Non-relativistic EFT and longevity of axion star

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K. Mukaida, M. Takimoto, M.Y., JHEP 1703 (2017) 122 (hep-ph/1612.07750)
J. Eby, K. Mukaida, M. Takimoto, L.C.R. Wijewardhana, M.Y., hep-ph/1808.xxxxx.

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PQ mechanism and axion

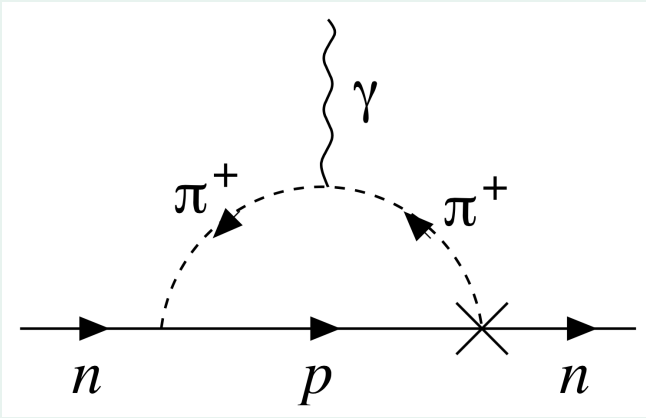
- Strong CP problem

$$\mathcal{L}_{\rm SM} \supset \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- Constraint from neutron EDM:

$$|\theta| \lesssim 10^{-(10-11)}$$

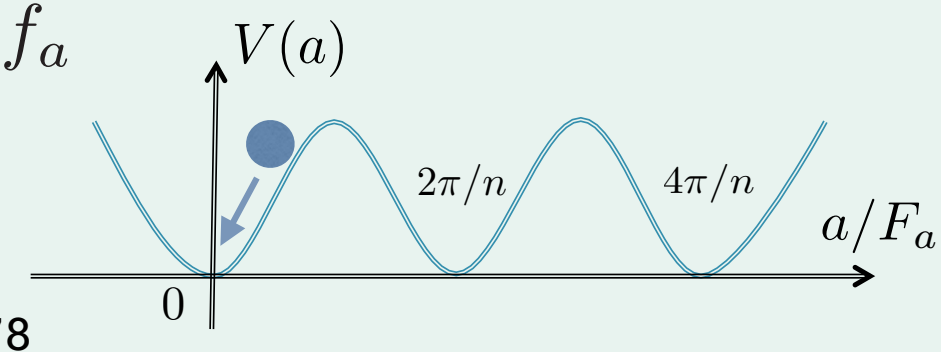
Baker et al. 06



Baluni 79,
Crewther, Vecchia,
Veneziano, Witten 80

- PQ mechanism: $\theta \to a(x)/f_a$

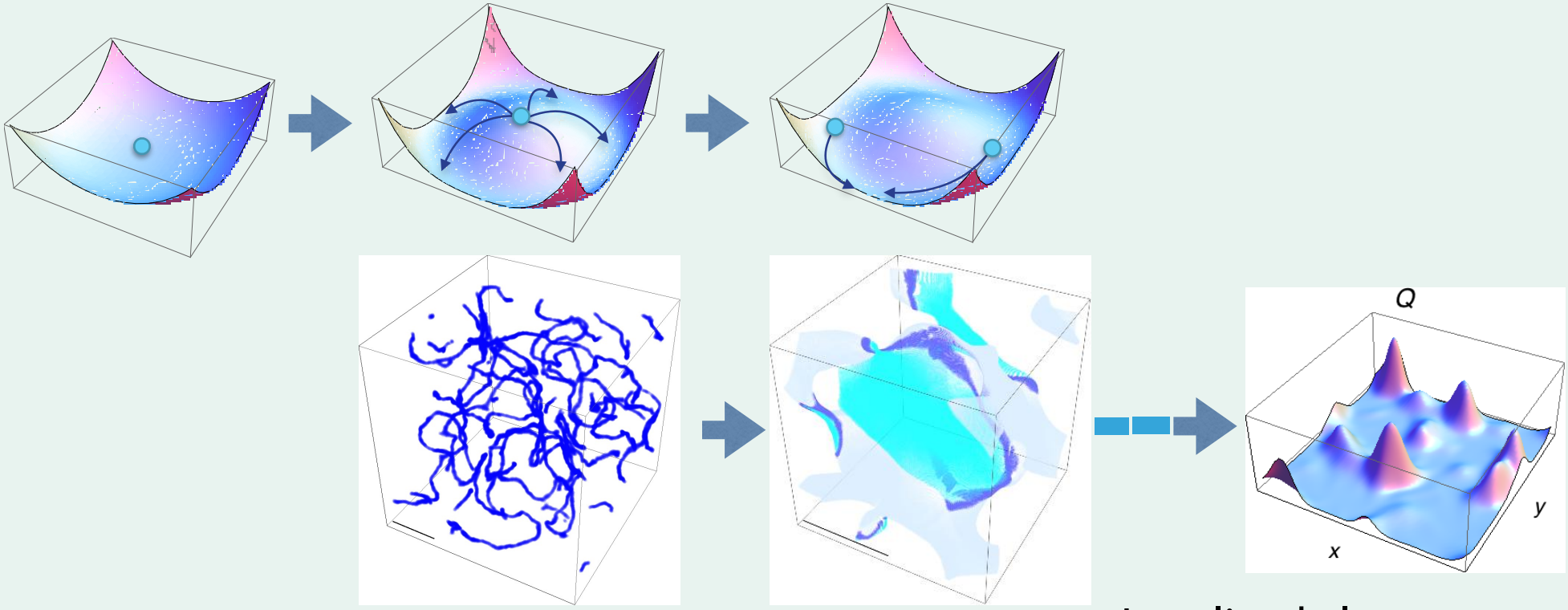
$$\theta_{\rm eff} = \langle a \rangle / f_a \approx 0$$



Peccei Quinn 77, Weinberg 78, Wilczek 78

PQ mechanism and axion

- Large density perturbations may be produced at the time of QCD phase transition.



Localized clumps may form at a later epoch.

Axion star

- Those kinds of localized clumps are called axion clumps, axion star, axiton, I-ball, oscillon, oscillaton, *e.t.c.*...
- They are localized quasi-stable condensates in real scalar field theories.

