Duration of Classicality of the Axion Dark Matter Condensate

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Dark Matter Axions

Axions: Solving the Strong CP Problem

$$\mathcal{L}_{\bar{\theta}} = \frac{\bar{g}^2}{16\pi^2}\bar{\theta} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \qquad \bar{\theta} = \theta + \mathrm{Arg}|\mathcal{M}| \leq 10^{-10}$$

Peccei and Quinn proposal: New Symmetry $U_{\mathrm{PQ}}(1)$

(Peccei y Quinn *Phys. Rev. Lett.*, 1977)

Axions could solve the Dark Matter problem if they were produced in the early universe with a non-thermal mechanism

Dark Matter Axions

$$T_{QCD} < T < f_a$$ → **Nambu-Goldstone boson**

$$T < T_{QCD}$$ → **The axion gets its mass**

$$t_1 \simeq 2\times10^{-7}\,\mathrm{sec}\left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{1/3}$$

$$m_a = 6\times10^{-6}\,\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

$$n(t) \sim \frac{4\times10^{47}}{\mathrm{cm}^3}\left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{5/3}\left(\frac{a(t_1)}{a(t)}\right)^3$$

$$\delta v(t) \sim \frac{1}{m_a t_1}\frac{a(t_1)}{a(t)}$$

Dark Matter Axions

Dark matter axions could form a Bose-Einstein condensate (BEC)

(Sikivie and Yang, Phys.Rev.Lett. 103 (2009) 111301)

(Saikawa and Yamaguchi, *Phys.Rev. D87 (2013) no.8, 085010*)

(Berges and Jaeckel, Phys.Rev. D91 (2015) no.2, 025020)

If Dark Matter axions form a Bose-Einstein condensate, they can explain the formation of caustic rings in galatic halos

BEC can only be described by quantum field equations

(Erken et al., Phys.Rev. D85 (2012) 063520)

Over what time scale is a classical description valid?

This time scale is defined as the "Duration of Classicality"

Classical vs. quantum description of interacting spin-0 bosons (a toy model approach)

$$H = \sum_j \omega_j a_j^\dagger a_j + \frac{1}{4} \sum_{jkln} \Lambda_{jk}^{ln} a_j^\dagger a_k^\dagger a_l a_n$$

$$i\dot{a}_j = \omega_j a_j + \frac{1}{2} \sum_{kln} \Lambda_{jk}^{ln} a_k^\dagger a_l a_n$$

