[Reconstructing flavoured leptoquark models for B-decay anomalies]

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[Outline]

[1] Leptoquarks in B decays?

[2] Flavour symmetries of the SM and beyond

[3] non-Abelian discrete flavour symmetries

[1] [Leptoquarks in B decays?]

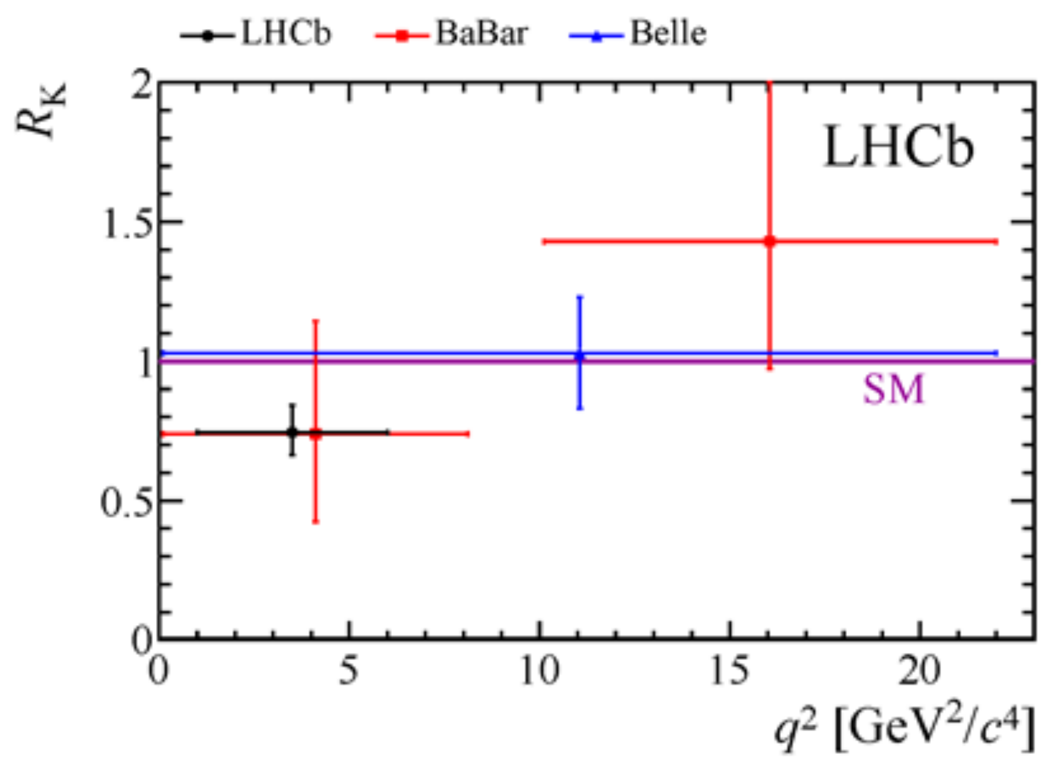
[Hints of new physics from B decays?]

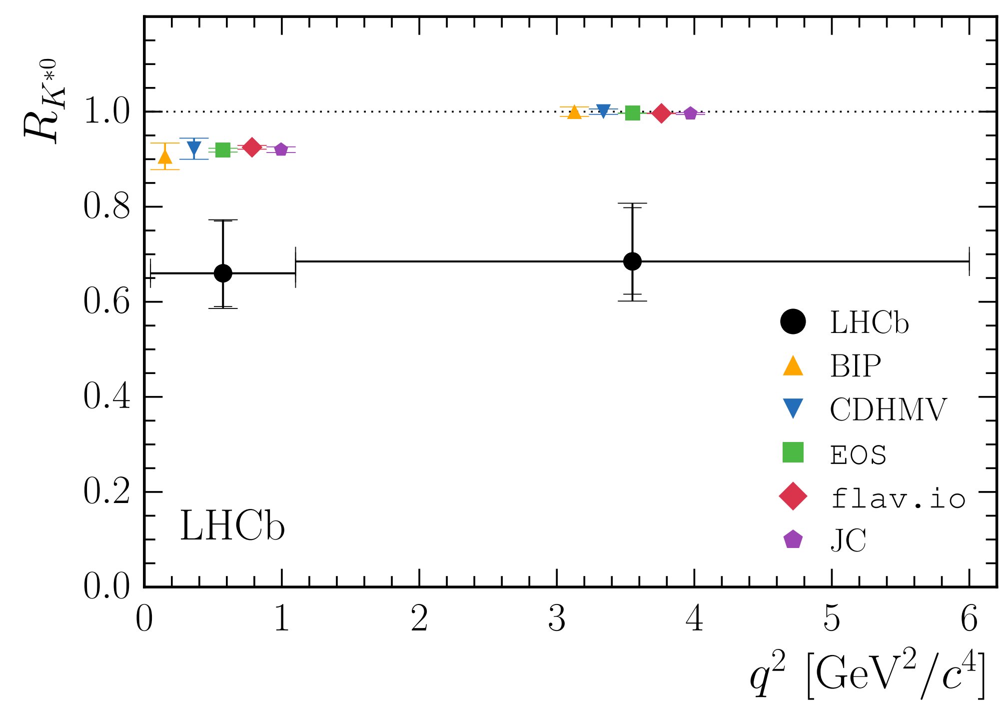
LHCb: 1406.6482
LHCb: 1705.05802
Albrecht: 1805.06243

