A comparison of gravitational acceleration measurement methods for undergraduate experiment

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Abstract. This research aimed to determine the acceleration due to gravity (g), by using the methods of free fall, simple pendulum, physical pendulum and an Atwood's machine in the undergraduate laboratory. The experiments were designed for students to explore, analyze the data and interpreting the results by using the principle of a Physics laboratory. The mean experimental values of acceleration due to the gravity of free fall, simple pendulum, physical pendulum and the Atwood's machine were 9.64 m/s^2, 9.67 m/s^2, 10.88 m/s^2 and 10.47 m/s^2, respectively.

1. Introduction

Acceleration due to the gravity of the Earth is denoted by the symbol g which is the acceleration of a body caused by the gravitational field acting on the body towards the center of the Earth, neglecting air resistance and mass. Gravitational acceleration (g) was first determined by Galileo Galilei in 1604. The law of gravitational attraction was formulated by Sir Isaac Newton (1642-1727) and published in 1687. The magnitude of g varies over the surface of the Earth in which many measurements were undertaken at different locations on the earth [1]. JS Clark [2] and AH Cook [3] measured the absolute g constants by using a reversible pendulum and free motion experiments, respectively. Both the experiments of g values were very close to the mean of g absolute determination by using other methods. After that, the absolute g value was determined by Cook [4] using the British Fundamental Gravity Station in the N.P.L. getting 981181.75 mgal and the National Bureau of Standards near Gaithersburg, Maryland getting 980.1018 cm/s^2 [5] with a standard deviation of 0.0005 cm/s^2. Recently, gravity was investigated to obtain the local acceleration due to gravity at the center of a test mass in the KRISS Watt balance system. The mean gravity value at the center of the test mass in the KRISS Watt balance was (979 832 568.9 ± 5.0) μGal at a reference height of (1.3893±0.001) m [6]. The numerical value of the acceleration of gravity is most accurately known as 9.8 m/s^2. On the moon, the gravitational acceleration is one-sixth of the earth equal to 1.63 m/s^2.

Recently, many researchers have been provided gravitational acceleration computations by using innovative technologies for instance, the measurement of Earth's gravity field after the gravity field and steady-state ocean circulation explorer mission by the European Space Agency program [7], and they proposed a method to extract the gravitational acceleration from additional accelerations based on a simplified walking model [8]. In teaching undergraduates in the Physics laboratory, that acceleration due to gravity can be measured by using free fall [9], simple pendulum, physical pendulum, projectile motion and Atwood's machine. In Thailand, the gravitational acceleration of the earth (*g*) was measured at the nearby Songkhla province, southern Thailand by the National Institute of Metrology at 9.78120 m/s^2 [10-11]. Furthermore, experiments on free falling objects through the air and liquid media were captured by a conventional digital camera and analyzed by using a video analysis open source called Tracker which was determined at Walailak University, Nakhon Si Thammarat. The average *g* value from all the repeated experiments was 10.06 m/s^2 which deviate from the standard acceleration due to gravity by only 2.62% [11].

In this study, the methods used to investigate local gravitational acceleration at Chumphon province, latitude of 10° 24' 0" N and longitude of 99° 4' 0" E, were by motion of free fall, simple pendulum, physical pendulum and Atwood's machine to provide valuable experience of measuring gravitational acceleration in the undergraduate experiments at King Mongkut's Institute of Technology Ladkrabang Prince of Chumphon Campus, South of Thailand, to allow students to study comparisons of acceleration due to gravity in a Physics laboratory.

