

Status of Neutrino Masses and Mixing

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Abstract

We present a very concise summary of the status of our knowledge and understanding of neutrino masses and mixing.

1 Basic experimental facts

That neutrinos have a mass has been established by experiments on neutrino oscillations that measure differences of squared masses and mixing angles [1]. Two distinct oscillation frequencies have been at first measured in solar and atmospheric neutrino oscillations and later confirmed by experiments on earth, like KamLAND and K2K. A signal corresponding to a third mass difference was claimed by the LSND experiment but not confirmed by KARMEN and recently by MiniBooNE. Two well separated differences need at least three neutrino mass eigenstates involved in oscillations. Conversely the three known neutrino species can be sufficient. At least two ν 's must be massive while, in principle, the third one could still be massless. In the following we will assume the simplest picture with three active neutrinos, CPT invariance and no sterile neutrinos. The mass eigenstates involved in solar oscillations are m_1 and m_2 and, by definition, $|m_2| > |m_1|$, so that $\Delta m_{sun}^2 = |m_2|^2 - |m_1|^2 > 0$. The atmospheric neutrino oscillations involve m_3 : $\Delta m_{atm}^2 = |\Delta m_{31}^2|$ with $\Delta m_{31}^2 = |m_3|^2 - |m_1|^2$ either positive (normal hierarchy) or negative (inverse hierarchy). The present data are compatible with both cases. The degenerate spectrum occurs when the average absolute value of the masses is much larger than all mass squared differences: $|m_i|^2 \gg \Delta m_{hk}^2$. With the standard set of notations and definitions [1] the present data are summarised in Table 1.

Oscillation experiments only measure differences of squared masses and do not provide information about the absolute neutrino mass scale. Limits on that are obtained [1] from the endpoint of the tritium beta decay spectrum, from cosmology and from neutrinoless double beta decay ($0\nu\beta\beta$). From tritium we have an absolute upper limit of 2.2 eV (at 95% C.L.) on the mass of electron antineutrino, which, combined with the observed oscillation frequencies under the assumption of three CPT-invariant light neutrinos, represents also an upper bound on the masses of the other active neutrinos. Complementary information on the sum of neutrino masses is also provided by the galaxy power spectrum combined with measurements of the cosmic microwave background anisotropies. According to recent analyses of the most reliable data [4] $\sum_i |m_i| < 0.60 \div 0.75$ eV (at 95% C.L.) depending on the retained data (the numbers for the sum have to be divided by 3 in order to obtain a limit on the mass of each neutrino). The

Table 1: Fit to neutrino oscillation data

Quantity	ref. [2]	ref. [3]
$(\Delta m_{sun}^2) (10^{-5} \text{ eV}^2)$	$7.67^{+0.16}_{-0.19}$	$7.65^{+0.23}_{-0.20}$
$\Delta m_{atm}^2 (10^{-3} \text{ eV}^2)$	$2.39^{+0.11}_{-0.08}$	$2.40^{+0.12}_{-0.11}$
$\sin^2 \theta_{12}$	$0.312^{+0.019}_{-0.018}$	$0.304^{+0.022}_{-0.016}$
$\sin^2 \theta_{23}$	$0.466^{+0.073}_{-0.058}$	$0.50^{+0.07}_{-0.06}$
$\sin^2 \theta_{13}$	0.016 ± 0.010	$0.010^{+0.016}_{-0.011}$

discovery of $0\nu\beta\beta$ decay would be very important because it would establish lepton number violation and the Majorana nature of ν 's, and provide direct information on the absolute scale of neutrino masses. As already mentioned the present limit from $0\nu\beta\beta$ (with large ambiguities from nuclear matrix elements) is about $|m_{ee}| < (0.3 \div 0.8)$ eV [4] (see eq. (3)).

