Cross-Polarized Wave Generation in a Nonlinear Hyperbolic Metamaterial

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Abstract. A generation of cross-polarized wave (XPW) in nonlinear hyperbolic metamaterials (NHMM), which are composed of periodic arrangement of gold (Au) and barium difluoride (BaF_2) layers with subwavelength thickness for exhibiting anisotropy of permittivity and third-order nonlinearity, has been investigated numerically. This cubic nonlinear effect is described by degenerate four-wave mixing (DFWM) of three linearly polarized fields and one produced field, which has linear polarization in orthogonal direction. By managing the fill-factor value of the NHMM, the nearly phase-matched condition based on hyperbolic phase-matching (HPM) technique are achieved implicitly. We found that the conversion efficiencies of XPW generation as a function of incident angle at various pumping intensities are maximized at optimal incident angle.

1. Introduction

Cross-polarized wave (XPW) generation is a degenerated four-wave mixing (DFWM) process in nonlinear medium providing anisotropy of the real part of third-order nonlinear susceptibility tensor $\chi^{(3)}_{ijkl}$ in which a linearly polarized incident field is nonlinearly converted to an orthogonal polarized field [1]. In other words, a strong pump field component $E_{\parallel}(\omega)$ is launched to interact with anisotropic nonlinearity $\chi^{(3)}_{ijkl}$ in the nonlinear medium, then a new field component $E_{\perp}(\omega)$, with polarization perpendicularly oriented to of incident field, is possibly generated. In accordance with energy conservation, the XPW interaction can be represented as: $\omega_{\perp} = \omega_{\parallel} + \omega_{\parallel} - \omega_{\parallel}$. Due to material dispersion, the wave-vector mismatch corresponding to XPW interaction is dominated and expressed as: $\Delta k = k_{\perp} - k_{\parallel} - k_{\parallel} + k_{\parallel}$. To achieve efficient XPW generation, the wave-vector mismatch Δk should be reduced to zero for maintaining constructive interference between interacting waves and enlarging coherent length by various techniques of phase-matching.

A few decade ago, artificial optical structures i.e. photonic crystals and metamaterials containing nonlinear materials have been proposed to improve efficiency of nonlinear optical effects. Since, they have been created the nearly phase-matched condition with band-edge phase-matching, quasi-phase-matching, negative-index phase-matching, and epsilon-near-zero phase-matching [2]. Recently, a kind of metamaterials named hyperbolic metamaterials (HMM) have been widely studied about controllable dispersion property by managing types, shapes, and dimensions of composed materials. Since, composed material dimension is reduced into deep sub-wavelength level, so the HMM are considered

as anisotropic metamaterials with single optical axis or uniaxial structure. They can provide dispersion relation which may be elliptic or hyperbolic function [3]. Consequently, the phase-matching of interacting ordinary and extra-ordinary waves, which lie on different dispersion curves may be possibly achieved at optimal incident angle in the same way as birefringence phase-matching in ordinary anisotropic nonlinear mediums. This method is called hyperbolic phase-matching (HPM) [3].
In this study, the enhancement of XPW generation from HMM, which composed of periodic arrangement of metal (Au) and nonlinear dielectric (BaF_2) ultrathin layers and called nonlinear hyperbolic metamaterials (NHMM), have been investigated numerically. By using HPM technique, the optimal NHMM gives a nearly phase-matched point for XPW process that indicated by intersection between two isofrequency contours of pump and XPW fields. Therefore, the conversion efficiency of XPW generation can be maximized when the nearly phase-matched condition is occurred at incident angle 21.40^o and pump pulse intensity 200 GW/cm^2.

