Baseline-dependent neutrino oscillations in asymmetrically-warped spacetimes

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Abstract
We study the possibility to explain the LSND anomaly by means of resonant active-sterile neutrino oscillations in an asymmetrically-warped spacetime. In such extra-dimensional scenarios sterile neutrinos take shortcuts through the extra dimension, which results in new types of resonances in the oscillation probability.

1 Introduction

In theories with large extra dimensions, the Standard Model particles are typically confined to the 3+1 dimensional Minkowskian brane, which is embedded in an extra-dimensional bulk. Singlets under the gauge group such as sterile neutrinos however are allowed to travel freely in the bulk as well as on the brane. In the context of such scenarios it has been argued that the LSND neutrino oscillation anomaly [1] and the MiniBooNE null result [2] might be explained by a brane-bulk resonance in active-sterile neutrino oscillations [3]. The resonance arises due to the additional phase difference $\delta(Ht) = t\delta H + H\delta t$ induced when the sterile neutrinos take temporal shortcuts through an extra dimension. Thus, in these models there are two sources of phase difference, the standard one $t\delta H = L\Delta m^2/2E$, and a new one $Ht\,(\delta t/t)$ arising from temporal shortcuts through the bulk available to gauge singlet quanta. The two phase differences may beat against one another to produce resonant phenomena. The relative difference in active and sterile neutrino travel times is encapsulated in the shortcut parameter $\epsilon = \left(t^{\text{brane}} - t^{\text{bulk}}\right)/t^{\text{brane}} = \delta t/t$. In the brane-bulk system the shortcut in the extra-dimension can be parametrized by an effective potential with nonzero sterile-sterile component in the flavor space Hamiltonian

$$H_{\text{eff}} = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} - \frac{1}{2E}\begin{pmatrix} 0 & 0 \\ 0 & 2E^2\epsilon \end{pmatrix}. \tag{1}$$

Diagonalization of the two-state system introduces a new effective mixing angle $\tilde{\theta}$ and a resonance energy E_{res} given by

$$\sin^2 2\tilde{\theta} = \frac{\sin^2 2\theta}{\sin^2 2\theta + \cos^2 2\theta\left[1 - \frac{E^2}{E^2_{\text{res}}}\right]^2} \qquad , \qquad E_{\text{res}} = \sqrt{\frac{\Delta m^2 \cos 2\theta}{2\epsilon}}. \tag{2}$$

There are three distinct energy domains: Below the resonance energy vacuum mixing is recovered, at the resonance energy the effective mixing becomes maximal ($\tilde{\theta} = \pi/4$), while above the resonance energy active and sterile neutrinos decouple and oscillations are suppressed. Such is the picture for a sterile neutrino traversing a unique shortcut through the bulk.

2 The shortcut model and its phenomenology

We introduce a metric for the brane-bulk system, namely an asymmetrically-warped 5D spacetime

$$d\tau^2 = dt^2 - \sum_{i=1}^{3} e^{-2k|u|}\left(dx^i\right)^2 - du^2. \tag{3}$$

The warp factor shrinks the space dimensions of the brane, but leaves the time and bulk dimension t and u unaffected. Thus, the basic feature of this model [4] is that longer neutrino travel times associated with longer baselines on the brane allow the off-brane geodesic of the sterile neutrino to penetrate deeper into the bulk and thus experience a greater warp. As a consequence, the shortcut parameter for the sterile neutrino increases with the baseline, corresponding in turn to a decreasing resonance energy in the effective Hamiltonian of the two-neutrino system. In addition, there are higher energy/longer baseline resonances resulting from additional classical geodesics. It turns out that there exists a countably infinite number of geodesics, which corresponds to possible paths of the sterile neutrino, giving rise to a countably infinite number of resonances. In a semi-classical approach to path-integral quantum mechanics one must then perform a path-integral-weighted sum over all amplitudes resulting from each of the geodesics.

Whether or not the higher resonances coming from the additional geodesics contribute significantly depends on the initial distribution of sterile neutrino velocities transverse to the brane. While the initial momenta are mostly on the brane, the uncertainty principle requires a nonzero momentum component transverse to the brane as well. We assume a normalized Gaußian distribution for the momentum component of the sterile neutrino along the extra dimension with a width σ, which is related to the thickness of the brane.

In the "Near Zone", defined as baselines short on the scale of the warp factor k^{-1}, the resonance condition is that the product of baseline and energy, LE, be an integer multiple of a fundamental resonant value $(LE)_{\text{res}} \propto k^{-1}\sqrt{\Delta m^2}$. That the brane-bulk resonances in the "Near Zone" expansion depend on the product of the energy and baseline, rather than on the energy alone as with the MSW matter-resonance, is a novel feature of our model.