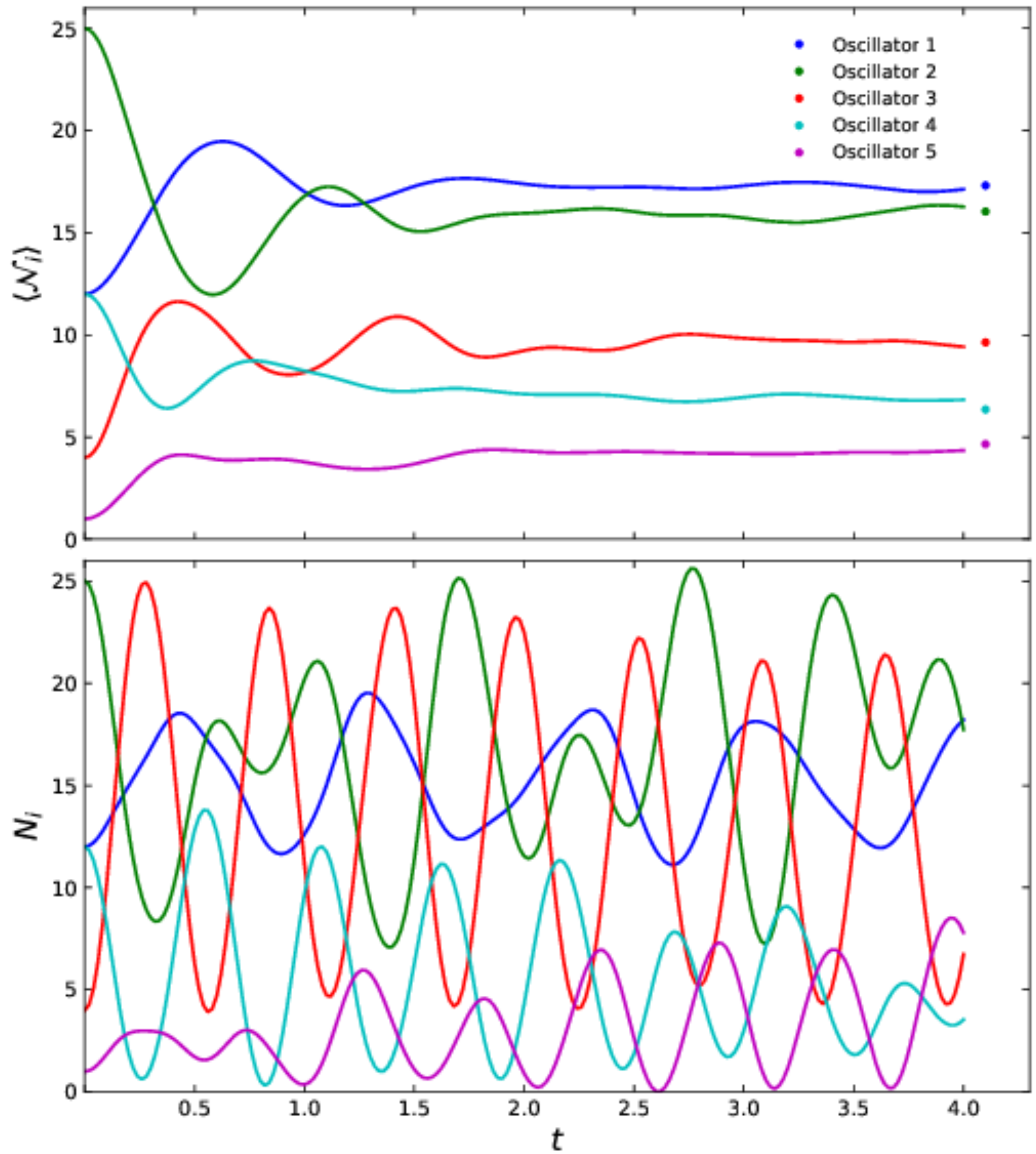
$$i\dot{A}_j = \omega_j A_j + \frac{1}{2} \sum_{kln} \Lambda_{jk}^{ln} A_k^* A_l A_n$$

Quantum equations (operators)

Classical equations (C-numbers)

Classical vs. quantum description of interacting spin-0 bosons (a toy model approach)

$$\left|\mathcal{N}_1(0), \mathcal{N}_2(0), \ldots, \mathcal{N}_5(0)\right\rangle = \left|12,\ 25,\ 4,\ 12,\ 1\right\rangle$$



