

The study on temperature dependent superfluid density of anisotropic superconductors

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Abstract. The most of superconductors are highly anisotropic superconductors, then the measurement on the superfluid density properties of superconductors are complicated and depended on directions. In this study, the superfluid density of anisotropic superconductors was studied by semiclassical approach. We are interested in the spin-singlet superconducting state with the anisotropic spherical Fermi surface, and the anisotropic gap function with the spherical ellipse shape in weak-coupling limit. After some calculation, we can derive the temperature dependent formula of the superfluid density near zero temperature for the ab- and c- spatial components. The numerical calculation fit to the experimental data of CaAlSi superconductor was shown. We found that our model can fit well, then the CaAlSi superconductor shows the anisotropic superfluid density.

1. Introduction

The most applications of superconductors are related to magnetic field, so the behavior of superconductors influenced by magnetic field had been extensively studied. There are many kinds of superconductors which use in an experiment such as Fe-based superconductors that can give the very high magnetic field [1]. However, these superconductors are highly shown the anisotropic properties under the magnetic field [2]. The theoretical model is needed for helping to predict the magnetic properties of these superconductors. In this paper, we are interested in influence of the magnetic properties on the superfluid density of anisotropic superconductors. The knowledge on the superfluid density should link to pairing state, the London penetration depth, nonlinear Meissner effect and vortex state of superconductivity.

2. Model and calculations

The semiclassical approach to the superfluid density proposed by Chandrasekhar and Einzel [3] was used as the main ideal in our calculation. This method can provide all three spatial components of the superfluid density limited to the electronic transport, pure coherent, and the scattering effect. The supercurrent $\vec{j}_s(r)$ and the vector potential $\vec{A}(r)$ are related by the response tensor R given by,

$$R_{ij} = \frac{e^2}{4\pi^3 hc} \oint dS_k \frac{\bar{v}_F^i \bar{v}_F^j}{|\bar{v}_F|} \left(1 + 2 \int_{\Delta(k)}^{\infty} \left(\frac{\partial f(E)}{\partial E} \right) \frac{N(E)}{N(0)} dE \right) \quad (1)$$

Where R_{ij} is the response tensor, $f(E)$ is the Fermi function, $E = \sqrt{\varepsilon^2 + \Delta(k)^2}$ is the quasiparticle energy with ε is the energy at Fermi energy level, $\Delta(k)$ is the energy gap depend on momentum space, $N(E)/N(0) = E/\sqrt{E^2 - \Delta(k)^2}$ is the density of states at the Fermi level, and v_F^i are the components of Fermi velocity \bar{v}_F .

As derived from Maxwell's equations and all three spatial components, we obtain the superfluid density of a 3D spherical Fermi surface described as [4]

$$\rho_{bb}^{aa} = 1 - \frac{3}{4\pi T} \int_0^1 (1-z^2) \int_0^{2\pi} \left(\frac{\cos^2(\varphi)}{\sin^2(\varphi)} \right) \int_0^{\infty} \cosh^{-2} \left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T, k)}}{2T} \right) d\varepsilon d\varphi dz \quad (2)$$

$$\rho_{cc} = 1 - \frac{3}{2\pi T} \int_0^1 z^2 \int_0^{2\pi} \cos^2(\varphi) \int_0^{\infty} \cosh^{-2} \left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T, k)}}{2T} \right) d\varepsilon d\varphi dz \quad (3)$$

Where $z = \cos\theta$, ρ_{bb}^{aa} are the normalized superfluid density for ab- spatial component, and ρ_{cc} is the normalized superfluid density for c- spatial component.

To include the effect of the anisotropic Fermi surface, let the energy gap has the form as

$$\Delta(T, k) = \Delta(T) f(k) \quad (4)$$

Where $\Delta(T)$ is the energy gap s-wave BCS function, and $f(k)$ is the anisotropic function of Fermi surface.

The energy gap $\Delta(T)$ is obeyed the weak-coupling limit that electron-phonon interaction is the main mechanism of the Cooper pair occurrence [5]. In this study, we are interested in the temperature-dependent energy gap near zero temperature. The BCS gap equation with the anisotropic energy gap has been set up to calculate approximately for near zero-temperature energy gap. Then, we have been obtained the temperature dependent s-wave energy gap with the anisotropic function as

$$\Delta(T) = \Delta(0) \left(1 - \sqrt{\frac{2\pi T}{\Delta(0) f(k)}} \cdot \exp\left(-\frac{\Delta(0) f(k)}{T}\right) \right) \quad (5)$$

Here $\Delta(0)$ is the zero-temperature energy gap, and this equation is valid for near zero-temperature.

To calculate especially on the shape of Fermi sphere, we use the spherical ellipse Fermi surface that was introduced by Hass and Maki [6]. The anisotropic function in the spherical coordinate was given as,

$$f(\theta) = \frac{1 + a' z^2}{1 + a'} \quad (6)$$

Where $z = \cos\theta$, and a' is the elliptical constant parameter that the coordinate is reduced to be dependent only on azimuthally angle.

