CERN

A Hybrid T-A Field Formulation for the Magnetoquasistatic Analysis of HTS Magnets

L. Bortot[1,2], M. Mentink[1], S. Schoeps[2], A. Verweij[1]

1 CERN 2 TECHNISCHE UNIVERSITÄT DARMSTADT

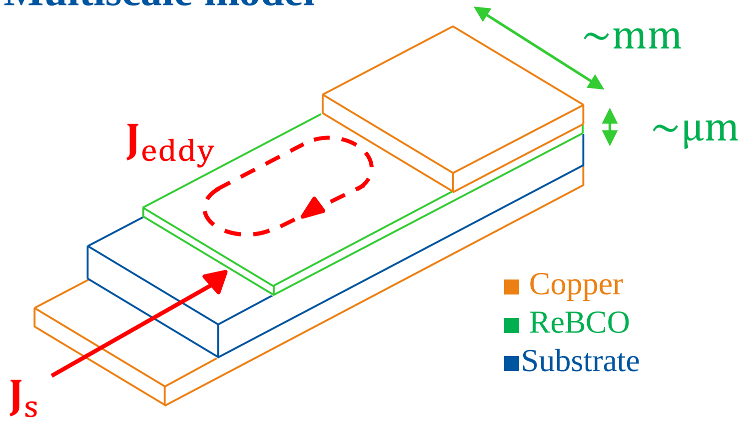
Special Thanks:
B. Auchmann, F. Grilli (KIT), M. Maciejewski,
M. Prioli, E. Ravaioli, J. Van Nugteren

This work has been sponsored by the Wolfgang Gentner Programme of the German Federal Ministry of Education and Research (grant no. 05E12CHA)

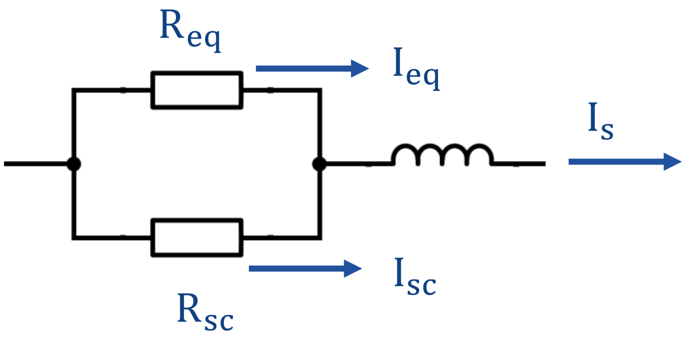
Rationale

- 20+ Tesla dipoles for future high-energy particle accelerators
- Simulation of the electrodynamics in HTS tapes and cables (then magnets, and circuits)

1) Multiscale model



3) Current sharing regime

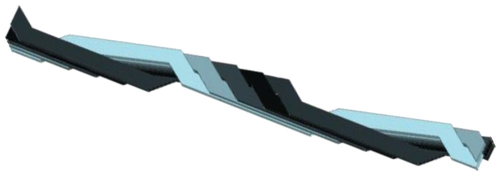


2) HTS resistivity

Nonlinear, field dependent, anisotropic

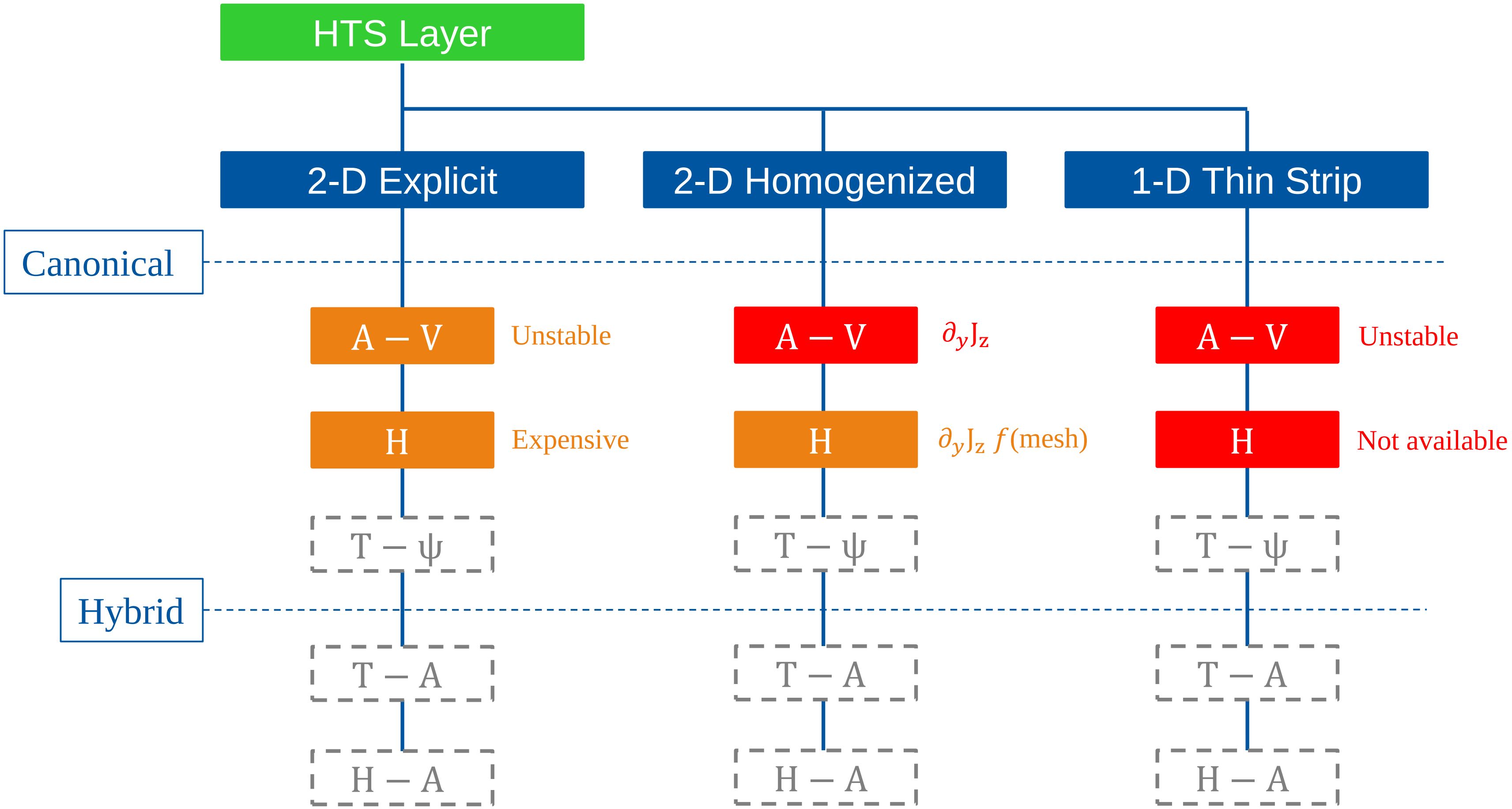
$$\sigma_{SC}^{-1} = \frac{E_c}{J_c(B)}\left(\frac{J}{J_c(B)}\right)^{n-1}$$