2 Majorana Neutrinos and the See-Saw Mechanism

Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that the lepton number L conservation is probably violated and that ν 's are Majorana fermions. In this case the smallness of neutrino masses can be naturally explained as inversely proportional to the very large scale where L is violated, of order the grand unification scale M_{GUT} or maybe, for the lightest among them, the Planck scale M_{Pl} . If neutrinos are Majorana particles, their masses arise from the generic dimension-five non renormalizable operator of the form:

$$O_5 = \frac{(Hl)_i^T \lambda_{ij} (Hl)_j}{M} + h.c. \quad , \quad (1)$$

with H being the ordinary Higgs doublet, l_i the $SU(2)$ lepton doublets, λ a matrix in flavour space, M a large scale of mass and a charge conjugation matrix C between the lepton fields is understood.

Neutrino masses generated by O_5 are of the order $m_\nu \approx v^2/M$ for $\lambda_{ij} \approx O(1)$, where $v \sim O(100 \text{ GeV})$ is the vacuum expectation value of the ordinary Higgs. A particular realization leading to comparable masses is the see-saw mechanism [5], where M derives from the exchange of heavy neutral objects of weak isospin 0 or 1. In the simplest case the exchanged particle is the ν_R and the resulting neutrino mass matrix reads (1st type see-saw):

$$m_\nu = m_D^T M^{-1} m_D \quad . \quad (2)$$

As one sees, the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large Majorana mass. For $m_\nu \approx \sqrt{\Delta m_{atm}^2} \approx 0.05$ eV and $m_\nu \approx m_D^2/M$ with $m_D \approx v \approx 200$ GeV we find $M \approx 10^{15}$ GeV which indeed is an impressive indication that the scale for lepton number violation is close to M_{GUT} . Thus probably neutrino masses are a probe into the physics near M_{GUT} . This argument, in my opinion, strongly discourages models where neutrino masses are generated near the weak scale and are suppressed by some special mechanism.

3 Importance of Neutrinoless Double Beta Decay

Oscillation experiments cannot distinguish between Dirac and Majorana neutrinos. The detection of neutrino-less double beta decay would provide direct evidence of L non conservation, and the Majorana nature of neutrinos. It would also offer a way to possibly disentangle the 3 cases of degenerate, normal or inverse hierarchy neutrino spectrum. The quantity which is bound by experiments on $0\nu\beta\beta$ is the 11 entry of the ν mass matrix, which in general, from $m_\nu = U^* m_{diag} U^\dagger$, is given by:

$$|m_{ee}| = |(1 - s_{13}^2) (m_1 c_{12}^2 + m_2 s_{12}^2) + m_3 e^{2i\phi} s_{13}^2| \quad (3)$$

where $m_{1,2}$ are complex masses (including Majorana phases) while m_3 can be taken as real and positive and ϕ is the U_{PMNS} phase measurable from CP violation in oscillation experiments. Starting from this general formula it is simple to derive the bounds for degenerate, inverse hierarchy or normal hierarchy mass patterns shown in Fig.1.

In the next few years a new generation of experiments will reach a larger sensitivity on $0\nu\beta\beta$ by about an order of magnitude. If these experiments will observe a signal this would indicate that the inverse hierarchy is realized, if not, then the normal hierarchy case remains a possibility.

