THE MASS RATIOS PARAMETRIZATION

ULISES SALDANA (TTP-KIT)
FLASY 2018, BASEL, SWITZERLAND

Karlsruher Institut für Technologie

Institut für Theoretische Teilchenphysik

US + K. Tame
arXiv:1804.04578

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Q: What do you hope people take away from your Doodle?

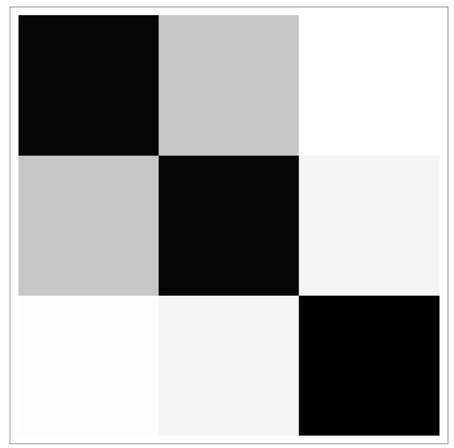
A: If this doodle doesn't boost the team into the final it will at least boost our sausage sales.

after a great conference dinner…

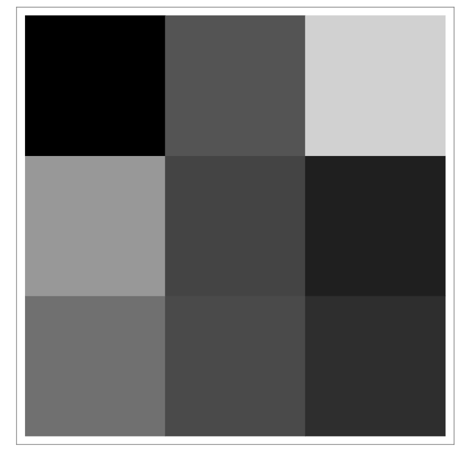
I had to rethink everything (a difficult task after drinking wine)…

How to make as clear as possible my selling point?

1\.



Quarks



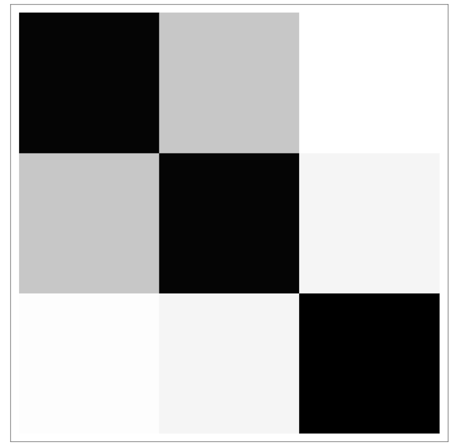
Leptons

2\.

$$\mathbf{V}_q = \mathbf{V}_q\left(\frac{m_u}{m_c}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_s}{m_b}\right)$$

$$\mathbf{U}_\ell = \mathbf{U}_\ell\left(\frac{m_e}{m_\mu}, \frac{m_\mu}{m_\tau}, \frac{m_{\nu l}}{m_{\nu 2}}, \frac{m_{\nu 2}}{m_{\nu h}}\right)$$

3\. The mass ratios parametrization



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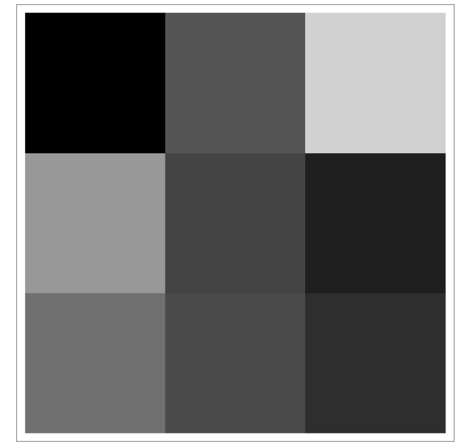


Leptons

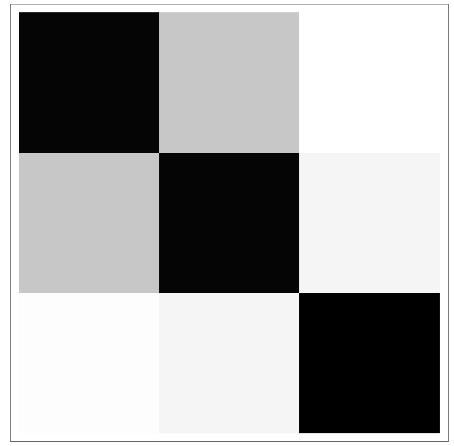
The mass ratios parametrization

"Philosophical"

What would it require?
(Conditions)
What would it mean?
(Implications)

Model building

Particular examples
e.g., texture zeros
flavour symmetries



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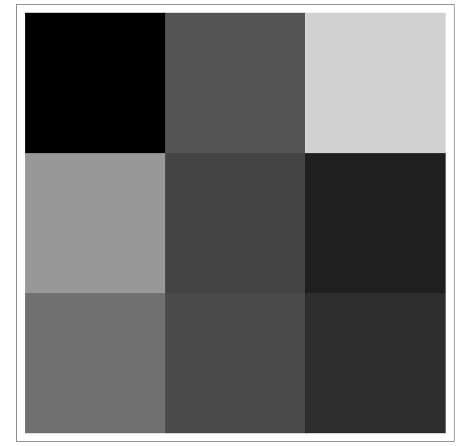

Leptons

The mass ratios parametrization

"Philosophical"

What would it require?
(Conditions)
What would it mean?
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Model building

Particular examples
e.g., texture zeros
flavour symmetries

4. Is it possible (the SM has too many arbitrary parameters)?
5. Can we understand Q&L mixing through the same explanation?

CONTENT

- the bottom line
- fermion mass ratios as mixing parameters?
- natural fermion masses?
- angles & mass ratios?
- an example
- conclusions

the bottom line

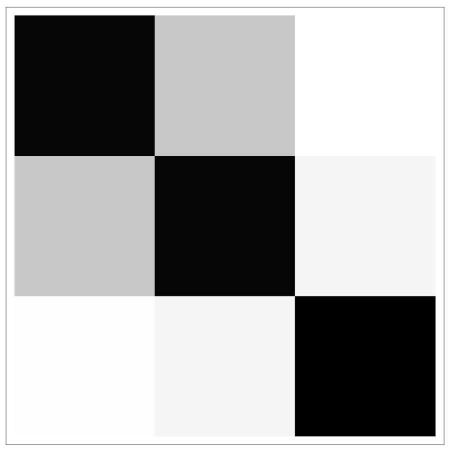
Ulises Saldana

Karlsruher Institut für Technologie

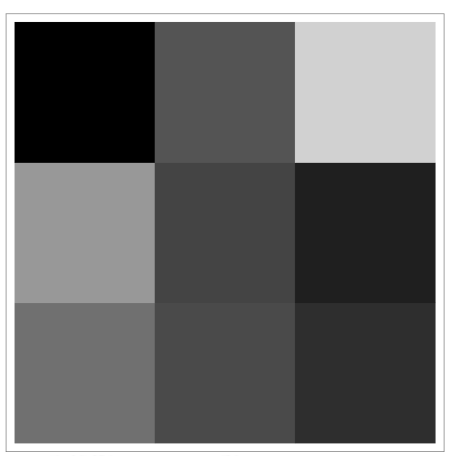
Institut für Theoretische Teilchenphysik

See talks by:
S. King, J. Valle, C. Nishi,
A. Yuu, P. Harrison, J. Penedo,
C. Hagedorn, Y. Shimizu, G. Ding,
A. Romanino, C. Kaur, R. Ziegler,
F. Ferruglio, M. Tanimoto, A. Titov, T. Tatsuishi

the bottom line



$Quarks$



$Leptons$

Why 3 fermion generations?

The Flavour Puzzle
(1968-2018)
50 Years Anniversary

See talks by:
A. Carcamo Hernandez
J. Leite

$$m_1 \ll m_2 \ll m_3$$

$$m_t \gg m_f$$

$$m_e \gg m_\nu$$

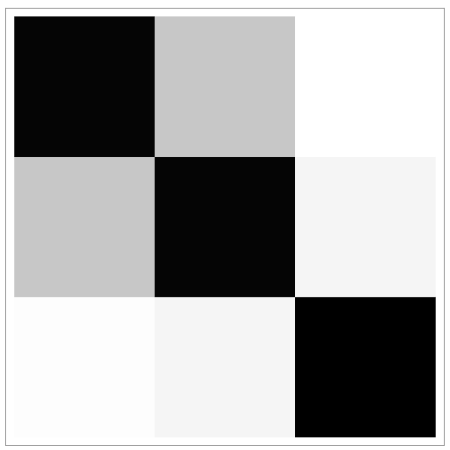
See talk by: R. Cepedello

Dirac or Majorana?