4) Complex cable geometry

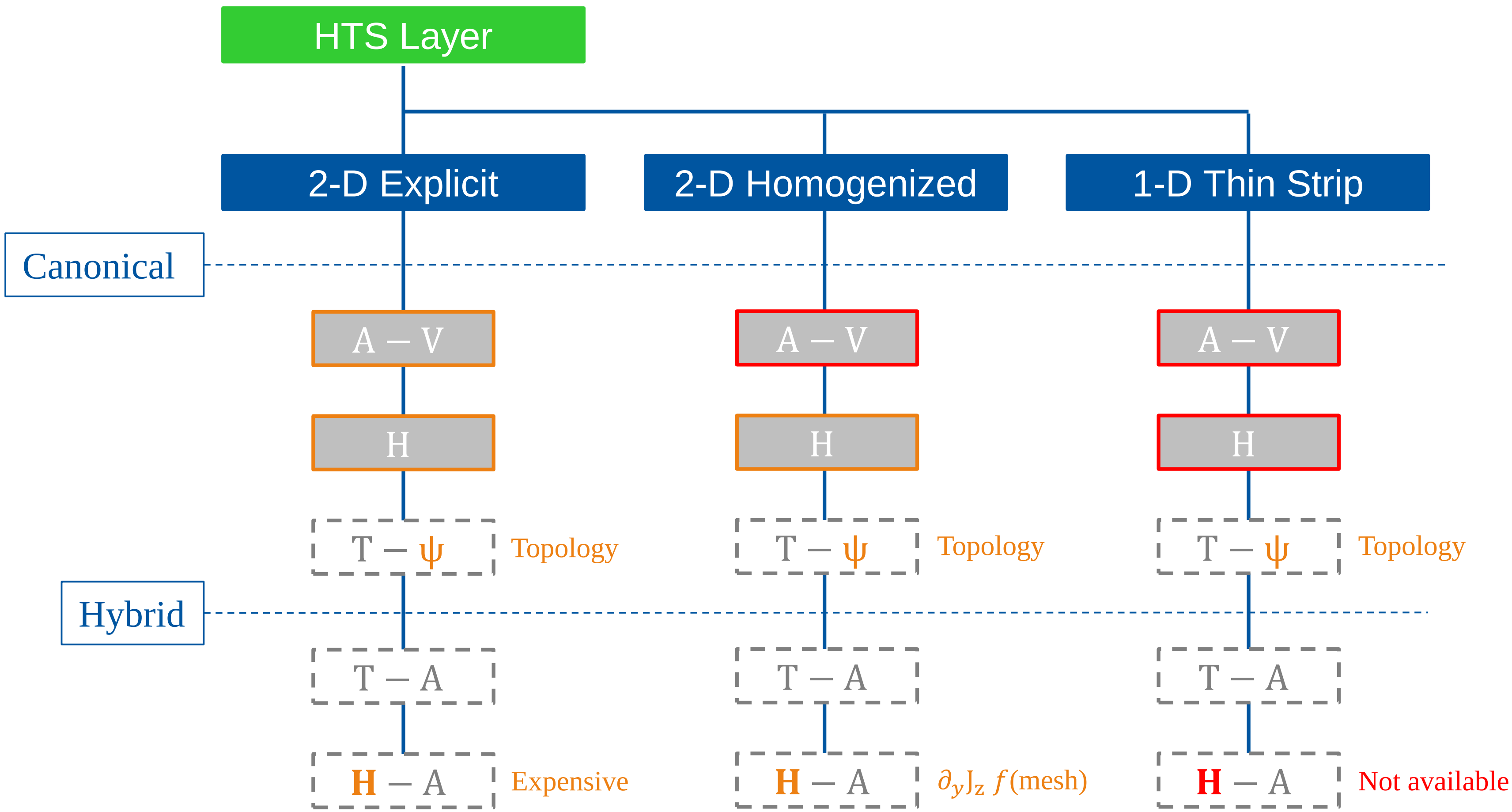


An ideal numerical formulation should be
accurate, robust, computationally fast

Last Time… Link



…Some Steps Forward



- A unstable
- H expensive
- ψ complex for nontrvial geometries

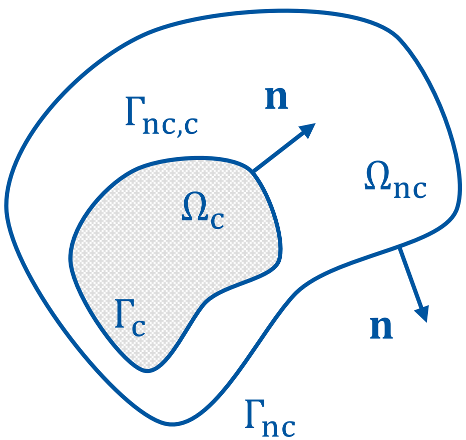
→ One does not simply choose a T-A hybrid form (Semicit.)

Hybridisation via Domain decomposition

The following approach might answer the simulation needs:

HTS Layer — 1-D Thin Strip — T – A

Domain decomposition:



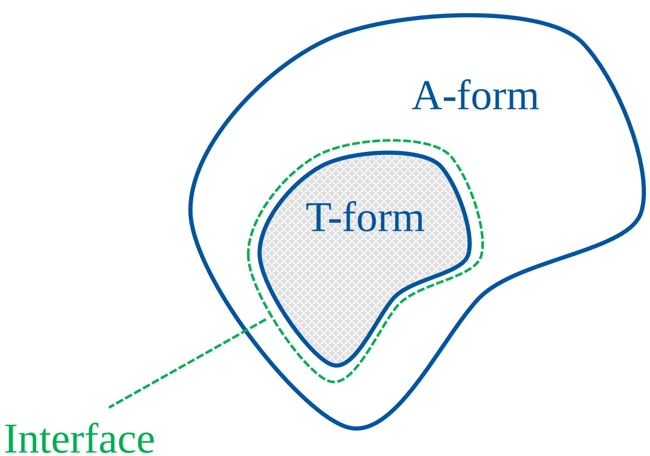
Domains Ω_{nc} , $\Omega_c \in \mathbb{R}^3$,

- Ω_{nc} : $\sigma = 0$ (e.g. iron yoke)
- Ω_c : $\mu = \mu_0$ (e.g. magnet coil)

Boundaries $\Gamma_{nc}, \Gamma_c \in \mathbb{R}^2$

Interface $\Gamma_{nc,c} \in \mathbb{R}^2$

Hybrid form:



Outline

- Fundamentals of Vector Fields Theory
- Hybrid T-A Field Formulation
- Numerical Implementation
- Applications
- Conclusions and Outlook

01 - Fundamentals

T-A Form in a Nutshell

Reformulation of Maxwell equations in terms of current (**T**) and magnetic (**A**) vector potentials.

What is needed:

Maxwell Equations

- Magnetoquasistatic Hypothesis
- Uniqueness of Solution

Vector Potentials

- Helmholtz decomposition (curl + divergence)
- Interface conditions
- Gauge fixing

Discretization technique

- Finite Element Method

Numerical solver

- Galerkin Method (Weighted residuals)

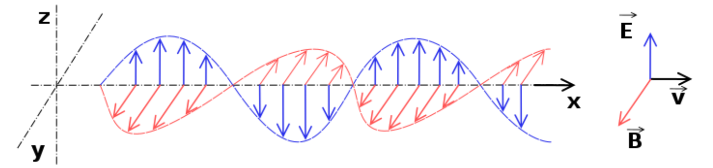
.. and, of course, a volunteer sorcerer

Maxwell Equations

In vacuum (*):

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial_t \mathbf{E}$$
$$\nabla \cdot \mathbf{E} = \rho \varepsilon_0^{-1}$$
$$\nabla \cdot \mathbf{B} = 0$$

+ material laws $\mathbf{B} = \mu_0 \mathbf{H}$, $\mathbf{D} = \varepsilon_0 \mathbf{E}$, $\mathbf{J} = \sigma \mathbf{E}$



Solution for $\mathbf{J} = \mathbf{0}, \rho = 0$

Symbols:

- $\mathbf{E}, \mathbf{D}$ electric field strength / density
- $\mathbf{H}, \mathbf{B}$ magnetic field strength / density
- $\rho, \mathbf{J}$ electric charge / current density
- μ_0 vacuum magnetic permeability
- ε_0 vacuum electric permittivity
- σ electric conductivity

Features:

- 4 independent variables (x, y, z, t)
- 2 equations (Faraday, Ampere-Maxwell) in 6 unknowns $B_{x,y,z}$ $E_{x,y,z}$
- 2 time-boundary conditions (Gauss laws)
- known field sources $(\mathbf{J}, \rho)$

(*) The vacuum hypothesis makes the equations linear and elegant.
Relaxing it would make the math more complex, without adding any new concept.

Magnetoquasistatic Hypothesis

- Dimensional analysis for arbitrary vector field **F**

$$F = f \cdot \mathcal{F}$$
$$\nabla F \approx F/\ell$$
$$\partial_t F \approx F/\tau$$

$f, \mathcal{F}$	reference quantity / non dimensional vector
ℓ, τ	characteristic spatial dimension / time constant
$c = (\varepsilon\mu)^{-1/2}$	speed of light

- Ampere-Maxwell Law: $J = J_f + J_d$ (free and displacement currents). One can obtain [1]:

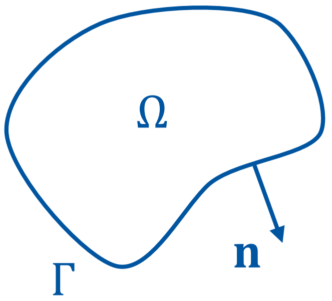
$$\frac{H_d}{H_f} \approx \left(\frac{\ell}{\tau c}\right)^2, \qquad \frac{J_d}{J_f} \approx \frac{\varepsilon}{\tau\sigma}$$

- If, compared to the dynamics of the device
  - $\tau \gg \ell/c$ "instantaneous" light propagation
  - $\tau \gg \varepsilon/\sigma$ "instantaneous" charge relaxation

Then $J_d = \partial_t \mathbf{D} \approx 0$
(Always true for small, conductive devices at power frequencies)

[1] Le Bellac, M, et al.. "Galilean electromagnetism." *Il Nuovo Cimento B (1971-1996).*

Uniqueness of Solution



Domain $\Omega \in \mathbb{R}^3$ with Γ as contour

Poynting vector: $\mathbf{P} = \mathbf{E} \times \mathbf{H}$

Conservation of energy: $\nabla \cdot \mathbf{P} = -\mathbf{E} \cdot \partial_t \mathbf{D} - \mathbf{H} \cdot \boldsymbol{\partial}_{\mathbf{t}} \mathbf{B} - \mathbf{E} \cdot \mathbf{J}$

Uniqueness Theorem, using the properties of **P** (e.g. [1]):

$\mathbf{E}, \mathbf{B}$ unique on Ω if

- $\mathbf{E}_0, \mathbf{B}_0$ known on Ω at $t = t_0$
- $\mathbf{E} \times \mathbf{n}$ OR $\mathbf{H} \times \mathbf{n}$ known on Γ, $\forall t$

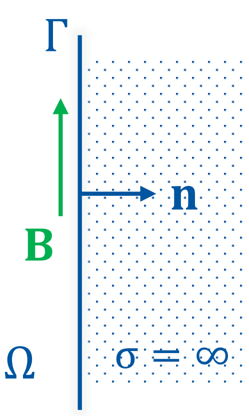
Two boundary conditions (BC) of practical importance, PEW and PMW

Perfect Electric Wall

$\sigma = \infty$

$\mathbf{E} \times \mathbf{n} = 0$

$(\mathbf{B} \cdot \mathbf{n} = 0)$

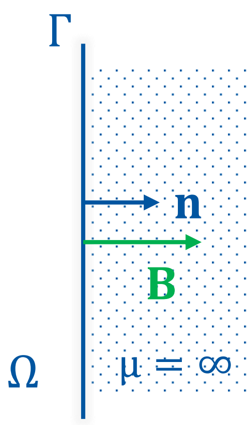


Perfect Magnetic Wall

$\mu = \infty$

$\mathbf{H} \times \mathbf{n} = 0$

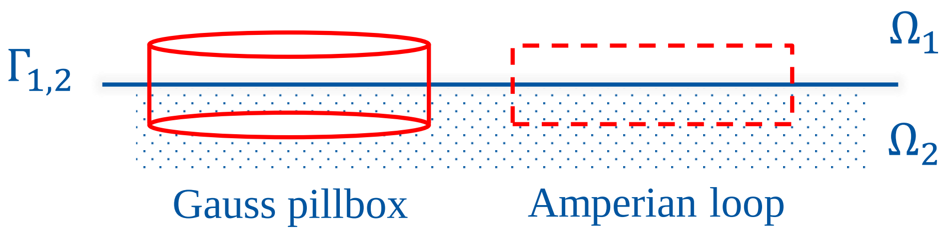
$(\mathbf{B} \times \mathbf{n} = 0)$

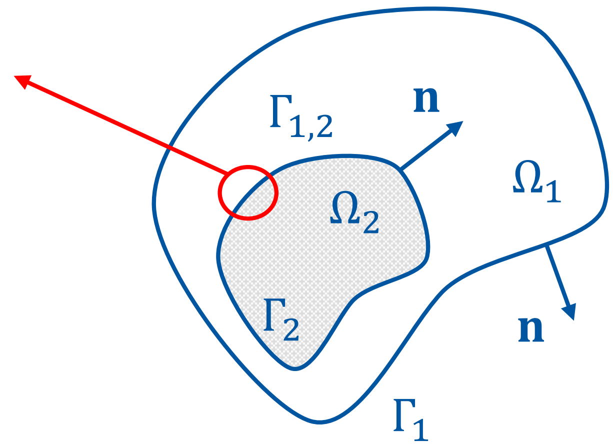


[1] Jones, D. S. *The theory of electromagnetism*. Elsevier, 2013.