- Lepton non-universal signatures hinted at from clean B-decay ratio observables:

$$R_H = \frac{\int \frac{d\Gamma(B\to H\mu^+\mu^-)}{dq^2}\, dq^2}{\int \frac{d\Gamma(B\to He^+e^-)}{dq^2}\, dq^2}$$

$$R_H = \frac{\mathcal{B}(B\to H\mu^+\mu^-)}{\mathcal{B}(B\to HJ/\psi(\to\mu^+\mu^-))} \Bigg/ \frac{\mathcal{B}(B\to He^+e^-)}{\mathcal{B}(B\to HJ/\psi(\to e^+e^-))}$$





$$R_K = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$$

$$R_{K^*} = \begin{cases} 0.66\,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.03\,(\text{syst}) & \text{for } 0.045 < q^2 < 1.1 \ \text{GeV}^2/c^4 \\ 0.69\,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.05\,(\text{syst}) & \text{for } 1.1 \ \ < q^2 < 6.0 \ \text{GeV}^2/c^4 \end{cases}$$

- Lepton universality tensions at ~2-3 σ also observed in b -> c transitions ($R_{D(*)}$).
- Combined global fits yield even higher (~4 σ) degrees of tension…

[EFT pulls and leptoquark solutions]

- NP effects can be parameterized within the Weak Effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha}{4\pi}\sum_i C_i^\ell \mathcal{O}_i^\ell + \text{h.c.}$$

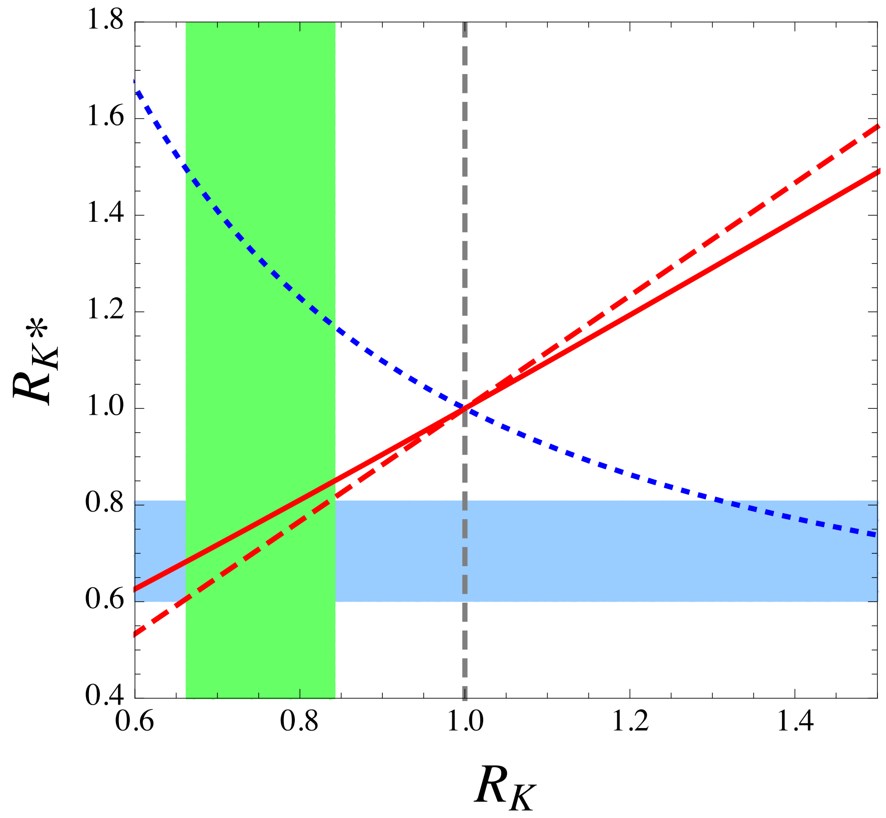
$$\mathcal{O}_9^\ell = (\bar{s}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell), \quad \mathcal{O}_9^{\prime\ell} = (\bar{s}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu \ell),$$
$$\mathcal{O}_{10}^\ell = (\bar{s}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell), \quad \mathcal{O}_{10}^{\prime\ell} = (\bar{s}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$$