2. Effective Medium Approximation and Hyperbolic Phase-Matching

In this work, a nonlinear hyperbolic metamaterial (NHMM) is composed of a periodical arrangement of ultra-thin conductive and dielectric layers along *z*-axis as shown in Fig. 1a. Electromagnetic response of composed materials can be described by homogenous and isotropic permittivity. The permittivity of conductive layer (gold: Au) is described by Drude model: $\varepsilon_{\mathrm{Au}}\left(\omega\right)=\varepsilon_{\infty}-\omega_{pm}^2/(\omega^2+i\Gamma_m\omega)$, where $\varepsilon_{\infty}=9.8$ is permittivity of Au at low frequency, $\omega_{pm}=1.36\times10^{16}$ rad/s is plasma frequency and $\Gamma_m=1.05\times10^{14}$ rad/s is damping factor, respectively. Then, the permittivity of dielectric layer (barium fluoride: BaF_2) is given by Sellmeier dispersion formula as reference 4. According to material layers being reduced into deep-subwavelength thickness, so the NHMM can be considered as an anisotropic media (uniaxial structure) characterized by an effective permittivity tensor, which given from effective medium approximation (EMA) [3]:

$$\bar{\boldsymbol{\varepsilon}}=\begin{bmatrix}\varepsilon_{xx} & 0 & 0\\ 0 & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz}\end{bmatrix},\quad \varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{\parallel}=p\varepsilon_{\mathrm{Au}}+(1-p)\varepsilon_{\mathrm{BaF_2}},\quad \varepsilon_{zz}=\varepsilon_{\perp}=\left(\frac{p}{\varepsilon_m}+\frac{1-p}{\varepsilon_d}\right)^{-1}, \tag{1}$$

where $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are transverse and longitudinal permittivity tensor components and $p = d_m/(d_m + d_d)$ is filling factor of each unit-cell, which calculated from thicknesses of Au (d_m) and BaF_2 (d_d) layers.

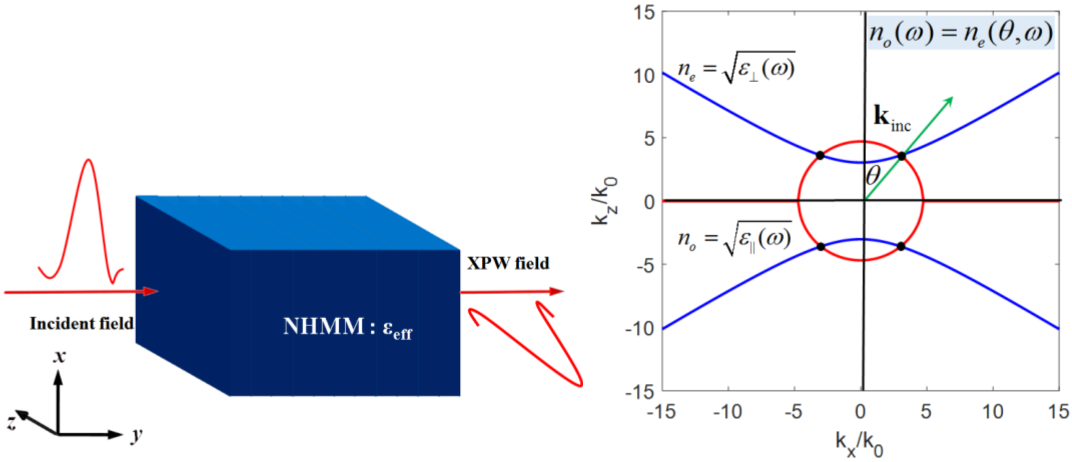
In the NHMM, a nearly phase matched condition can be achieved by using hyperbolic dispersion and choosing the appropriate polarization of the interacting waves. This configuration is called hyperbolic phase-matching (HPM) technique. In accordance with classical electromagnetic theory of plane wave propagation in uniaxial structure, the refractive index for ordinary (*o*) and extra-ordinary (*e*) waves are [2-3]:

$$n_o(\omega)=\left(\mathrm{Re}[\varepsilon_{\parallel}(\omega)]\right)^{1/2},\quad n_e(\theta,\omega)=\left(\frac{\cos^2\theta}{\mathrm{Re}[\varepsilon_{\parallel}(\omega)]}+\frac{\sin^2\theta}{\mathrm{Re}[\varepsilon_{\perp}(\omega)]}\right)^{-1/2}, \tag{2}$$

where θ is propagating angle of interacting wave with respect to optical axis. Eq. (2) gives the dispersion formula for the NHMM as:

$$\frac{k_x^2}{\mathrm{Re}[\varepsilon_{\perp}(\omega)]}+\frac{k_z^2}{\mathrm{Re}[\varepsilon_{\parallel}(\omega)]}=k_0^2, \tag{3}$$