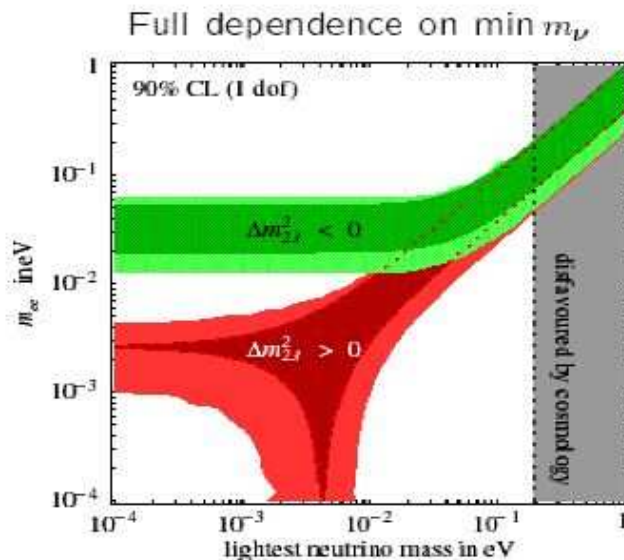


Fig. 1: A plot [6] of m_{ee} in eV, the quantity measured in neutrino-less double beta decay, given in eq.(3), versus the lightest neutrino mass m_1 , also in eV. The upper (lower) band is for inverse (normal) hierarchy.

4 Baryogenesis via Leptogenesis from Heavy ν^c Decay

In the Universe we observe an apparent excess of baryons over antibaryons. It is appealing that one can explain the observed baryon asymmetry by dynamical evolution (baryogenesis) starting from an initial state of the Universe with zero baryon number. For baryogenesis one needs the three famous Sakharov conditions: B violation, CP violation and no thermal equilibrium. In the history of the Universe these necessary requirements have possibly occurred at different epochs. Note however that the asymmetry generated by one epoch could be erased at following epochs if not protected by some dynamical reason. In principle these conditions could be verified in the SM at the electroweak phase transition. B is violated by instantons when kT is of the order of the weak scale (but B-L is conserved), CP is violated by the CKM phase and sufficiently marked out-of-equilibrium conditions could be realized during the electroweak phase transition. So the conditions for baryogenesis at the weak scale in the SM superficially appear to be present. However, a more quantitative analysis [7] shows that baryogenesis is not possible in the SM because there is not enough CP violation and the phase transition is not sufficiently strong first order, unless the Higgs mass is below a bound which by now is excluded by LEP. In SUSY extensions of the SM, in particular in the MSSM, there are additional sources of CP violation and the bound on m_H is modified but also this possibility has by now become at best marginal after the results from LEP2.

If baryogenesis at the weak scale is excluded by the data it can occur at or just below the GUT scale, after inflation. But only that part with $|B - L| > 0$ would survive and not be erased at the weak scale by instanton effects. Thus baryogenesis at $kT \sim 10^{10} - 10^{15}$ GeV needs B-L violation and this is also needed to allow m_ν if neutrinos are Majorana particles. The two effects could be related if baryogenesis arises from leptogenesis then converted into baryogenesis by instantons [8]. The decays of heavy Majorana neutrinos (the heavy eigenstates of the see-saw mechanism) happen with violation of lepton number L, hence also of B-L and can well involve a sufficient amount of CP violation. Recent results on neutrino masses are compatible with this elegant possibility. Thus the case of baryogenesis through leptogenesis has been boosted by the recent results on neutrinos.

5 Models of Neutrino Mixing

After KamLAND, SNO and the upper limits on the absolute value of neutrino masses not too much hierarchy in the spectrum of neutrinos is indicated by experiments:

$$r = \Delta m_{sol}^2 / \Delta m_{atm}^2 \sim 1/30. \quad (4)$$

Precisely $r = 0.032_{-0.005}^{+0.006}$ at 3σ 's [2, 3]. Thus, for a hierarchical spectrum, $m_2/m_3 \sim \sqrt{r} \sim 0.2$, which is comparable to the Cabibbo angle $\lambda_C \sim 0.22$ or $\sqrt{m_\mu/m_\tau} \sim 0.24$. This suggests that the same hierarchy parameter (raised to powers with $o(1)$ exponents) may apply for quark, charged lepton and neutrino mass matrices. This in turn indicates that, in the absence of some special dynamical reason, we do not expect quantities like θ_{13} or the deviation of θ_{23} from its maximal value to be too small. Indeed it would be very important to know how small the mixing angle θ_{13} is and how close to maximal θ_{23} is.