Thermal states

$$\tau = \frac{1}{\Gamma}$$

$$\Gamma \sim \Lambda\sqrt{I\mathcal{N}}$$

I **number of relevants interactions**

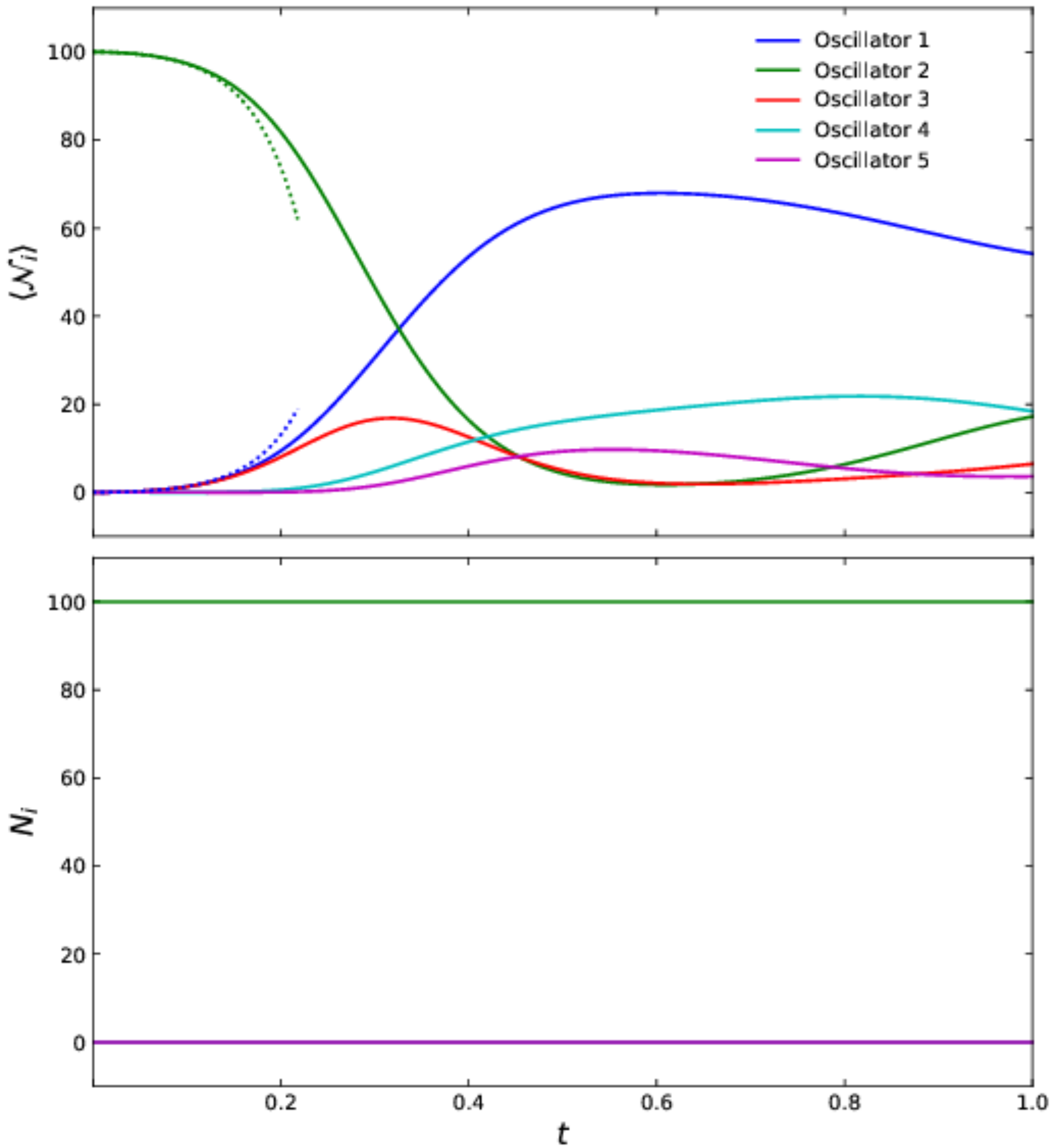
Λ **typical interaction strength**

$\mathcal{N}$ **typical occupation numbers**

(Sikivie and Todarello, Phys.Lett. B770 (2017) 331-334)

Classical vs. quantum description of interacting spin-0 bosons (a toy model approach)

$$\left|0,\, N,\, 0,\, 0,\, 0\right\rangle = \left|0,\, 100,\, 0,\, 0,\, 0\right\rangle$$



$$2 + 2 \rightarrow 1 + 3$$

state 2 jumps in pairs to states 1 and 3

$$\Gamma \sim \left|\Lambda^{22}_{13}\right| N/\log(N)$$

$$\left\langle \mathcal{N}_1 \right\rangle \sim \left\langle \mathcal{N}_3 \right\rangle \sim e^{|\Lambda^{22}_{13}|Nt}$$

Outline

- How to find the duration of classicality in quantum field theory?
- Duration of classicality of the homogeneous classical condensate with atractive contact interaction
- Duration of classicality of the homogeneous classical condensate with gravitational self-interaction
- Duration of classicality of the inhomogeneous classical condensate with a repulsive contact interaction

How to find the duration of classicality in quantum field theory?

$$\psi(\mathbf{x},t) = \sum_{\mathbf{k}} u^{\mathbf{k}}(\mathbf{x},t) a_{\mathbf{k}}(t)$$

$$\left[\psi(\mathbf{x},t), \psi(\mathbf{y},t)\right] = 0$$

$$\left[\psi(\mathbf{x},t), \psi(\mathbf{y},t)^{\dagger}\right] = \delta^3(\mathbf{x}-\mathbf{y})$$

$u^{\mathbf{k}}(\mathbf{x},t)$ **are an orthonormal and complete set of wavefunctions**

$$\int_V d^3x \; u^{\mathbf{k}}(\mathbf{x},t)^* u^{\mathbf{k}'}(x,t) = \delta_{\mathbf{k}}^{\mathbf{k}'}$$

$$\sum_{\mathbf{k}} u^{\mathbf{k}}(\mathbf{x},t)^* u^{\mathbf{k}}(\mathbf{y},t) = \delta^3(\mathbf{x}-\mathbf{y})$$

How to find the duration of classicality in quantum field theory?