where k_x and k_z are transverse and longitudinal components of the wave-vector and $k_0 = \omega/c$ is free-space wave-vector. In case of $\varepsilon_{\perp}>0$ and $\varepsilon_{\parallel}<0$ (type-I) or $\varepsilon_{\perp}<0$ and $\varepsilon_{\parallel}>0$ (type-II), Eq. (3) is hyperbolic function. But when all permittivity components are all positive, Eq. (3) is elliptic function. For real value of $\varepsilon_{\mathrm{Au}}$ and $\varepsilon_{\mathrm{BaF_2}}$, the hyperbolic dispersion is achieved when $(\varepsilon_{\mathrm{BaF_2}}/\varepsilon_{\mathrm{Au}})\geq(-d_m/d_d)$. Furthermore, Eq. (3) also provides an isofrequency contour formed by conic section about z-axis. An intersection between two isofrequency contours for each interacting waves due to XPW generation is represented a nearly phase-matched point. The nearly phase-matched point gives an optimal propagating angle for: $\mathbf{k}_{\mathrm{XPW}}(\omega)-\mathbf{k}_{\mathrm{inc}}(\omega)-\mathbf{k}_{\mathrm{inc}}(\omega)+\mathbf{k}_{\mathrm{inc}}(\omega)=0$ or $n_e(\theta,\omega)=n_o(\omega)$, as well. Fig. 1b represents an isofrequency contour of collinear *o-o-o-e* XPW configuration, which means that three input *o*-waves producing one output *e*-wave, from the NHMM with fill-factor p = 0.3. The intersection of two contours as shown in Fig. 1b gives the angle θ, which corresponds to the right propagating angle of incident field respecting to z-axis along the NHMM and creates phase-matched condition. By using Eq. (2), the magnitude of θ is about 50.33° at λ = 1,000 nm. Hence, the optimal incident angle θ_i of the incident field can be directly determined by Snell's law and equal to 21.40°.



(a) (b)

Figure 1. (a) A schematic of the nonlinear hyperbolic metamaterial (NHMM) stack and collinear cross-polarized wave (XPW) generating configuration. (b) Isofrequency contours for *o* and *e*-waves at ω corresponding with collinear *o-o-o-e* XPW configuration represented by red solid curve and blue solid curve, respectively. The nearly phase-matched point (black dots) is obtained from the intersection between two isofrequency contours. Moreover, the intersection also gives right propagating angle of incident *o*-wave with respect to the optical axis.

3. Conversion Efficiency of XPW Generation

The calculation of the XPW generation can be completed by solving the nonlinear-wave equation as Eq. (4):

$$\nabla^2\mathbf{E}-\frac{\bar{\boldsymbol{\varepsilon}}}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2\mathbf{P}^{NL}}{\partial t^2}, \quad (4)$$

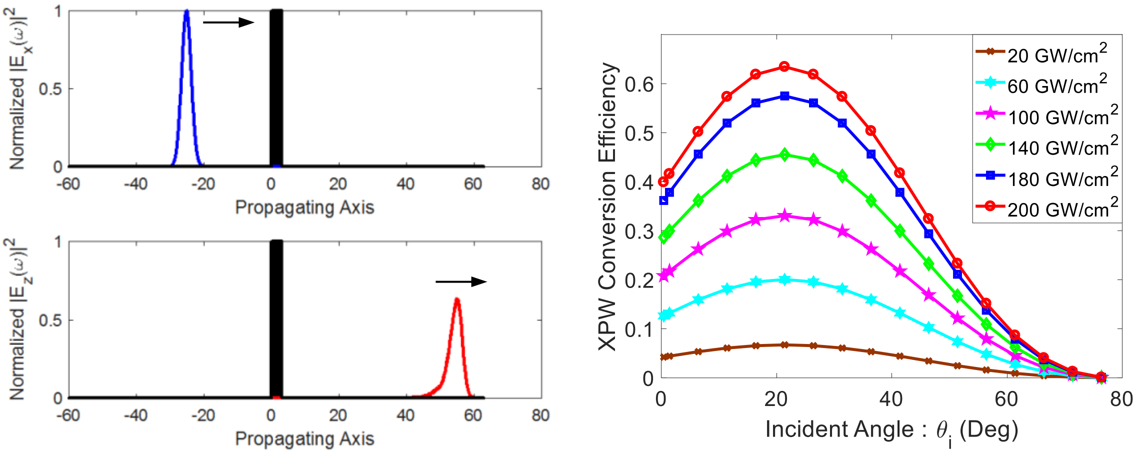
where $\mathbf{E}$ = $[E_x\ E_y\ E_z]^T$ is electric field vector and each field components in Cartesian coordinates can be written in complex form: $E_i(z,t)=1/2\{A_i(z)\exp[i(kz-\omega t)]+A_i^*(z)\exp[-i(kz-\omega t)]\}$. Meanwhile, the nonlinear polarization in the i direction for $\mathbf{P}^{NL}$ can be written as [5]:

$$P_i^{NL}=\varepsilon_0\sum_{jkl}\chi_{ijkl}^{(3)}E_jE_kE_l, \quad (5)$$

where the i, j, k, l are the x, y, z and ε_0 is permittivity in vacuum. Due to XPW field being generated from nonlinearity of BaF_2 only, then the $\chi_{ijkl}^{(3)}$ is a real tensor with only two independent components $\chi_{xxxx}^{(3)}=1.9\times10^{-22}$ and $\chi_{xxzz}^{(3)}=2.85\times10^{-23}$ m^2/V^2 [1]. The time-evolution of XPW interaction is achieved by solving Eq. (4) with a fast Fourier transform-pulse propagation method (FFT-PPM) in pulse regime [6]. A snap-shot of time-evolution of transverse components of incident pump pulse ($|E_{x0}|^2$) with intensity 200 GW/cm^2 (blue pulse), and of generated XPW pulse ($|E_z(L)|^2$) with intensity 126.42 GW/cm^2 (red pulse) are shown in figure 2a, respectively. Finally, the XPW conversion efficiency as a function of incident angle and different pumping levels is calculated by

$$\eta_{\mathbf{XPW}}=|E_z(L)|^2/|E_{x0}|^2, \quad (6)$$

and illustrated in figure 2b. From this figure, the generated XPW conversion efficiency is maximized (63.43%) when incident angle is optimized at 21.4° at pumping intensity 200 GW/cm^2 as shown with the red solid line and circle markers in Fig. 2b. A maximum XPW conversion efficiency at this incident angle indicates that the phase-match condition are satisfied by HPM in the sample structure. Moreover, the maximum conversion efficiency is decreased by reducing level of pump pulse intensity. Due to the inclusion of Kerr-nonlinearity inside the NHMM, so the XPW generation is directly amplified by high intensity pump field.



(a) (b)

Figure 2. (a) A snap-shot of incident pump pulse (blue) and output XPW pulse (red) from the NHMM with $p = 0.3$ that are achieved by using FFT-PPM. The vertical axis is normalized intensity of interacting pulses, and the horizontal axis is normalized longitudinal position. The black stripe is the NHMM. (b) The conversion efficiency of generated XPW pulse as a function of incident angle and pumping levels.

4. Conclusions

In this study, the XPW generation from the NHMM with $p = 0.3$ has been demonstrated numerically. By using engineered dispersion of the structure, the HPM technique permits the phase-matched point corresponding to XPW with *o-o-o-e* configuration (indicated by intersection of two isofrequency contours), which can be achieved by incident angle tuning. Consequently, an optimal incident angle is $\theta_i = 21.40°$. By using FFT-PPM algorithm, the output XPW pulse with intensity 126.86 GW/cm^2 is emitted when the nonlinearity of the NHMM is interacted with oblique incident pump pulse with intensity 200 GW/cm^2 and $\theta_i = 21.40°$. The conversion efficiency of XPW generation is 0.6343 at this moment. When incident angle is varied from this angle, the conversion efficiency is obviously decreased. Moreover, the XPW conversion efficiency is also decreased by reducing input pump pulse intensity since the NHMM includes Kerr-nonlinearity (intensity-dependent parameters) that directly contributes to strengthen of XPW response.

References

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