See talks by:
R. Srivastava, S. Centelles, C. Dib, F. Deppich

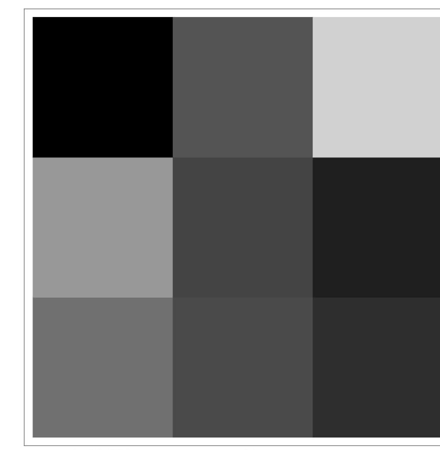
the bottom line

The problem of mixing,



Quarks

Why is the CKM so hierarchical and close to triviality?
Why do the quark mixing angles behave as 12 >> 23 >> 13?
Why is the Kobayashi-Maskawa phase the way it is?
Why lepton mixing is so different to quark mixing?
Why the PMNS matrix is so anarchical?
Why the 23 lepton mixing angle is almost maximal?
Why is the Dirac CP phase the way it is?
What about Majorana phases?

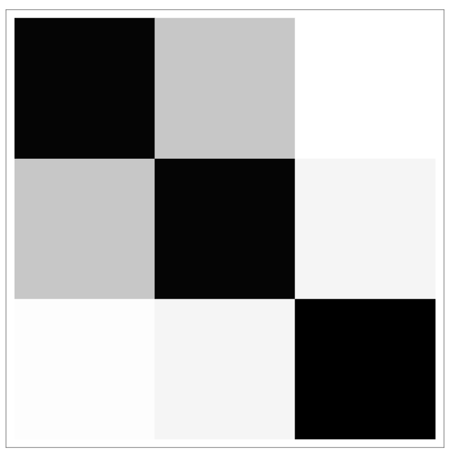


Leptons

Can both mixings be understood under the same explanation?
Do they share any similarities?
Yes, the 1-3 angle is the smallest one.
How to understand all of these?

the bottom line

The problem of mixing,



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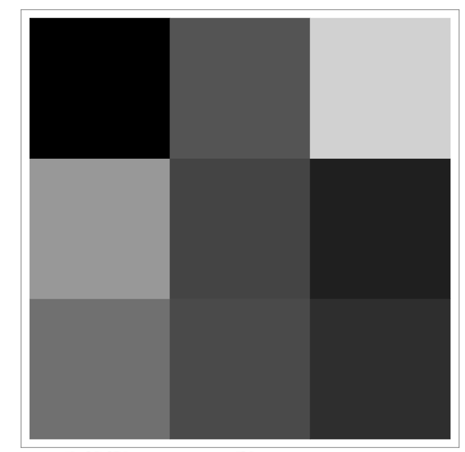

Leptons

the bottom line

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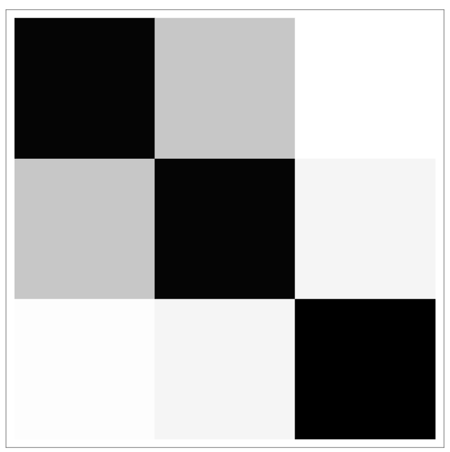

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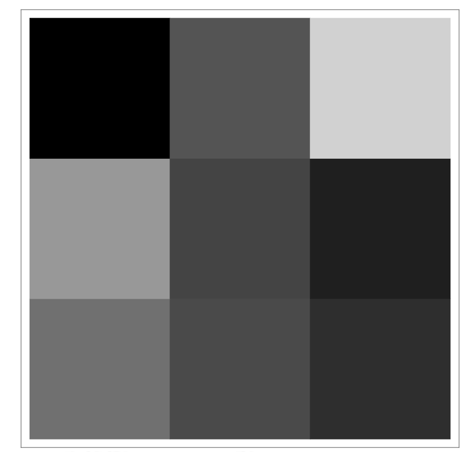


Leptons

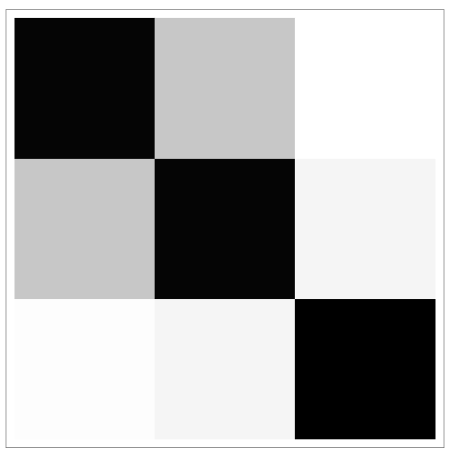
could it be solved if…

$$\mathbf{V}_q = \mathbf{V}_q\left(\frac{m_u}{m_c}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_s}{m_b}\right) \qquad \mathbf{U}_\ell = \mathbf{U}_\ell\left(\frac{m_e}{m_\mu}, \frac{m_\mu}{m_\tau}, \frac{m_{\nu l}}{m_{\nu 2}}, \frac{m_{\nu 2}}{m_{\nu h}}\right) \; ?$$

A new mixing parametrization

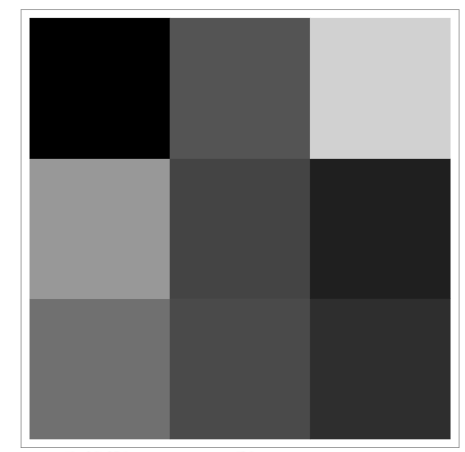
the bottom line

The problem of mixing,



Quarks

Can we do it?



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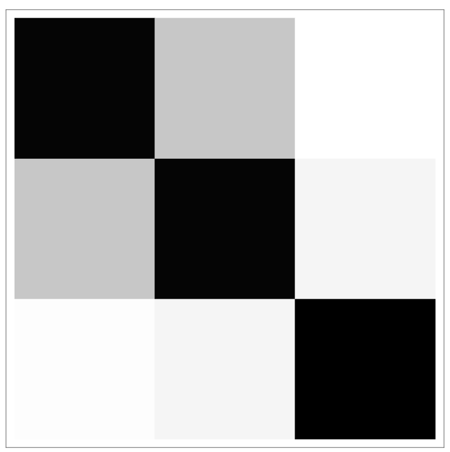
A new mixing parametrization

the bottom line

The problem of mixing,

W. G. Hollik + US, NPB892 (2015) 364
US + K. Tame, arXiv:1804.04578



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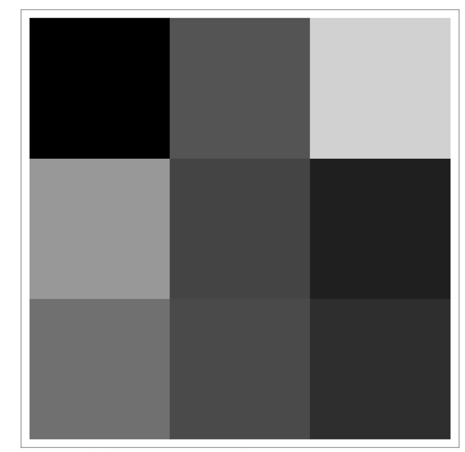

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A new mixing parametrization

fermion mass ratios as mixing parameters?

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Ulises Saldana

Institut für Theoretische Teilchenphysik

Physics Letters B

Volume 28, Issue 2, 11 November 1968, Pages 128-130

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Weak self-masses, Cabibbo angle, and broken $SU_2 \times SU_2$

R. Gatto *, G. Sartori, M. Tonin

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$$\tan^2 \theta_c = \frac{m_d}{m_s}$$

https://doi.org/10.1016/0370-2693(68)90150-0

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fermion mass ratios as mixing parameters?

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fermion mass ratios as mixing parameters?

W. G. Hollik + US, NPB892 (2015) 364

- Number of independent mass ratios

for example, $$\frac{m_1}{m_3} = \frac{m_1}{m_2}\frac{m_2}{m_3}$$ $$6 \to 4$$

- Always holds?

$$1 < n \leq 3$$

- Track its origin to the mass matrices: $$\mathbf{V} = \mathbf{L}_u \mathbf{L}_d^\dagger$$

-

fermion mass ratios as mixing parameters?

W. G. Hollik + US, NPB892 (2015) 364

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for example, $$\frac{m_1}{m_3} = \frac{m_1}{m_2}\frac{m_2}{m_3}$$ $6 \to 4$

- Always holds?

$$1 < n \leq 3$$

- $$\mathbf{V}_q\left(\frac{m_u}{m_c}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_s}{m_b}\right) = \mathbf{L}_u\left(\frac{m_u}{m_c}, \frac{m_c}{m_t}\right)\mathbf{L}_d^\dagger\left(\frac{m_d}{m_s}, \frac{m_s}{m_b}\right)$$

- $$\mathbf{L}_f\mathbf{M}_f\mathbf{M}_f^\dagger\mathbf{L}_f^\dagger = \mathbf{\Sigma}_f^2$$

fermion mass ratios as mixing parameters?