$\Psi(\mathbf{x}, t)$ → **some solution of the classical equations**

$$u^{\mathbf{k}}(\mathbf{x}, t) = \frac{1}{\sqrt{N}} \Psi(\mathbf{x}, t) e^{i\mathbf{k}\cdot\chi(\mathbf{x},t)}$$

$$\frac{d^3N}{d\chi^3} = \frac{n(\mathbf{x}, t)}{J(\mathbf{x}, t)} = n_0$$

$$\int_V d^3x \; u^{\mathbf{k}}(\mathbf{x}, t)^* u^{\mathbf{k}'}(\mathbf{x}, t) = \frac{1}{V_0} \int_{V_0} d^3\chi \; e^{i(\mathbf{k}'-\mathbf{k})\cdot\chi} = \delta_{\mathbf{k}}^{\mathbf{k}'}$$

$$a_{\mathbf{k}}(t) = \sqrt{N}\delta_{\mathbf{k}}^{\mathbf{0}} + b_{\mathbf{k}}(t)$$

$$b_{\mathbf{k}}(0) \left| \Psi \right\rangle = 0$$

Duration of classicality of the homogeneous classical condensate with atractive contact self-interaction

Atractive contact interaction

$$\Psi = \sqrt{n_0} e^{-i\delta\omega t}$$

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}\psi^\dagger\psi\psi$$

$$\delta\omega = \frac{\lambda n_0}{8m^2}$$

$$\chi(\mathbf{x}, t) = \mathbf{x}$$

$$a_{\mathbf{k}}(t) = \sqrt{N}\delta_{\mathbf{k}}^{\mathbf{0}} + b_{\mathbf{k}}(t)$$

$$i\partial_t b_{\mathbf{k}} = \left(\frac{k^2}{2m} + \delta\omega\right) b_{\mathbf{k}} + \delta\omega b_{-\mathbf{k}}^\dagger$$

$$b_{\mathbf{k}}(0)\left|\Psi\right\rangle = 0$$

Duration of classicality of the homogeneous classical condensate with atractive contact self-interaction

$$\lambda < 0 \qquad k < k_J = \sqrt{\frac{|\lambda| n_0}{2m}}$$

→ **Instability**

$$b_{\mathbf{k}} \sim e^{s(k)t}$$

$$N_{ev}(t) = \sum_{k<k_J} \langle \Psi | \, b_{\mathbf{k}}^{\dagger}(t) b_{\mathbf{k}}(t) \, | \Psi \rangle$$

$$s(k) = \frac{k}{2m}\sqrt{k_J^2 - k^2}$$

Time of Classicality t_c $\qquad N_{ev}(t_c) \sim N$

$$t_c \sim \frac{2m}{k_J^2} \sim \frac{1}{2\delta\omega}$$

Duration of classicality of the homogeneous classical condensate with gravitation self-interaction

Gravitation self-interaction

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + m\varphi\psi \qquad \varphi(\mathbf{r},t) = -Gm\int_V d^3r' \, \frac{\psi(\mathbf{r}',t)^\dagger \psi(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}$$

$$\partial_t n_0 + 3Hn_0 = 0$$

$$\Psi(\mathbf{r},t) = \sqrt{n_0(t)}\, e^{i\frac{1}{2}mH(t)r^2} \qquad H(t)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3} m n_0(t)$$

$$H = \frac{\dot{a}}{a}$$

Duration of classicality of the homogeneous classical condensate with gravitation self-interaction

critical universe $K = 0$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\chi(\mathbf{r}, t) = \frac{\mathbf{r}}{a(t)}$$