We see from Table(1) [2, 3] that within measurement errors the observed neutrino mixing matrix is compatible with the so called Tri-Bimaximal (TB) form [9]. The best measured neutrino mixing angle θ_{12} is just about 1σ below the TB value $\tan^2 \theta_{12} = 1/2$, while the maximal value for θ_{23} is well inside the $1-\sigma$ interval and θ_{13} is still compatible with zero (see table 1). Thus, one possibility is that one takes this coincidence seriously and only considers models where TB mixing is automatically a good first approximation. Alternatively one can assume that the agreement of the data with TB mixing is accidental. Indeed there are many models that fit the data and yet TB mixing does not play any role in their architecture.

The TB mixing matrix (in a particular phase convention) is given by:

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (5)$$

The TB mixing matrix suggests that mixing angles are independent of mass ratios (while for quark mixings relations like $\lambda_C^2 \sim m_d/m_s$ are typical). In fact in the basis where charged lepton masses are diagonal, the effective neutrino mass matrix in the TB case is given by $m_\nu = U_{TB} \text{diag}(m_1, m_2, m_3) U_{TB}^T$:

$$m_\nu = m_1 M_1 + m_2 M_2 + m_3 M_3 \quad . \quad (6)$$

where:

$$M_1 = \frac{1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \quad M_2 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_3 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \quad (7)$$

The eigenvalues of m_ν are m_1, m_2, m_3 with eigenvectors $(-2, 1, 1)/\sqrt{6}$, $(1, 1, 1)/\sqrt{3}$ and $(0, 1, -1)/\sqrt{2}$, respectively. The expression in eq.(6) can be reproduced in models with sequential dominance or with form dominance, discussed by S. King and collaborators [10].

As we see the most general neutrino mass matrix corresponding to TB mixing, in the basis of diagonal charged leptons, is of the form:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}, \quad (8)$$

This is a symmetric, 2-3 symmetric matrix with $a_{11} + a_{12} = a_{22} + a_{23}$.

As we said, one possibility is that one takes this coincidence seriously and considers models where TB mixing is a good first approximation. In a series of papers [11–17] it has been pointed out that a broken flavour symmetry based on the discrete group A_4 appears to be particularly suitable to reproduce this specific mixing pattern in Leading Order (LO). Other solutions based on alternative discrete or continuous flavour groups have also been considered [18–20], but the A_4 models have a very economical and attractive structure, e.g. in terms of group representations and of field content.

We recall that A_4 , the group of even permutations of 4 objects, can be generated by the two elements S and T obeying the relations (a "presentation" of the group):

$$S^2 = (ST)^3 = T^3 = 1 \quad . \quad (9)$$

The 12 elements of A_4 are obtained as: $1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST$. The inequivalent irreducible representations of A_4 are $1, 1', 1''$ and 3 . It is immediate to see that one-dimensional unitary representations are given by:

$$\begin{aligned} 1 \quad S = 1 \quad T = 1 \\ 1' \quad S = 1 \quad T = e^{i4\pi/3} \equiv \omega^2 \\ 1'' \quad S = 1 \quad T = e^{i2\pi/3} \equiv \omega \end{aligned} \quad (10)$$

The three-dimensional unitary representation, in a basis where the element T is diagonal, is given by:

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad . \quad (11)$$

Note that the generic mass matrix for TB mixing in eq.(8) can be specified as the most general matrix that is invariant under $\mu - \tau$ symmetry, implemented by the unitary matrix $A_{\mu\tau}$:

$$A_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (12)$$

and under the S transformation:

$$m = SmS, \quad m = A_{\mu\tau}mA_{\mu\tau} \quad (13)$$

where S is given in eq.(11). This observation plays a role in leading to A_4 as a candidate group for TB mixing, because S is a matrix of A_4 , but $A_{\mu\tau}$ is not and μ - τ symmetry has to be separately implemented. In A_4 models the 2-3 symmetry is maintained by imposing that there are no flavons transforming as $1'$ or $1''$ that break A_4 with two different VEV's (in particular one can assume that there are no flavons in the model transforming as $1'$ or $1''$). It is also clear that a generic diagonal charged lepton matrix $m_e^\dagger m_e$ is characterized by the invariance under T , or $T^\dagger m_e^\dagger m_e T = m_e^\dagger m_e$.