Putting all the pieces together, the probability of oscillation including the weights mentioned above is given by

$$P(\nu_{\text{a}} \to \nu_{\text{s}}) = \left| \sum_{n=1}^{\infty} \Delta n \, e^{iS_{\text{cl}}(n)} \, \frac{vn}{(n^2+v^2)^{3/2}} \, \left[\sqrt{\frac{2}{\pi}} \, \frac{\beta E}{\sigma} \, e^{-\frac{(\beta E v)^2}{2\sigma^2 (n^2+v^2)}} \right] \sin 2\tilde{\theta}_n \, \sin \frac{L\delta \tilde{H}_n}{2} \right|^2 . \qquad (4)$$

In order to select only the geodesics (of quantum mechanical width $S_{\text{cl}}/\hbar$) which cross the brane at baseline length L, the measure Δn is introduced; $e^{iS_{\text{cl}}}$ is the path-integral weight with S_{cl} being the classical action. $v = kL/2$ is the dimensionless "scaling variable" of the system; β the velocity of the sterile neutrinos. The eigenvalue difference of the effective Hamiltonian is given by $\delta\tilde{H}_n$. Finally the sum takes all different modes, labeled by integers n, into account. Fig. 1 displays the oscillation probability for a certain choice of parameters.

The phenomenology of the new LE-resonance can be treated in analogy to the simpler case of a resonance energy. Instead of defining three distinct energy domains, we are led to define three distinct LE domains: Above the resonance active and sterile neutrinos decouple; at the resonance mixing becomes maximal and below the resonance vacuum oscillations prevail.

Since, in addition, higher-LE resonances are suppressed, and active-sterile neutrino mixing is suppressed for LE above the resonant values, sterile neutrinos taking shortcuts in the extra-dimensional bulk decouple from active neutrinos in long-baseline experiments. Thus, no active-sterile mixing is expected in atmospheric data, in MINOS [5], CDHS [6] or SuperK [7].

One possible phenomenology of our shortcut model would thus be a resonant value of LE between the LSND and MiniBooNE values of 2.5×10^{-3} km GeV and 2.5×10^{-1} km GeV, respectively. With such a resonant value of LE, active-sterile vacuum oscillation, or even the resonance, could explain the LSND excess, while no observable active-sterile mixing would be expected in MiniBooNE, or in any other longer-LE experiments. In explanations proposed so-far for the LSND and MiniBooNE anomalies assuming baseline-independent mixing, difficulties accommodating longer-baseline data were encountered. These difficulties do not immediately extend to scenarios with warped extra dimensions, as developed here. In fact, the failure of previous models to reconcile short baseline data such as LSND with

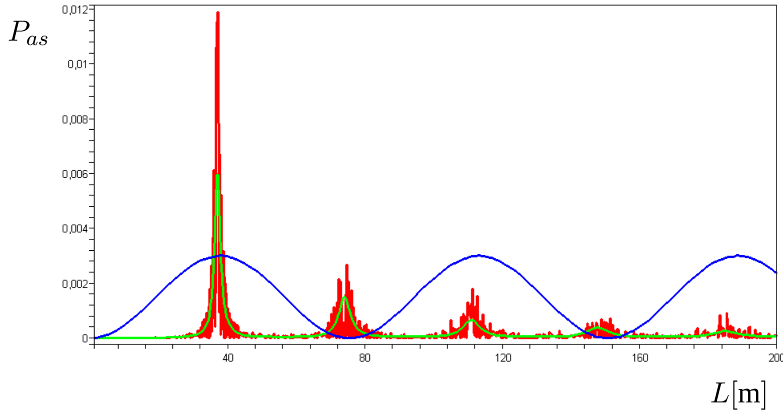

Fig. 1: Oscillation probability as a function of the experimental baseline, for the Gaußian distribution (red and green curves). The green curve presents the phase-averaged oscillation probability, and the sinusoidal blue curve presents the probability as given by the standard 4D vacuum formula for oscillations between sterile and active neutrinos. Parameter choices are $\sin^2 2\theta = 0.003$, $k = 5/(10^8$ m$)$, $E = 15$ MeV, $\Delta m^2 = 64$ eV^2, and $\sigma = 100$ eV. The resulting value of $(LE)_{\text{res}}$ is 550 m MeV. For our choice of E, the resonance peaks are found at the multiples $L = n\,(LE)_{\text{res}}/E = 37n$ m, $n = 1, 2, 3, \ldots$, with the principal resonance corresponding to $n = 1$.

longer baseline data might be construed as favoring the extra-dimensional shortcut scenario. Finally, we mention that the bulk shortcut scenario might even relieve some of the remaining tension between the LSND and KARMEN [8] experiments. Since LSND has almost twice the baseline of KARMEN, the bulk shortcut model opens more parameter space for accommodating the two experiments.

Our model relies on metric shortcuts and therefore does not discriminate between particles and antiparticles. It will thus be difficult to accommodate the MiniBooNE claims that an excess of flavor changing events exists in the neutrino channel [2] but not in the antineutrino channel [9]. It might still be possible though to explain the MiniBooNE data by incorporating non-standard matter effects [10] or CPT violation [11].

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