Interface conditions

Domains $\Omega_1, \Omega_2 \in \mathbb{R}^3$ with Γ_1, Γ_2 as contour and $\Gamma_{1,2}$ as interface
Magnetic charge / current densities ignored (weakly related with the known universe)





- Gauss pillbox for flux conservation

$$(\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{n} = 0$$
$$(\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{n} = \sigma_q$$

- Amperian loop for potential conservation

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{n} = \mathbf{K_s}$$
$$(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{n} = 0$$

$\sigma_q, \mathbf{K_s}$ surface electric charge / current density.

Interface conditions [1] (IC) must always hold true!

[1] Vágó, I. "On the interface and boundary conditions of electromagnetic fields." *Periodica Polytechnica Electrical Engineering* (1994).

Helmholtz decomposition

- If $\mathbf{F} \in \mathbb{R}^3$ well-behaving field (sufficiently smooth, rapidly decaying at $\mathbf{r} \to \infty$) then [1] :

  $\mathbf{F} = \mathbf{F}_\mathrm{T} + \mathbf{F}_\mathrm{L}$
  $\mathbf{F}_\mathrm{T}$ curling, non diverging (i.e. $\nabla \cdot \mathbf{F}_\mathrm{T} = 0$)
  $\mathbf{F}_\mathrm{L}$ diverging, non curling (i.e. $\nabla \times \mathbf{F}_\mathrm{L} = 0$)

- Vice-versa, given a scalar field $\phi \in \mathbb{R}^3$ and a solenoidal vector field $\mathbf{A} \in \mathbb{R}^3$, both well behaving, then it exists a field **F** such that

  $\nabla \cdot \mathbf{F} = \phi, \ \nabla \times \mathbf{F} = \mathbf{A}$
  → F determined by knowing its curl and divergence

- Curiosity: What if $\nabla \cdot \mathbf{F} = 0, \nabla \times \mathbf{F} = 0$?

  $\nabla \times \nabla\phi = 0 \to \mathbf{F} = -\nabla\phi$
  $\nabla \cdot (-\nabla\phi) = 0 \ \to \nabla^2\phi = 0$

  - Laplacian (relaxed) nature of the field
  - "Hidden" in both $\mathbf{F}_\mathrm{T}$ and $\mathbf{F}_\mathrm{L}$, and determined only by BC.
  - Caveat: A non-curling, non-diverging field can still contain energy!

[1] Arfken, G. B., et al. "Mathematical methods for physicists." (1999).

Potentials – Gauge invariance

$\mathbf{B}, \mathbf{E}$ fields fulfil Helmholtz criteria, rewritten as

$$\mathbf{B} = \nabla \times \mathbf{A}_\mathrm{B} - \nabla \phi_\mathrm{B} - \partial_t \mathbf{A}'_\mathrm{B}$$
$$\mathbf{E} = \nabla \times \mathbf{A}_\mathrm{E} - \nabla \phi_\mathrm{E} - \partial_t \mathbf{A}'_\mathrm{E}$$

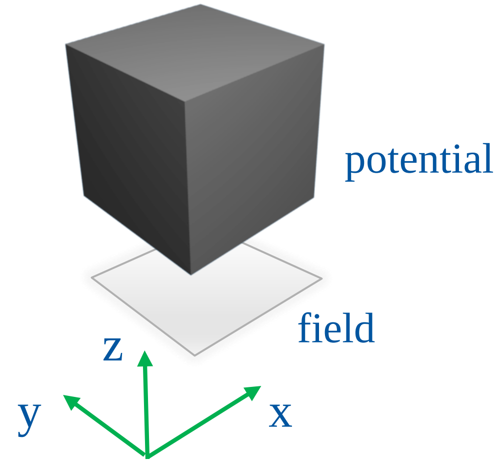
Potentials gauging (fixing the “integration constants”):

- 6 new equations (traditionally $\mathbf{A}_\mathrm{E} = 0$, $\mathbf{A}_\mathrm{B} = \mathbf{A}'_\mathrm{E}$)
- BC for ϕ on Γ, $\nabla \cdot$ for $\mathbf{A}$ on Ω
- (IC reformulated in terms of potentials)

Any gauge is fine! (though some are “numerically” better)

e.g. classic Coulomb gauge $\nabla \cdot \mathbf{A}_\mathrm{B} = 0,\ \phi_\mathrm{B} = 0$

Example:



Invariance to:
- z coordinate
- axial rotations of $\pi/2$

- Why potentials? (*)
  - More variables, equations, conditions
  - IC: **B**,**D** tangent and **H**,**E** normal are discontinuous.
  - potentials continuous, discontinuities embedded in their derivative

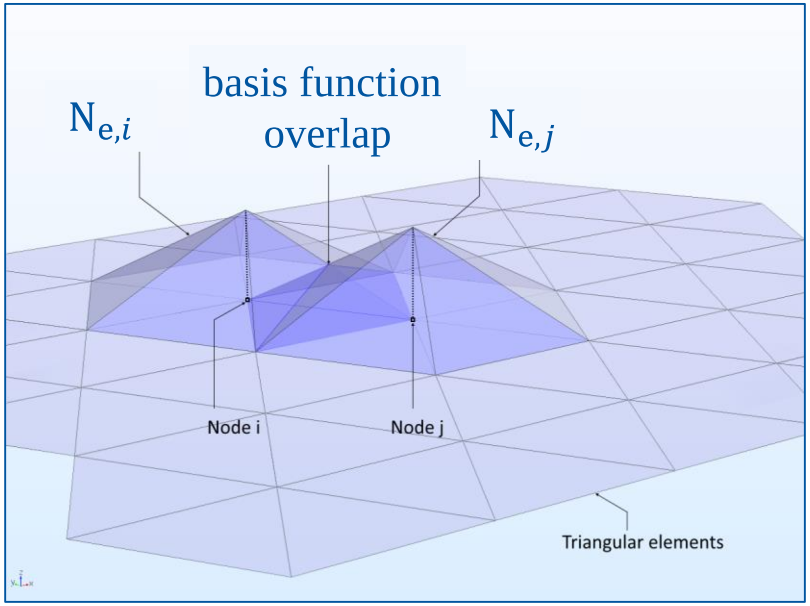
(*) Broadly speaking, potentials are a consequence of the way we interpret the universe. See for example the excellent article: Hammond, Percy. "The role of the potentials in electromagnetism." COMPEL (1999).

Discretization Technique

Fundamental lemma of calculus of variations [1] (variational formulation):

$$f = 0 \quad \Leftrightarrow \quad \int f \cdot w \, d\Omega = 0 \quad \forall w \in C_0(\mathbb{R})$$

$f = 0$ generic field equation (e.g. Laplacian)
w weighting (test) function: continuous, vanishing at infinity



Source: COMSOL blog

FEM approach (e.g.[2]):

1. $f \approx F \cdot N_e$
2. $w = N_e \rightarrow$ Galerkin method
3. We solve $\int (F \cdot N_e) \cdot N_e \, d\Omega = R$ (R=residual) looking for R_{min}
4. Discretization (equations assembled per node)
5. Algebraic problem $[N_e] \cdot F = 0$
6. Numerical solver (Newton-Raphson)

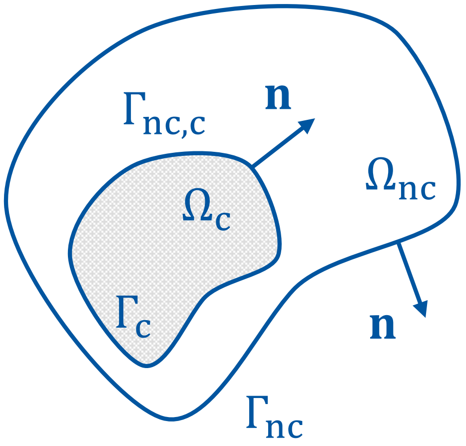
N.B.
If $\Omega_{N_e} \rightarrow 0$, then $F \cdot N_e \rightarrow f$

[1] Jost, Jurgen, Jürgen Jost, and Xianqing Li-Jost. Calculus of variations. Vol. 64. Cambridge University Press, 1998.
[2] Sayas, Francisco-Javier. "A gentle introduction to the Finite Element Method." Lecture notes, University of Delaware (2008).

