A Realistic U(2) Model of Flavor

based on arXiv: 1805.07341 with M. Linster

$$m_{\{u,d,e,\nu\}} \sim \begin{pmatrix} 0 & \varepsilon^2 & 0 \\ \varepsilon^2 & \varepsilon^2 & \{\varepsilon, \varepsilon^2, \varepsilon, \varepsilon^2\} \\ 0 & \{\varepsilon, \varepsilon, \varepsilon^2, \varepsilon^2\} & \{1, \varepsilon, \varepsilon, \varepsilon^2\} \end{pmatrix}$$

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Outline

Realistic extension of original U(2) models by Barbieri & al

R. Barbieri, G. Dvali, L. Hall '96, R. Barbieri, L. Hall, A. Romanino '97, ...

Differences:

- viable CKM
- does not need SUSY
- Neutrinos

U2

Quarks & Charged Leptons | Dirac Neutrinos in U(2) | Majorana Neutrinos in $D_6 \times U(1)$ | U(2) Axiflavon | Summary

The Model

Classic ***FLASY*** Model based on $U(2)_F \equiv SU(2)_F \times U(1)_F$

SM quantum numbers

- compatible with SU(5) GUT
  $\mathbf{10} = Q, U, E \quad \bar{\mathbf{5}} = L, E$
- generations transform as **2+1**
  $\mathbf{10}_i = \mathbf{10}_a + \mathbf{10}_3$
- need to specify 4 $U(1)_F$ charges
  $X_{10_3} = 0 \quad X_{10_a} = X_{\bar{5}_a} = X_{\bar{5}_3} = 1$

U(2) Breaking with Flavons

- 2 flavons in **2+1** with charge -1
  $\phi = \mathbf{2}_{-1} \qquad \chi = \mathbf{1}_{-1}$
- VEVs slightly below cutoff Λ
  $\langle\phi\rangle = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix} \qquad \langle\chi\rangle = \varepsilon_\chi \Lambda$

$\epsilon_\phi \sim 0.03 \qquad \epsilon_\chi \sim 0.01$

	$\mathbf{10}_a$	$\bar{\mathbf{5}}_a$	$\mathbf{10}_3$	$\bar{\mathbf{5}}_3$	H	ϕ_a	χ
$SU(2)_F$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)_F$	1	1	0	1	0	-1	-1

[old models: SO(10)]

Up-Quark Sector

Invariant Yukawa Lagrangian needs Flavon insertions

$$\mathcal{L}_u = \frac{\lambda_{11}^u}{\Lambda^6}\chi^4(\phi_a^* Q_a)(\phi_b^* U_b)H + \frac{\lambda_{12}^u}{\Lambda^2}\chi^2 \epsilon_{ab} Q_a U_b H + \frac{\lambda_{13}^u}{\Lambda^3}\chi^2(\phi_a^* Q_a)U_3 H$$
$$+ \frac{\lambda_{22}^u}{\Lambda^2}(\epsilon_{ab}\phi_a Q_b)(\epsilon_{cd}\phi_c U_d)H + \frac{\lambda_{23}^u}{\Lambda}(\epsilon_{ab}\phi_a Q_b)U_3 H + \frac{\lambda_{31}^u}{\Lambda^3}\chi^2 Q_3(\phi_a^* U_a)H$$
$$+ \frac{\lambda_{32}^u}{\Lambda}Q_3(\epsilon_{ab}\phi_a U_b)H + \lambda_{33}^u Q_3 U_3 H\,,$$