Oscillon: Bogolyubsky, Makhankov 76, Segur, Kruskal 87, Gleiser 94, Copeland, Gleiser, Muller 95,
I-ball: Kasuya, Kawasaki, Takahashi 03,
Oscillaton: Kaup 68, Ruffini, Bonazzola 69, Colpi, Shapiro, Wasserman 86,
Axion stars: Hogan, Rees 88, Kolb, Tkachev 93, Seidel, Suen 93,

Axion star

- Those kinds of localized clumps are called axion clumps, axion star, axiton, I-ball, oscillon, oscillaton, *e.t.c.*...
- They are localized quasi-stable condensates in real scalar field theories.
- They can be described by non-relativistic effective field theories.

$$\phi(x,t) = \frac{1}{\sqrt{2m_\phi}}\left[e^{-i\,m_\phi\,t}\psi(x,t) + e^{i\,m_\phi\,t}\psi^*(x,t)\right]$$

$$\mathcal{L}_{\rm eff} = \frac{i}{2}\left(\dot{\psi}\psi^* - \psi\dot{\psi}^*\right) - \frac{1}{2m_\phi}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi - V_{\rm int}(\psi)$$

Braaten, Mohopatra, Zhang '16, '18
Namjoo, Guth, Kaiser '17

- EoM is similar to the Schrodinger eq.

Non-relativistic effective field theory

- The "Standard" method was recently sophisticated by Namjoo, Guth, Kaiser, where high-frequency modes of the *new* field are integrated out:

$$\phi(t,\boldsymbol{x}) = \frac{1}{\sqrt{2m_\phi}}\mathcal{P}^{-1/2}\left[e^{-im_\phi t}\psi(t,\boldsymbol{x}) + e^{im_\phi t}\psi^*(t,\boldsymbol{x})\right] \qquad \mathcal{P} \equiv \sqrt{1-\frac{\boldsymbol{\nabla}^2}{m_\phi^2}}$$

$$\psi(x,t) = \sum_{\nu=0,\pm1,\pm2,\ldots} \psi_\nu(x,t)e^{i\nu m_\phi t}$$

Namjoo, Guth, Kaiser '17

- We proposed a new method, where high-frequency modes of the *original* field are integrated out:

$$\phi(t,\boldsymbol{x}) = \left[e^{-i\omega t}\psi_1(t,\boldsymbol{x}) + \text{h.c.}\right] + \delta\phi$$

$$\delta\phi \equiv \sum_{n\geq 2} e^{-in\omega t}\psi_n + \text{h.c.},$$

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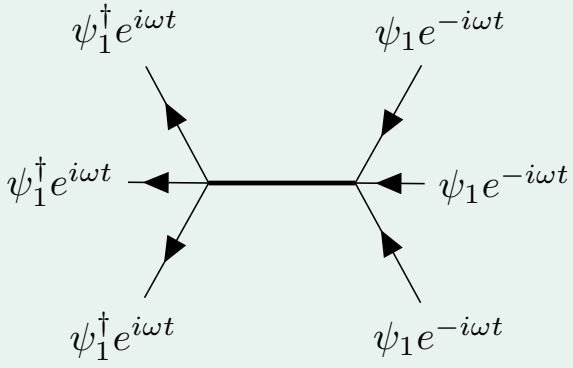
- Degrees of freedom are doubled.

Corrections from relativistic modes

- Non-relativistic effective action is written as

$$S_{\rm NR}[\psi_1] = \int_x \left[\left| \dot{\psi}_1 - i\omega\psi_1 \right|^2 + |\nabla\psi_1|^2 - m_\phi^2 |\psi_1|^2 - V_{\rm eff}(|\psi_1|) \right] + i\Gamma$$

- The effective potential can be calculated by integrating out relativistic modes.



$$= -\int_x \frac{\lambda_4^2}{(3!)^2} \psi_1^{\dagger 3} \frac{1}{9\omega^2 - m_\phi^2 + 6i\omega\partial_t + \boldsymbol{\nabla}^2 - \partial_t^2} \psi_1^3$$

$$\simeq -\int_x \frac{\lambda_4^2}{8(3!)^2 m_\phi^2} \left\{ |\psi_1|^4 \left[|\psi_1|^2 - \frac{9}{4}\psi_1 \left(\frac{\omega}{m_\phi} - 1 + \frac{i\overleftrightarrow{\partial_t}}{2m_\phi} \right) \psi_1 \right] - \psi_1^{\dagger 3} \frac{\boldsymbol{\nabla}^2}{8m_\phi^2} \psi_1^3 \right\}$$

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Axion star as a projection of Q-ball

- The real part of EFT has a U(1) global symmetry, so we can define a conserved charge.

$$Q[\psi_1] = i \int_x \left[\psi_1^\dagger \left(\partial_t - i\omega \right) \psi_1 - \psi_1 \left(\partial_t + i\omega \right) \psi_1^\dagger \right]$$

$$E[\psi_1] = \int_x \left[\left| \dot{\psi}_1 - i\omega\psi_1 \right|^2 + \left| \nabla\psi_1 \right|^2 + m_\phi^2 \left| \psi_1 \right|^2 + V_{\rm eff}(|\psi_1|) \right]$$

- Lowest energy configuration with a fixed charge is kwon as Q-ball.

  We would like to minimize $E[\psi_1] - \omega' \left(q - Q[\psi_1] \right)$

  Lagrange multiplier

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Axion star as a projection of Q-ball

- Lowest energy configuration with a fixed charge is kwon as Q-ball.

  - The stable solution should satisfy

$$\nabla^2\psi + \left(\omega^2 - m_\phi^2(1-2\Psi)\right)\psi - \partial_{\psi^\dagger} V_{\rm eff} = 0$$

$$\frac{\nabla^2\Psi}{4\pi G} = -2m_\phi^2\,|\psi|^2$$

Gravitational potential

  - These are similar to the Euclidean EoM for instanton solution and can be solved numerically.

  - Condition for stability against small perturbations:

$$\frac{\omega}{Q}\frac{{\rm d}Q}{{\rm d}\omega} < 0 \qquad \frac{\partial E}{\partial Q} = \omega$$

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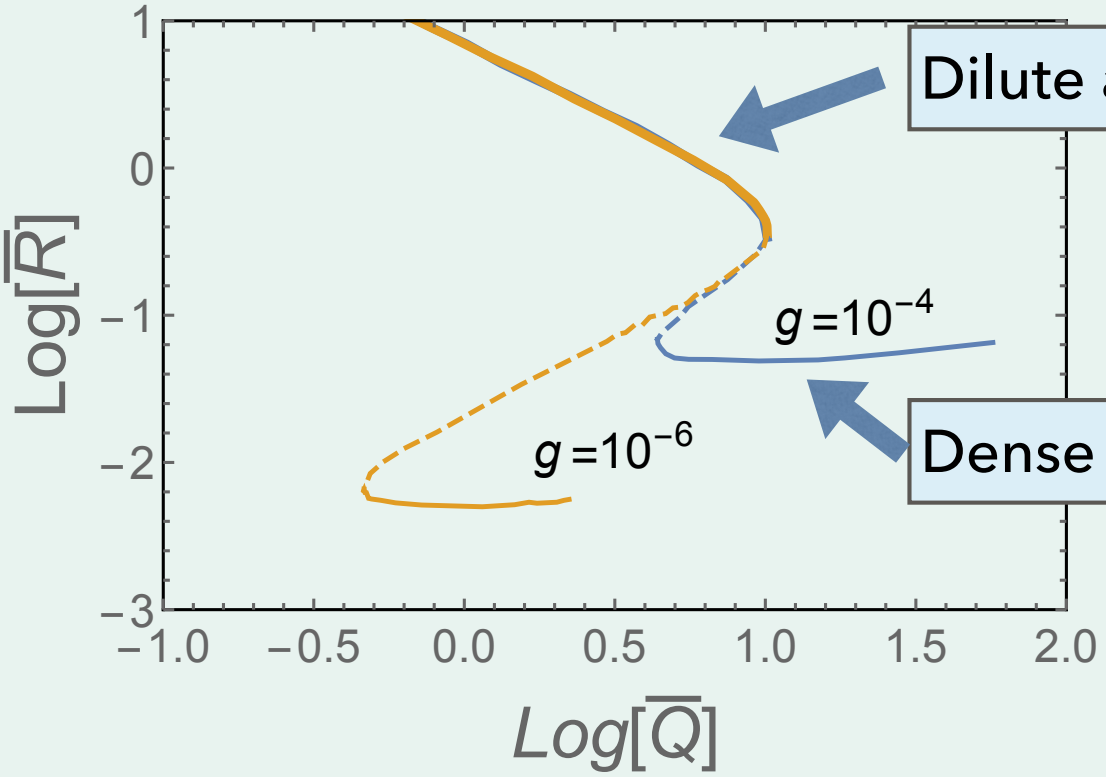
Axion star as a projection of Q-ball