02 - Hybrid T-A field formulation

Domain decomposition

Domains Ω_{nc} , $\Omega_c \in \mathbb{R}^3$, Ω_{nc} : $\sigma = 0$, Ω_c : $\mu = \mu_0$
Γ_{nc}, Γ_c as contour and $\Gamma_{nc,c}$ as interface



- Equations on Ω_{nc}

$\rho = 0, \mathbf{J} = \mathbf{0}$ (no sources)
$\mathbf{B} = \nabla \times \mathbf{A}$ (magnetic vector potential)
$\mathbf{E} = -\partial_t \mathbf{A}$ (Faraday law)
$\nabla \cdot \mathbf{A}_B = 0,\ \phi_B = 0 + \phi_E = 0$ (radiation gauge [1]) (*)

$$\nabla \times \mu^{-1} \nabla \times \mathbf{A} = \mathbf{0} \quad \text{on } \Omega_{nc}$$
$$\mathbf{A} \times \mathbf{n} = 0 \quad \text{on } \Gamma_{nc}$$

- Equations on Ω_c

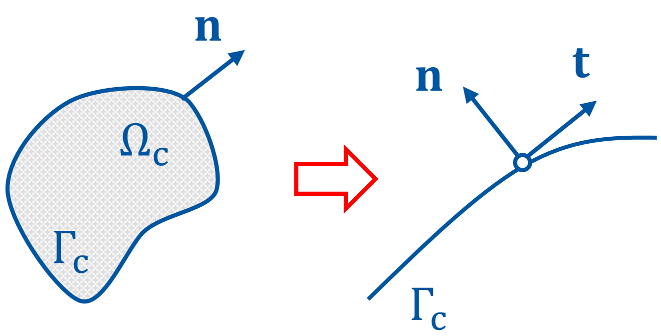
$\mathbf{H} = \mathbf{T} - \nabla\psi$ (ψ is the magnetic scalar potential [2])
$\nabla \times \mathbf{T} = \mathbf{J}$ (electric vector potential [3])
$\nabla \cdot \mathbf{T} = \nabla^2\psi$ (Gauss law)

$$\nabla \times \sigma^{-1} \nabla \times \mathbf{T} = -\mu_0 \partial_t (\mathbf{T} - \nabla\psi) \quad \text{on } \Omega_c$$
$$\psi = f(x, y, z, t) \quad \text{on } \Gamma_{nc}$$

[1] Arfken, G. B., et al. "Mathematical methods for physicists." (1999).
[2] Biro, O., et al. "On the use of the magnetic vector potential in the finite-element analysis of three-dimensional eddy currents." *IEEE Trans Mag* (1989).
[3] Carpenter, C. J. "Comparison of alternative formulations of 3-dimensional magnetic-field and eddy-current problems at power frequencies." *Proceedings of the Institution of Electrical Engineers*. 1977.

(*) Unclear naming, among the others: Coulomb-Weyl, Coulomb-Hamilton. Coulomb-Gibbs, Coulomb-temporal

Thin Strip approximation



$\Omega_c \rightarrow \Gamma_c \in \mathbb{R}^2$, $\mathbf{J} \cdot \mathbf{n} = \mathbf{0}$, $\mathrm{J} \in \mathbb{R}^2$

$\psi = 0$ on Γ_{nc} (gauge choice, ψ on a surface)

$\mathbf{T}$ as stream function, $\mathbf{T} = \mathrm{T}\,\mathbf{n}$:

$\nabla \times \mathbf{T} = \nabla \times (\mathrm{T}\,\mathbf{n}) = \mathrm{T}\,(\nabla \times \mathbf{n}) + \nabla\mathrm{T} \times \mathbf{n}$

but $\nabla \times \mathbf{n} = 0$ (true for any surface unit normal vector)

hence $\nabla \times \mathbf{T} = \nabla\mathrm{T} \times \mathbf{n}$ (*)

- Equations for Γ_c

  $\nabla \times \sigma^{-1}(\nabla\mathrm{T} \times \mathbf{n}) = -\mu_0 \partial_t(\mathrm{T}\,\mathbf{n})$

  $\nabla \times \mathbf{T} = \mathbf{J}$

  $\nabla \cdot \mathbf{T} = 0$ remember Helmholtz, well posed field

- IC on $\Gamma_{nc,c}$

  Formulations "welded" via the continuity of $\mathbf{B} \cdot \mathbf{n}$ and $\mathbf{E} \times \mathbf{n}$, in terms of T and A

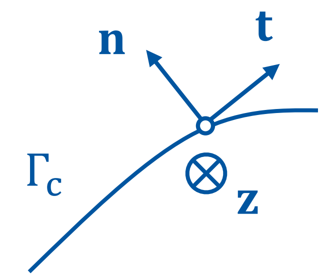
  $\mu_0 \partial_t \mathrm{T}\,\mathbf{n} = \partial_t(\mathbf{B} \cdot \mathbf{n})\mathbf{n} = \partial_t(\nabla \times \mathbf{A} \cdot \mathbf{n})\mathbf{n}$

  $\sigma^{-1}(\nabla\mathrm{T} \times \mathbf{n}) = \mathbf{E} \times \mathbf{n} = -\partial_t \mathbf{A} \times \mathbf{n}$

(*) Carpenter (1977) relied on $\nabla \times \mathbf{T}$.
Rodger (1988) introduced $\partial_t(\nabla\mathrm{T} \times \mathbf{n})$, where ∂_t brings symmetry to the weak form.
Biro (1992) used $\partial_t(\nabla \times \mathrm{T}\,\mathbf{n})$, a hybrid version of Carpenter- Rodger
Zhang (2017) followed Carpenter with $\nabla \times \mathbf{T}$, but he claimed no IC are needed in his approach.

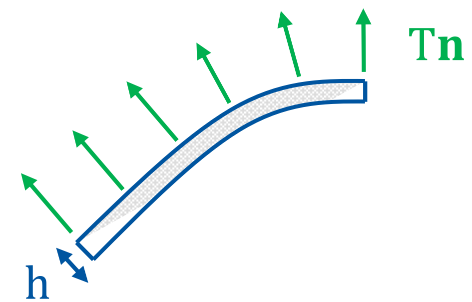
External Source: Current Excitation

$\Omega_c \rightarrow \Gamma_c \in \mathbb{R}^2, \mathbf{J}\cdot\mathbf{n} = \mathbf{0}, \mathbf{J} \in \mathbb{R}^2$



External current excitation i_s. One can show that [1]:

$$\begin{aligned} i_s &= \int \mathbf{J} \cdot \mathbf{z} \; d\Omega_c \\ &= \int \nabla \times \mathbf{T} \cdot \mathbf{z} \; d\Omega_c \text{ (Stokes)} \\ &= \int \mathbf{T} \cdot \mathbf{t} \; d\Gamma_c \\ &= \int (\mathrm{T}\mathbf{n}) \cdot \mathbf{t} \; d\Gamma_c \quad \text{(stream function)} \end{aligned}$$



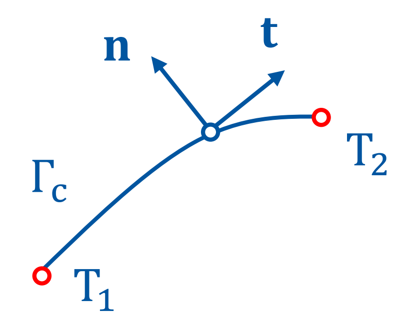
Now, $(\mathrm{T}\mathbf{n}) \cdot \mathbf{t} = 0 \;\; \forall$ point, except edges

$$\int (\mathrm{T}\mathbf{n}) \cdot \mathbf{t} \; d\Gamma_c = h(T_1 - T_2)$$

Two Dirichlet conditions per tape:

$T_1 = \alpha, \alpha \in \mathbb{R}$

$T_2 = i_s/h - T_1$



Stokes + thin strip allows to

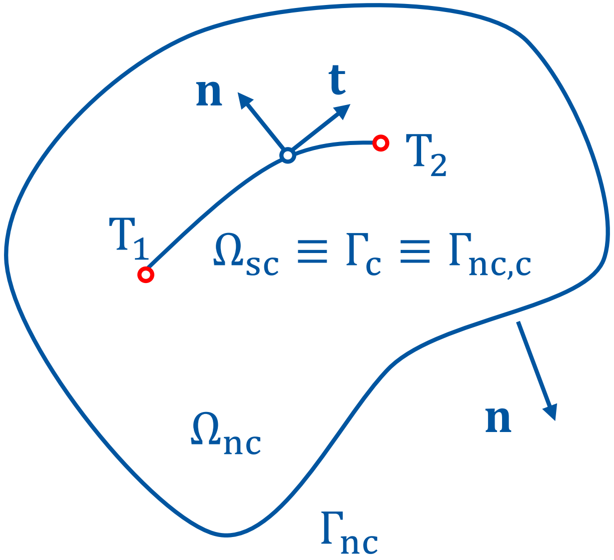
Surface integral → two scalar, linear equations

CERN

[1] Carpenter, C. J. "Comparison of alternative formulations of 3-dimensional magnetic-field and eddy-current problems at power frequencies." *Proceedings of the Institution of Electrical Engineers*. 1977.

To Sum Up…

Hybrid T-A form – Thin Strip Approximation
$\Omega_c \rightarrow \Gamma_c \in \mathbb{R}^2$, $\mathbf{J} \cdot \mathbf{n} = \mathbf{0}$, $\mathbf{J} \in \mathbb{R}^2$



- Equations on Ω_{nc}

  $\nabla \times \mu^{-1} \nabla \times \mathbf{A} = \mathbf{0}$

  $\mathbf{A} \times \mathbf{n} = \mathbf{0}$ on Γ_{nc} (PEW)

  with gauge

  $\nabla \cdot \mathbf{A} = 0, \quad \phi = 0$

- Equations on Γ_c

  $\nabla \times \sigma^{-1}(\nabla \mathrm{T} \times \mathbf{n}) = -\mu_0 \partial_t \mathrm{T}\, \mathbf{n}$

  with gauge

  $\nabla \cdot \mathbf{T} = 0, \quad \psi = 0$

- Equations on interface $\Gamma_{nc,c}$

  $\mu_0 \partial_t \mathrm{T}\, \mathbf{n} = \partial_t (\nabla \times \mathbf{A} \cdot \mathbf{n})\, \mathbf{n}$

  $\nabla \mathrm{T} \times \mathbf{n} = -\sigma \partial_t \mathbf{A} \times \mathbf{n}$

- External source

  $i_{source} = h(T_1 - T_2)$

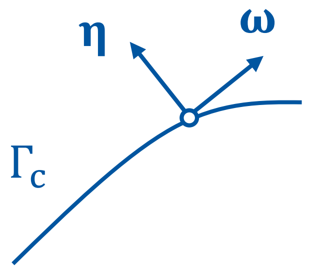
Compatible with the STEAM co-sim framework [1]:

- Current-driven, via i_{source}
- Flux linkage as $\varphi(\mathbf{A})$

[1] Garcia, Idoia Cortes, et al. "Optimized field/circuit coupling for the simulation of quenches in superconducting magnets." *IEEE Journal on Multiscale and Multiphysics Computational Techniques* 2 (2017): 97-104.

