Backward Third-Harmonic Pulse Generation in a One-Dimensional PIM/NIM Structure

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Abstract. In this work, a set of couple-mode equations for describing a backward third harmonic generation (BTHG) in a one-dimensional periodic structure of positive-index material (PIM) layers and third-order nonlinear negative-index material (NIM) layers was analysed using multiple-scale method. Due to the negative-index phase matching and band-edge field enhancement the intensities of backward third-harmonic pulsed generated from the PIM/NIM periodic structure is increased for 100 time of those generated from a single NIM medium.

1. Introduction

Metamaterials are artificial electromagnetic structures providing unnatural properties, which cannot be found in nature such as negative refraction or reversal of Snell's law, reversal of Doppler shift, and backward Cherenkov radiation [1]. The key feature of metamaterials is having negative refractive index, so they are often called NIM due to their simultaneous negative permittivity and permeability. NIMs have become an interesting research topics because of the previous experimental demonstration of photonics crystal with negative refractive index at optical and infrared frequencies [2]. Various types of NIM have been developed to create potential applications. The nonlinear optical effects can take the benefits of negative-index of NIM for improving efficiency of nonlinear frequency conversions [3-4]. The nonlinear NIMs can possibly create the nearly phase-matched conditions by a method called "negative-index phase-matching (NIPM)", for counter-propagating energy flow waves when fundamental frequency and converted frequency waves lie in the negative-index and the positive-index frequency domain, respectively [4].

In this paper, an enhancement of BTHG effect in a one-dimensional PIM/NIM (1D-PIM/NIM) structure by using NIPM technique and band-edge field enhancement is demonstrated numerically. Our 1D-PIM/NIM structure is composed of periodically alternating of NIM embedded in third-order nonlinearity material layer and linear PIM layer as illustrated in figure 1. The time-evolution of incident FF and generated backward TH pulse from 1D-PIM/NIM structure is obtained by numerical integrating coupled-mode equations (CMEs) with a fast Fourier transform-beam propagation (FFT-BPM). Finally, the BTHG conversion efficiency would be calculated at different pumping level of FF fields and compared with the efficiency of an equivalent-length NIM slab.

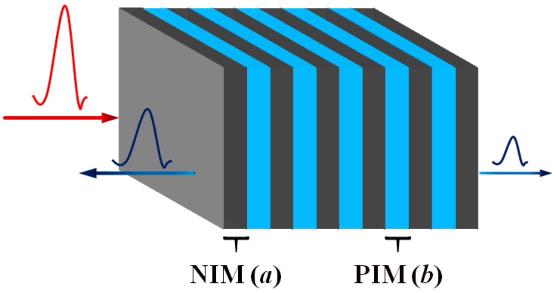


Figure 1. The illustration of BTHG mechanism in a 1D-PIM/NIM structure.

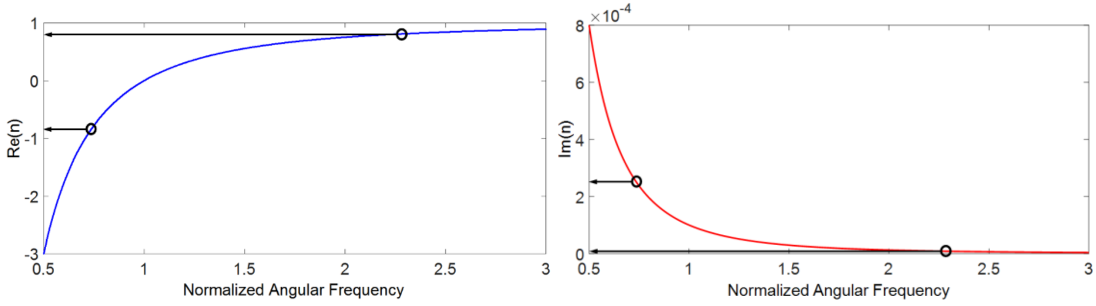
2. Related Theories

2.1. Negative-index phase-matching

The refractive index of NIM is dispersive and its dispersion relation can be expressed by a Drude model for allowing the negative energy density [4] as below:

$$n(\omega) = 1 - \frac{(\omega_p^2 / \omega_0^2)}{\omega^2 + i\Gamma\omega}, \tag{1}$$

where ω is the angular frequency of incident fields, ω_0 is the reference angular frequency, ω_p is the plasma angular frequency, and Γ is the normalized damping factor.



(a) (b)

Figure 2. (a) Real (blue line) and (b) imaginary (red line) parts of refractive index for the Drude model with the normalized damping factor $\Gamma \approx 10^{-4}$. The cross-circles point to the magnitudes of the refractive indices at the FF and TH fields where the negative-index phase-matching (NIPM) is nearly satisfied.