- Numerical results for axion star



Dilute axion star

$$M \sim \left(\frac{M_P}{m}\right)^2 \frac{1}{R}$$

Dense axion star

$$M \sim \frac{m^2}{|\lambda_4|} R$$

$$g \equiv \frac{m_\phi^2}{M_{\rm Pl}^2 |\lambda_4|} = \left(\frac{f_a}{M_{\rm Pl}}\right)^2$$

See also Braaten, Mohapatra, Zhang 16, Schiappacasse, Hertzberg 17,

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Axion star as a projection of Q-ball

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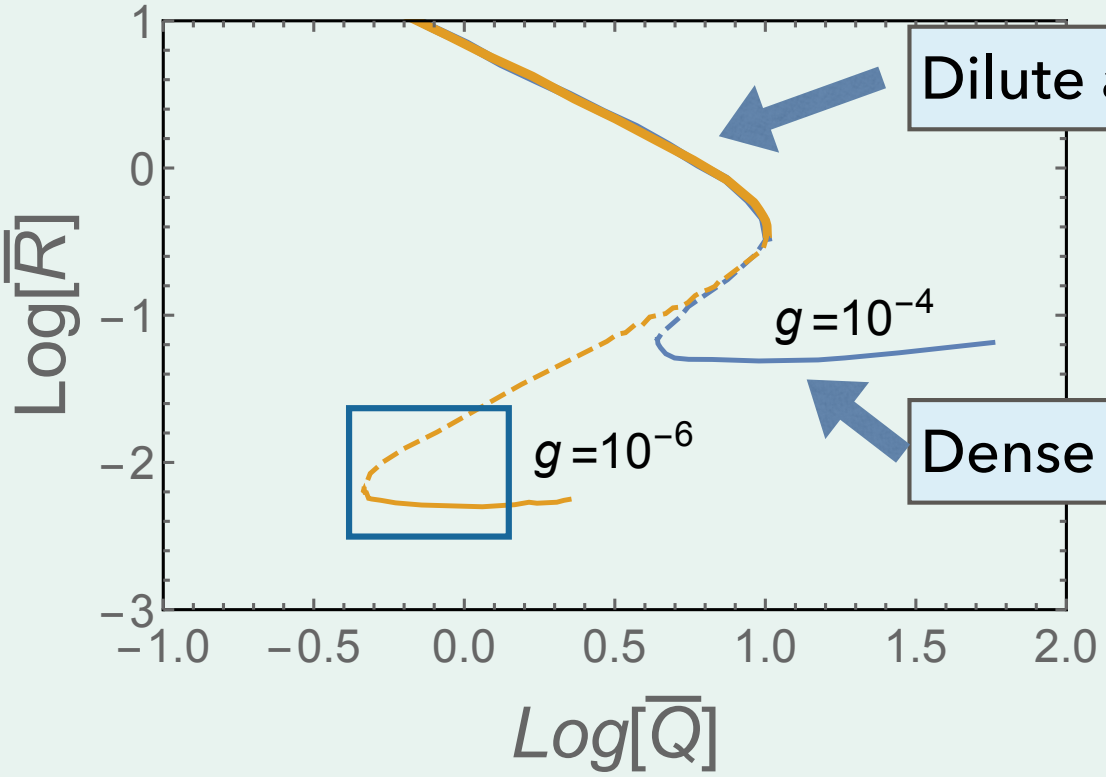