03 – Numerical Implementation

Formulation in 2D



The general T-A form is characterized for a 2D domain

Local reference frame (ω, η) on $\Gamma_c \rightarrow$ T: $T(\omega)$

Faraday law:

$$\nabla \times \sigma^{-1}(\nabla T \times \boldsymbol{\eta}) = -\mu_0 \partial_t T\, \boldsymbol{\eta}$$

Vector calculus identity:

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

Faraday law, left hand part:

$$\sigma^{-1}\nabla T(\nabla \cdot \boldsymbol{\eta}) - \boldsymbol{\eta}(\nabla \cdot \sigma^{-1}\nabla T) + (\boldsymbol{\eta} \cdot \nabla)\sigma^{-1}\nabla T - (\sigma^{-1}\nabla T \cdot \nabla)\boldsymbol{\eta}$$

1) $\nabla \cdot \boldsymbol{\eta} = 0$ — true for any surface unit normal vector
2) $(\boldsymbol{\eta} \cdot \nabla)\sigma^{-1}\nabla T = 0$ — $T \neq T(\eta)$
4) $(\sigma^{-1}\nabla T \cdot \nabla)\boldsymbol{\eta} = 0$ — $\boldsymbol{\eta} \neq \boldsymbol{\eta}(\omega)$

$$-\boldsymbol{\eta}(\nabla \cdot \sigma^{-1}\nabla T) = -\mu_0 \partial_t T\, \boldsymbol{\eta}$$

Elliptic partial differential equation of type $\nabla \cdot \alpha \nabla u = f$

The weak form is easily implementable in a numerical solver

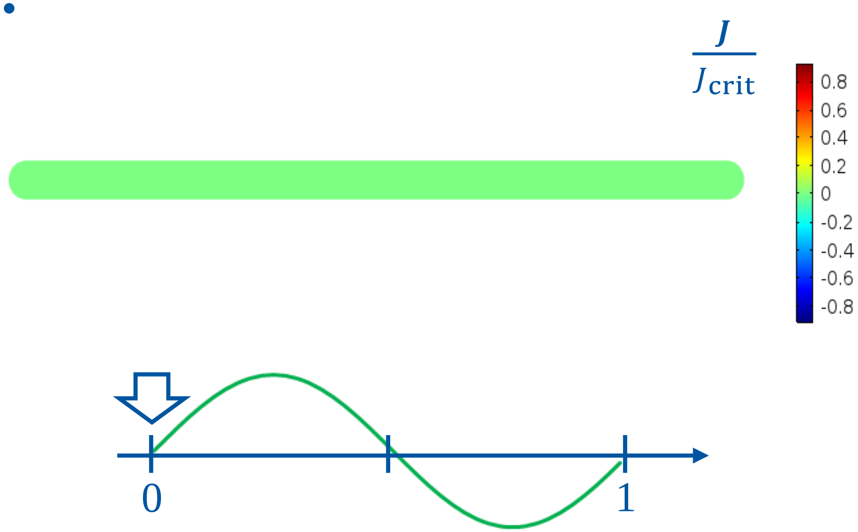
Validation

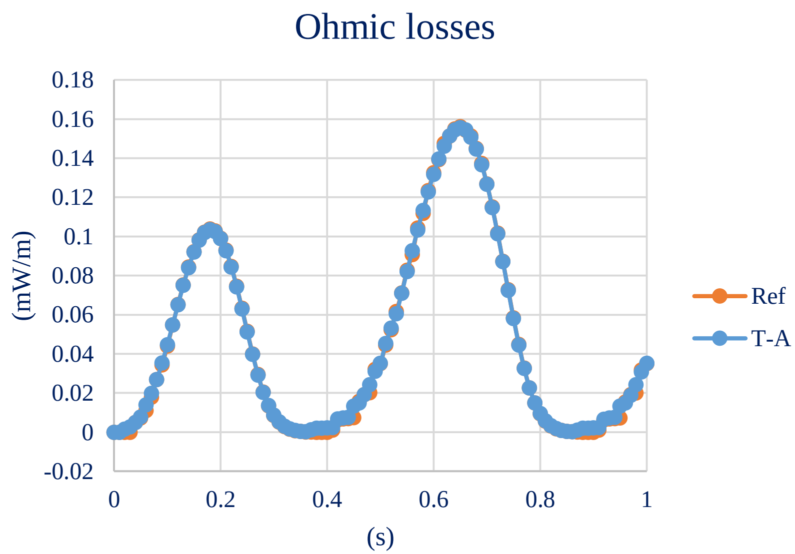
Active community in the field of HTS modeling

HTS MODELLING WORKGROUP

Reference models are available. Here, Link is used for crosscheck

- Single HTS tape in self-field
- Source: $I_s = I_0 \sin(2\pi t)$, $I_0 = 0.5 I_{crit}$ $t \in [0; 1]$
- $2e^3$ unknowns, simulation time 9 s
-

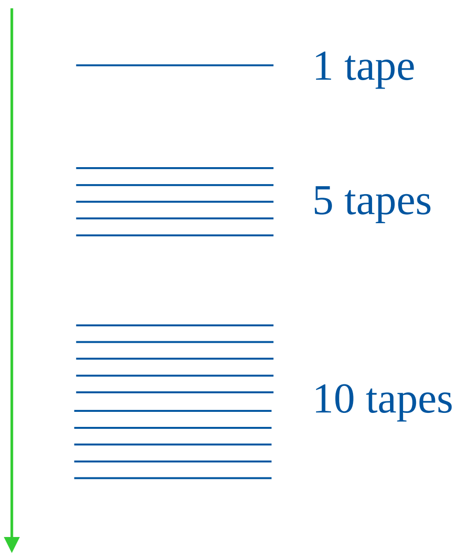




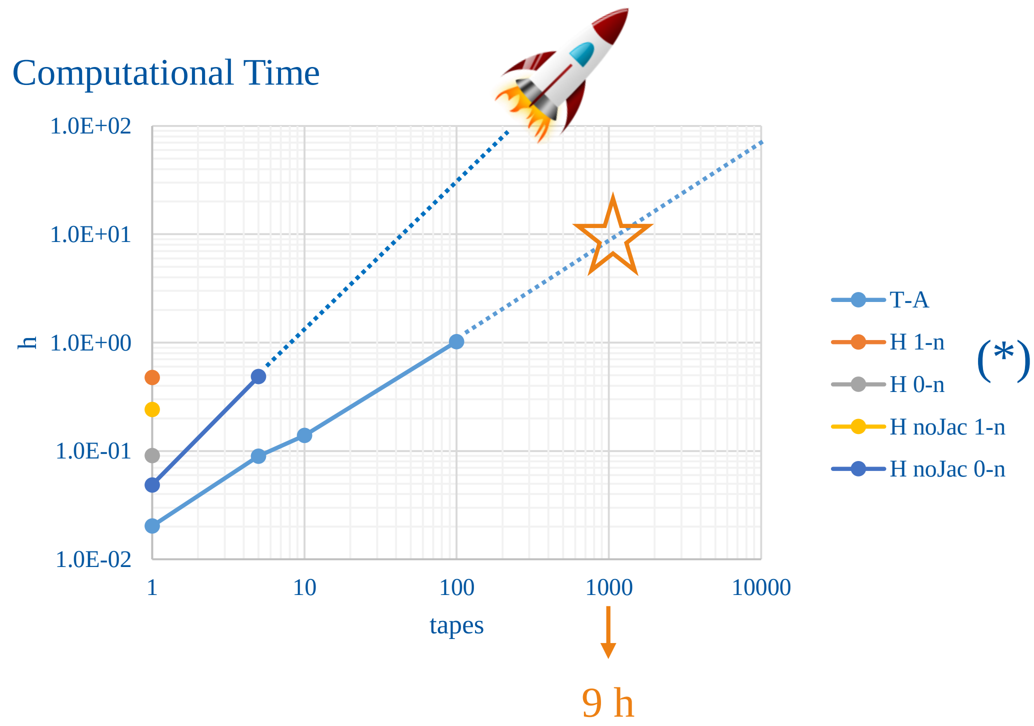
Scalability: H vs T-A Form

Forecasts on expected computational time (Disclaimer: forecasts may not match reality!)

Same physics



Increased computational cost



Results of qualitative analysis:
- H-form: well…
- T-A form: humm...