As seen in figure 2, the dispersion curve for real and imaginary parts of NIM's refractive index are represented by blue and red lines, respectively. For FF component at ω, the complex refractive index of NIM layer is $n(\omega) = -0.821200 + 0.0002458i$. Meanwhile, at 3ω, the complex index of NIM is $n(3\omega) = 0.820900 + 0.000008i$. Obviously, the negative refractive index of NIM layers at FF is used to satisfying the phase-matched condition with NIPM technique as follows:

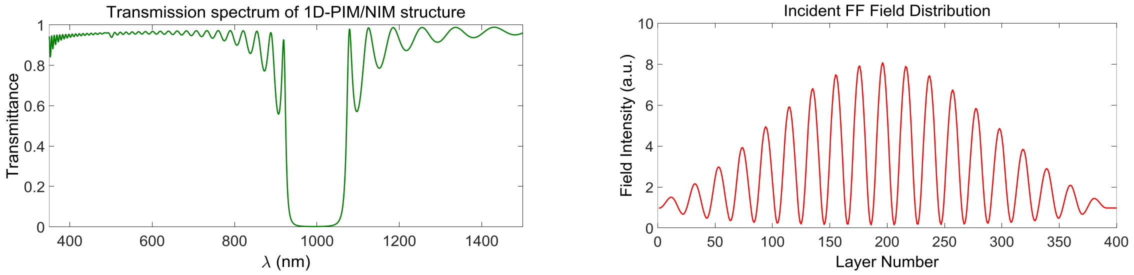
$$\Delta k = (k_3 - 3k_1) = 3\omega\frac{|n(3\omega)|}{c} - 3\omega\frac{|n(\omega)|}{c} \approx 10^{-3}. \tag{2}$$

The magnitude of Δk is very small and close to zero, so this phase-mismatch between FF and TH pulses is nearly phase-matched condition for the BTHG.

2.2. Band-edge field enhancement

Since, the 1D-PIM/NIM structure is periodic structure as shown in figure 1, so its refractive index is periodically modulated and not homogeneous. Thus, its transmission is quite difference from bulk mediums and have to be examined for ensuring that the FF and TH fields can transmit through this structure. In this study, NIM layer has refractive index corresponding to dispersion curves in figure 2 and PIM layer is assumed to be air layer, so each layer thickness can be determined as

$a = \lambda_0 / 4n_{NIM} = 300\ \text{nm}$ and $b = \lambda_0 / 4n_{PIM} = 250\ \text{nm}$, respectively, where reference wavelength λ_0 is 1,000 nm. By using transfer-matrix method (TMM), the transmission spectrum of the 1D-PIM/NIM structure can be calculated and illustrated as shown in figure 3a. The wavelength of FF pulse is usually chosen at the long-wavelength band-edge ($\lambda_\omega \approx 1{,}079\ \text{nm}$). At this wavelength, the band-edge field enhancement mechanism can be achieved because the local electric field distributing along the structure is resonant as shown in figure 3b [5-6]. By pumping with resonant FF field, therefore, the BTHG effect can be easily enhanced.

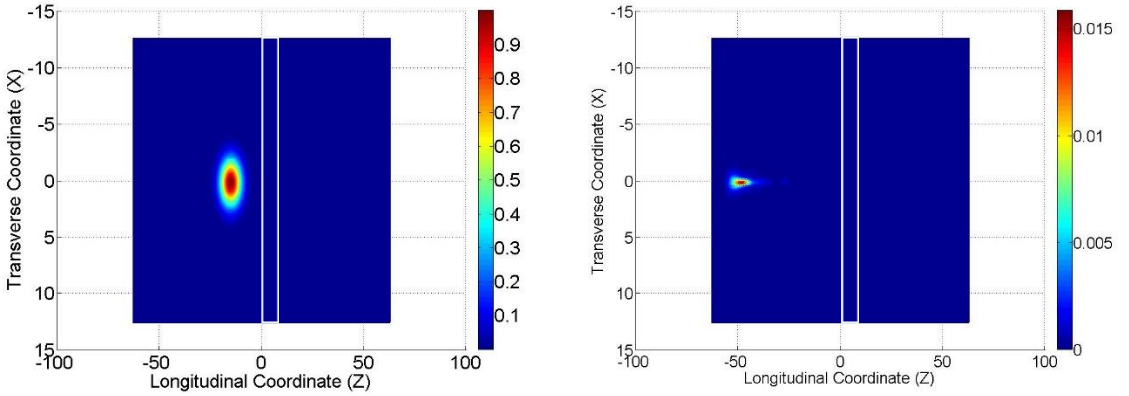


(a) (b)

Figure 3. (a) The transmission spectrum of the 1D-PIM/NIM structure. (b) The electric field distribution, when tuning the incidental FF field wavelength to the long-wavelength band-edge of photonic band-gap.

