1], the individuals know of the health status of every other individual in the network. However, in real world, the knowledge should only be limited to a certain distance. We introduce a more realistic rewiring method called the local rewiring method, where new links are limited to join susceptible individuals within their d -nearest neighbouring distance. The distance $d = 4$ is only considered in this work.

To study the effects of local versus global rewiring, we generate the Erdős–Rényi random network. Initially, an individual has an average of k neighbours so the total number of links (L) in the network is $kN/2$. The corresponding degree distribution, which is the distribution of the number of neighbours of each individual, is a Poissonian distribution with parameter k [1, 5]. We perform kinetic Monte Carlo simulations [6] on the random networks with $N = 10^4$ and $L = 2 \times 10^4$. Larger network sizes with the same k were also considered, however the major results of this paper do not depend strongly on the network size.

In the global rewiring scheme, taking into account the above dynamical rules and using the pair approximation, one could write down a system of three coupled ordinary differential equations for i , $[si]$ and $[ss]$ which represent, respectively, the fraction of infected nodes, the fraction of links joining susceptible-infected and the fraction of links joining susceptible-susceptible:

$$\begin{aligned} i' &= \frac{bk}{2}[si] - i \\ [ss]' &= 2[si] - bk \frac{[ss][si]}{1-i} + 2w[si] \\ [si]' &= -(b+1)[si] + \frac{bk}{2} \left(\frac{[ss][si]}{1-i} - \frac{[si]^2}{1-i} \right) + 2 - [ss] - 2[si] - w[si] \end{aligned} \quad (1)$$

Here prime indicates time derivative.

In this article we use the rescaled parameters $\tilde{b} = b/r$ and $\tilde{w} = w/r$, so that the time unit is that of the average recovery time r^{-1} . Henceforth, we shall omit tilde symbol for brevity. At the stationary state, the right-hand side of equations (1) are zero. This leads to two critical lines $w = bk - 1$ and $w = \frac{b^2(k-1)^2 + 4b}{4}$ that are represented in figure 1 by the solid (A) and dash (B) lines, respectively. The solid (C) and dash (D) lines are obtained from simulations of the model with local rewiring (here with $d = 4$). Three phases are possible: (a) the endemic steady state is stable (below the solid lines), (b) the disease-free steady state is stable (above the dash lines) and (c) two steady states exist, one endemic and the other disease-free (between the solid and the dash lines).

3. Effects of global and local rewiring

We consider three conditions: (1) in the endemic phase of global and local rewiring ($w = 2$), (2) in the disease-free phase of global rewiring but in the endemic phase of local rewiring ($w = 4$) and

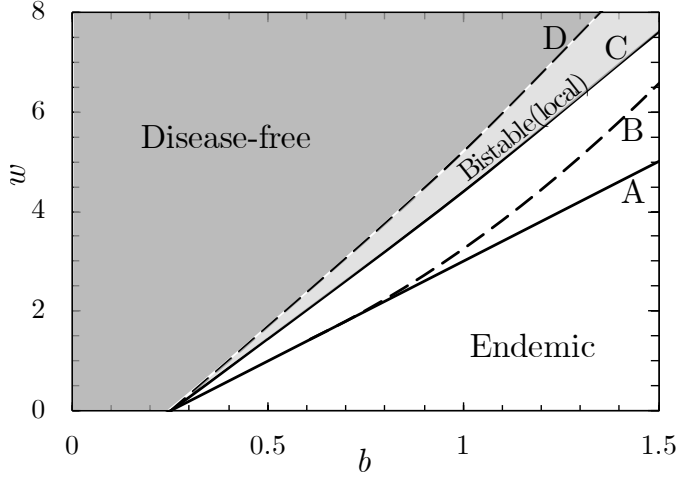


Figure 1. Phase diagram showing the dependence on the rewiring rate w and the infection rate b for global (A and B) and local (C and D) rewiring methods. Each method, the region below the solid line corresponds to endemic phase whereas the region above the dash line corresponds to disease-free phase. The V-shape region between them is the bistable phase. The value of $N = 10^4$ and $L = 2 \times 10^4$, and for the local rewiring distance $d = 4$ are chosen for the plot.

(3) in the disease-free phase of global and local rewiring ($w = 6$). All conditions are simulated at the same infection rate of $b = 1$.