Flavon vevs generate hierarchical Yukawa structure

$$Y_u \approx \begin{pmatrix} \lambda_{11}^u \varepsilon_\phi^2 \varepsilon_\chi^4 & \lambda_{12}^u \varepsilon_\chi^2 & \lambda_{13}^u \varepsilon_\phi \varepsilon_\chi^2 \\ -\lambda_{12}^u \varepsilon_\chi^2 & \lambda_{22}^u \varepsilon_\phi^2 & \lambda_{23}^u \varepsilon_\phi \\ \lambda_{31}^u \varepsilon_\phi \varepsilon_\chi^2 & \lambda_{32}^u \varepsilon_\phi & \lambda_{33}^u \end{pmatrix} \approx \begin{pmatrix} 0 & \lambda_{12}^u \varepsilon_\chi^2 & 0 \\ -\lambda_{12}^u \varepsilon_\chi^2 & \lambda_{22}^u \varepsilon_\phi^2 & \lambda_{23}^u \varepsilon_\phi \\ 0 & \lambda_{32}^u \varepsilon_\phi & \lambda_{33}^u \end{pmatrix}$$

drop sub-leading
corrections $\mathcal{O}(\epsilon_\phi^2) \sim 10^{-4}$

Quark and Charged Lepton Sector

Analogous structure in down- and charged lepton sector

$$Y_u \approx \begin{pmatrix} 0 & \lambda^u_{12}\varepsilon^2_\chi & 0 \\ -\lambda^u_{12}\varepsilon^2_\chi & \lambda^u_{22}\varepsilon^2_\phi & \lambda^u_{23}\varepsilon_\phi \\ 0 & \lambda^u_{32}\varepsilon_\phi & \lambda^u_{33} \end{pmatrix}, \qquad Y_d \approx \begin{pmatrix} 0 & \lambda^d_{12}\varepsilon^2_\chi & 0 \\ -\lambda^d_{12}\varepsilon^2_\chi & \lambda^d_{22}\varepsilon^2_\phi & \lambda^d_{23}\varepsilon_\phi\varepsilon_\chi \\ 0 & \lambda^d_{32}\varepsilon_\phi & \lambda^d_{33}\varepsilon_\chi \end{pmatrix}$$

$$Y_e \approx \begin{pmatrix} 0 & \lambda^e_{12}\varepsilon^2_\chi & 0 \\ -\lambda^e_{12}\varepsilon^2_\chi & \lambda^e_{22}\varepsilon^2_\phi & \lambda^e_{23}\varepsilon_\phi \\ 0 & \lambda^e_{32}\varepsilon_\phi\varepsilon_\chi & \lambda^e_{33}\varepsilon_\chi \end{pmatrix}.$$

1-2 mixing $\sim \epsilon^2_\chi/\epsilon^2_\phi$

2-3 mixing [d_R & e_L] $\sim \epsilon_\phi/\epsilon_\chi$

2-3 mixing [u_L, u_R, d_L, e_R] $\sim \epsilon_\phi \sim V_{cb}$

1-3 mixing = (1-2 mixing) × (2-3 mixing)

Analytics

Texture zeros accurately relate masses and mixing angles

$$|V_{us}| \approx \left| \sqrt{\frac{m_d}{m_s}} \sqrt{c_{23}^{Rd}} - e^{i(\phi_2-\phi_1)} \sqrt{\frac{m_u}{m_c}} \right|$$

2-3 mixing angle in RH down sector

free phase parameters

$$|V_{td}| \approx \sqrt{\frac{m_d}{m_s}} \sqrt{c_{23}^{Rd}} \left| |V_{cb}| - e^{i\phi_2} \frac{s_{23}^{Rd}}{c_{23}^{Rd}} \frac{m_s}{m_b} \right|$$

$$|V_{ub}| \approx \left| \sqrt{\frac{m_u}{m_c}} |V_{cb}| - e^{i\phi_1} \sqrt{\frac{m_d}{m_s}} \sqrt{c_{23}^{Rd}} \frac{s_{23}^{Rd}}{c_{23}^{Rd}} \frac{m_s}{m_b} \right|$$

In original U(2) models $s_{23}^{Rd} \sim V_{cb}$

$$|V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}$$

Off by more than 3σ: **need $s_{23}^{Rd} \sim 1$** to fix

(but get additional phase dependence)

see also Roberts, Ross, Romanino, Velasco-Sevilla, '01

Numerical Fit

Fit parameters $\{\lambda_{ij}^{u,d,e}, \varepsilon_\phi, \varepsilon_\chi\}$ with SM observables