The group A_4 has two obvious subgroups: G_S , which is a reflection subgroup generated by S and G_T , which is the group generated by T , which is isomorphic to Z_3 . If the flavour symmetry associated to A_4 is broken by the VEV of a triplet $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ of scalar fields, there are two interesting breaking pattern. The VEV

$$\langle \varphi \rangle = (v_S, v_S, v_S) \quad (14)$$

breaks A_4 down to G_S , while

$$\langle \varphi_T \rangle = (v_T, 0, 0) \quad (15)$$

breaks A_4 down to G_T . As we will see, G_S and G_T are the relevant low-energy symmetries of the neutrino and the charged-lepton sectors, respectively. Indeed we have already seen that the TB mass matrix is invariant under G_S and a diagonal charged lepton mass $m_e^\dagger m_e$ is invariant under G_T .

A crucial part of all serious A_4 models is the dynamical generation of this alignment in a natural way. In most of the models A_4 is accompanied by additional flavour symmetries, either discrete like Z_N or continuous like $U(1)$, which are necessary to eliminate unwanted couplings, to ensure the needed vacuum alignment and to reproduce the observed mass hierarchies. In the leading approximation A_4 models lead to exact TB mixing. Given the set of flavour symmetries and having specified the field content, the non leading corrections to the TB mixing arising from higher dimensional effective operators can be evaluated in a well defined expansion. In the absence of specific dynamical tricks, in a generic model, all the three mixing angles receive corrections of the same order of magnitude. Since the experimentally allowed departures of θ_{12} from the TB value $\sin^2 \theta_{12} = 1/3$ are small, at most of $\mathcal{O}(\lambda_C^2)$, with λ_C the Cabibbo angle, it follows that both θ_{13} and the deviation of θ_{23} from the maximal value are expected in these models to also be at most of $\mathcal{O}(\lambda_C^2)$. Note that λ_C is a convenient hierarchy parameter not only for quarks but also in the charged lepton sector with $m_\mu/m_\tau \sim 0.06 \sim \lambda_C^2$ and $m_e/m_\mu \sim 0.005 \sim \lambda_C^{3-4}$. A value of $\theta_{13} \sim \mathcal{O}(\lambda_C^2)$ is within the sensitivity of the experiments which are now in preparation and will take data in the near future (note that by $\mathcal{O}(\lambda_C^2)$ we mean numerically of order λ_C^2 ; as $\lambda_C \sim 0.22$ a linear term in λ_C with a smallish coefficient can easily be $\mathcal{O}(\lambda_C^2)$).

6 A_4 , quarks and GUT's

Much attention has been devoted to the question whether models with TB mixing in the neutrino sector can be suitably extended to also successfully describe the observed pattern of quark mixings and masses and whether this more complete framework can be made compatible with (supersymmetric (SUSY)) $SU(5)$ or $SO(10)$ grand unification. Early attempts of extending models based on A_4 to quarks [13, 25] and to construct grand unified versions [26] have not been satisfactory, e.g. do not offer natural mechanisms for mass hierarchies and/or for the vacuum alignment. A direct extension of the A_4 model to quarks leads to the identity matrix for V_{CKM} in the lowest approximation, which at first looks promising. But the corrections to it turn out to be strongly constrained by the leptonic sector, because lepton mixings are nearly TB, and, in the simplest models, are proven to be too small to accommodate the observed quark mixing angles [13]. Also, the quark classification adopted in these models is not compatible with A_4 commuting with $SU(5)$ (in ref. [27] an A_4 model compatible with the Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$ has been presented). Due to this, larger discrete groups have been considered for the description of quarks and for grand unified versions with approximate TB mixing in the lepton sector. A particularly appealing set of models is based on the discrete group T' , the double covering group of A_4 [28]. In ref. [29] a viable description was obtained, i.e. in the leptonic sector the predictions of the A_4 model are reproduced, while the T' symmetry plays an essential role for reproducing the pattern of quark mixing. But, again, the classification adopted in this model is not compatible with grand unification. Unified models based on the discrete groups T' [30], S_4 [31] and $\Delta(27)$ [32] have been discussed. Several models using the smallest non-abelian symmetry S_3 (which is isomorphic to D_3) can also be found in the recent literature [33].

