LOW-ENERGY ELECTRON-POSITRON COLLIDER TO SEARCH AND STUDY $(\mu^+\mu^-)$ BOUND STATE

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Dimuonium

- Dimuonium, bimuonium or true muonium is a lepton atom ($\mu^+\mu^-$).
- Dimuonium is pure QED system (no strong interaction, calculable).
- From 6 leptonic atoms ($e^+e^-$), ($\mu^+e^-$), ($\mu^+\mu^-$), ($\tau^+e^-$), ($\tau^+\mu^-$), ($\tau^+\tau^-$) only two ($e^+e^-$), ($\mu^+e^-$) were observed.
- Very compact (large $m_\mu$), more sensitive to new physics than other exotic atoms.
Why dimuonium?

• Observation of the new classical QED object.
• QED test in the new regime.
• Experimental challenge leads to development of new techniques.
• Tests of muon properties motivated by
  • $3.5\sigma$ difference between $(g-2)_\mu$ measurement and SM prediction
  • discrepancies in the proton charge radius in muonic hydrogen
  • Hints of lepton-universality violation in rare B decays (LHCb), $B^+\rightarrow K^+e^+e^-$ and $B^+\rightarrow K^+\mu^+\mu^-$
Some references

• V.N.Baier and V.S.Synakh, Bimuonium production in electron-positron collisions, SOVIET PHYSICS JETP, 14, № 5, 1962, pp.1122-1125
  • Properties of the bound state, probability of observation

  • Very large crossing angle in order to eliminate background

• H. Lamm and R.F. Lebed, True Muonium ($\mu^+\mu^-$) on the Light Front, arXiv 1311.3245v3, 12 Nov 2014
  • Spectrum

• H. Lamm, True muonium: the atom that has it all, arXiv 1509.09306v1, 30 Sep 2016
  • Novel properties
Dimuonium properties

- Mass
  \[ M_{\mu\mu} = 2 \times 105.7 \text{ MeV} - 1.4 \text{ keV} \]
- Bohr radius
  \[ R_{\mu\mu} = 512 \text{ fm} \]
  \[ R_{ee} = 106000 \text{ fm} \]
- Muon lifetime 2.2 \( \mu \)s
- \( ^3S_1 \) states have photon quantum numbers \( (J^P_C = 1^-) \); therefore could be produced in \( e^+e^- \) collisions
Dimuonium production cross section

- Production of $n^3S_1$ in the $e^+e^- \rightarrow (\mu^+\mu^-) \rightarrow e^+e^-$
- $1^3S_1$: $\sigma(m_{\mu\mu}) \approx \frac{12\pi}{m_{\mu\mu}^2} \sqrt{\frac{\pi \Gamma_{ee}}{8} \frac{\Gamma_{ee}}{\sigma_M}} \approx 0.2 \frac{\Gamma_{ee}}{\sigma_M}$

where $\sigma_M$ is center-of-mass energy spread

- For different collision schemes
  \[
  \frac{\Gamma_{ee}}{\sigma_M} = \frac{0.37 \times 10^{-6} \text{keV}}{(7 \div 400) \text{keV}} \approx (1 \div 50) \times 10^{-9}, \sigma(m_{\mu\mu}) = 0.23 \div 11 \text{nb}
  \]

- Background: elastic $e^+e^- \rightarrow e^+e^-$ scattering
  - For crossing angle $45^\circ \div 135^\circ$ $\sigma_{Bhabha} = 22000 \text{nb}$
  - $Background/signal = (3 \div 130) \times 10^3$
  - Background suppression is possible if decay point is separated from the origin point ($decay \ path 1^3S_1: c\tau = 540 \mu m$)
Head-on e+e- collision

$E_{\text{beam}} = 100 \div 150$ MeV
Collision monochromatization a la Reniery:
10 keV invariant mass resolution
$L \approx 10^{30} \text{ cm}^{-2}\text{s}^{-1}$ ($\sim 50 (\mu^+\mu^-)/\text{hour}$).

Observation of the dimuonium by searching for X-rays from $(\mu^+\mu^-)$ Bohr transitions such as $2P \rightarrow 1S$ (J.W. Moffat).