Use quality measure for "O(1)-ness" of $\lambda_{ij}^{u,d,e}$

$$\chi^2_{\mathcal{O}(1)} = \sum_{\lambda_{ij}^p} \frac{\left(\log(|\lambda_{ij}^p|)\right)^2}{2\cdot 0.55^2} \longrightarrow \lambda_{ij}^{u,d,e} \text{ with 95\% prob. in [1/3, 3]}$$

e.g. for single parameter $\lambda = \{3, 5, 7, 10, 50, 100\}$

get contribution $\Delta\chi^2_{\mathcal{O}(1)} = \{2, 4, 6, 9, 25, 35\}$

Fit satisfactory if

$\chi^2_{\mathcal{O}(1)} \leq \#\text{pars}$

$\chi^2 \leq \#\text{obs}$

Fit	ε_ϕ	ε_χ	min $\lvert\lambda_{ij}^{u,d,\ell}\rvert$	max $\lvert\lambda_{ij}^{u,d,\ell}\rvert$	χ^2	$\chi^2_{\mathcal{O}(1)}$
$QL1_{\mathbb{R}}$	0.019	0.008	$1/3.1$	2.7	1.7	7.8
$QL2_{\mathbb{R}}$	0.023	0.008	$1/2.7$	2.8	12	5.4

Neutrino Sector

Majorana neutrinos don't work because leading order 1-2 entry in Weinberg operator vanishes due to SU(2) antisymmetrization

$\epsilon_{ab} L_a L_b H H = 0$

Do Dirac Neutrinos with $N_i = N_a + N_3$ & $X_a^N = X_3^N$

$$m_\nu^D \approx v\, \varepsilon_\chi^{X_a^N - 1} \begin{pmatrix} 0 & \lambda_{12}^\nu \varepsilon_\chi^2 & 0 \\ -\lambda_{12}^\nu \varepsilon_\chi^2 & \lambda_{22}^\nu \varepsilon_\phi^2 & \lambda_{23}^\nu \varepsilon_\phi \varepsilon_\chi \\ 0 & \lambda_{32}^\nu \varepsilon_\phi \varepsilon_\chi & \lambda_{33}^\nu \varepsilon_\chi^2 \end{pmatrix}$$

get anarchic structure; smallness from largish U(1) charges

Neutrino Fit & Predictions

Good fit just for Normal Ordering

Fit	X_a^N	X_3^N	ε_ϕ	ε_χ	min $\lvert\lambda_{ij}^{u,d,e,\nu}\rvert$	max $\lvert\lambda_{ij}^{u,d,e,\nu}\rvert$	χ^2	$\chi^2_{\mathcal{O}(1)}$
QLν_D-1 (NO)	6	6	0.026	0.012	1/2.9	2.6	0.5	10
QLν_D-2 (NO)	6	6	0.024	0.013	1/2.6	2.2	18	9
QLν_D-3 (NO)	5	5	0.022	0.006	1/3.1	3.8	1.0	13
QLν_D-4 (NO)	5	5	0.021	0.006	1/2.5	2.4	18	9

Predict overall mass scale from scanning over succesful fits

Quantity	Range [meV]	Preferred values [meV]
$\sum m_i$	$58 - 110$	$60 - 65$
m_β	$8 - 26$	$9 - 10$

No chance with KATRIN, automatically satisfy PLANCK bound

$$m_\beta = \sqrt{\sum_i m_i^2 |U_{ei}|^2}$$

Majorana Neutrinos from $D_6 \times U(1)$

Consider dihedral $D_6 = D_3 \times Z_2$ instead of SU(2)

	$\mathbf{10}_a$	$\overline{\mathbf{5}}_a$	$\mathbf{10}_3$	$\overline{\mathbf{5}}_3$	H	ϕ_a	χ
$D_3 \times Z_2$	$\mathbf{2}_-$	$\mathbf{2}_-$	$\mathbf{1}_+$	$\mathbf{1}_+$	$\mathbf{1}_+$	$\mathbf{2}_-$	$\mathbf{1}_+$
$U(1)_F$	1	1	0	1	0	-1	-1

$$(\psi \otimes \phi)_{\mathbf{1}} = \psi_1\phi_2 + \psi_2\phi_1$$

$$(\psi \otimes \phi \otimes \chi)_{\mathbf{1}} = \psi_1\phi_1\chi_1 + \psi_2\phi_2\chi_2$$