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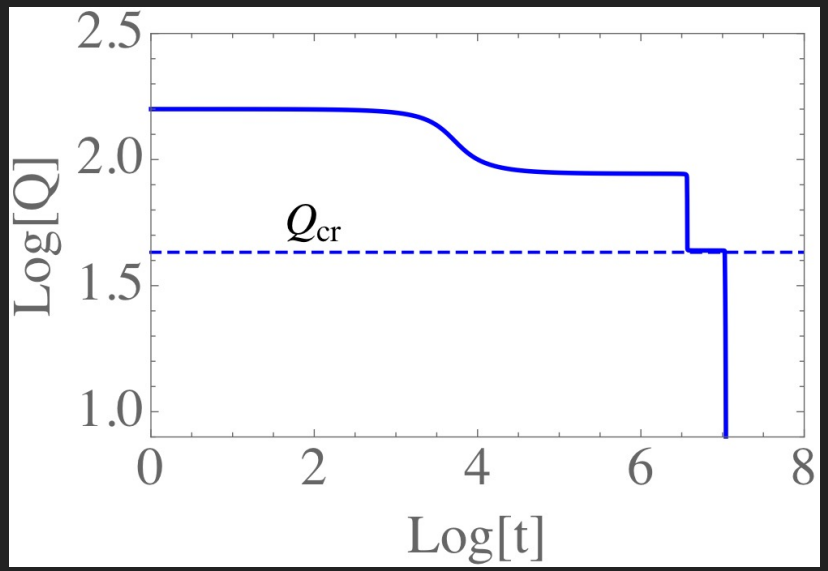
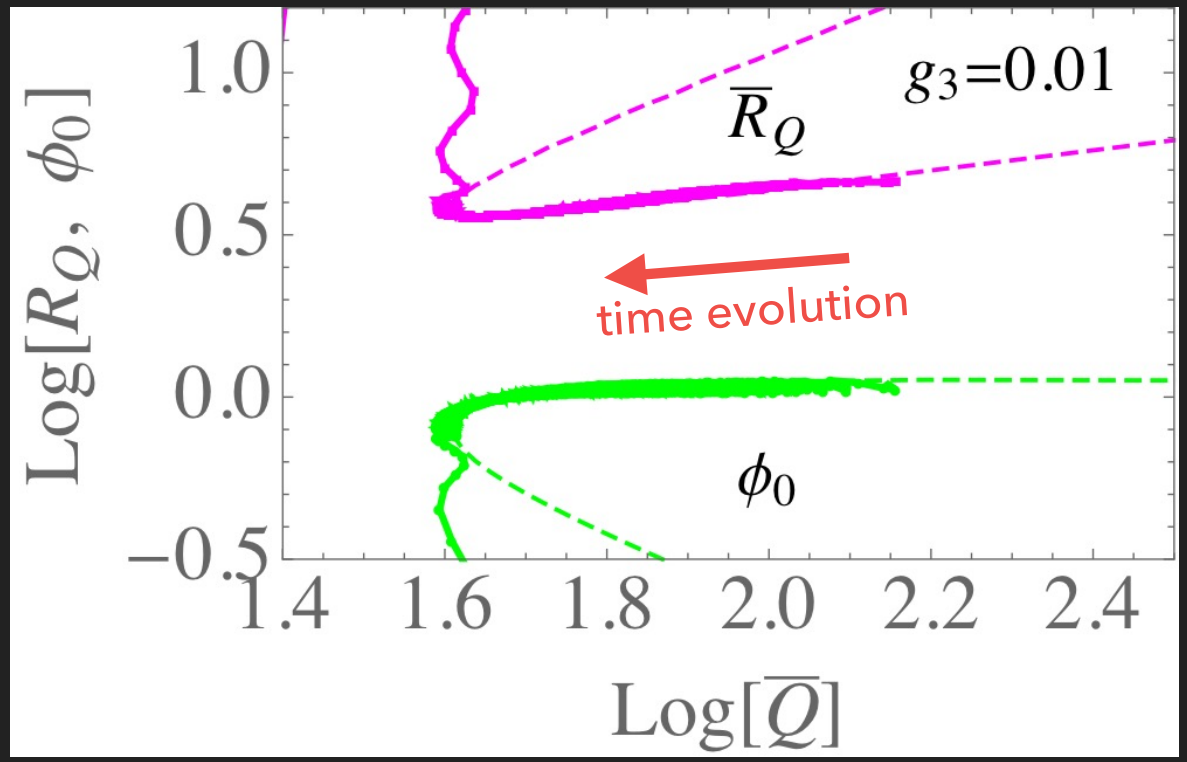
Time evolution of dense axion star

- We have checked that the result is consistent with full numerical simulations:





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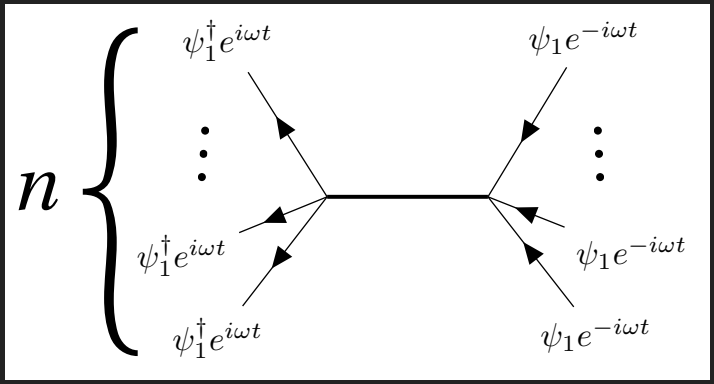
Decay rate of axion star

- Classical decay can be calculated from the imaginary part of EFT.



$$i\Gamma_{n\to 1} = -\pi i \frac{\lambda_{n+1}^2}{(n! m_\phi^{n-3})^2} \left|\overline{\psi}_1\right|^{2n} \int_{\boldsymbol{p}} |j_n(\boldsymbol{p})|^2 \, \delta\left(\boldsymbol{p}^2 - n^2\omega^2 + m_\phi^2\right),$$