2. Experimental Apparatus
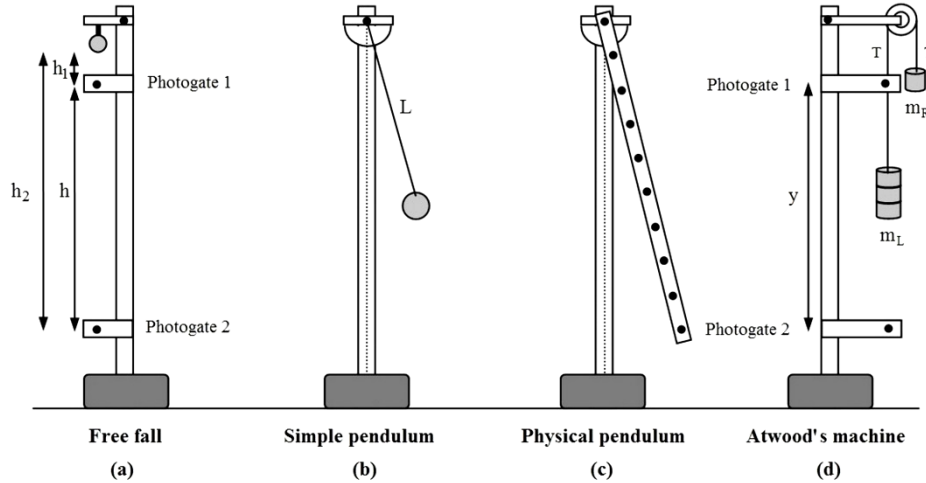
Figure 1. Free body diagram of the experiments set up to measure gravitational acceleration.

2.1. Free fall
In principle, *g* can be measured by using a smart timer to detect the time *t* for a metallic ball falling between photogate 1 and photogate 2 known as distance *h* as shown in figure 1(a), acceleration can be computed by applying the simple formula as equation (1).

$$g = \frac{\left[2(h_2 - h_1)^2\right]}{t_{av}^2(h_1 + h_2 + 2\sqrt{h_1 h_2})}, \tag{1}$$

where h_1 is the distance from the metal ball to photogate 1, h_2 is the distance from the metal ball to photogate 2, t_{av} is the average time of the metal ball to fall between the two photogates.

2.2. Simple pendulum

Using the simple pendulum to measure the acceleration due to gravity, the apparatus was constructed from string (length L) connected to a metal ball and a snooker ball, which were attached to the aluminum bar as shown in figure 1(b). This experiment used an oscillation angle of less than 10°. In the simple pendulum experiment, digital timer was used to detect the time of the ball swing of 10 revolutions with five repetitions. The period of an oscillation is

$$T = 2\pi \sqrt{\frac{L}{g}}. \tag{2}$$

The acceleration due to gravity can be calculated by equation (3)

$$g = \frac{4\pi^2}{slope}. \tag{3}$$

Where L is the length of the string, the *slope* can be calculated from the linear fitting curve as shown in figure 2.

2.3. Physical pendulum

A physical pendulum consists of a rigid body that undergoes fixed axis rotation about a fixed point. In this experiment, we used 100 cm length metal ruler length with the holes for rotation points as shown in figure 1(c). The physical pendulum experiment was setup the same as the simple pendulum, digital timer was used to detect the time for the metal ruler to complete 10 revolutions, which estimated from five repetitions, in 5 cm phases, from 5 to 95 cm. This experiment also used an oscillation angle of less than 10° the same as the simple pendulum. The obtained g value was computed by using equation (4).

$$g = \frac{4\pi^2}{T^2}\left(x_1 + x_2\right) \tag{4}$$

where T is the period oscillates of the physical pendulum, x_1 and x_2 are the parameters from the graph as shown in figure 4.

2.4. Atwood's machine

The Atwood's machine is a simple machine that consists of a pulley which has two suspended weights of unequal mass. The mass difference between the two hanging masses m_1 and m_2 determines the net force acting on the system of both masses. This net force accelerates both of the hanging masses; the heavier mass is accelerated downward and the lighter mass is accelerated upward. In the free body diagram of the Atwood's machine, T is the tension of the string, m_R is the lighter mass, m_L is the heavier mass. The time of the masses motion (t) measured was the time taken to travel from photogate 1 to photogate 2 this is known as distance y in figure 1(d) and the g value can be obtained from equation (5a) or (5b).

$$g = M \times slope \tag{5a}$$

$$g = (m_L + m_R + m_P) \times slope, \tag{5b}$$

where $M = m_L + m_R + m_P$ is the net masses of the system (g), m_L is the left mass (g), m_R is the right mass (g) and m_P is pulley mass (g).