3. Time-Evolution of Backward Third-Harmonic Pulsed Generation

In order to model a BTHG effect from a 1D-PIM/NIM structure in pulse regime, a completed set of CMEs for electric and magnetic fields is developed by using MSA approach as explained in Ref [5-6]. The time-evolution of backward TH electric fields are computed by applying the fast Fourier transform-beam propagation method (FFT-BPM) method to solve the CMEs [6-7]. The appropriate initial condition must be applied to this problem by determining the initial Gaussian-shape FF field with intensity 1 GW/cm^2 (as shown in figure 4a) is launched into left-side of 20-periods 1D-PIM/NIM structure only and initial values of other fields is zero. Figure 4b shows the BTHG response of the structure giving the output backward TH pulse with intensity ~15 MW/cm^2.



(a) (b)

Figure 4. (a) Initial Gaussian-shape FF pulse with normalized intensity at time step *t* = 0. (b) The snapshot of the generated backward TH pulses at total time step *t* = 400. Note that: the clear rectangular at origin of *z*-coordinate represents the location of 1D-PIM/NIM structure.

Since, NIM layers are cubic nonlinearity $\chi^{(3)} = \chi_E^{(3)} = \chi_M^{(3)} \approx 10^{-18}\ \text{m}^2/\text{V}^2$ for both electric and magnetic fields. These nonlinear coefficients depend on the intensity of input FF and other coupling fields (self-phase and cross-phase modulations). When the high intensity FF field interacts with the nonlinearity of the structure, it contributes to BTHG response. Consequently, the backward TH pulse intensity are directly increased following the increased nonlinearity of 1D-PIM/NIM structure.

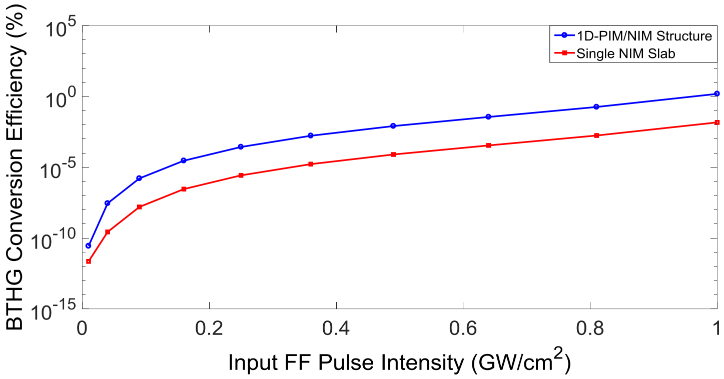


Figure 5. Comparison of the conversion efficiency of BTHG process from 1D-PIM/NIM structure and single nonlinear NIM medium with same length.

Finally, the conversion efficiencies of BTHG effect of 1D-PIM/NIM structure is computed by using the percentage of output backward TH pulse and input FF pulse ratio at different level of FF intensity as shown in figure 5. This figure also shows the comparison between the BTHG conversion efficiencies of the 1D-PIM/NIM structure (blue line with circle-marker) and single nonlinear NIM medium (red line with square-marker). Both curves suggest that these efficiencies are increased with the intensity level of FF pulse and maximized at 1 GW/cm^2. Both medium give high conversion efficiency since the phase-matched condition was satisfied by NIPM technique. But maximum conversion efficiency of backward TH pulse from 1D-PIM/NIM structure is enhanced by a factor of 100 greater than backward TH pulse from single NIM slab because of band-edge field enhancement mechanism that being provided only in periodic structure.

4. Conclusions

The BTHG effect in a 20-periods 1D-PIM/NIM structure is numerically investigated in this study. To enhance the strength nonlinear effect in 1D-PIM/NIM structure, the refractive index of NIM layer at ω and 3ω, which satisfy nearly phase-matched condition with NIPM method are optimized, and then FF pulse wavelength is chosen at 1,079 nm for creating band-edge field enhancement. Numerical results show that, the maximum output intensity of backward TH pulse (~15 MW/cm^2) is achieved at FF pulse intensity 1.0 GW/cm^2. Moreover, the maximum efficiency of 1D-PIM/NIM structure (~1.5 %) is boosted up to 100 times greater than single NIM medium because of existing of both negative-index phase matching and band-edge field enhancement.

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