W. G. Hollik + US, NPB892 (2015) 364

- Numbe

$6 \to 4$

- Always

- $\mathbf{V}_q$

-

fermion mass ratios as mixing parameters?

fermion mass ratios as mixing parameters?

Too many spurious parameters!

natural fermion masses?

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[5] A. Antaramian, L.J. Hall and A. Rasin, Phys. Rev. Lett. **69** (1992) 1871; L.J. Hall and S. Weinberg, Phys. Rev. **D48** (1993) R979.

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Nuclear Physics B

Volume 420, Issue 3, 6 June 1994, Pages 468-504

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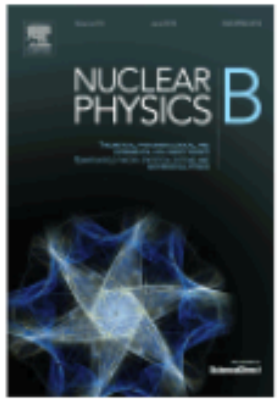

Mass matrix models: the sequel

Miriam Leurer [a], Yosef Nir [a], Nathan Seiberg [b]

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https://doi.org/10.1016/0550-3213(94)90074-4

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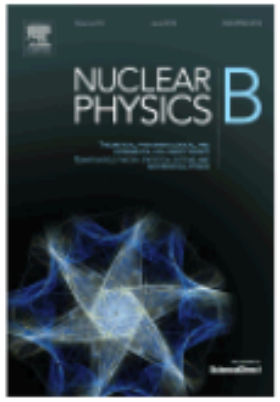

Mass matrix models: the sequel

Miriam Leurer [a], Yosef Nir [a], Nathan Seiberg [b]

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content

As articulated by 'tHooft [4], small numbers are natural only if an exact symmetry is acquired when they are set to zero ("naturalness"). Therefore, both the smallness of the quark sector parameters and the hierarchy among them may be related to a symmetry – a horizontal symmetry $\mathcal{H}$ that acts on the quarks (for recent discussions, see e.g. [5]). Such

natural fermion masses?

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natural fermion masses?

US + K. Tame, arXiv:1804.04578

Dine, 1501.01035
Giudice, 1710.07663

In the weak interaction basis,

$$\mathbf{Y}_f = 0 \qquad f = u, d, e$$

$$\mathcal{G}_F = U_L^Q(3) \times U_R^u(3) \times U_R^d(3) \times U_L^E(3) \times U_R^e(3)$$

$$m_f \ll \Lambda_F \sim \mathcal{O}(1 \text{ TeV})$$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

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$$m_f \ll \Lambda_F \sim \mathcal{O}(1\ \text{TeV})$$

But, what about…?

$$m_1 \ll m_2 \ll m_3$$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

$$m_1 \ll m_2 \ll m_3$$

Could we also say the lightest generations are natural compare to the heaviest one?

$$m_1, m_2 \ll m_3$$

And what about the first two generations?

$$m_1 \ll m_2$$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

$$m_1^2 \ll m_2^2 \ll m_3^2$$

Weak Basis		Mass Basis
$\mathbf{M}\mathbf{M}^\dagger = \mathbf{L}^\dagger \mathbf{\Sigma}^2 \mathbf{L}$	$m_1, m_2 \to 0$	$\mathbf{W} \neq 1$ $U_R^a(2) \times U_R^b(2)$
	$m_1 \to 0$	$\mathbf{W} \neq 1$ $U_R^a(1) \times U_R^b(1)$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

$$m_1^2 \ll m_2^2 \ll m_3^2$$

Weak Basis

Mass Basis

$$\mathbf{M}\mathbf{M}^\dagger = \mathbf{L}^\dagger \mathbf{\Sigma}^2 \mathbf{L}$$

$$m_1, m_2 \to 0$$

$$\mathbf{W} \neq 1$$

$$\boxed{U_R^a(2) \times U_R^b(2)}$$

What about the left handed parts?

$$\mathbf{W} \neq 1$$

$$m_1 \to 0$$

$$\boxed{U_R^a(1) \times U_R^b(1)}$$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

Mass Basis

$$m_1^2 \ll m_2^2 \ll m_3^2$$

$$\mathbf{W} = \mathbf{W}\left(\frac{m_1^a}{m_3^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_3^b}, \frac{m_2^b}{m_3^b}\right)$$

?

Barbieri et al, 1203.4218
Blankenburg et al, 1204.0688
Barbieri et al, 1206.1327
Buras et al, 1206.3878
Barbieri et al, 1512.01560
...

Limits:

$m_1, m_2 \to 0$ $\quad \mathbf{W}(0,0,0,0) = 1 \quad U^3(2)$

$m_1 \to 0$ $\quad \mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23} \quad U^3(1)$

$m_3 \to \infty$ $\quad \mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

$$m_u^2 :: m_c^2 :: m_t^2 = 10^{-10} :: 10^{-6} :: 1$$
$$m_d^2 :: m_s^2 :: m_b^2 = 10^{-6} :: 10^{-4} :: 1$$

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$$\mathbf{V}_{CKM} \approx 1 + \mathbf{V}_{12}(\frac{m_d}{m_s})$$

Limits:

$$m_1, m_2 \to 0 \quad \Rightarrow \quad \mathbf{W}\,(0,0,0,0) = 1$$

$$m_1 \to 0 \qquad \mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$

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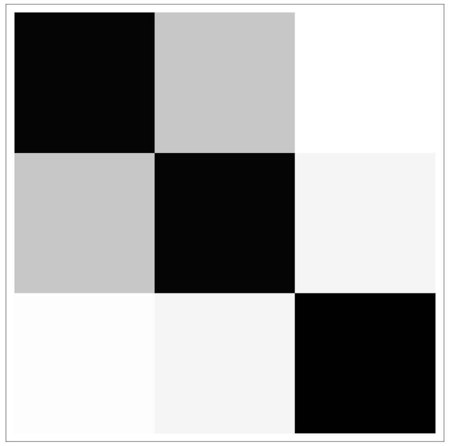

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natural fermion masses?

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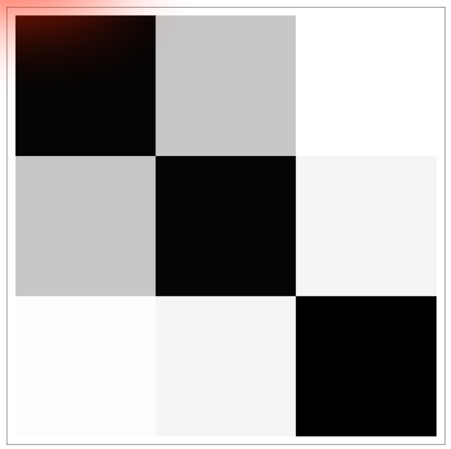

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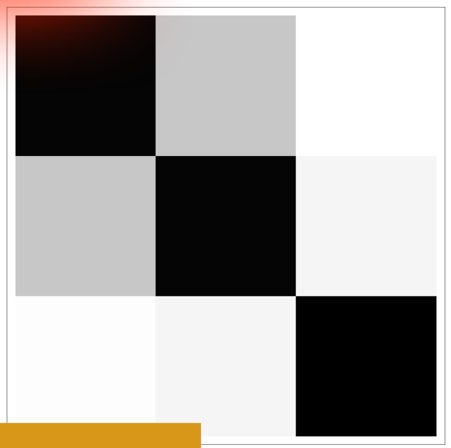


The approach is correctly helping us to understand!

$$m_3 \to \infty \quad \mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$$

natural fermion masses?

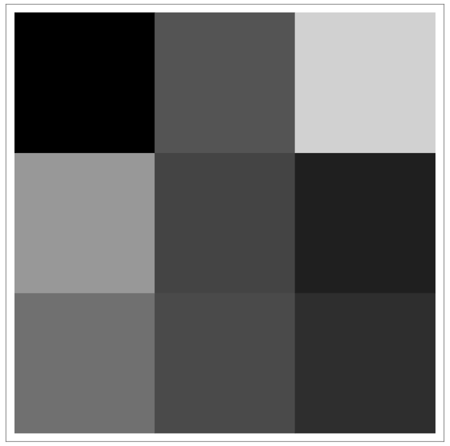
US + K. Tame, arXiv:1804.04578

$$m_e^2 :: m_\mu^2 :: m_\tau^2 = 10^{-6} :: 10^{-2} :: 1$$

Limits:

$$m_1, m_2 \to 0 \qquad \mathbf{W}\,(0,0,0,0) = 1$$

$$m_1 \to 0 \qquad \mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$



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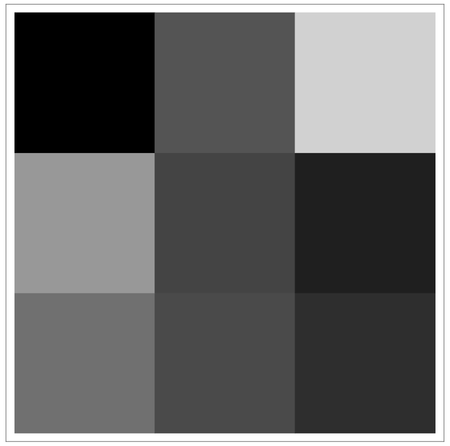

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natural fermion masses?