3. Results and discussion

In the experiments, the acceleration due to gravity (g) from free fall, simple pendulum, physical pendulum and Atwood's machine was determined. In the first experiment, the data collected for analysis of the free-falling analysis was the time when the steel ball was dropped and was measured by photogates, this had 10 repetitions as shown in figure 1(a), in order to evaluate the mean value of the falling ball times. The acceleration due to gravity calculated by equation (1) was 9.64 m/s^2, the percentage error was 0.41%. The second experiment, the relationship between of oscillation in the simple pendulum, the slope of the graph between the period square motion and the length of the string of the snooker ball and small the steel ball was shown in figure 2. The slope of the snooker ball was 4.1339 s^2/m and the small steel ball of 4.0939 s^2/m, the acceleration due to gravity can be calculated via equation (3) to be 9.55 m/s^2 and 9.64 m/s^2, respectively. The percentage error of the snooker ball was 2.96% and the small steel ball was 1.73%. The third experiment, the experiment studied the physical pendulum to determine the gravitational acceleration by using a 100 cm long of the steel rod. Figure 3 shows the period of oscillation and the length of the pivot hole. Finally, the obtained average g value from the Atwood's machine experiment was 10.47 m/s^2 which differed from the standard of 6.73%.
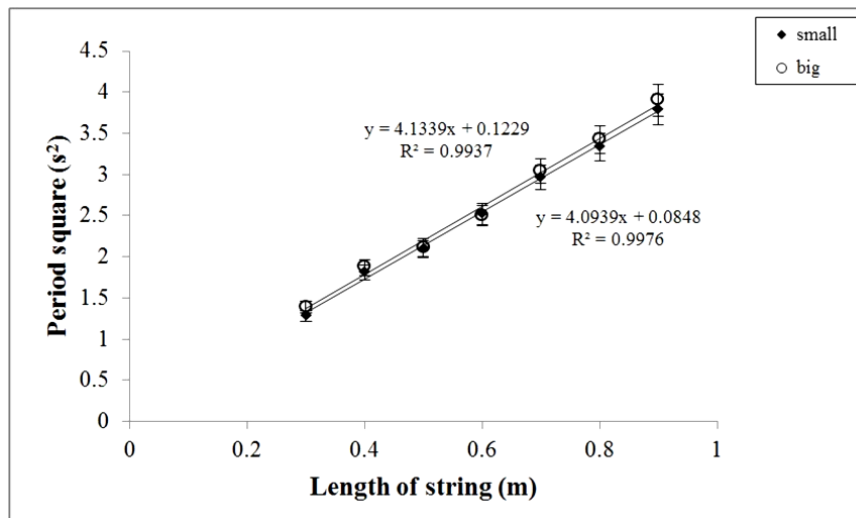

Figure 2. Graph of the period square and the length of the string use in the simple pendulum.

The variation of swing period responds to the pendulum string length, the slope of a linear fitting curve of the small ball and the big ball is shown in figure 2. The obtained g values were 9.55 m/s^2 and 9.64 m/s^2, respectively. However, the average g value from the experiment's five repetitions as shown in table 1 which differs from the standard of 1.41%.