Failed due to large background.

Large crossing angle proposed by S.J. Brodsky and R.F. Lebed.

Large angle beam crossing

Invariant mass

$$\langle M \rangle = 2E_0 \cos \theta - \frac{E_0}{2} \cos \theta \left[ \sigma^2_\delta + \sigma^2_{px} + \sigma^2_{py} \frac{\cos 2\theta}{(\cos \theta)^2} \right]$$

Invariant mass resolution

$$\sigma^2_M = 2E_0^2 \left[ \sigma^2_\delta (\cos \theta)^2 + \sigma^2_{px} (\sin \theta)^2 \right]$$

Luminosity ($\varphi = \sigma_z \tan \theta / \sigma_x$)

$$\mathcal{L}_0 = \frac{N_1 N_2}{4\pi \sigma_y \sigma_x \sqrt{1 + \varphi^2}} f_0 N_b \approx \frac{N_1 N_2}{4\pi \sigma_y \sigma_z \tan \theta} f_0 N_b$$

Peak production rate

$$\dot{N}_{\mu\mu} \approx \frac{\Gamma_{\mu\mu} \sigma_{\mu\mu} \mathcal{L}_0}{2\sqrt{\pi} \sigma_M}$$
Background

Decay length \((\mu^+\mu^- (1^3S_1) \rightarrow e^+e^-)\)

\[ \frac{OA}{l} = c \tau_{0,\mu\mu} \beta_{\mu\mu} \gamma_{\mu\mu} = c \tau_{0,\mu\mu} \tan \theta \]

Background: density of beam particles

\[ N_1 \propto \exp(-n_x^2/2) \]

\[ n_x = \frac{AB}{\sigma_x} = \frac{l \cos \theta}{\sigma_x} = \frac{c \tau_{0,\mu\mu}}{\sigma_x} \sin \theta \]

Signal to background ratio

\[ \frac{\dot{N}_{\mu\mu}}{\dot{N}_{ee}} \propto \exp \left[ \frac{c^2 \tau_{0,\mu\mu}^2}{\sigma_x^2} \sin^2 \theta \right] \]

\[ \frac{\sqrt{\sigma_\delta^2 \cos^2 \theta + (\sigma_x^2 / \beta_x^2) \sin^2 \theta}}{\sigma_x^2} \]

9
Beam-beam effects with large crossing angle

Beam-beam tuneshift

$$\xi_z = -\frac{N r_e}{2\pi\gamma} \frac{\alpha}{|\alpha|\sigma_\delta \sigma_z} \frac{\varphi^2}{1 + \varphi^2}$$

Hamiltonian

$$\mathcal{H} = -\alpha \frac{p_z^2}{2} - \frac{\nu_s^2}{\alpha R^2} \frac{z^2}{2} - \frac{2\xi_z \nu_s z^2}{\alpha R^2}$$

Population limit for $\alpha > 0$

$$N < \frac{2\pi R \gamma \alpha \sigma_\delta^2}{r_e} \frac{\sigma_z^2}{2}$$

$\alpha < 0$ has been studied at KEKB and at DAΦNE, no large currents, no luminosity due to microwave instability
Accelerator requirements

• Large positive momentum compaction (small circumference)
• Large crossing angle with small vertical beta function gives high luminosity (similar to crab waist)
• Large crossing angle $75^\circ$ provides comfortable beam energy ($e^+$ production) and decay length
  • beam energy $E_b = 408$ MeV
  • decay length $l \left( \mu^+ \mu^- (1^3 S_1) \right) = 2$ mm
• Higher signal to noise ratio requires $\sigma_x < c \tau_{0,\mu\mu} = 0.54$ mm
• Horizontal beam divergence contributes significant part in invariant mass resolution; therefore, low horizontal emittance
• Reverse of the beam direction provides $15^\circ$ crossing angle and allows to study c.m. energy range from $\eta$ to $\eta'$ mesons (550-960 MeV)
Collider: overview

\[ \Pi = 23 \text{ m} \]