Substituting Eq.(4), Eq.(5) and Eq.(6) into Eq.(2) and Eq.(3), we can get the superfluid density of a 3D spherical Fermi surface of anisotropic superconductor for the ab- and c- spatial components.

$$\rho_{bb}^{aa} = 1 - \frac{3}{4} \sqrt{\frac{8\pi\Delta(0)}{T}} \int_0^1 (1-z^2) \sqrt{\frac{1+a'z^2}{1+a'}} \cdot e^{-\frac{\Delta(0)}{T} \left(\frac{1+a'z^2}{1+a'} \right)} dz \quad (7)$$

$$\rho_{cc} = 1 - \frac{3}{2} \sqrt{\frac{8\pi\Delta(0)}{T}} \int_0^1 z^2 \sqrt{\frac{1+a'z^2}{1+a'}} \cdot e^{-\frac{\Delta(0)}{T} \left(\frac{1+a'z^2}{1+a'} \right)} dz \quad (8)$$

The near zero-temperature approximation of superfluid density can be found as

$$\rho_{bb}^{aa} = 1 - \frac{3}{4} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \begin{aligned} & \left(4A^2 \left(1 - \frac{1}{2} \left(\frac{a'}{1+a'} \right) \right) - 2A \left(1 - \left(\frac{a'}{1+a'} \right) \right) - \frac{3}{2} \left(\frac{a'}{1+a'} \right) \right) \cdot \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{A}) \cdot e^A}{8A^2 \sqrt{A}} \\ & + \frac{1}{2A} - \frac{1}{4A} \left(\frac{a'}{1+a'} \right) \left(1 - \frac{3}{2A} \right) \end{aligned} \right\} \quad (9)$$

$$\rho_{cc} = 1 - \frac{3}{2} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \begin{aligned} & \left(2A - \left(\frac{a'}{1+a'} \right) A + \frac{3}{2} \left(\frac{a'}{1+a'} \right) \right) \cdot \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{A}) \cdot e^A}{8A^2 \sqrt{A}} \\ & - \frac{1}{2A} \left(1 + \left(\frac{a'}{1+a'} \right) \frac{3}{4A} \right) \end{aligned} \right\} \quad (10)$$

Where $A = \frac{\Delta(0)}{T} \cdot \left(\frac{a'}{1+a'} \right)$. These equations are the temperature dependent superfluid density of anisotropic superconductors with elliptical Fermi sphere.

3. Results and discussion

For comparing to the experimental data, the numerical calculations of superfluid density in the ab- and c- spatial components were done. Our numerical results can fit well with the experimental data of CaAlSi superconductor [7]. The results were shown in Figure 1.

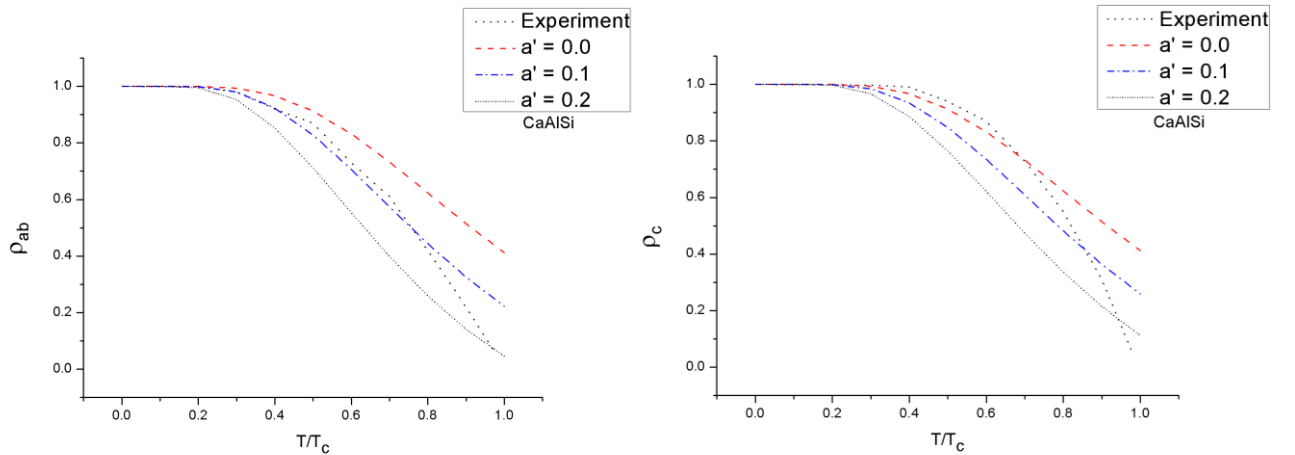


Figure 1. The superfluid density of two components ρ_{ab} and ρ_c of CaAlSi superconductor with critical temperature at 7.0 K and $2\Delta(0)/T_c = 4.2$ [8-10].

The CaAlSi superconductor has the critical temperature in range 6.2 to 7.0 K, and the band structure shows highly hybridized 3D form. The 3D s-wave BCS gap with deviations from the isotropic gap have been reported [8-10], and the measurement of gap-to- T_c ratio, $2\Delta(0)/T_c = 4.2$, has been measured by ARPES [11]. In Figure 1. The superfluid density of two components of CaAlSi superconductor with critical temperature at 7.0 K and $2\Delta(0)/T_c = 4.2$. The ab- and c-spatial components at a' is 0.0, 0.1, and 0.2 were shown. We found that the ab- and c-spatial components fit well at $a' = 0.1$ and $a' = 0.0$, respectively. Finally, we found that the CaAlSi superconductor shows the anisotropy in ab-plane, and the isotropy in c-direction of the superfluid density.

4. Conclusion

The semiclassical approach was used for studying the superfluid density of anisotropic superconductors. Our model was included the spin-singlet superconducting state with the spherical ellipse Fermi surface. We derived the temperature dependent formula of the superfluid density near zero-temperature for the ab- and c- spatial components. The numerical calculation that fit to the experimental data of CaAlSi superconductor was shown. We found that our model can fit well with the experimental data of the CaAlSi superconductor with the anisotropic superfluid density.

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