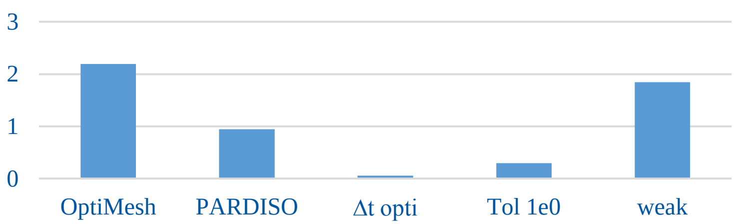
(*) 1-n, 0-n: different mesh
noJac: no Jacobian update

Scalability: T-A Form Optimization

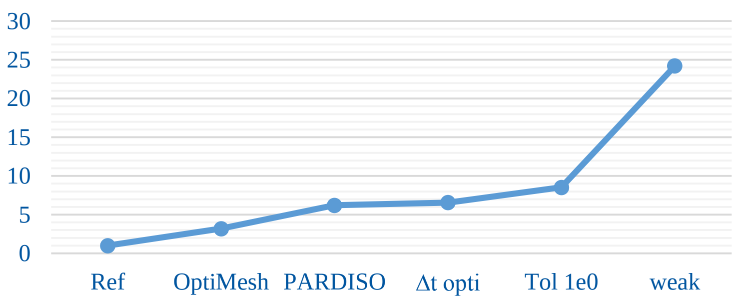
Optimization implemented on:

1. Mesh - OptiMesh
2. Solver - PARDISO, Δt opti, tol $1e^0$
3. Formulation - Weak form b-PDE

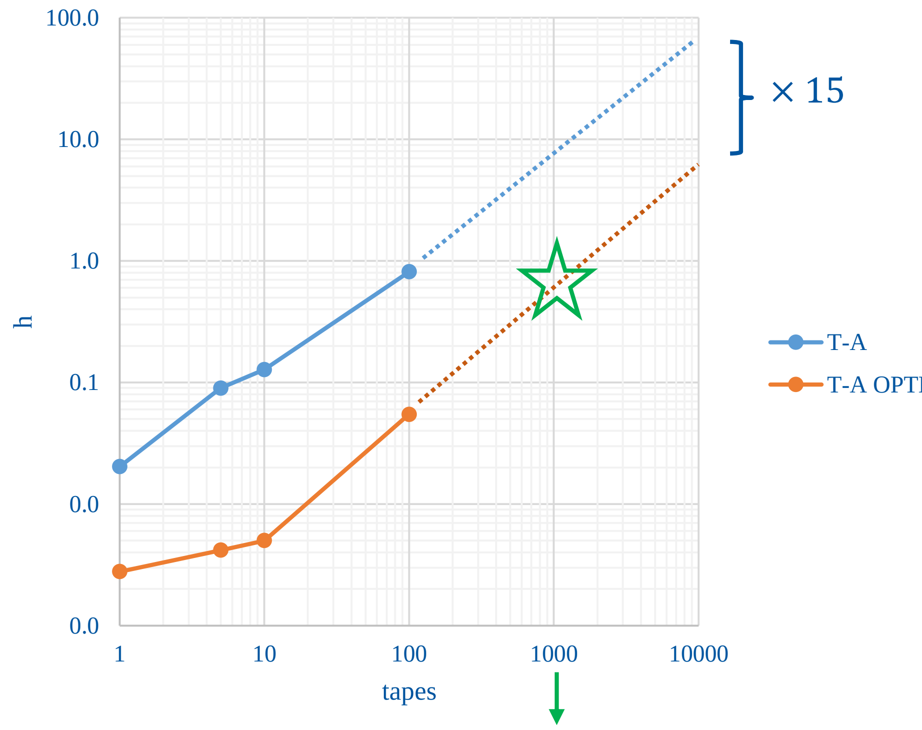
Incremental Speed-up



Cumulative Speed-up



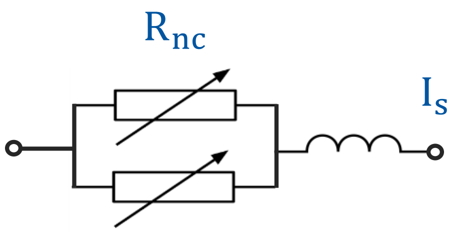
Computational Time



0.5 h
Way better!

Current Sharing in Tape

In HTS, the Stekly approximation [1] is no longer valid [2] (slow quench propagation):



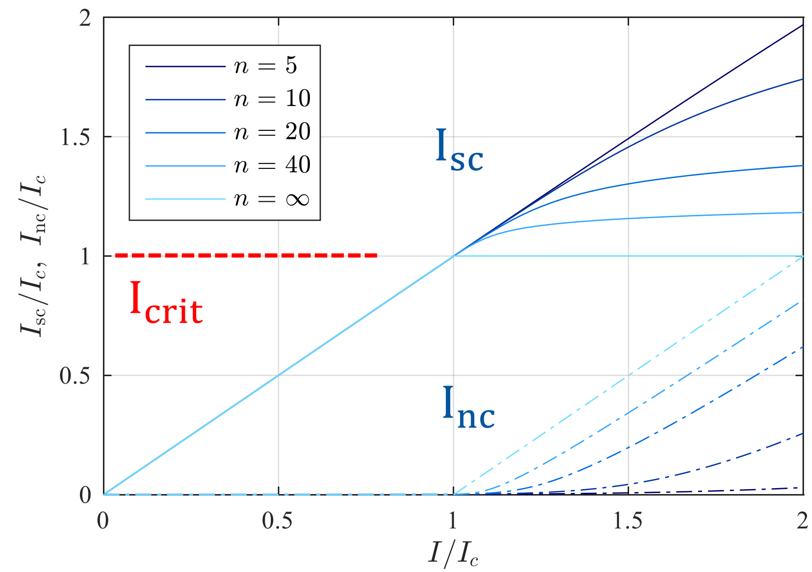
$$I_s = I_{sc} + I_{nc}$$

$$R_{sc}(I_{sc}) = \frac{\ell_{sc}}{\Omega_{sc}} \int \frac{E_c}{J_c} \left(\frac{J}{J_c}\right)^{n-1} d\Omega$$

$$I_{sc} = \frac{R_{nc}}{R_{sc}(I_{sc}) + R_{nc}} (I_{sc} + I_{nc})$$

$$I_{nc} = \frac{R_{sc}(I_{sc})}{R_{sc}(I_{sc}) + R_{nc}} (I_{sc} + I_{nc})$$

- Stekly $\approx n = \infty$
- $n_{HTS} \approx 20$

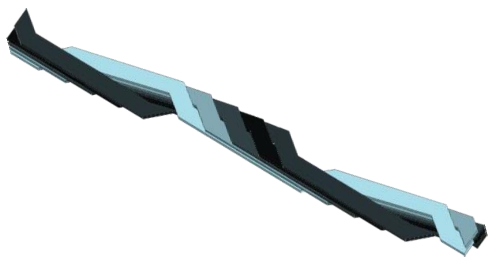


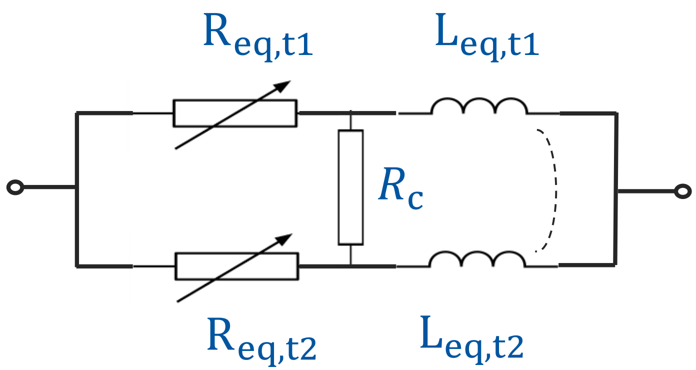
Implicit equations → Algebraic constraints in the solver

[1] Z. Stekly, J. Zar et al., "Stable superconducting coils," IEEE Trans.Nucl. Sci., vol. 12, no. 3, pp. 367–372, 1965.
[2] Van Nugteren, J. *High temperature superconductor accelerator magnets*. Diss. Twente U., Enschede, 2016.

Current Sharing in Roebel Cable

- Roebel cable (only 2 tapes represented in the network model)





$$I_s = \sum_{i=1}^{n} I_{ti}$$

$$I_{ti} = I_{sc,i} + I_{nc,i}$$

Full transposition assumption:

- $R_{eq,t1} = R_{eq,t2} = R_{eq}$
- $L_{eq,t1} = L_{eq,t2} = L_{eq}$

No current redistribution, (as $R_c = +\infty$), conservative
Even distribution of I_s between the tapes

Should be good for:

- Localized quenches (small normal zone, slow propagation velocity)
- Homogeneously distributed losses (e.g. quench-back)

Any better ideas?

Rationale (cont'd)

- 20+ Tesla dipoles for future high-energy particle accelerators
- Simulation of the electrodynamics in HTS tapes and cables (then magnets, and circuits)

Main challenges

1) Multiscale model
- Domain decomposition
- Thin strip approximation, model order reduction

2) HTS resistivity
- **T** vector potential for conductive domains

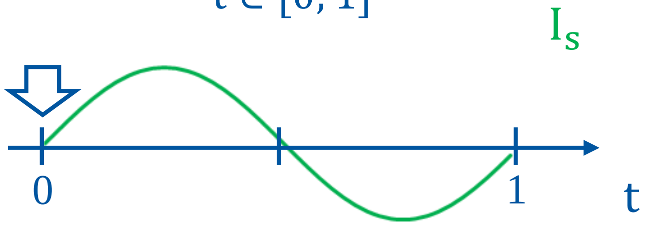
3) Current sharing regime in tape
- Algebraic constraints in the solver

4) Complex geometries
- Full transposition assumption

04 – Applications

Solenoid

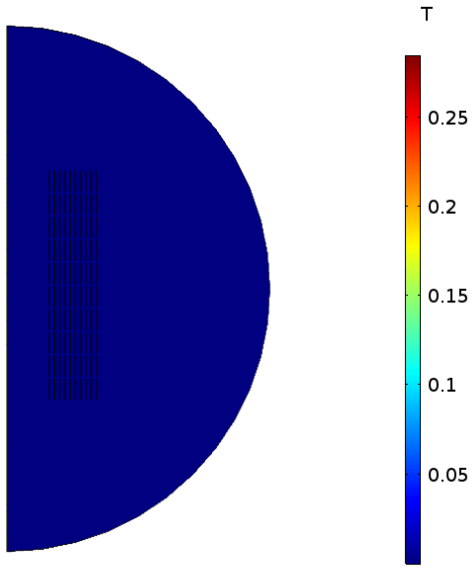
$$I_s = I_0 \sin(2\pi t)$$
$$I_0 = 0.8 I_{crit}$$
$$t \in [0; 1]$$



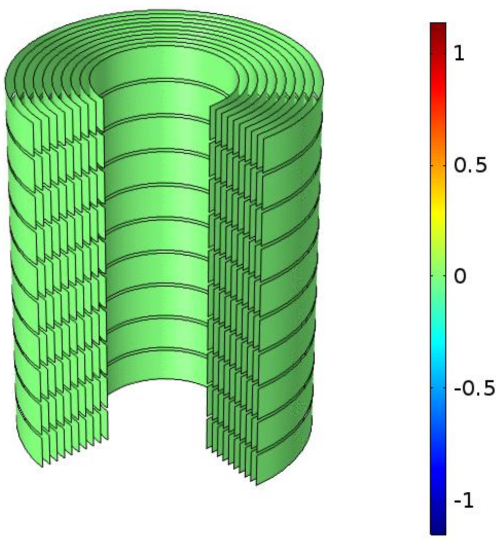
Model features

- 2D - Axisymmetric
- 100 tapes, aspect ratio $1e^4$
- $J_{crit,0} = 1e^{10}$ $[Am^{-2}]$
- $20e^3$ unknowns

Magnetic flux density (T)



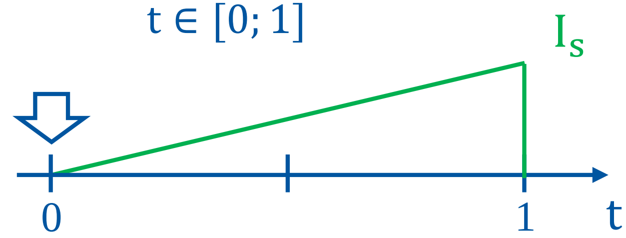
Current density (p.u.)