IP1

IP2

75°
Collider: overview
• Experimental chamber: flat box with 0.5-mm-thick beryllium windows on the top and on the bottom allowing passage of $e^\pm$ produced by the dimuonium atoms decay.
• Detector: tracking systems around the median plane, magnetic spectrometer
Interaction region

- QD0: permanent magnet, $G = -35 \text{T/m}$, $\varnothing 30\text{mm}$
- QD/QF1: electromagnet
## Collider: optics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>408 MeV</td>
</tr>
<tr>
<td>Circumference</td>
<td>23 m</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$6.4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Bunch intensity</td>
<td>$3.5 \times 10^{10}$ / 73 mA</td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>26 nm</td>
</tr>
<tr>
<td></td>
<td>90 nm (IBS)</td>
</tr>
<tr>
<td>Energy spread</td>
<td>$4 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$8.4 \times 10^{-4}$ (IBS)</td>
</tr>
<tr>
<td>$\beta_x / \beta_y$</td>
<td>200 mm / 2 mm</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$4 \times 10^{30}$ cm$^{-2}$s$^{-1}$, Nb=1</td>
</tr>
<tr>
<td></td>
<td>$8 \times 10^{31}$ cm$^{-2}$s$^{-1}$, Nb=20</td>
</tr>
</tbody>
</table>

![Graph showing beam parameters](image)
## Collider: parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency</td>
<td>338.98 MHz</td>
</tr>
<tr>
<td>Beam energy</td>
<td>408 MeV</td>
</tr>
<tr>
<td>RF harmonic</td>
<td>26</td>
</tr>
<tr>
<td>Invariant mass (M)</td>
<td>211.315 MeV</td>
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<tr>
<td>RF voltage</td>
<td>450 kV</td>
</tr>
<tr>
<td>RF acceptance</td>
<td>2%</td>
</tr>
<tr>
<td>σ(_M)/M</td>
<td>1.8×10^{-3}</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>1.71×10^{-2}</td>
</tr>
<tr>
<td>IP beam divergence</td>
<td>6.7×10^{-4} (hor)</td>
</tr>
<tr>
<td>Damping partition</td>
<td>1.6 (hor)</td>
</tr>
<tr>
<td>Energy spread</td>
<td>4×10^{-4}</td>
</tr>
<tr>
<td>1.4 (lon)</td>
<td>8.4×10^{-4} (IBS)</td>
</tr>
<tr>
<td>Damping times</td>
<td>17.3 ms (hor)</td>
</tr>
<tr>
<td>Beam-beam tune shift</td>
<td>2×10^{-6} (hor)</td>
</tr>
<tr>
<td>27.3 ms (ver)</td>
<td>1.2×10^{-3} (ver)</td>
</tr>
<tr>
<td>22.1 ms (lon)</td>
<td>-2×10^{-3} (lon)</td>
</tr>
<tr>
<td>Bunch length</td>
<td>5.4 mm</td>
</tr>
<tr>
<td>Beam size at IP</td>
<td>130 μm</td>
</tr>
<tr>
<td>11.6 mm (IBS)</td>
<td>0.7 μm</td>
</tr>
</tbody>
</table>
Dimuonium production and distribution

- Detection efficiency is about 50%
- $\beta\gamma\tau = 2.03 \text{ mm}$
- $\sigma_x(IP) = \sigma_x/\left(\sqrt{2} \cos \theta\right) = 350 \mu m$
- Detector vertex resolution is 300 $\mu m$
- Total $\sigma_{vtx} = 460 \mu m$
- 5$\sigma$ background suppression with vertex position $x>2.3$ mm

<table>
<thead>
<tr>
<th></th>
<th>$\mu^+\mu^-$ rate</th>
<th>1 hour</th>
<th>4 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total 1S/2S/3S</td>
<td>65/8.1/2.4</td>
<td>187k/23k/6.9k</td>
<td></td>
</tr>
<tr>
<td>$x &gt; 2.3 \text{ mm}$</td>
<td>1S/2S/3S</td>
<td>21/7/2.3</td>
<td>59k/20k/6.6k</td>
</tr>
</tbody>
</table>

\[ \Sigma n^3S_1 \]
\[ \Sigma n^3S_1 - 1^3S_1 \]
Experiments: what can we measure?