US + K. Tame, arXiv:1804.04578

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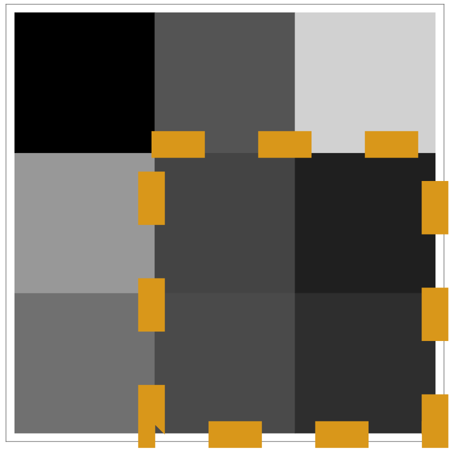

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natural fermion masses?

US + K. Tame, arXiv:1804.04578

Antusch et al, 0910.5127
Dorame et al, 1203.0155
Gehrlein et al, 1704.02371

$$m_e^2 :: m_\mu^2 :: m_\tau^2 = 10^{-6} :: 10^{-2} :: 1$$

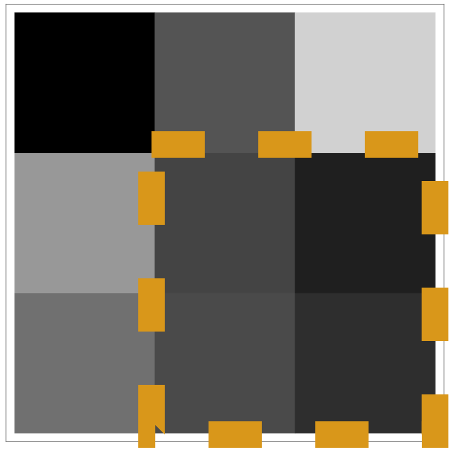
Mixing sum rules

$$\tan\Theta_{12}^\nu \simeq \tan\theta_{12}^{\rm PMNS}\,, \quad \sin\Theta_{13}^\nu \simeq \sin\theta_{13}^{\rm PMNS}\,, \quad \frac{-\Theta_{23}^\ell + \tan\Theta_{23}^\nu}{1+\Theta_{23}^\ell \tan\Theta_{23}^\nu} \simeq \tan\theta_{23}^{\rm PMNS}$$

Limits:

$m_1, m_2 \to 0$ $\qquad \mathbf{W}\,(0,0,0,0) = 1$

$m_1 \to 0$ $\qquad \mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$



$m_3 \to \infty$ $\qquad \mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

See Talk: C. Nishi

$\mu - \tau$ **reflection symmetry** Harrison, Scott, 0210197

$$\tan\Theta_{23}^{\nu} = 1$$

$$\frac{-\Theta_{23}^{\ell} + \tan\Theta_{23}^{\nu}}{1 + \Theta_{23}^{\ell}\tan\Theta_{23}^{\nu}} \simeq \tan\theta_{23}^{\mathrm{PMNS}}$$

$$\Theta_{23}^{\ell} \simeq 0.059$$

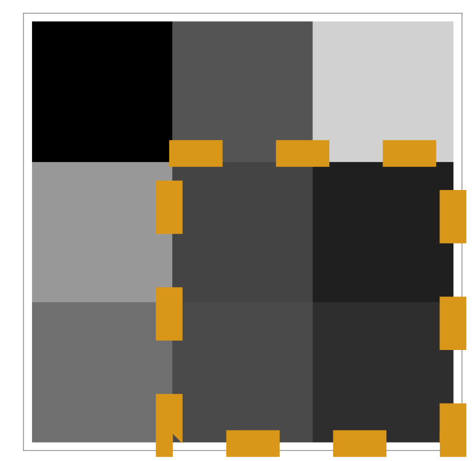
$$\Theta_{23}^{\ell} \simeq \frac{m_\mu}{m_\tau}$$

A first hint?

Limits:

$m_1, m_2 \to 0$ $\qquad$ $$\mathbf{W}\,(0,0,0,0) = 1$$

$m_1 \to 0$ $\qquad$ $$\mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$



$m_3 \to \infty$ $\qquad$ $$\mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$$

natural fermion masses?

US + K. Tame, arXiv:1804.04578

See Talk: C. Nishi

$\mu - \tau$ **reflection symmetry** Harrison, Scott, 0210197

$$\tan\Theta^{\nu}_{23} = 1$$

$$\frac{-\Theta^{\ell}_{23} + \tan\Theta^{\nu}_{23}}{1 + \Theta^{\ell}_{23}\tan\Theta^{\nu}_{23}} \simeq \tan\theta^{\rm PMNS}_{23}$$

$$\Theta^{\ell}_{23} \simeq 0.059$$

A first hint?

$$\Theta^{\ell}_{23} \sim \frac{m_\mu}{\ldots}$$

With so much arbitrariness…
How to unveil the relation to the masses?

Limits:

$$m_1, m_2 \to 0 \qquad \mathbf{W}(0,0,0,0) = 1$$

$$m_1 \to 0 \qquad \mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$

$$m_3 \to \infty \qquad \mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$$

angles & mass ratios?

Karlsruher Institut für Technologie

Ulises Saldana

Institut für Theoretische Teilchenphysik

angles & mass ratios?

W. G. Hollik + US, NPB892 (2015) 364
US + K. Tame, arXiv:1804.04578

$$m_1^2 \ll m_2^2 \ll m_3^2$$

$$\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \xrightarrow{m_3} \begin{pmatrix}0&0&0\\0&0&0\\0&0&\blacksquare\end{pmatrix} \xrightarrow{m_2} \begin{pmatrix}0&0&0\\0&\blacksquare&\blacksquare\\0&\blacksquare&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}_{23}]{} \begin{pmatrix}0&0&0\\0&\blacksquare&0\\0&0&\blacksquare\end{pmatrix} \xrightarrow{m_1}$$

$$\begin{pmatrix}\blacksquare&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}'_{23}]{} \begin{pmatrix}\blacksquare&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&0\\\blacksquare&0&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}_{13}]{} \begin{pmatrix}\blacksquare&\blacksquare&0\\\blacksquare&\blacksquare&0\\0&0&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}_{12}]{} \begin{pmatrix}\blacksquare&0&0\\0&\blacksquare&0\\0&0&\blacksquare\end{pmatrix}$$

$$\mathbf{L}_f = \mathbf{L}_{12}\mathbf{L}_{13}\mathbf{L}_{23}$$

angles & mass ratios?

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US + K. Tame, arXiv:1804.04578

$$m_1^2 \ll m_2^2 \ll m_3^2$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{m_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \blacksquare \end{pmatrix} \xrightarrow{m_2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare \end{pmatrix} \xrightarrow[\mathbf{L}_{23}]{} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \blacksquare & 0 \\ 0 & 0 & \blacksquare \end{pmatrix} \xrightarrow{m_1}$$

$$\begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \xrightarrow[\mathbf{L}'_{23}]{} \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & 0 \\ \blacksquare & 0 & \blacksquare \end{pmatrix} \xrightarrow[\mathbf{L}_{13}]{} \begin{pmatrix} \blacksquare & \blacksquare & 0 \\ \blacksquare & \blacksquare & 0 \\ 0 & 0 & \blacksquare \end{pmatrix} \xrightarrow[\mathbf{L}_{12}]{} \begin{pmatrix} \blacksquare & 0 & 0 \\ 0 & \blacksquare & 0 \\ 0 & 0 & \blacksquare \end{pmatrix}$$

$$\mathbf{L}_f = \mathbf{L}_{12}\mathbf{L}_{13}\mathbf{L}_{23}$$

angles & mass ratios?

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US + K. Tame, arXiv:1804.04578

Two fermion generations

$$\mathbf{m}\mathbf{m}^\dagger = \mathbf{L}^\dagger \Sigma^2 \mathbf{L}$$

Arbitrary

Known behaviour

$$m_1 \to 0$$
$$\mathbf{L}(0) = 1$$

$$m_1^2 \ll m_2^2$$
$$\mathbf{L}\left(\frac{m_1}{m_2}\right) \approx \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}$$

angles & mass ratios?

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Two fermion generations

$$\mathbf{m}\mathbf{m}^\dagger = \mathbf{L}^\dagger \Sigma^2 \mathbf{L}$$

Arbitrary → $\mathbf{m}\mathbf{m}^\dagger$

Known behaviour → $\mathbf{L}$

$$m_1 \to 0$$
$$\mathbf{L}(0) = 1$$

$$m_1^2 \ll m_2^2$$
$$\mathbf{L}(\frac{m_1}{m_2}) \approx \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}$$

$$\mathbf{m}\mathbf{m}^\dagger \approx m_2^2 \begin{pmatrix} \theta^2 + \frac{m_1^2}{m_2^2} & \theta(\frac{m_1^2}{m_2^2} - 1) \\ \theta(\frac{m_1^2}{m_2^2} - 1) & 1 + \theta^2 \frac{m_1^2}{m_2^2} \end{pmatrix}$$

angles & mass ratios?

US + K. Tame, arXiv:1804.04578

Two fermion generations

$$m_1 \to 0 \qquad \mathbf{L} = 1$$

$$m_2 \to \infty$$

$$\theta \sim (\frac{m_1}{m_2})^n$$

$$\mathbf{mm}^\dagger \approx m_2^2 \begin{pmatrix} \theta^2 + \frac{m_1^2}{m_2^2} & \theta(\frac{m_1^2}{m_2^2} - 1) \\ \theta(\frac{m_1^2}{m_2^2} - 1) & 1 + \theta^2 \frac{m_1^2}{m_2^2} \end{pmatrix}$$

angles & mass ratios?