As a result, the group A_4 was considered by most authors to be unsuitable to also describe quarks and to lead to a grand unified description. We have recently shown [15] that this negative attitude is not justified and that it is actually possible to construct a viable model based on A_4 which leads to a grand unified theory (GUT) of quarks and leptons with TB mixing for leptons. At the same time our model offers an example of an extra dimensional GUT in which a description of all fermion masses and mixings is attempted. The model is natural, since most of the small parameters in the observed pattern of masses and mixings as well as the necessary vacuum alignment are justified by the symmetries of the model. The formulation of $SU(5)$ in extra dimensions has the usual advantages of avoiding large Higgs representations to break $SU(5)$ and of solving the doublet-triplet splitting problem. A see-saw realization in terms of an A_4 triplet of right-handed neutrinos ν_R ensures the correct ratio of light neutrino masses with respect to the GUT scale. In our model extra dimensional effects directly contribute to determine the flavour pattern, in that the two lightest tenplets T_1 and T_2 are in the bulk (with a doubling T_i and T'_i ,

$i = 1, 2$ to ensure the correct zero mode spectrum), whereas the pentaplets F and T_3 are on the brane. The hierarchy of quark and charged lepton masses and of quark mixings is determined by a combination of extra dimensional suppression factors for the first two generations and of the $U(1)$ charges, while the neutrino mixing angles derive from A_4 . The choice of the transformation properties of the two Higgses H_5 and H'_5 is also crucial. They are chosen to transform as two different A_4 singlets 1 and 1'. As a consequence, mass terms for the Higgs colour triplets are not directly allowed at all orders and their masses are introduced by orbifolding, à la Kawamura [34]. Finally, in this model, proton decay is dominated by gauge vector boson exchange giving rise to dimension six operators. Given the relatively large theoretical uncertainties, the decay rate is within the present experimental limits. In conclusion, the model is shown to be directly compatible with approximate TB mixing for leptons as well as with a realistic pattern of fermion masses and of quark mixings in a SUSY $SU(5)$ framework.

7 Bimaximal Mixing and S_4

Alternatively one can assume that the agreement of TB mixing with the data is accidental. Indeed there are many models that fit the data and yet TB mixing does not play a role in their architecture. For example, in ref. ([21]) there is a list of Grand Unified $SO(10)$ models with fits to the neutrino mixing angles that show good agreement with the data although most of them have no relation with TB mixing. Similarly for models based on $SU(5) \otimes U(1)$ [1]. Another class of examples is found in ref. ([22]). However, in most cases, for this type of models different mixing angles could also be accommodated by simply varying the fitted values of the parameters. Assuming that the agreement of TB mixing with the data is accidental, we observe that the present data do not exclude a larger value for θ_{13} , namely $\theta_{13} \sim \lambda_C$, than generally implied by models with approximate TB mixing. In fact, two recent analyses of the available data lead to $\sin^2 \theta_{13} = 0.016 \pm 0.010$ at 1σ [2] and $\sin^2 \theta_{13} = 0.010^{+0.016}_{-0.011}$ at 1σ [3], which are compatible with both options. If experimentally it is found that θ_{13} is near its present upper bound, this could be interpreted as an indication that the agreement with the TB mixing is accidental. Then a scheme where instead the Bimaximal (BM) mixing is the correct first approximation could be relevant. The BM mixing matrix is given by:

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (16)$$

In the BM scheme $\tan^2 \theta_{12} = 1$, to be compared with the latest experimental determination: $\tan^2 \theta_{12} = 0.45 \pm 0.04$ (at 1σ) [2, 3], so that a rather large non leading correction is needed such that $\tan^2 \theta_{12}$ is modified by terms of $\mathcal{O}(\lambda_C)$. This is in line with the well known empirical observation that $\theta_{12} + \lambda_C \sim \pi/4$, a relation known as quark-lepton complementarity [23], or similarly $\theta_{12} + \sqrt{m_\mu/m_\tau} \sim \pi/4$. No compelling model leading, without parameter fixing, to the exact complementarity relation has been produced so far. Probably the exact complementarity relation is to be replaced with something like $\theta_{12} + \mathcal{O}(\lambda_C) \sim \pi/4$ or $\theta_{12} + \mathcal{O}(m_\mu/m_\tau) \sim \pi/4$ (which we could call "weak" complementarity), as in models where the large ν mixings arise from the diagonalisation of charged leptons. Along this line of thought, we have used the expertise acquired with non Abelian finite flavour groups to construct a model [24] based on the permutation group S_4 which naturally leads to the BM mixing at LO. We have adopted a supersymmetric formulation of the model in 4 space-time dimensions. The complete flavour group is $S_4 \times Z_4 \times U(1)_{FN}$. In LO, the charged leptons are diagonal and hierarchical and the light neutrino mass matrix, after see-saw, leads to the exact BM mixing. The model is built in such a way that the dominant corrections to the BM mixing, from higher dimensional operators in the superpotential, only arise from the charged lepton sector and naturally inherit λ_C as the relevant expansion parameter. As a result the mixing angles deviate from the BM values by terms of $\mathcal{O}(\lambda_C)$ (at most), and weak

complementarity holds. A crucial feature of the model is that only θ_{12} and θ_{13} are corrected by terms of $\mathcal{O}(\lambda_C)$ while θ_{23} is unchanged at this order (which is essential to make the model agree with the present data).

8 Conclusion

In the last decade we have learnt a lot about neutrino masses and mixings. A list of important conclusions have been reached. Neutrinos are not all massless but their masses are very small. Probably masses are small because neutrinos are Majorana particles with masses inversely proportional to the large scale M of lepton number violation. It is quite remarkable that M is empirically not far from M_{GUT} , so that neutrino masses fit well in the SUSY GUT picture. Also out of equilibrium decays with CP and L violation of heavy RH neutrinos can produce a B-L asymmetry, then converted near the weak scale by instantons into an amount of B asymmetry compatible with observations (baryogenesis via leptogenesis) [8]. It has been established that neutrinos are not a significant component of dark matter in the Universe. We have also understood there there is no contradiction between large neutrino mixings and small quark mixings, even in the context of GUT's.

This is a very impressive list of achievements. Coming to a detailed analysis of neutrino masses and mixings a very long collection of models have been formulated over the years. With continuous improvement of the data and more precise values of the mixing angles most of the models have been discarded by experiment. By now, besides the detailed knowledge of the entries of the V_{CKM} matrix we also have a reasonable determination of the neutrino mixing matrix U_{P-MNS} . It is remarkable that neutrino and quark mixings have such a different qualitative pattern. One could have imagined that neutrinos would bring a decisive boost towards the formulation of a comprehensive understanding of fermion masses and mixings. In reality it is frustrating that no real illumination was sparked on the problem of flavour. We can reproduce in many different ways the observations but we have not yet been able to single out a unique and convincing baseline for the understanding of fermion masses and mixings. In spite of many interesting ideas and the formulation of many elegant models the mysteries of the flavour structure of the three generations of fermions have not been much unveiled.

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