• From the fit of the decay vertex distribution
  • dimuonium production rate ($\Gamma_{ee}$) of 1S (1% for $10^7$ s), 2S(5%), 3S(15%)
  • dimuonium decay lengths with the same accuracy

• Dimuonium interaction with a thin foil (30μm Al) allows
  • measurement of the breakup probability
  • measurement 1S-2P transition probabilities
    • 2P lifetime

• Laser spectroscopy
  • $\Delta E(2S-2P)$ (laser $\lambda \approx 100\mu m$)
    • 2P lifetime
Experiments: $e^+ e^- \rightarrow \mu^+ \mu^-$ near threshold

Coulomb interaction in the final state leads to nonzero cross section at the threshold; therefore,

- Background-free measurement of the cross section near the threshold, requires magnetic spectrometer
- Precision measurement of the SSSG-factor
- C.M. energy and its spread calibration
- The same technique may be used for $e^+ e^- \rightarrow \pi^+ \pi^-$

![Graph showing cross section vs. energy with Born cross section, radiation correction, and energy spread with $\delta=400$ keV]
Experiments: 15° crossing angle

- This region (c.m. 550-960 MeV) of $\rho$ and $\omega$ resonances is important for SM $(g-2)_\mu$ calculation
- $e^+e^- \rightarrow \pi^+\pi^-$ cross section measurement with unlimited statistics
- Precision measurements of other hadronic cross sections ($e^+e^- \rightarrow \pi^+\pi^-\pi^0, \pi^0\gamma, \eta\gamma, \pi^0\pi^0\gamma, 4\pi, \cdots$)
- Rare processes $e^+e^- \rightarrow \eta, \eta'$
- Two-photon processes $\gamma\gamma \rightarrow \pi^0, \pi\pi, \eta$
- Measurement of meson-photon transition form factors
### Reverse beam: 15° crossing angle

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>283.59 MeV (η)</td>
<td>495.78 MeV (η′)</td>
</tr>
<tr>
<td>Invariant mass (M)</td>
<td>547.86 MeV</td>
<td>957.76 MeV</td>
</tr>
<tr>
<td>$\sigma_M$ (σ_M/M)</td>
<td>420 keV (7.7×10^{-4})</td>
<td>580 keV (6.1×10^{-4})</td>
</tr>
<tr>
<td>Energy spread</td>
<td>2.8×10^{-4} / 10.6×10^{-4} (IBS)</td>
<td>4.8×10^{-4} / 8.4×10^{-4} (IBS)</td>
</tr>
<tr>
<td>IP beam divergence (hor)</td>
<td>8.3×10^{-4}</td>
<td>7.1×10^{-4}</td>
</tr>
<tr>
<td>Horizontal emittance</td>
<td>11.4 nm / 105 nm (IBS)</td>
<td>34.8 nm / 75 nm (IBS)</td>
</tr>
<tr>
<td>Bunch length</td>
<td>3.7 mm / 14.2 mm (IBS)</td>
<td>6.3 mm / 11 mm (IBS)</td>
</tr>
<tr>
<td>Beam-beam $\zeta$ (h/v/l)</td>
<td>3×10^{-4} / 1.4×10^{-2} / 2×10^{-3}</td>
<td>3×10^{-4} / 1.3×10^{-2} / 2×10^{-3}</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>1.67×10^{-2}</td>
<td>1.71×10^{-2}</td>
</tr>
<tr>
<td>Luminosity (Nb=1 / 20)</td>
<td>3.3×10^{31} / 6.6×10^{32}</td>
<td>5.2×10^{31} / 1×10^{33}</td>
</tr>
</tbody>
</table>
Conclusion

• Collider to observe and study bound state of ($\mu^+\mu^-$)
  • two rings
  • large crossing angle
  • circumference 23 m
  • not expensive to build and operate
  • luminosity $8 \times 10^{31}$ cm$^{-2}$s$^{-1}$
• Reverse of the beam allows to perform experiments in 500-1000 MeV central mass energy range
• Details are in https://arxiv.org/abs/1708.05819
• We are preparing technical design and plan to make a decision by the end of the year

We are open for collaboration and experiments proposals