US + K. Tame, arXiv:1804.04578

Two fermion generations

$$m_1 \to 0 \qquad \mathbf{L} = 1$$
$$m_2 \to \infty$$

$$\theta \sim (\frac{m_1}{m_2})^n$$

$$\frac{\mathbf{m}\mathbf{m}^\dagger}{m_2^2} \sim \begin{pmatrix} \theta & \theta \\ \theta & 1 \end{pmatrix} \quad \begin{pmatrix} \theta^2 & \theta \\ \theta & 1 \end{pmatrix}$$

Different kinds of structures
all consistent with the hierarchy!!

$$\theta \sim \frac{m_1}{m_2} \qquad \theta \sim (\frac{m_1}{m_2})^{1/n}$$

angles & mass ratios?

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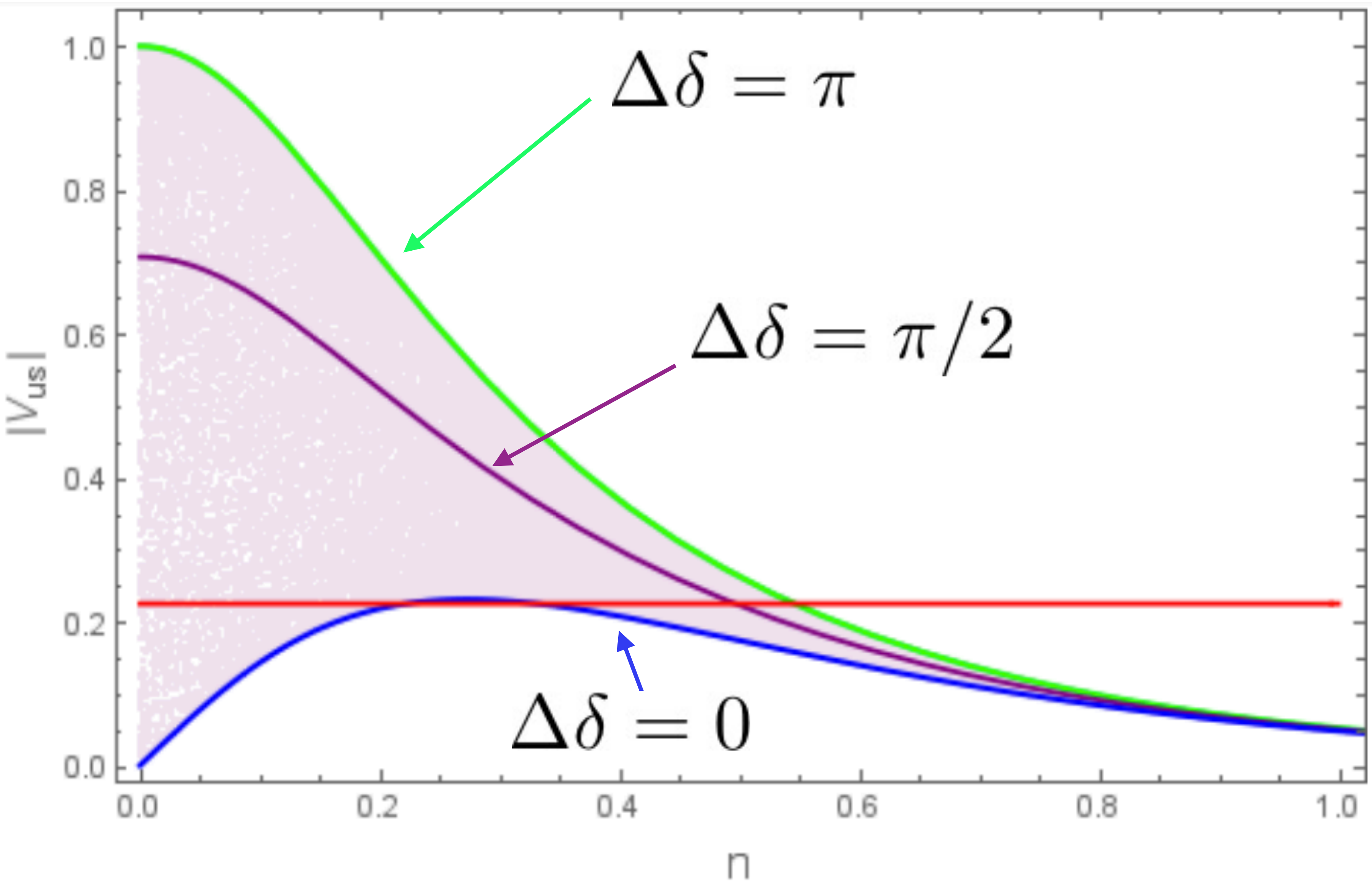
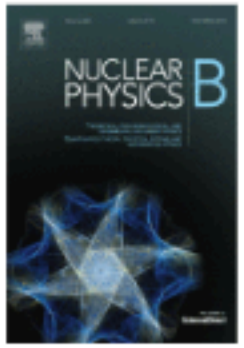


$$|V_{us}| = \sqrt{\frac{\left(\frac{m_u}{m_c}\right)^{2n} + \left(\frac{m_d}{m_s}\right)^{2n} - 2\left(\frac{m_u}{m_c}\right)^{n}\left(\frac{m_d}{m_s}\right)^{n}\cos(\delta_u - \delta_d)}{\left(1+\left(\frac{m_u}{m_c}\right)^{2n}\right)\left(1+\left(\frac{m_d}{m_s}\right)^{2n}\right)}}$$



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The double mass hierarchy pattern: Simultaneously understanding quark and lepton mixing

Wolfgang Gregor Hollik [a], Ulises Jesús Saldaña Salazar [a, b]

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an example

an example

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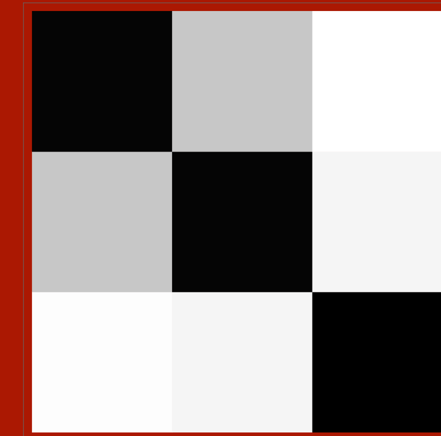

$$\delta_{\rm CP}^{\rm q,th} \approx \arctan\left[\sqrt{\frac{\frac{m_d}{m_s}(1+\frac{m_d}{m_s})}{\frac{m_u}{m_c}(1+\frac{m_u}{m_c})}}\right],$$

$$\frac{m_d}{m_s} \ll \frac{m_u}{m_c}$$

$$\sin\theta_{12}^{\rm q,th} \approx \sqrt{\frac{m_d/m_s + m_u/m_c}{(1+m_d/m_s)(1+m_u/m_c)}},$$

$$\sin\theta_{23}^{\rm q,th} \approx -\left(\sqrt{\frac{m_u}{m_t}} + \sqrt{\frac{m_d}{m_b}} + \sqrt{\frac{m_c}{m_t}} - \sqrt{\frac{m_s}{m_b}}\right),$$

$$\sin\theta_{13}^{\rm q,th} \approx \sin\theta_{23}^{\rm q,th}\sqrt{\frac{m_u}{m_c}} - \left(\sqrt{\frac{m_u}{m_t}} + \frac{m_c}{m_t} - \sqrt{\frac{m_d}{m_b}} + \sqrt{\frac{m_d m_s}{m_b^2}} + \frac{m_s}{m_b}\right).$$

$$\sin\theta_{12}^{\rm q,th} = 0.22 \pm 0.01\,, \qquad \sin\theta_{13}^{\rm q,th} = 0.003 \pm 0.001\,,$$

$$\sin\theta_{23}^{\rm q,th} = 0.039 \pm 0.003\,, \qquad \delta_{\rm CP}^{\rm q,th} = (62^{+28}_{-30})^\circ\,.$$

$$m_1^2 \ll m_2^2 \ll m_3^2$$

Low rank approximation theorem

an example

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US + K. Tame, arXiv:1804.04578

$$\frac{m_{\nu_1}}{m_{\nu_2}} = \frac{\sin^2\theta_{12}^{\rm PMNS}\left(1+\frac{m_e}{m_\mu}\right) - \frac{m_e}{m_\mu}}{1-\sin^2\theta_{12}^{\rm PMNS}\left(1+\frac{m_e}{m_\mu}\right)}$$

$$\Delta m_{21}^2 \quad \Delta m_{31}^2$$

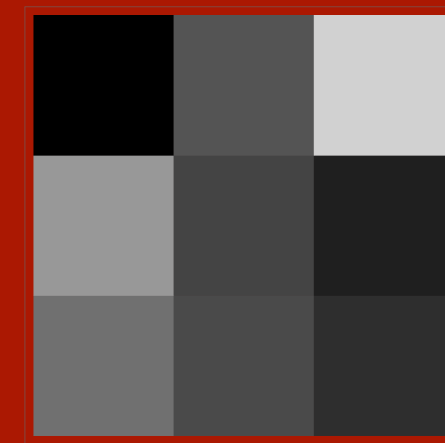
$$m_{\nu_1} = (4.2\pm 0.5)\ \text{meV},\ m_{\nu_2} = (9.6\pm 0.2)\ \text{meV},\ m_{\nu_3} = (50.1\pm 0.3)\ \text{meV}$$