$$j_n(\boldsymbol{p}) \equiv \int_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left[j(\boldsymbol{x})\right]^n$$

$$1/T_{\rm typ} \equiv \left[\frac{\dot{Q}}{m_\phi Q}\right]_{n\to 1} \simeq -\frac{4\pi^4}{n^2}\sqrt{\frac{n^2-1}{\pi}}\sqrt{\delta_{V_n}}\left(\frac{\delta_{V_n}}{\delta_x}\right)^{3/2} \exp\left(-\frac{n^2-1}{n\delta_x}\right).$$

Hertzberg 10,
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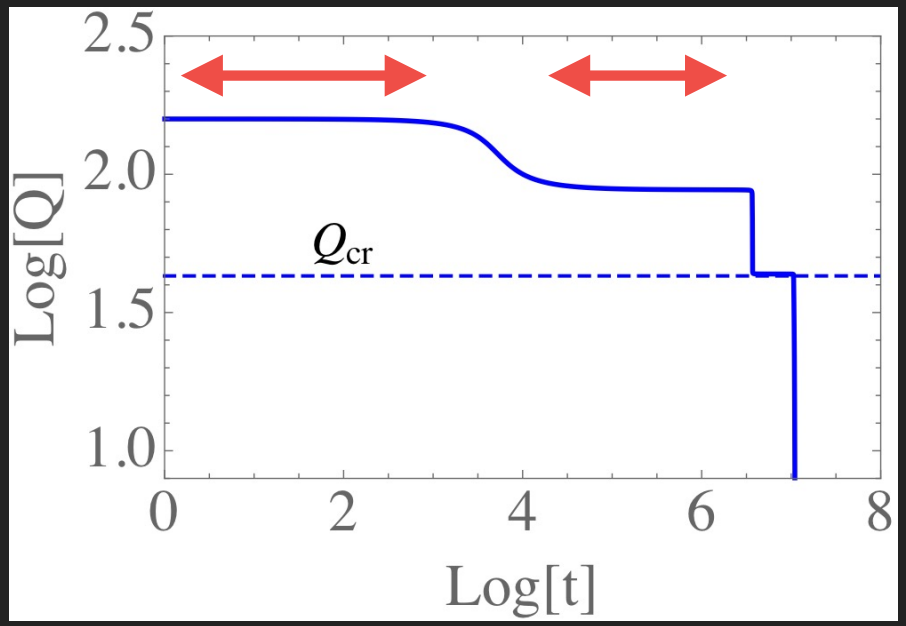
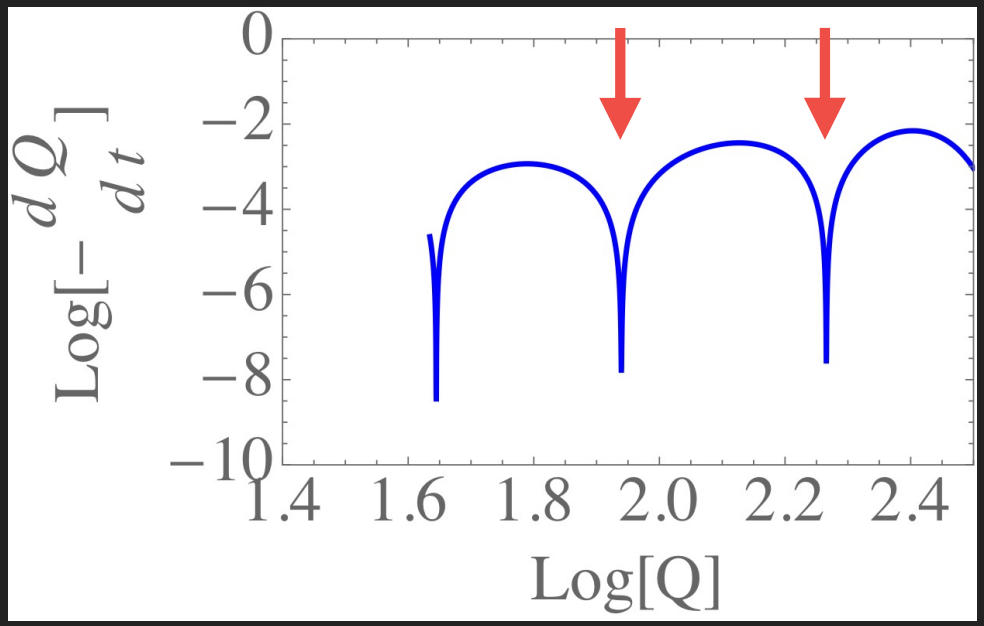
- There are resonances at several scales, where the lifetime is enhanced.