Mimics U(2) structure except for symmetric 1-2: Weinberg operator fixed from charged lepton sector

$$m_u \approx v \begin{pmatrix} 0 & \lambda^u_{12}\varepsilon_\chi^2 & 0 \\ \lambda^u_{12}\varepsilon_\chi^2 & \lambda^u_{22}\varepsilon_\phi^2 & \lambda^u_{23}\varepsilon_\phi \\ 0 & \lambda^u_{32}\varepsilon_\phi & \lambda^u_{33} \end{pmatrix}, \qquad m_d \approx v \begin{pmatrix} 0 & \lambda^d_{12}\varepsilon_\chi^2 & 0 \\ \lambda^d_{12}\varepsilon_\chi^2 & \lambda^d_{22}\varepsilon_\phi^2 & \lambda^d_{23}\varepsilon_\phi\varepsilon_\chi \\ 0 & \lambda^d_{32}\varepsilon_\phi & \lambda^d_{33}\varepsilon_\chi \end{pmatrix}$$

$$m_e \approx v \begin{pmatrix} 0 & \lambda^e_{12}\varepsilon_\chi^2 & 0 \\ \lambda^e_{12}\varepsilon_\chi^2 & \lambda^e_{22}\varepsilon_\phi^2 & \lambda^e_{23}\varepsilon_\phi \\ 0 & \lambda^e_{32}\varepsilon_\phi\varepsilon_\chi & \lambda^e_{33}\varepsilon_\chi \end{pmatrix}, \qquad m_\nu \approx \frac{v^2}{M} \begin{pmatrix} 0 & \lambda^\nu_{12}\varepsilon_\chi^2 & 0 \\ \lambda^\nu_{12}\varepsilon_\chi^2 & \lambda^\nu_{22}\varepsilon_\phi^2 & \lambda^\nu_{23}\varepsilon_\phi\varepsilon_\chi \\ 0 & \lambda^\nu_{23}\varepsilon_\phi\varepsilon_\chi & \lambda^\nu_{33}\varepsilon_\chi^2 \end{pmatrix}$$

Majorana Neutrino Fit

Neutrino mass matrix automatically anarchic: better fit with less parameters

Fit	ε_ϕ	ε_χ	min $\lvert\lambda_{ij}^{\mathrm{u,d},\ell}\rvert$	max $\lvert\lambda_{ij}^{\mathrm{u,d},\ell}\rvert$	χ^2	$\chi^2_{\mathcal{O}(1)}$	M [10^{11} GeV]
QLν_M-1	0.025	0.009	1/2.8	2.1	0.7	7.9	4.1
QLν_M-2	0.024	0.009	1/2.6	1.9	18	6.3	3.3

Predict overall mass scale from scanning over succesful fits

Quantity	Range [meV]	Preferred values [meV]
$\sum m_i$	$59 - 78$	60, 70
m_β	$8 - 15$	$9 - 10,\ 11 - 12$
$m_{\beta\beta}^{\max}$	$3 - 16$	5, 9

Similar to Dirac case, neutrinoless double beta decay hopeless

$m_{\beta\beta} = \lvert\sum U_{ei}^2\, m_i\rvert$

Testability

- Neutrino sector predictions
- Low-energy UV completion?
- Full-fledged SU(5) model?
- **The U(2) Axiflavon**

$U(1)_F$ spontaneously broken and has QCD anomaly:
Goldstone is QCD axion solving Strong CP [“axiflavon”]

Wilczek ’82; Ema, Hamaguchi, Moroi, Nakayama ’16; Calibbi, Goertz, Redigolo, RZ, Zupan ‘16