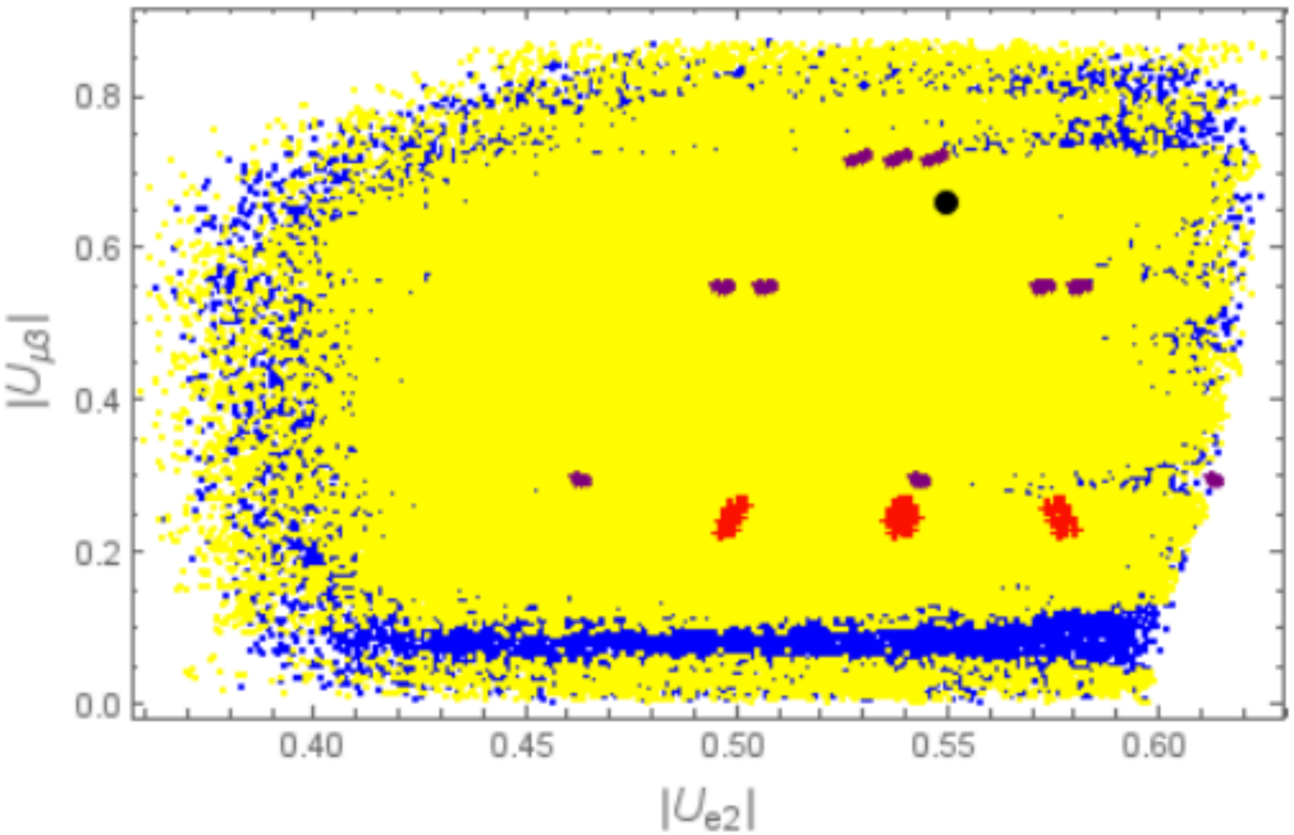
$$|\mathbf{U}_{\rm PMNS}^{\rm th}| = \begin{pmatrix} 0.83\pm 0.01 & 0.53\pm 0.01 & 0.14\pm 0.01 \\ 0.38^{+0.26}_{-0.15} & 0.59^{+0.25}_{-0.49} & 0.71\pm 0.28 \\ 0.40^{+0.14}_{-0.27} & 0.61^{+0.29}_{-0.23} & 0.68^{+0.29}_{-0.62} \end{pmatrix} \quad J_\ell^{\rm th} = -(0.03^{+0.01}_{-0.02})$$

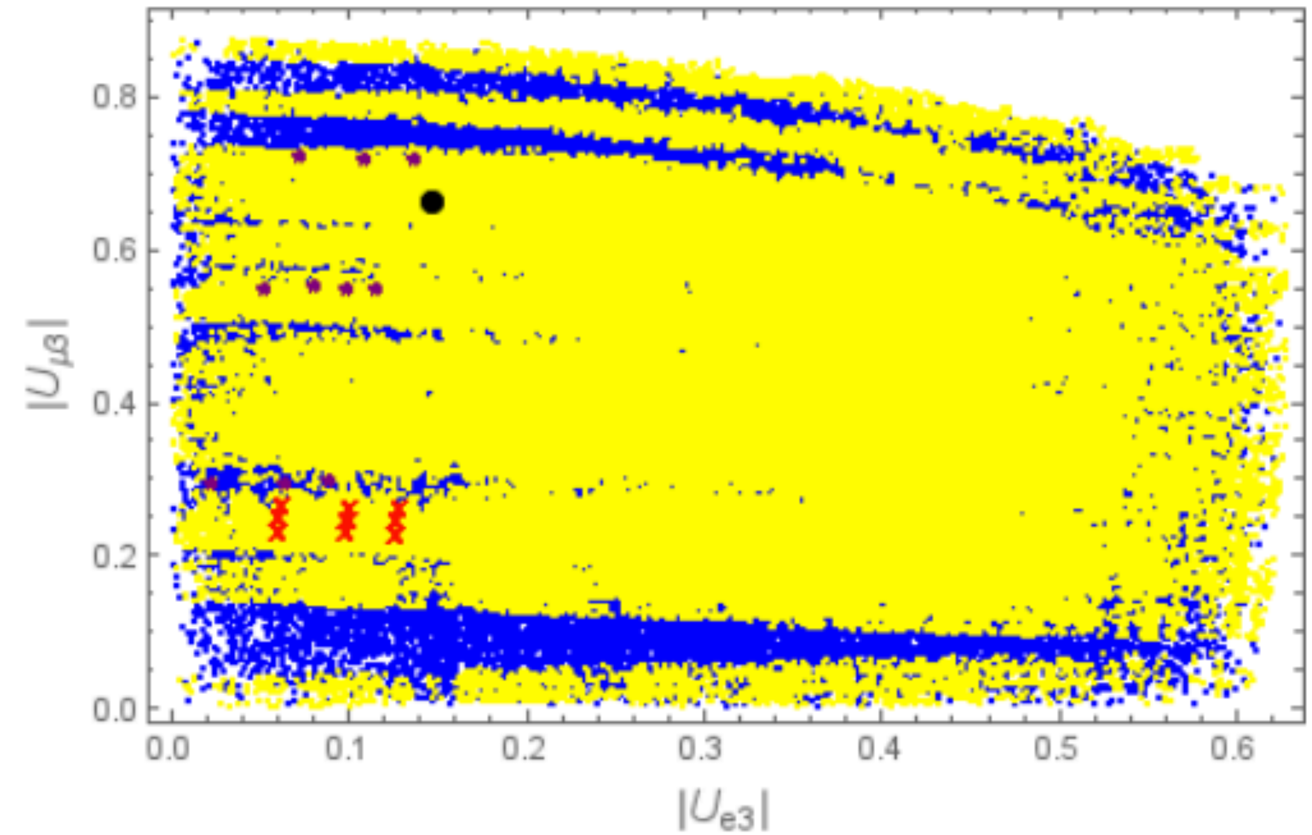
$$\sin^2\theta_{12}^{\ell,\rm th} = 0.54\pm 0.01\ , \quad \sin^2\theta_{13}^{\ell,\rm th} = 0.14\pm 0.01\ , \quad \sin^2\theta_{23}^{\ell,\rm th} = 0.72\pm 0.28$$

an example

US + K. Tame, arXiv:1804.04578

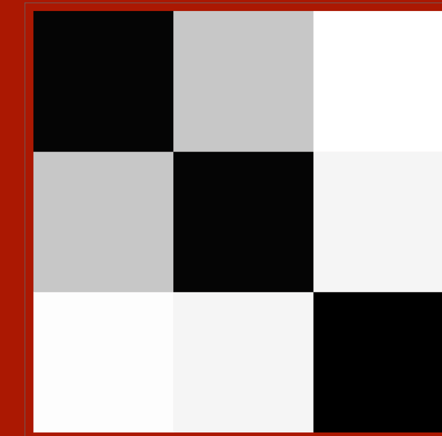






an example

US + K. Tame, arXiv:1804.04578



$$\delta_{\mathrm{CP}}^{\mathrm{q,th}} \approx \arctan\left[\sqrt{\frac{\frac{m_d}{m_s}\left(1+\frac{m_d}{m_s}\right)}{\frac{m_u}{m_c}\left(1+\frac{m_u}{m_c}\right)}}\right],$$

$$\sin\theta_{12}^{\mathrm{q,th}} \approx \sqrt{\frac{m_d/m_s + m_u/m_c}{(1+m_d/m_s)(1+m_u/m_c)}},$$

$$\sin\theta_{23}^{\mathrm{q,th}} \approx -\left(\sqrt{\frac{m_u}{m_t}}+\sqrt{\frac{m_d}{m_b}}+\sqrt{\frac{m_c}{m_t}}-\sqrt{\frac{m_s}{m_b}}\right),$$