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Summary

- We provided a new scheme for NREFT to study axion stars.

Advantages in our EFT:

- Relativistic corrections are smaller
- Relativistic corrections can be calculated from Feynman diagrams
- It is easy to calculate the axion star configuration
- Reason of (approximate) stability is clear
- It is straightforward to include the effect of gravity
- Lifetime can be calculated from the imaginary part of EFT

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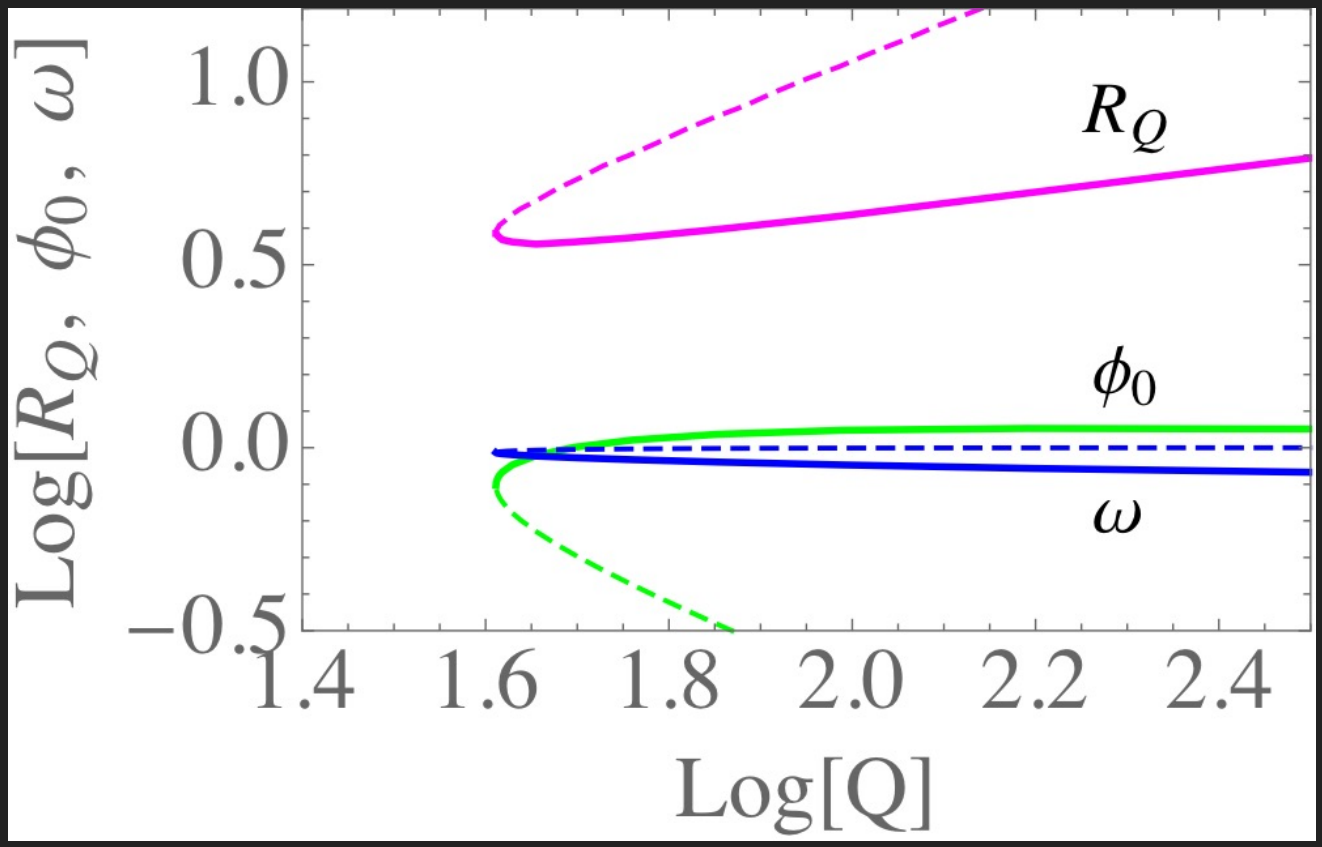
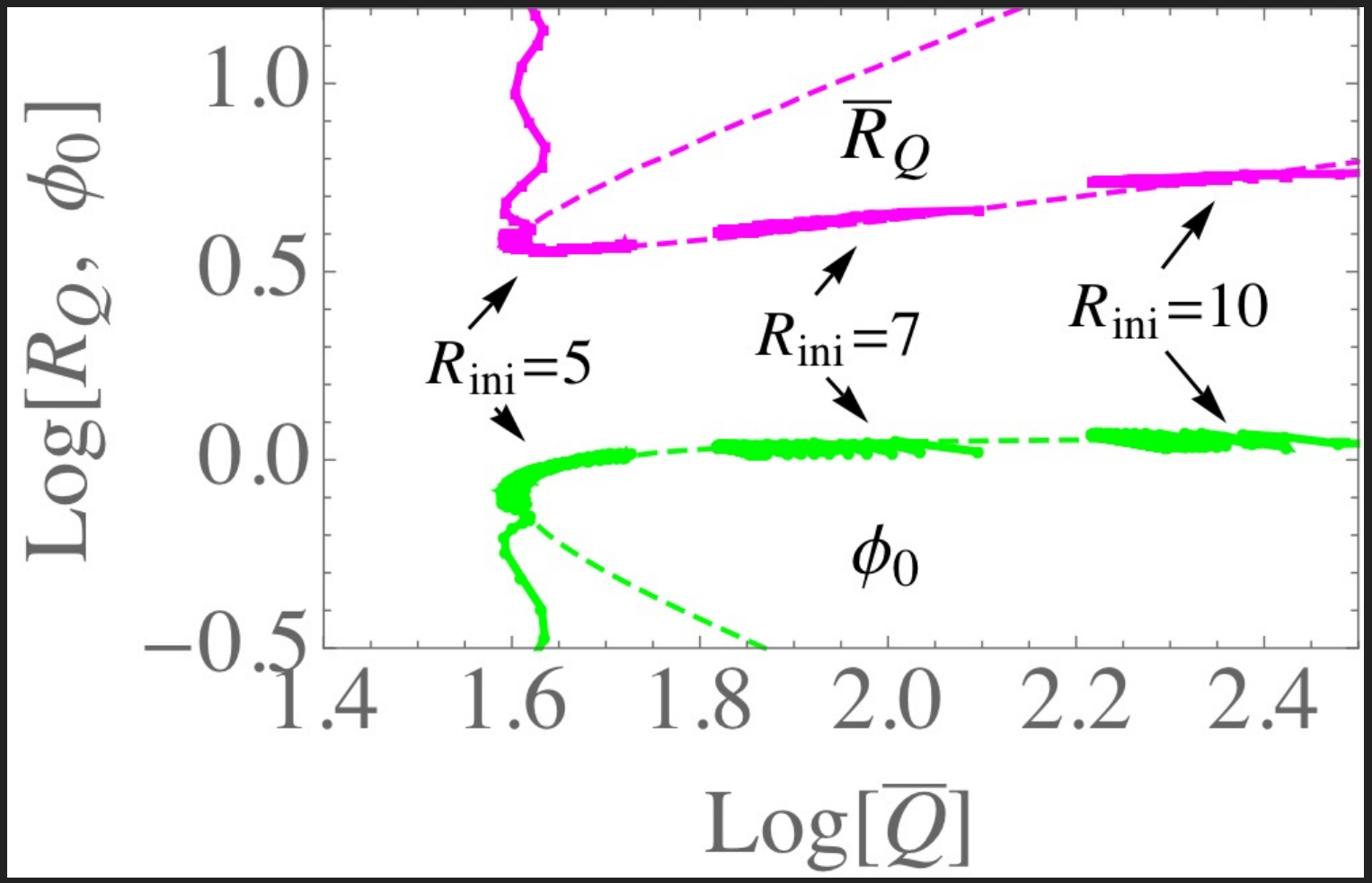
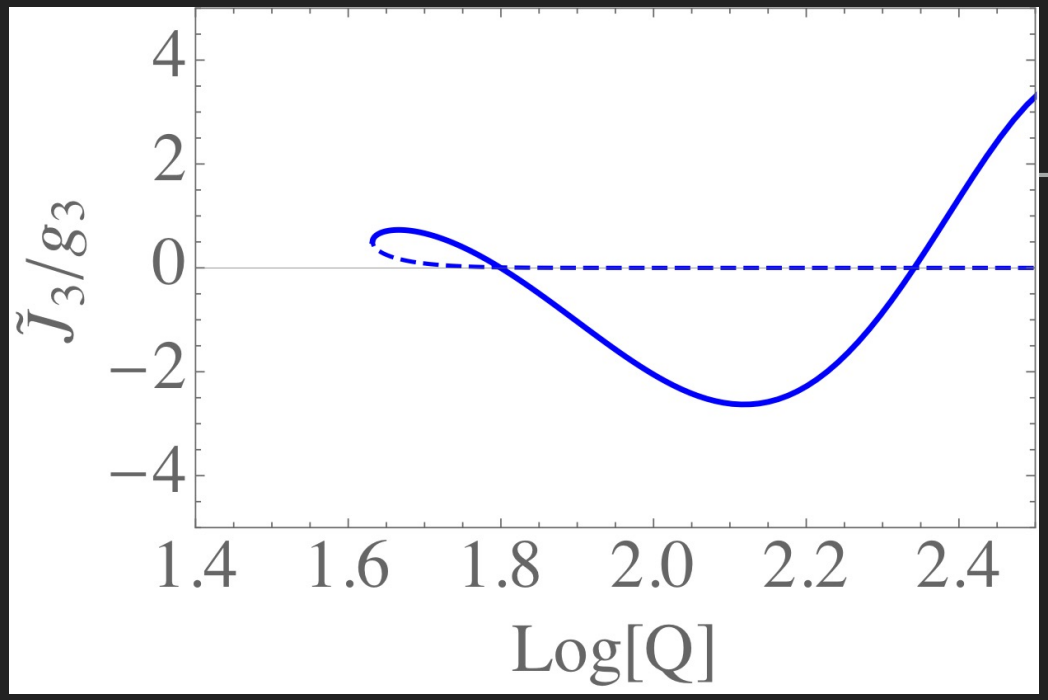
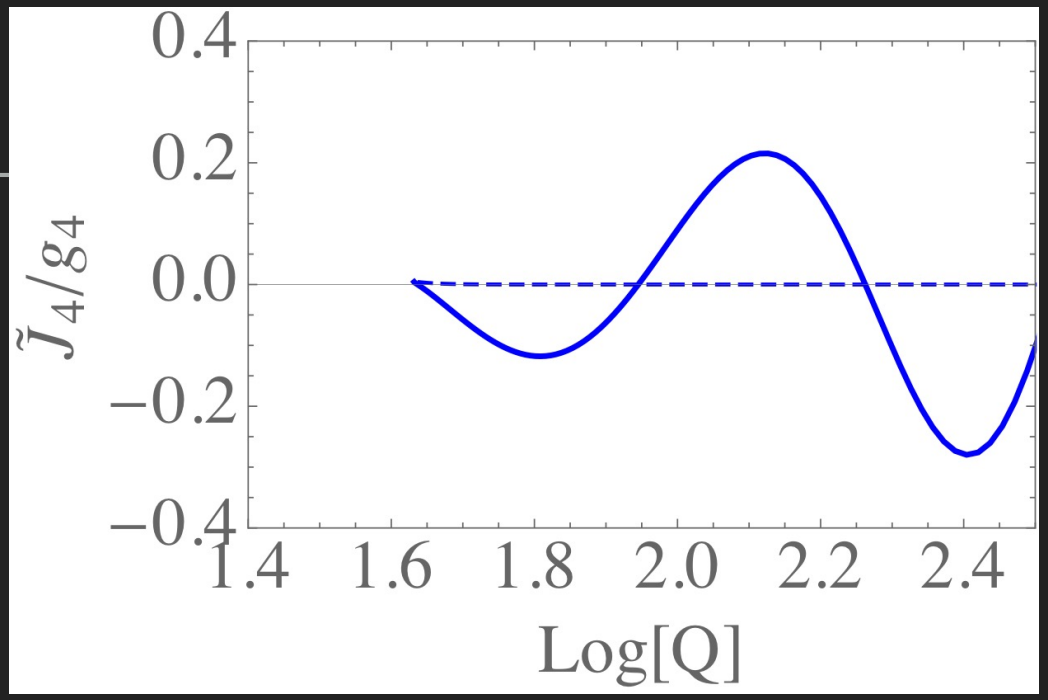
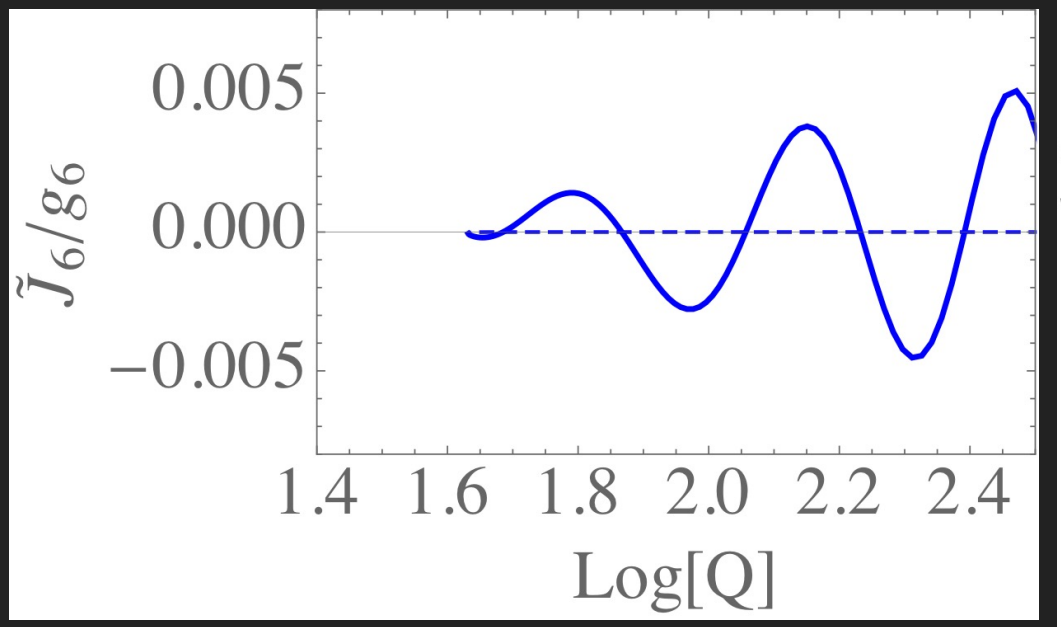
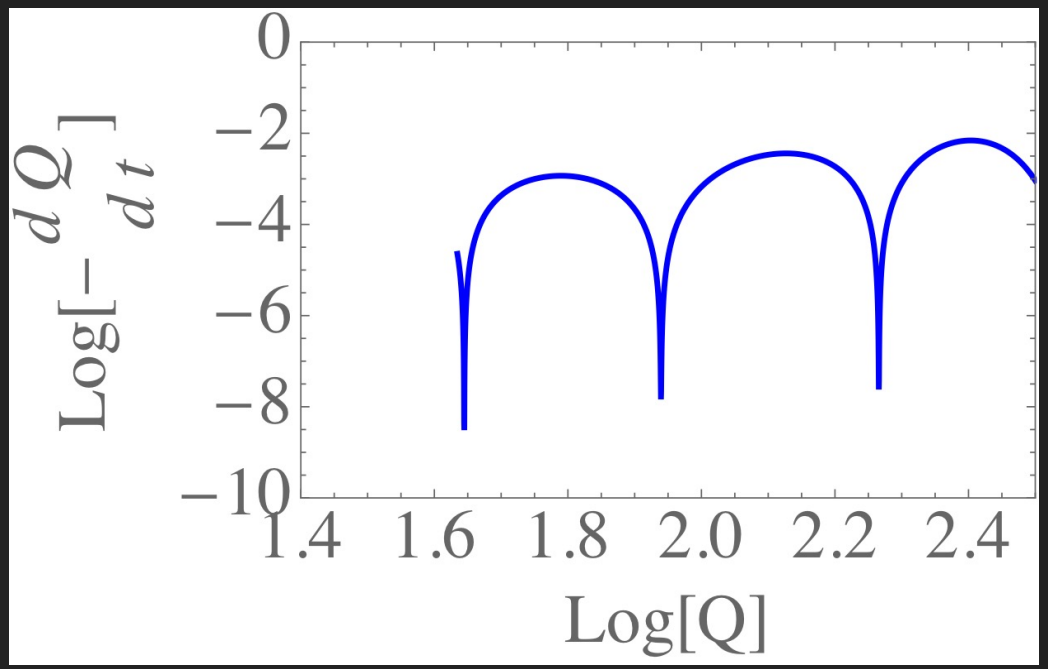