Predict flavor-violating axion couplings, fix cutoff by Axion DM

The U(2) Axiflavon

➔ see yesterday's talk by Chun

$$\mathcal{L}_a = \frac{\partial_\mu a}{2f_a}\bar{f}_i\gamma^\mu\left[C^V_{f_if_j} + C^A_{f_if_j}\gamma_5\right]f_j + \frac{E}{N}\frac{a(x)}{f_a}\frac{\alpha_{\rm em}}{8\pi}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

U(1)$_F$ charges give anomaly coefficients

$$N_{\rm DW} = 2N = 9$$

$$E/N = 8/3 \quad \text{[SU(5)]}$$

Flavon breaking scale (~cutoff) sets mass scale

$$f_a \sim \sqrt{\epsilon_\chi^2 + \epsilon_\phi^2}\,\Lambda$$

$$m_a = 5.7\,\mu\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)$$

Yukawa structure gives axion-fermion couplings

$$C^V_{f_if_j} = (V_{fL})_{ki}X_{f_k}(V_{fL})^*_{kj}$$

$$C^A_{f_if_j} = (V_{fR})^*_{ki}X_{f^c_k}(V_{fR})_{kj}$$

The U(2) Axiflavon

$$C^V_{u_iu_j} = \frac{\varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}\,, \quad C^A_{u_iu_j} = \frac{2\delta_{ij} - \varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}\,,$$
$$C^V_{d_id_j} = \frac{\varepsilon^d_{L,ij}}{9}\,, \quad C^A_{d_id_j} = \frac{2\delta_{ij} - \varepsilon^d_{L,ij}}{9}\,,$$
$$C^V_{e_ie_j} = -\frac{\varepsilon^e_{R,ij}}{9}\,, \quad C^A_{e_ie_j} = \frac{2\delta_{ij} - \varepsilon^e_{R,ij}}{9}\,,$$

$$\varepsilon^u_L \sim \varepsilon^u_R \sim \varepsilon^d_L \sim \varepsilon^e_R \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

SU(2)/D6 protects s-d transitions, strongest bound from SN

Coupling	$m_a^{\max}/C$ [eV]	$m_a^{\max,\mathrm{U(2)}}$ [eV]	$f_a^{\min,\mathrm{U(2)}}$ [GeV]	Constraint
C^V_{bs}	$9.1 \cdot 10^{-2}$	16	$3.6 \cdot 10^5$	$B^+ \to K^+ a$ [28]
C^V_{sd}	$1.7 \cdot 10^{-5}$	0.58	$9.8 \cdot 10^6$	$K^+ \to \pi^+ a$ [29]
C^A_{ee}	$3.1 \cdot 10^{-3}$	0.014	$4.1 \cdot 10^8$	WD Cooling [30]
C_N	$3.5 \cdot 10^{-3}$	0.0092	$6.2 \cdot 10^8$	SN1987A [31]

U(2) Axiflavon Phenomenology

U(1)$_F$ broken before inflation:

$$\Omega_{DM} h^2 \approx 0.12 \left(\frac{6\,\mu\text{eV}}{m_a} \right)^{1.165} \theta^2$$