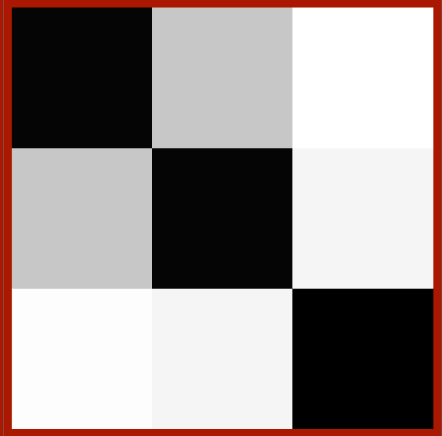
$$\sin\theta_{13}^{\mathrm{q,th}} \approx \sin\theta_{23}^{\mathrm{q,th}}\sqrt{\frac{m_u}{m_c}} - \left(\sqrt{\frac{m_u}{m_t}}+\frac{m_c}{m_t}-\sqrt{\frac{m_d}{m_b}}+\sqrt{\frac{m_d m_s}{m_b^2}}+\frac{m_s}{m_b}\right).$$

$$\mathbf{W}\,(0,0,0,0) = 1$$

Limits?

$$\mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$$

$$\mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$



an example

US + K. Tame, arXiv:1804.04578

$$|\mathbf{V}_{\rm ub}^{\rm th}| \approx |\mathbf{V}_{\rm cb}^{\rm th}| \sqrt{\frac{m_u}{m_c}} - \left(\sqrt{\frac{m_u}{m_t}} + \frac{m_c}{m_t} - \sqrt{\frac{m_d}{m_b}} + \sqrt{\frac{m_d m_s}{m_b^2}} + \frac{m_s}{m_b} \right)$$

$$|\mathbf{V}_{\rm ub}| \simeq \sqrt{\frac{m_d m_s}{m_b^2}} - \sqrt{\frac{m_u m_c}{m_t^2}} = (4.2^{+0.2}_{-0.3}) \times 10^{-3}$$

$$|\mathbf{U}_{e3}| \simeq \sqrt{\frac{m_{\nu_1} m_{\nu_2}}{m_{\nu_3}^2}} - \sqrt{\frac{m_e m_\mu}{m_\tau^2}} = 0.12 \pm 0.01 \ .$$

$$\mathbf{W}\,(0,0,0,0) = 1$$

Limits?

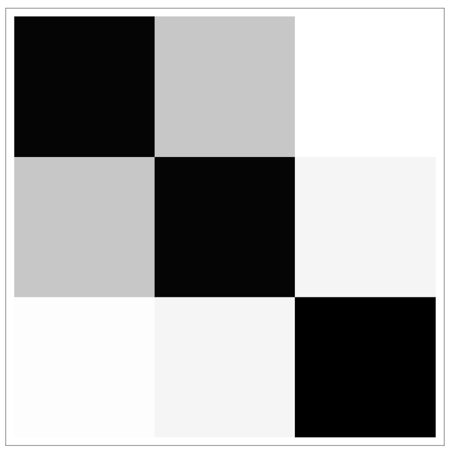
$$\mathbf{W}\left(\frac{m_1^a}{m_2^a}, 0, \frac{m_1^b}{m_2^b}, 0\right) = \mathbf{W}_{12}$$

$$\mathbf{W}\left(0, \frac{m_2^a}{m_3^a}, 0, \frac{m_2^b}{m_3^b}\right) = \mathbf{W}_{23}$$

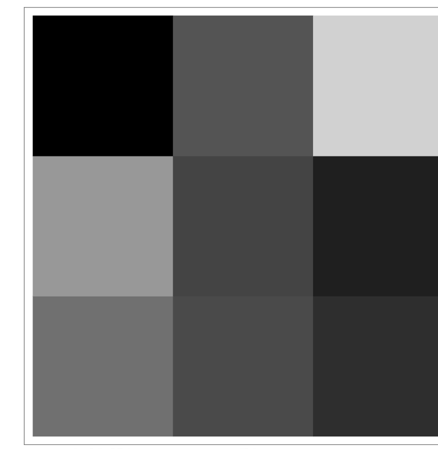
conclusions

conclusions

The problem of mixing,



$Quarks$



$Leptons$

Why is the CKM so hierarchical and close to triviality?
Why do the quark mixing angles behave as 12 >> 23 >> 13?
Why is the Kobayashi-Maskawa phase the way it is?
Why lepton mixing is so different to quark mixing?
Why the PMNS matrix is so anarchical?
Why the 23 lepton mixing angle is almost maximal?
Why is the Dirac CP phase the way it is?
What about Majorana phases?

$$\mathbf{V}_q = \mathbf{V}_q\left(\frac{m_u}{m_c}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_s}{m_b}\right)$$

$$\mathbf{U}_\ell = \mathbf{U}_\ell\left(\frac{m_e}{m_\mu}, \frac{m_\mu}{m_\tau}, \frac{m_{\nu l}}{m_{\nu 2}}, \frac{m_{\nu 2}}{m_{\nu h}}\right)$$

Can both mixings be understood under the same explanation?
Do they share any similarities?
Yes, the 1-3 angle is the smallest one.
How to understand all of these?

$$m_u^2 :: m_c^2 :: m_t^2 = 10^{-10} :: 10^{-6} :: 1$$
$$m_d^2 :: m_s^2 :: m_b^2 = 10^{-6} :: 10^{-4} :: 1$$

$$m_e^2 :: m_\mu^2 :: m_\tau^2 = 10^{-6} :: 10^{-2} :: 1$$
$$m_{\nu l}^2 :: m_{\nu 2}^2 :: m_{\nu h}^2 = 10^{-2.5} :: 10^{-2} :: 1$$

Naturalness could be the bridge...

Thanks for your attention!

"Art does not reproduce the visible; rather, it makes visible."

Paul Klee
Swiss Artist

Backup slides

How to introduce CPV?

$$\mathbf{L}_{12}, \mathbf{L}_{13}, \mathbf{L}_{23}$$

e.g.,

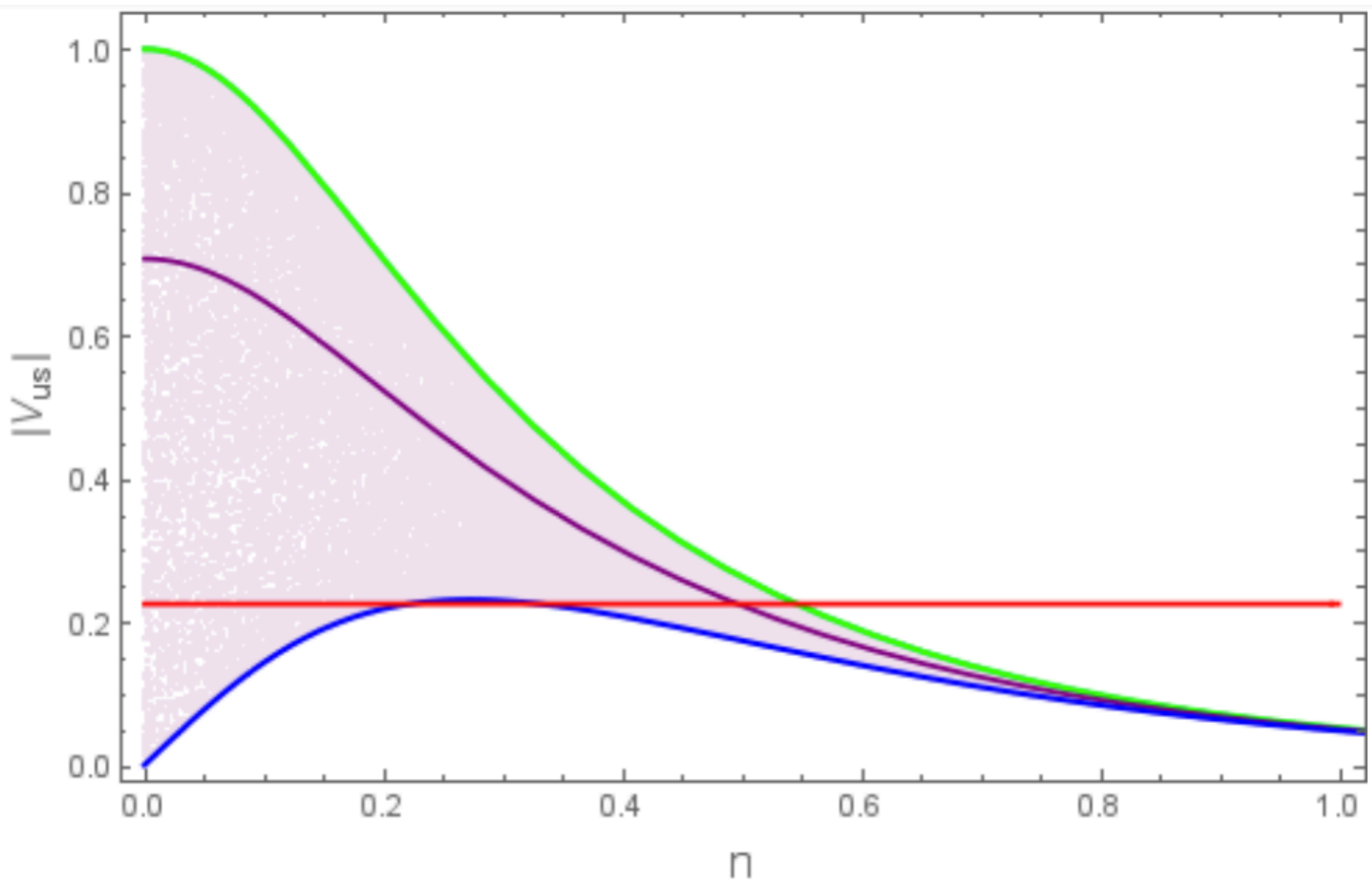
$$\mathbf{L}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\delta} \\ 0 & -\sin\theta e^{i\delta} & \cos\theta \end{pmatrix}$$