Log[R_Q, ϕ_0, ω]
1.0
0.5
0.0
−0.5
1.4
1.6
1.8
2.0
2.2
2.4
Log[Q]
R_Q
ϕ_0
ω

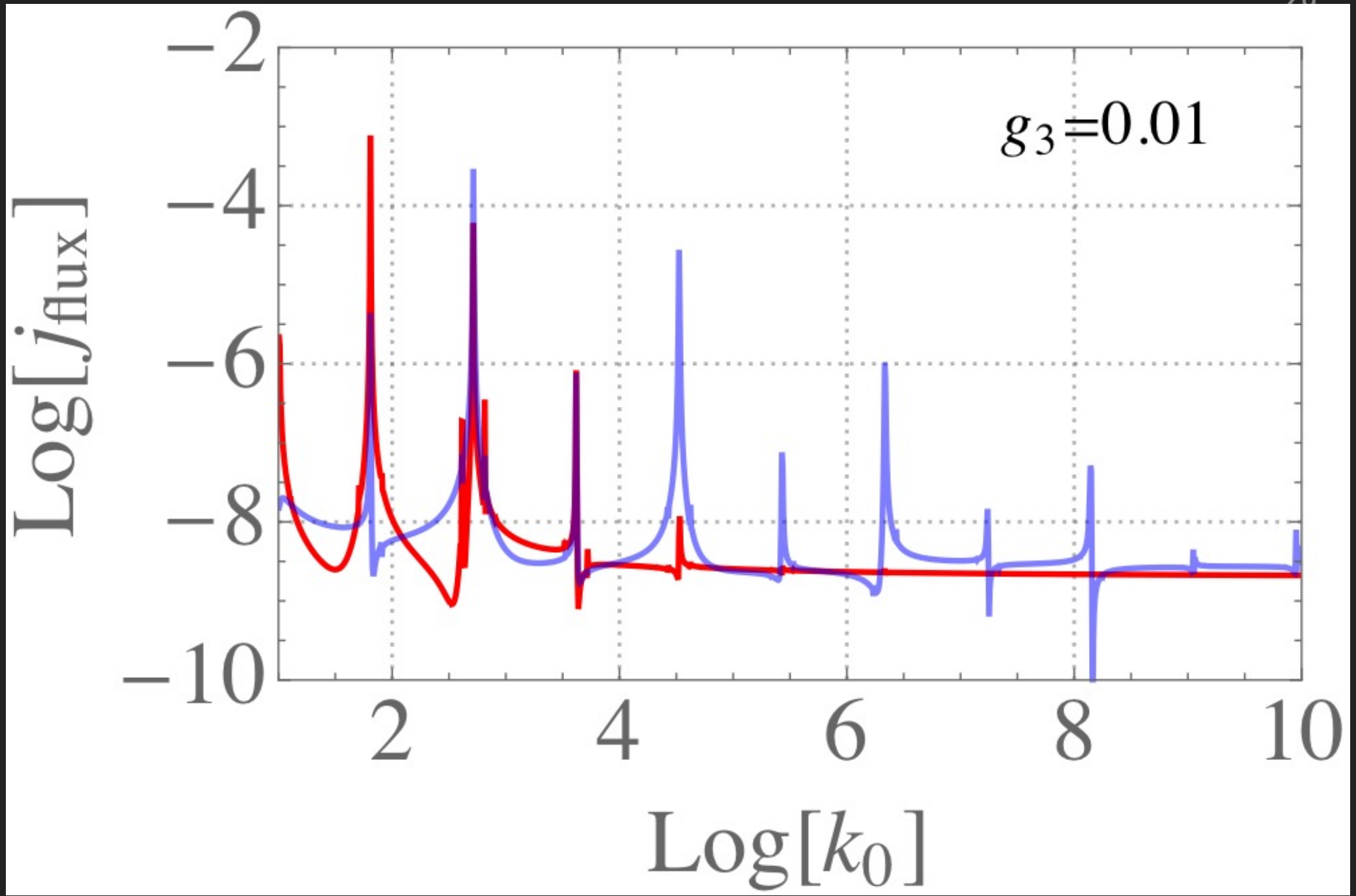

1.0
0.5
0.0
−0.5
1.4
1.6
1.8
2.0
2.2
2.4
Log[R_Q, ϕ_0]
Log[$\overline{Q}$]
$\overline{R}_Q$
ϕ_0
$R_{\rm ini}$=5
$R_{\rm ini}$=7
$R_{\rm ini}$=10

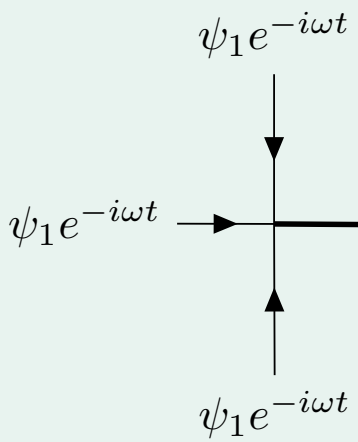

$\tilde{J}_3/g_3$
4
2
0
−2
−4
1.4
1.6
1.8
2.0
2.2
2.4
Log[Q]


$\tilde{J}_4/g_4$
0.4
0.2
0.0
−0.2
−0.4
1.4
1.6
1.8
2.0
2.2
2.4
Log[Q]

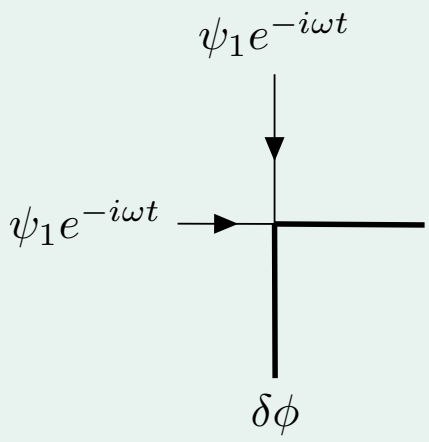

$\tilde{J}_6/g_6$
0.005
0.000
−0.005
1.4
1.6
1.8
2.0
2.2
2.4
Log[Q]


$\mathrm{Log}[-\frac{dQ}{dt}]$
0
−2
−4
−6
−8
−10
1.4
1.6
1.8
2.0
2.2
2.4
Log[Q]

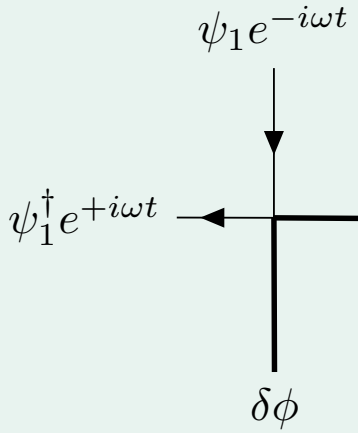

g_3=0.01
Log[j_{flux}]
−2
−4
−6
−8
−10
2
4
6
8
10
Log[k_0]



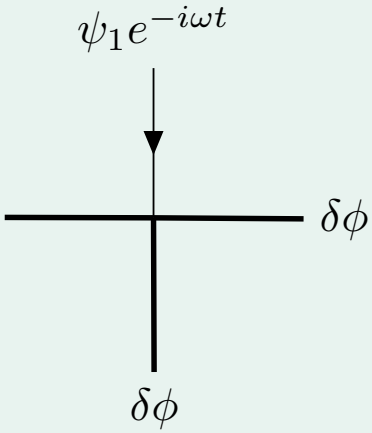
$$= \frac{\lambda_4}{3!}\psi_1^3 e^{-3i\omega t}\delta\phi,$$



$$= \frac{3\lambda_4}{2\cdot 3!}\psi_1^2 e^{-2i\omega t}\delta\phi^2,$$



$$= \frac{3\lambda_4}{2\cdot 3!}\left|\psi_1\right|^2\delta\phi^2,$$



$$= \frac{\lambda_4}{3!}\psi_1 e^{-i\omega t}\delta\phi^3,$$

$$M \sim \left(\frac{M_P}{m}\right)^2 \frac{1}{R}$$

$$R_c \sim \frac{\sqrt{|\lambda_4|}\, M_P}{m^2}$$

$$M \sim \frac{m^2}{|\lambda_4|}\, R$$

$$\psi = \frac{m_\phi^2}{M_{\rm Pl}\,|\lambda_4|}\bar{\phi},$$

$$r = \frac{\sqrt{|\lambda_4|}M_{\rm Pl}}{m_\phi^2}\bar{r},$$

$$Q = \frac{M_{\rm Pl}}{m_\phi\sqrt{|\lambda_4|}}\bar{Q},$$

$$g \equiv \frac{m_\phi^2}{M_{\rm Pl}^2\,|\lambda_4|} = \left(\frac{f_a}{M_{\rm Pl}}\right)^2$$