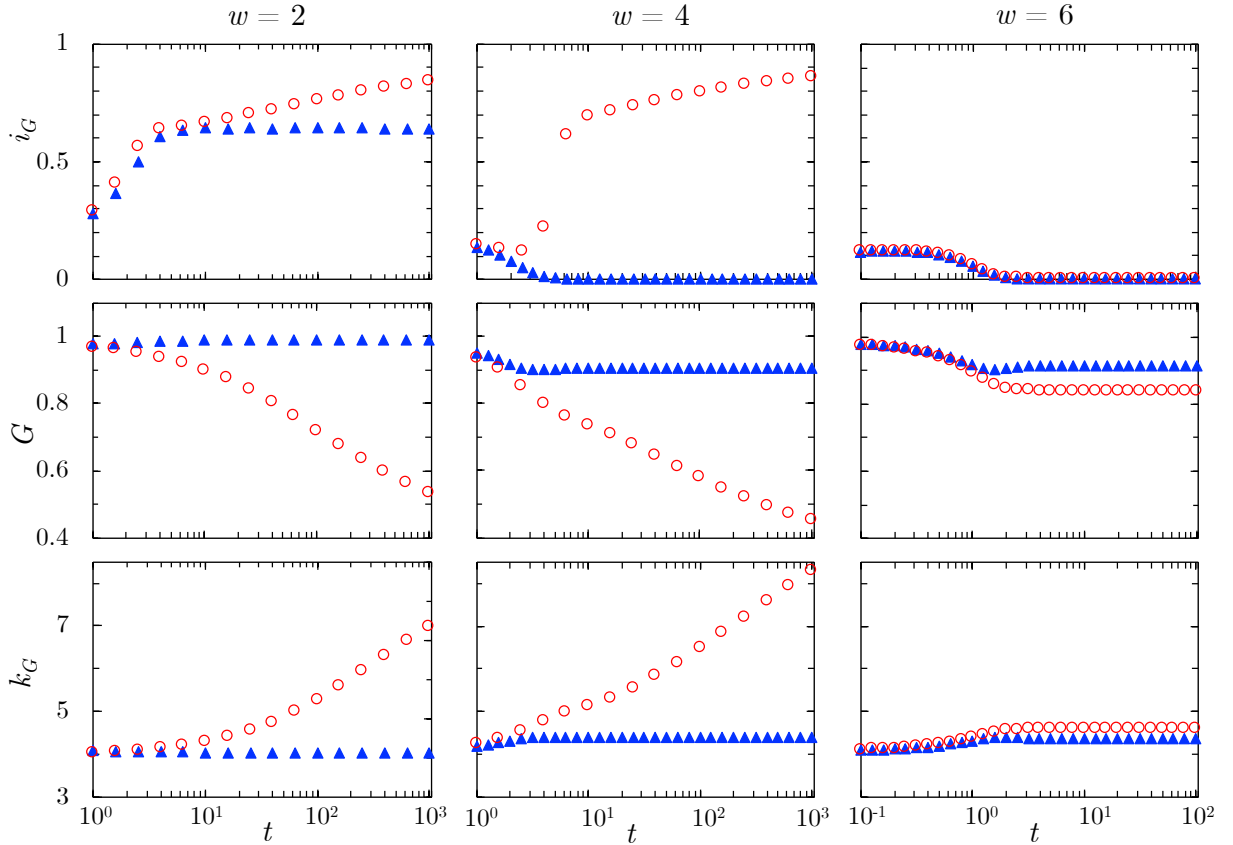


Figure 2. Simulation results from 3 different values of w , i.e., $w = 2$ (left), $w = 4$ (middle) and $w = 6$ (right). For each value of w , the triangles and circles represent, respectively, results from global and local rewiring. Plotted from top to bottom are the time evolution of the infected fraction in the largest component (i_G), the largest component fraction (G) and the degree of the largest component (k_G).

Let us first consider the network in an endemic condition for both global and local rewiring (figure 2, left column). In this case an epidemic can persist. For the global rewiring, the largest component size and the degree of the largest component do not quite change from their initial values. The rewiring mechanism tries to isolate the infected nodes from the largest component, and due to the global rewiring rules the recovered nodes can reconnect to the largest component. For the local rewiring, the largest component reduces in size but not to zero because globally the number of links is conserved. The largest component will stop shrinking when all of its nodes are nearly completely connected and then the rewiring does not isolate the infected node out of the component. For other components such as small components, their evolution is typically of size one. Its evolution is therefore trivial. So that we are interested in the largest component rather than the whole network.

Now, consider the disease-free condition for both global and local rewiring (figure 2, right column). In this case the fraction of the infected nodes goes to zero at the steady state and the size of the largest component decreases slightly, whereas the degree of the largest component increases slightly. Under this condition rewiring always reduces the number of links that are accessible for epidemic spreading by isolating the infected nodes and therefore the prevalence of the disease, i.e., the infected fraction, is gradually reduced and finally disappears.

Finally, consider $w = 4$ where the global rewiring gives the disease-free phase, but the local rewiring gives the endemic phase. Even the same rewiring rate but with limited amount of the knowledge of health status within the neighbouring distance, $d = 4$, the local rewiring cannot prevent an epidemic spreading. The effect of the value of d to epidemic spreading was also considered. If d increases the capability to prevent disease spreading may be better but still less effective than the global rewiring. This is because the local rewiring can only reconnect to susceptible nodes within the same component. This makes the largest component, which has high connectivity, be more vulnerable to attack.

4. Conclusion

We study the effects of the local and global rewiring as a strategy to prevent epidemic spreading. In the global rewiring the susceptible nodes can rewire to another susceptible nodes in the network whereas our proposed method called local rewiring allows the nodes to rewire only within their neighbouring distance. This is more practical in real life. One crucial feature of the local rewiring is that with same rewiring rate in disease-free zone of local and global rewiring, the local rewiring needs less information of attempted rewiring individuals to avoid disease spreading. Further study is required to pinpoint the region of rewiring rates. Our result may be applicable to practical use in trying to limit or control the spreading of an outbreak in systems where information is not shared globally.

Acknowledgments

This research is supported by the 90th Anniversary of Chulalongkorn University, Ratchadaphisek Somphot Fund. The authors thank to National e-Science Infrastructure Consortium for providing computing resources that have contributed to the research results reported within this paper. URL: <http://www.e-science.in.th>

References

- [1] Gross T, D'Lima C J D and Blasius B 2006 *Phys. Rev. Lett.* **96** 208701
- [2] Zanette D H and Risau-Gusman S 2008 *J. Biol. Phys.* **34** 135
- [3] Shaw L B and Schwartz I B 2008 *Phys. Rev. E* **77** 066101
- [4] Gross T and Blasius B 2008 *J. Royal Soc. Interface* **5** 259
- [5] Newman M E J 2010 *Networks: An Introduction* (Oxford: Oxford University Press)
- [6] Serebrinsky S A 2011 *Phys. Rev. E* **83** 037701