Simulation time: 200 s

Roebel cable

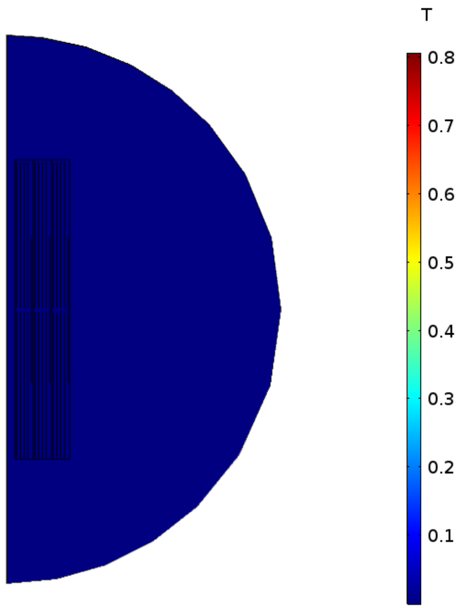
$I_s = I_0 t$
$I_0 = 1.5 I_{crit}$
$t \in [0; 1]$



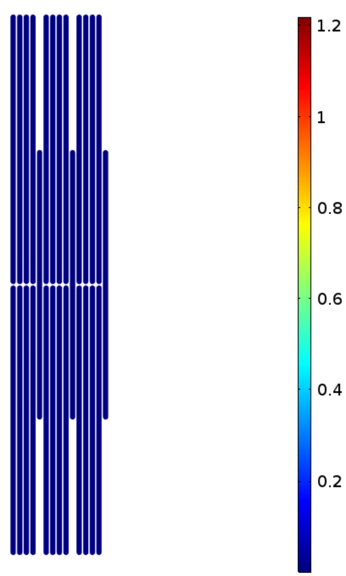
Model features

- 2D
- 3 cables (27 tapes), aspect ratio $1e^4$
- $J_{crit,0} = 1e^{10}$ $[Am^{-2}]$
- $12e^3$ unknowns

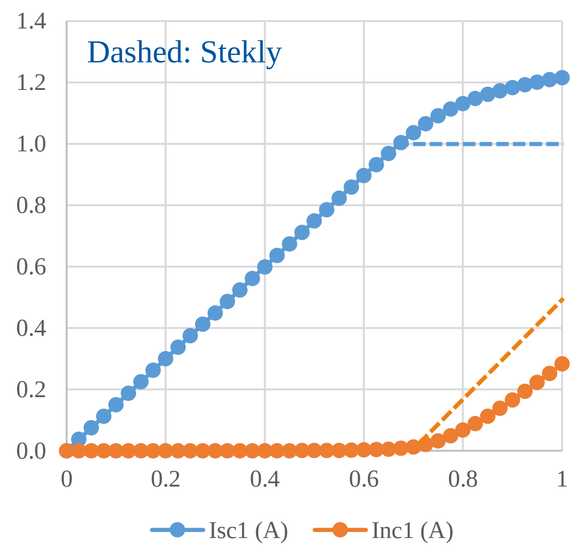
Magnetic flux density (T)



Current density (p.u.)

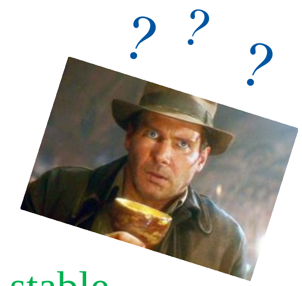


Current (p.u.) - tape 01



Simulation time: 120 s

Hybrid T-A: Summary and Outlook



Formulation

1. Field and interface equations
2. Thin line approximation

Implementation

1. $\mathbb{R}^2$ domain
2. Current sharing regime
3. Applications (solenoids, Roebel cables)

⇨

- Numerically stable
- Computationally efficient
- Reasonably simple

What is next

- Rigorous mathematical assessment (e.g. de Rahm currents)
- HTS material database
- Thermal equations
- Crosscheck with other codes
- 2D model of FRESCA2 + FEATHER2 insert
- FEM 2 LUMPED modeling, for circuital analysis
- Co-simulation interface
- Automatic model generation (SIGMA-HTS module)
- 3D modelling (equations are in place)
- …

Thank you
for your attention!

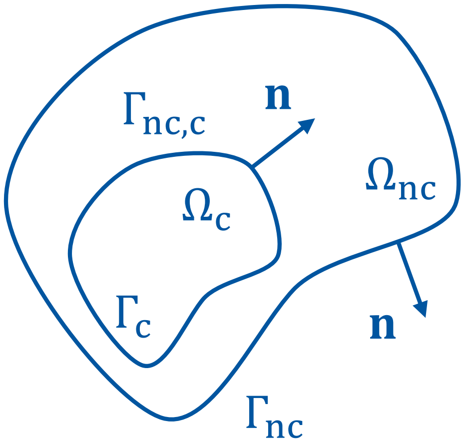
CERN

www.cern.ch

Annex 01 – A form

Domain decomposition

Domains Ω_{nc} , $\Omega_c \in \mathbb{R}^3$, $\Omega_{nc}:\ \sigma = 0$, $\Omega_c:\ \mu = \mu_0$
Γ_{nc}, Γ_c as contour and $\Gamma_{nc,c}$ as interface



$\mathbf{B} = \nabla \times \mathbf{A}$ (magnetic vector potential)
$\mathbf{E} = -\partial_t \mathbf{A}$ (Faraday law)
$\nabla \cdot \mathbf{A}_B = 0,\ \phi_B = 0 + \phi_E = 0$ (radiation gauge)

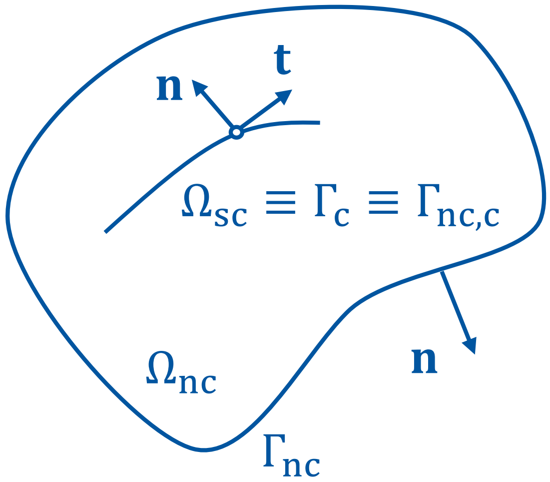
- Equations on Ω_{nc}
  $\rho = 0, \mathbf{J} = \mathbf{0}$ (no sources)
  $\nabla \times \mu^{-1} \nabla \times \mathbf{A} = \mathbf{0}$ on Ω_{nc}
  $\mathbf{A} \times \mathbf{n} = 0$ PEW on Γ_{nc}

- Equations on Ω_c
  $\rho = 0, \mathbf{J} = \sigma \mathbf{E}$
  $\nabla^2 \mathbf{A} = \mu_0 \sigma \partial_t \mathbf{A}$ on Ω_c

- Equations on interface $\Gamma_{nc,c}$
  $(\nabla \times \mathbf{A}_1 - \nabla \times A_2) \cdot \mathbf{n} = 0$
  $(\mu_1^{-1} \nabla \times \mathbf{A}_1 - \mu_2^{-1} \nabla \times \mathbf{A}_2) \times \mathbf{n} = \mathbf{0}$
  $\partial_t(\mathbf{A}_1 - \mathbf{A}_2) \times \mathbf{n} = 0$

A form – Thin Line Approximation

$\Omega_c \rightarrow \Gamma_c \in \mathbb{R}^2, \mathbf{J}\cdot\mathbf{n} = \mathbf{0}, \mathbf{J} \in \mathbb{R}^2$



- Equations on Ω_{nc}

  $\nabla \times \mu^{-1} \nabla \times \mathbf{A} = \mathbf{0}$

  $\mathbf{A} \times \mathbf{n} = \mathbf{0}$ PEW on Γ_{nc}

- Equations on Γ_c

  $\mathbf{A} = A\mathbf{t}$

  $\nabla^2(A\mathbf{t}) = \mu_0 \sigma \partial_t (A\mathbf{t})$ on Ω_c

- Equations on interface $\Gamma_{nc,c}$

  $(\nabla \times \mathbf{A}_1 - \nabla \times \mathbf{A}_2)\cdot\mathbf{n} = 0$

  $(\mu_1^{-1} \nabla \times \mathbf{A}_1 - \mu_2^{-1} \nabla \times \mathbf{A}_2) \times \mathbf{n} = h\mathbf{J}$

  $\partial_t(\mathbf{A}_1 - \mathbf{A}_2) \times \mathbf{n} = 0$

- Field source

  $i_{source} = h \int \sigma \partial_t (A\mathbf{t})\, d\Gamma_c$

Annex 02 – H form

Domain decomposition

Domains Ω_{nc} , $\Omega_c \in \mathbb{R}^3$, Ω_{nc} : $\sigma = 0$, Ω_c : $\mu = \mu_0$
Γ_{nc}, Γ_c as contour and $\Gamma_{nc,c}$ as interface