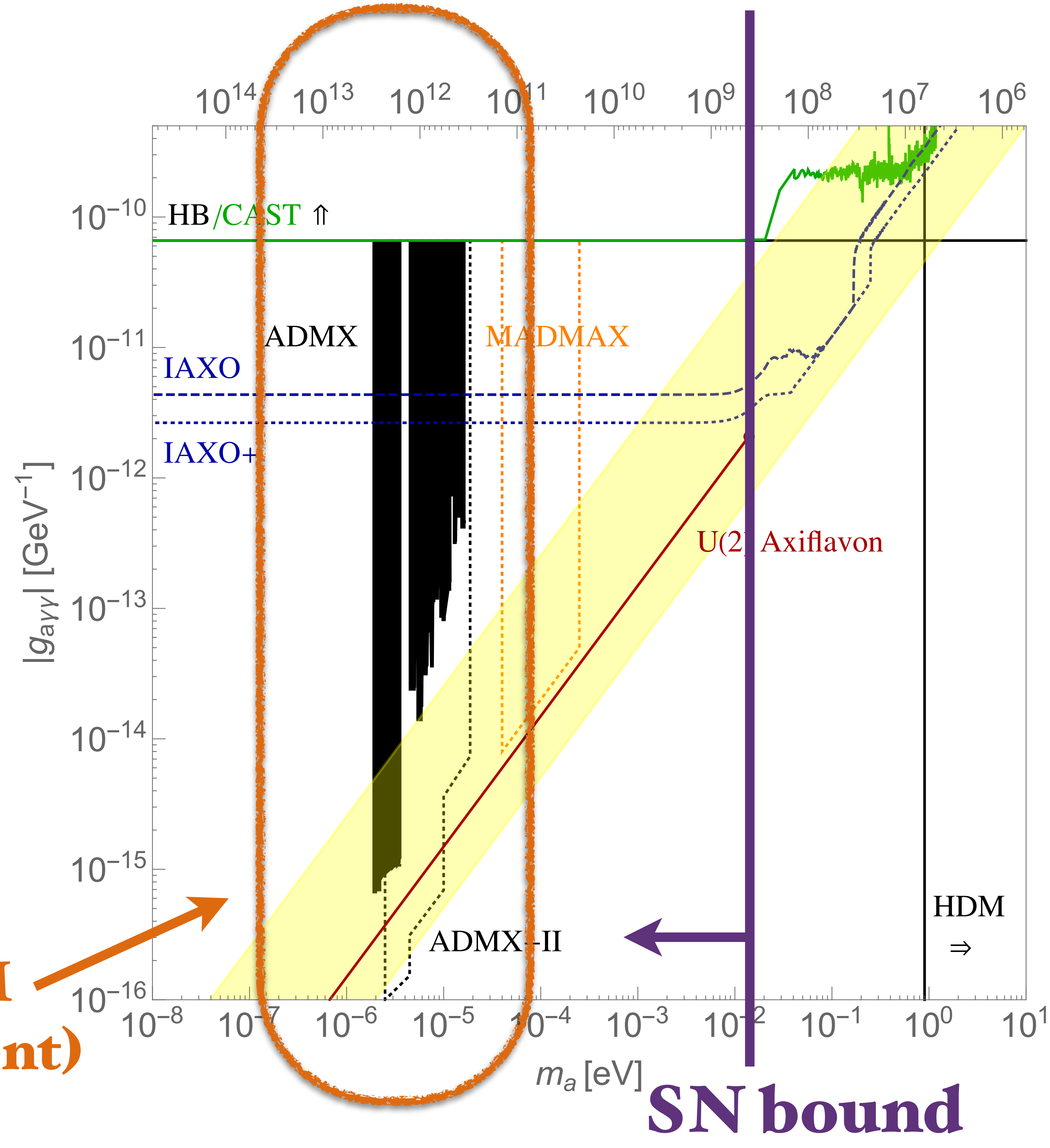
with misalignment angle

$$\theta \in [-\pi, \pi]$$

Axion DM gives preferred range for cutoff

$$\Lambda \sim (10^{13} \div 10^{15})\,\text{GeV}$$

→ see talk by Ringwald



Summary

- Simple, realistic Flavor Model from U(2), works without SUSY, compatible with SU(5)

- Dirac Neutrinos in U(2); Majorana Neutrinos in $D_6 \times U(1)_F$, which are automatically anarchic; can predict absolute neutrino mass scale

- Naturally get QCD Axion as DM from $U(1)_F$; fixes breaking /cutoff scale; SU(2) strongly suppresses flavor violation