The average period time of the oscillations can be obtained by using the linear fitting curve of the physical pendulum as shown in figure 3 was 1.65 s. The x_1 and x_2 from figure 3 were 0.44 and 0.24 m, respectively. The average acceleration due to gravity was determined from the five repetitions of the physical pendulum and computed by using equation (4) was 10.88 m/s^2, the standard error was 10.91%.
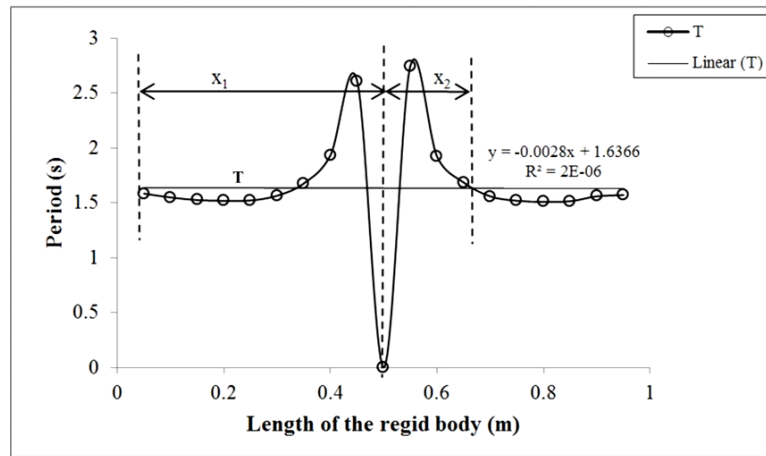
Figure 3. The period values varied from physical pendulum pivot points.
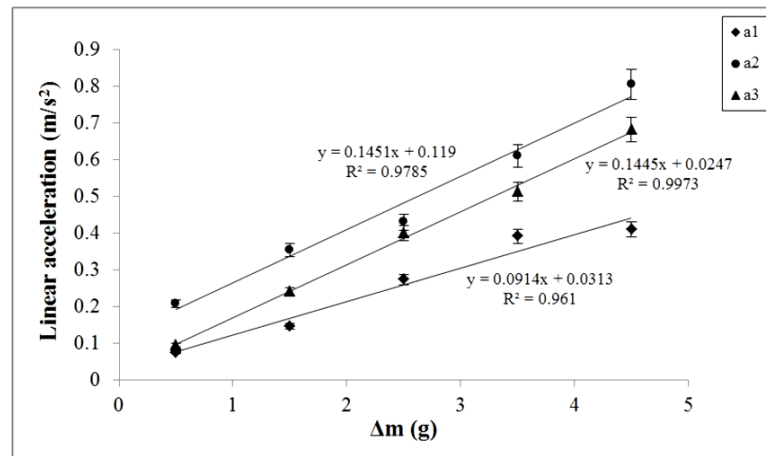
Figure 4. Linear fit graph of the Atwood's machine to determine the slope for calculating *g*.

The *g* value of the Atwood's machine can be computed by using equation (5a) or (5b). Figure 4 shows the slopes of the linear fit graph of three differences mass, the obtained average *g* value from the Atwood's machine experiment was 10.47 m/s^2 which differed from the standard of 6.73%. All the experiments of the gravitational acceleration measurement were set up for undergraduate study at King Mongkut's Institute of Technology Ladkrabang, Prince of Chumphon Campus, Chumphon province, Thailand.

Table 1. The gravitational acceleration values of 4 methods.

Time/Methods	g (m/s^2)			
	Free fall	Simple pendulum	Physical pendulum	Atwood's machine
1	9.52	9.55	12.70	7.56
2	10.00	9.64	10.17	12.70
3	9.76	9.82	10.09	11.89
4	9.33	9.74	10.51	9.78
5	9.60	9.61	10.95	10.44
Mean	9.77	9.67	10.88	10.47
Percentage Error	0.41%	1.43%	10.91%	6.73%

The results, shown in figure 1, demonstrate that the physical pendulum had greater percentage error than the Atwood's machine, the free fall, and the simple pendulum. As the results of the physical pendulum had a fixed point of rotation which had little friction force, therefore, this parameter affected the gravitational acceleration the same as the Atwood's machine in which the free fall and the simple pendulum were not found to have.

4. Conclusions

The values of the gravitational acceleration experiments were determined by using four methods, the results showed that the acceleration results due to gravity values of free fall, simple pendulum, physical pendulum and Atwood's machine had the percentage errors of 0.41%, 1.43%, 10.91% and 6.73%, respectively. Therefore, the percentage error of the free fall was the least percentage error and the physical pendulum had the highest percentage error owing to the fixed point of rotation which had a little friction force.

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