- Equations on Ω_{nc}

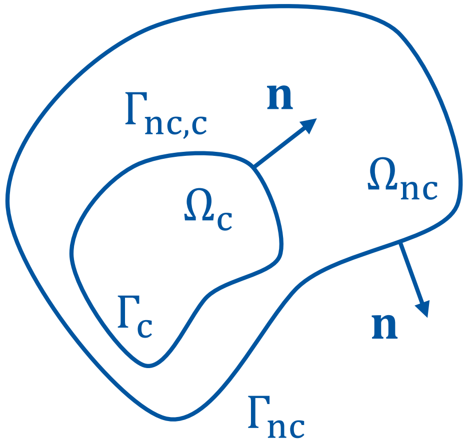
  $\rho = 0, \mathbf{J} = \mathbf{0}$ (no sources)
  $\nabla \times \sigma \nabla \times \mathbf{H} - \mu \partial_t \mathbf{H} = \mathbf{0}$ on Ω_{nc}
  $\nabla \cdot (\mu \mathbf{H}) = 0$ on Ω_{nc}
  $\mathbf{E} \times \mathbf{n} = 0$ PEW on Γ_{nc}
  N.B. numerically, $\sigma \neq 0 \ \forall \Omega$

- Equations on Ω_c

  $\rho = 0, \mathbf{J} = \nabla \times \mathbf{H}$
  $\nabla \times \sigma \nabla \times \mathbf{H} - \mu \partial_t \mathbf{H} = \mathbf{0}$ on Ω_c
  $\nabla \cdot (\mu \mathbf{H}) = 0$ on Ω_c

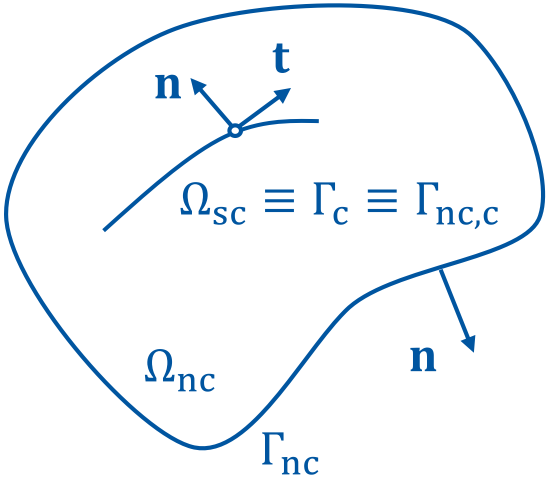
- Equations on interface $\Gamma_{nc,c}$

  $(\mu_1 \mathbf{H}_1 - \mu_2 \mathbf{H}_2) \cdot \mathbf{n} = 0$
  $(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{n} = h\mathbf{J}$
  $(\sigma_1 \nabla \times \mathbf{H}_1 - \sigma_2 \nabla \times \mathbf{H}_2) \times \mathbf{n} = 0$



H form – Thin Line Approximation

$\Omega_c \rightarrow \Gamma_c \in \mathbb{R}^2$, $\mathbf{J} \cdot \mathbf{n} = 0$, $\mathbf{J} \in \mathbb{R}^2$



- Equations on Ω_{nc}

  $\rho = 0$, $\mathbf{J} = \mathbf{0}$ (no sources)

  $\nabla \times \sigma \nabla \times \mathbf{H} - \mu \partial_t \mathbf{H} = \mathbf{0}$ on Ω_{nc}

  $\nabla \cdot (\mu \mathbf{H}) = 0$ on Ω_{nc}

  $\mathbf{E} \times \mathbf{n} = 0$ PEW on Γ_{nc}

  N.B. numerically, $\sigma \neq 0 \ \forall \Omega$

- Equations on Γ_c

  $\rho = 0$, $\mathbf{J} = \nabla \times \mathbf{H}$

  $\nabla \times \sigma \nabla \times \mathbf{H} - \mu \partial_t \mathbf{H} = \mathbf{0}$ on Γ_c

  $\nabla \cdot (\mu \mathbf{H}) = 0$ on Γ_c

- Interface $\Gamma_{nc,c}$

  $(\mu_1 \mathbf{H}_1 - \mu_2 \mathbf{H}_2) \cdot \mathbf{n} = 0$

  $(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{n} = h\mathbf{J}$

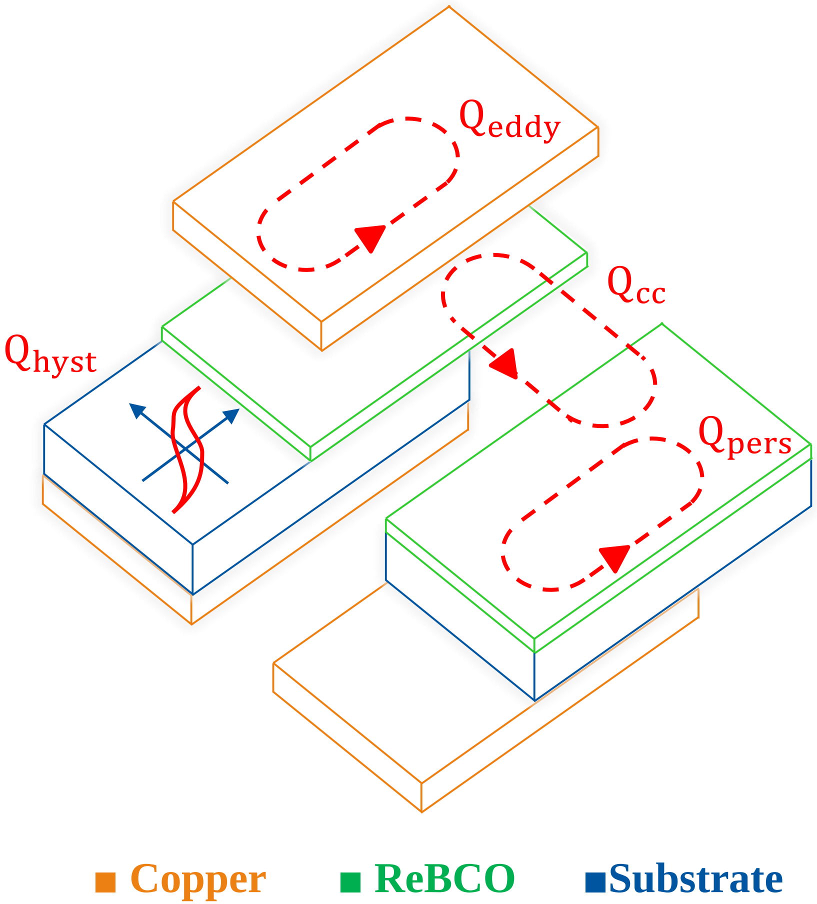
  $(\sigma_1 \nabla \times \mathbf{H}_1 - \sigma_2 \nabla \times \mathbf{H}_2) \times \mathbf{n} = 0$

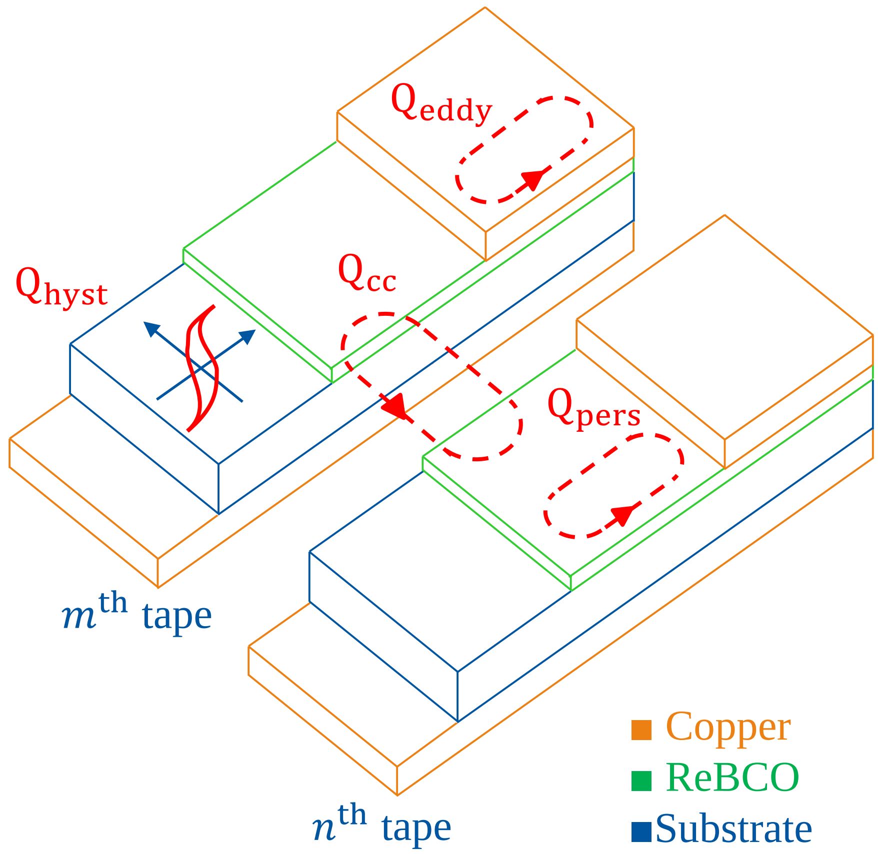
Incompatible conditions:
on Γ_c, **H** cannot be both
divergence-free and discontinuous!

- Field source

  $i_{source} = \int \nabla \times \mathbf{H} \ d\Gamma_c$

Backup Slides


Q_{eddy}
Q_{cc}
Q_{hyst}
Q_{pers}
■ Copper
■ ReBCO
■ Substrate


Q_{eddy}
Q_{cc}
Q_{hyst}
Q_{pers}
m^{th} tape
n^{th} tape
■ Copper
■ ReBCO
■ Substrate