Backup

Fit Results

Parameter	QLν_D-1	QLν_D-2	QLν_D-3	QLν_D-4	QLν_M-1	QLν_M-2
λ^u_{12}	0.902	0.843	3.831	1.162	-1.633	-1.176
λ^u_{22}	1.187	-1.047	1.859	1.148	1.339	1.112
λ^u_{23}	2.222	-2.175	-2.138	-1.799	2.127	1.925
λ^u_{32}	-1.103	-1.419	1.511	2.422	1.196	1.615
λ^u_{33}	0.787	0.779	-0.787	0.786	0.787	0.785
δ_{33}	-0.640	-0.720	-3.948	-1.097	-3.837	-3.988
λ^d_{12}	0.479	-0.479	2.165	2.173	-0.888	0.976
λ^d_{22}	-1.000	-1.156	-1.075	-0.972	-0.973	0.976
λ^d_{23}	0.913	-0.786	-1.304	-1.155	1.073	0.985
λ^d_{32}	-0.355	0.401	0.414	0.423	0.365	-0.394
λ^d_{33}	0.665	0.651	1.394	1.497	-0.902	-0.948
λ^ℓ_{12}	0.402	-0.376	-1.752	-1.758	-0.801	0.856
λ^ℓ_{22}	0.987	-1.134	1.821	2.052	1.306	1.497
λ^ℓ_{23}	0.343	0.381	0.393	-0.414	-0.368	0.391
λ^ℓ_{32}	-0.992	-1.132	1.175	1.193	-1.198	1.294
λ^ℓ_{33}	0.432	-0.399	-0.945	0.992	-0.503	-0.536
λ^ν_{12}	0.882	-1.416	0.938	1.006	2.130	-1.873
λ^ν_{22}	-0.994	-1.303	0.325	0.398	-0.844	-0.760
λ^ν_{23}	-2.588	-1.074	-1.505	1.681	1.137	-1.078
λ^ν_{32}	1.065	-0.704	0.601	0.680	$\parallel$	$\parallel$
λ^ν_{33}	0.952	-1.572	-0.890	0.891	-0.489	-0.655
$X^{\rm N}_a$	6	6	5	5		
$X^{\rm N}_3$	6	6	5	5		
$v/M \times 10^9$					-0.421	-0.520
ε_ϕ	0.026	0.024	0.022	0.021	0.025	0.024
ε_χ	0.012	0.013	0.006	0.006	0.009	0.009

Flavon Potential

$$\phi = 2_-[-1]\,, \qquad \chi = 1_+[-1]\,, \qquad \psi = 1_-[+1]\,,$$

$$\begin{aligned} V_{\rm scal} = & \, m_\chi^2|\chi|^2 + \left(m_\phi^2 + \kappa_\chi|\chi|^2 + \kappa_\psi|\psi|^2\right)(\tilde{\phi}\cdot\phi) + m_\psi^2|\psi|^2 \\ & + \frac{\lambda_1}{4}(\tilde{\phi}\cdot\tilde{\phi})(\phi\cdot\phi) + \frac{\lambda_2}{2}(\tilde{\phi}\cdot\tilde{\phi}\cdot\phi\cdot\phi) + \lambda_3|\chi^2||\psi|^2 + \frac{\lambda_\chi}{2}|\chi|^4 + \lambda_\psi|\psi|^4 \\ & + \left[\frac{\kappa_1}{2}\psi\psi\,(\phi\cdot\phi) + \frac{\kappa_2}{2}\chi^*\chi^*\,(\phi\cdot\phi) + \frac{1}{2}\lambda_{\chi\psi}\psi\psi\chi\chi + \rho\psi(\tilde{\phi}\cdot\phi\cdot\phi) + \text{h.c.}\right] \end{aligned}$$

$$m_\phi^2 = -2m^2\,, m_\chi^2 = -3/10m^2\,, m_\psi^2 = 2m^2\,, \lambda_1 = 1\,, \lambda_2 = 1/9\,, \lambda_\chi = 1\,, \kappa_\chi = -1/8$$
$$\lambda_3 = 2/3\,, \lambda_{\chi\psi} = -1/20\,, \kappa_1 = -1/3\,, \kappa_\psi = 7/10\,, \rho = -1/20\,, \lambda_\psi = 9/10\,, \kappa_2 = 1/2000$$

$$v_1 = 4.6m\,, \qquad v_2 = -3.6\cdot 10^{-4}\,m\,, \qquad v_\chi = 1.7m\,, \qquad v_\psi = -2.1\cdot 10^{-5}\,m$$