$$n(t) = n_0 \left(\frac{t_0}{t}\right)^2$$

$$a_{\mathbf{k}}(t) = \sqrt{N}\delta_{\mathbf{k}}^{\mathbf{0}} + b_{\mathbf{k}}(t)$$

$$b_{\mathbf{k}}(0)\left|\Psi\right\rangle = 0$$

$$i\partial_t b_{\mathbf{k}} = \left(\frac{k^2}{2m}\left(\frac{t_0}{t}\right)^{4/3} - \frac{2m}{3k^2t_0^2}\left(\frac{t_0}{t}\right)^{2/3}\right)b_{\mathbf{k}} - \frac{2m}{3k^2t_0^2}\left(\frac{t_0}{t}\right)^{2/3} b_{-\mathbf{k}}^{\dagger}$$

Duration of classicality of the homogeneous classical condensate with gravitation self-interaction

$$t \gg t_k = \frac{k^6 t_0^4}{(2m)^3}$$ → **Instability**

$$\left\langle N_{\mathbf{k}}(t) \right\rangle = \langle \Psi \left| \, b_{\mathbf{k}}^{\dagger}(t) b_{\mathbf{k}}(t) \, \right| \Psi \rangle \approx \frac{1}{10}\sqrt{\frac{2}{3}}\left(\frac{t}{t_k}\right)^2$$

$$N_{ev}(t) = \sum_{k<k_J} \left\langle N_{\mathbf{k}}(t) \right\rangle \sim 0.26 N G m^2 \sqrt{m t_0}\left(\frac{t}{t_0}\right)^2$$

$$t_c \sim t_0 \frac{1}{\left(G m^2 \sqrt{m t_0}\right)^{1/2}}$$

Duration of classicality of the inhomogeneous classical condensate with repulsive contact self-interaction

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}\psi^\dagger\psi\psi$$

$$\Psi(\mathbf{x},t) = \sqrt{n_0}e^{-i\delta\omega t} + \Psi_1(\mathbf{x},t)$$

$$\omega(p) = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2\delta\omega\right)}$$

$$\delta\omega = \frac{\lambda n_0}{8m^2}$$

$$n(\mathbf{x},t) = n_0 + \delta n \cos(\mathbf{p}\cdot\mathbf{x} - \omega(p)t)$$

$$\chi(\mathbf{x},t) = \mathbf{x} - \frac{i}{2}\frac{\delta n}{n_0}\frac{\mathbf{p}}{p^2}e^{i(\mathbf{p}\cdot\mathbf{x}-\omega(p)t)}$$

Duration of classicality of the inhomogeneous classical condensate with repulsive contact self-interaction

$$a_{\mathbf{k}}(t) = \sqrt{N}\delta^{\mathbf{0}}_{\mathbf{k}} + b_{\mathbf{k}}(t) \qquad b_{\mathbf{k}}(0)\,\big|\Psi\rangle = 0$$

$$i\partial_t b_{\mathbf{k}} = \left(\frac{k^2}{2m} + \delta\omega\right) b_{\mathbf{k}} + \delta\omega b^{\dagger}_{-\mathbf{k}} + \frac{\delta\omega}{2}\frac{\delta n}{n_0} e^{-i\omega(p)t}\left[P(\mathbf{k},\mathbf{p})b_{\mathbf{k}-\mathbf{p}} + Q(\mathbf{k},\mathbf{p})b^{\dagger}_{-\mathbf{k}+\mathbf{p}}\right]$$

$$-\delta\omega\frac{\delta n}{n_0} < \omega(p) - \omega(k) - \omega(|\mathbf{k}-\mathbf{p}|) < \delta\omega\frac{\delta n}{n_0}$$

➡ **Instability**

$$b_{\mathbf{k}} \sim e^{s(k)t} \qquad s(k) \sim \delta\omega\frac{\delta n}{n_0}$$

$$N_{ev}(t) = \sum_{k<k_J}\left\langle b^{\dagger}_{\mathbf{k}}(t) b_{\mathbf{k}}(t)\right\rangle \qquad t_c \sim \frac{1}{2\delta\omega}\frac{n_0}{\delta n}$$

What is the next step?

We want to use this idea to calculate the time of classicality for an inhomogeneous sef-gravitating field, which is the relevant case for Dark Matter Axions