Real mass matrices and/or purely imaginary:

Masina, Savoy, 0603101
Antusch, King, Luhn, Spinrath 1103.5930

$$\Delta\delta = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$$

Minimal-Maximal CP conserving mixing & CPV mixing



$$|V_{us}| = \sqrt{\frac{(\frac{m_u}{m_c})^{2n} + (\frac{m_d}{m_s})^{2n} - 2(\frac{m_u}{m_c})^n(\frac{m_d}{m_s})^n \cos(\delta_u - \delta_d)}{\left(1 + (\frac{m_u}{m_c})^{2n}\right)\left(1 + (\frac{m_d}{m_s})^{2n}\right)}}$$

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PDG 2017
nuFit 2018, de Salas, Forero, Ternes, Tortola, Valle 2018

state-of-the-art in mixings

$$|\mathbf{V}_{\rm CKM}| = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$

$$\sin\theta_{12}^{\rm CKM} = 0.22506 \pm 0.00050\,, \qquad \sin\theta_{13}^{\rm CKM} = 0.00357 \pm 0.00015\,,$$
$$\sin\theta_{23}^{\rm CKM} = 0.0411 \pm 0.0013\,, \qquad \text{and} \qquad \delta_{\rm CP}^{\rm CKM} = (71.6^{+1.3}_{-1.0})^\circ.$$

$$|\mathbf{U}_{\rm PMNS}| = \begin{pmatrix} 0.799 \to 0.844 & 0.516 \to 0.582 & 0.141 \to 0.156 \\ 0.242 \to 0.494 & 0.467 \to 0.678 & 0.639 \to 0.774 \\ 0.284 \to 0.521 & 0.490 \to 0.695 & 0.615 \to 0.754 \end{pmatrix}$$

$$\sin^2\theta_{12}^{\rm PMNS} = 0.307^{+0.013}_{-0.012}\,, \qquad \sin^2\theta_{13}^{\rm PMNS} = 0.02206 \pm 0.00075\,,$$
$$\sin^2\theta_{23}^{\rm PMNS} = 0.538^{+0.033}_{-0.069}\,, \qquad \text{and} \qquad \delta_{\rm CP}^{\rm PMNS} = (234^{+43}_{-31})^\circ\,.$$

an example

US + K. Tame, arXiv:1804.04578

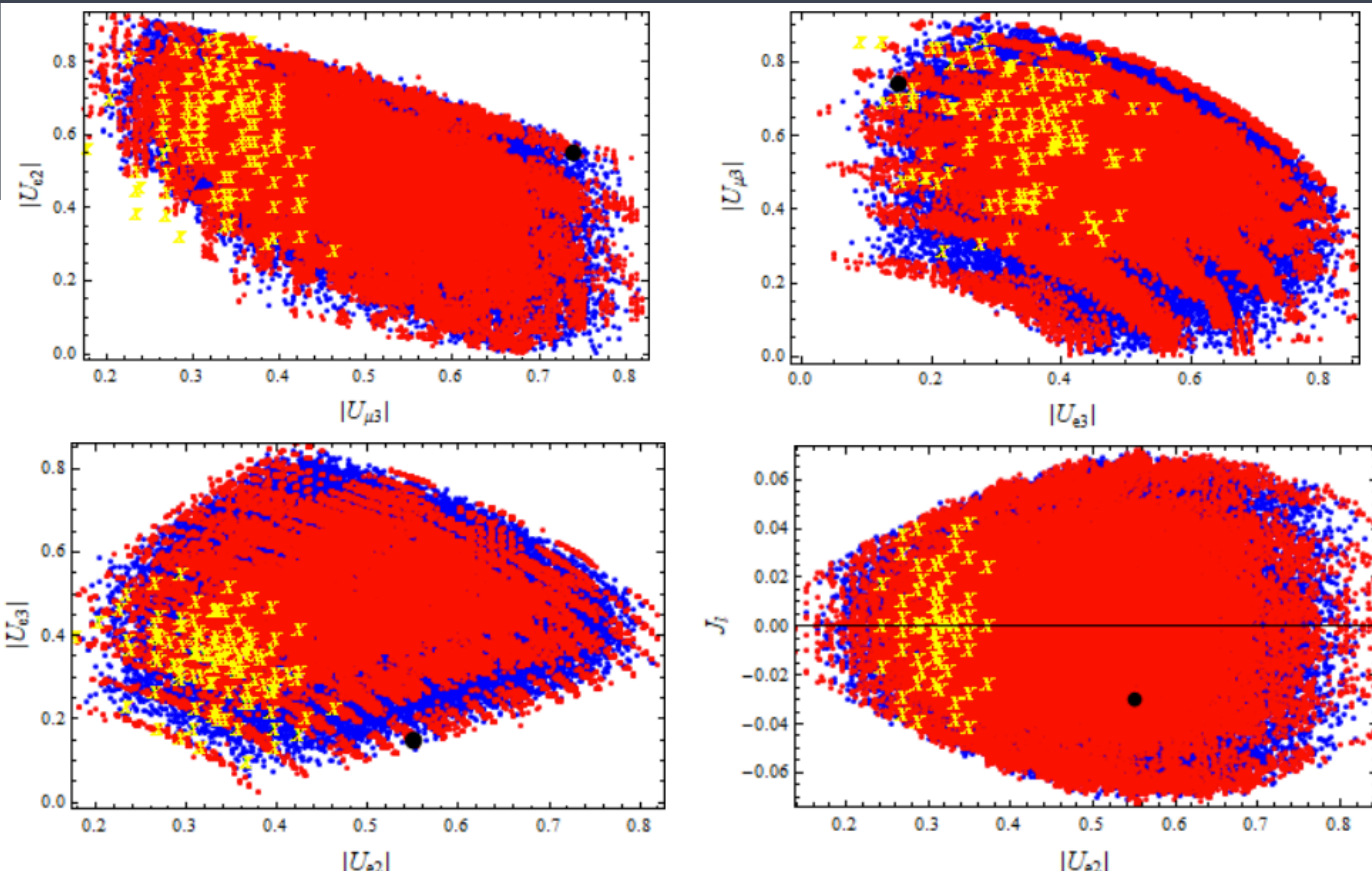
Inverted Ordering case:

$$\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \xrightarrow{m_{\nu_2}} \begin{pmatrix}0&0&0\\0&\blacksquare&0\\0&0&0\end{pmatrix} \xrightarrow{m_{\nu_1}} \begin{pmatrix}\blacksquare&\blacksquare&0\\\blacksquare&\blacksquare&0\\0&0&0\end{pmatrix} \xrightarrow[\mathbf{L}_{12}]{} \begin{pmatrix}\blacksquare&0&0\\0&\blacksquare&0\\0&0&0\end{pmatrix} \xrightarrow{m_{\nu_3}}$$

$$\begin{pmatrix}\blacksquare&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}'_{12}]{} \begin{pmatrix}\blacksquare&0&\blacksquare\\0&\blacksquare&\blacksquare\\\blacksquare&\blacksquare&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}_{23}]{} \begin{pmatrix}\blacksquare&0&\blacksquare\\0&\blacksquare&0\\\blacksquare&0&\blacksquare\end{pmatrix} \xrightarrow[\mathbf{L}_{13}]{} \begin{pmatrix}\blacksquare&0&0\\0&\blacksquare&0\\0&0&\blacksquare\end{pmatrix}$$

$$m_{\nu_2} = (49.7 \pm 0.3)\ \text{meV}\ , \quad m_{\nu_1} = (48.9 \pm 0.3)\ \text{meV}$$

$$m_{\nu_3} = \begin{cases}(1.4 \pm 0.3)\ \text{meV} & \delta_\nu = 0\\(8.8 \pm 0.3)\ \text{meV} & \delta_\nu = \pi\\(1.1 \pm 0.3)\ \text{meV} & \delta_\nu = \frac{(3)\pi}{2}\end{cases}$$


0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$|U_{e2}|$
$|U_{\mu3}|$
$|U_{\mu3}|$
$|U_{e3}|$
$|U_{e3}|$
$|U_{e2}|$
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
J_l
$|U_{e2}|$

an example

US + K. Tame, arXiv:1804.04578