$$C_9^\ell = \frac{1}{2}(C_{LL}^\ell + C_{LR}^\ell), \quad C_{10}^\ell = \frac{1}{2}(C_{LR}^\ell - C_{LL}^\ell)$$
$$C_9^{\prime\ell} = \frac{1}{2}(C_{RL}^\ell + C_{RR}^\ell), \quad C_{10}^{\prime\ell} = \frac{1}{2}(C_{RR}^\ell - C_{RL}^\ell)$$

See global fits from D'Amico et al: 1704.05438, Altmannshofer et al.: 1704.05435, …

- **Leptoquarks** offer concrete and obvious model opportunities satisfying EFT fits: GH, Nisandzic: 1704.05444 + …

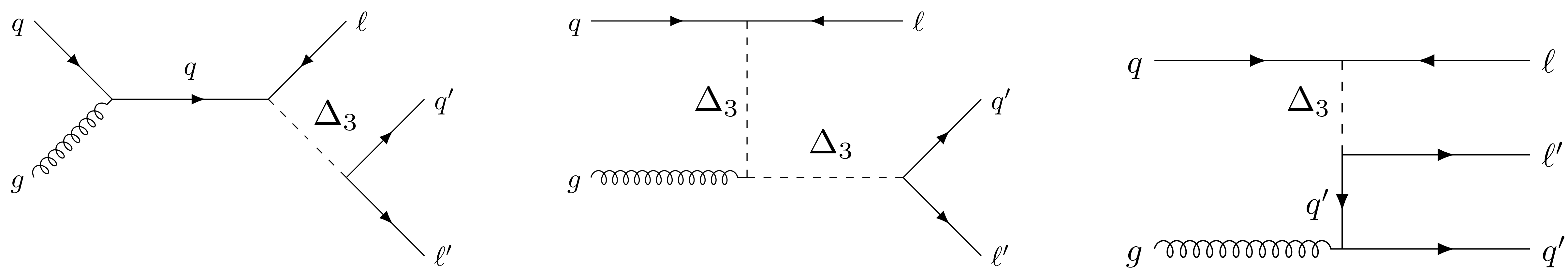
	representation	C_{AB}	Relation	$R_{K^{(*)}}$
$\tilde{S}_2$	$(3,2,1/6)$	C_{RL}	$C_9' = -C_{10}'$	$R_K < 1,\ R_{K^*} > 1$
Δ_3	$(\bar{3},3,1/3)$	$C_{LL}^{\rm NP}$	$C_9 = -C_{10}$	$R_K \simeq R_{K^*} < 1.$
S_2	$(3,2,7/6)$	C_{LR}	$C_9 = C_{10}$	$R_K \simeq R_{K^*} \simeq 1$
$\tilde{S}_1$	$(\bar{3},1,4/3)$	C_{RR}	$C_9' = C_{10}'$	$R_K \simeq R_{K^*} \simeq 1$



We explore the scalar triplet (see **solid red line** in figure) today. Also note that vector LQs still viable…

- Note that MANY other models also explored (Z', sterile neutrinos, gauged flavour, B-L, …)…see e.g. Crivellin, Fuentes-Martin, Greljo, Isidori: 1611.02703 and later talks as well…

[Extending the SM with a scalar leptoquark]



GH, Loose, Nisandzic: hep-ph/1801.09399

- A phenomenologically viable option is a (Lorentz) scalar (SU(2)) triplet leptoquark:

$$\Delta_3 \sim (\bar{3}, 3, 1/3) \qquad \mathcal{L} \supset y^{LL}_{3,ij} \bar{Q}^{C\,i,a}_L \epsilon^{ab} (\tau^k \Delta^k_3)^{bc} L^{j,c}_L + z^{LL}_{3,ij} \bar{Q}^{C\,i,a} \epsilon^{ab} ((\tau^k \Delta^k_3)^\dagger)^{bc} Q^{j,c}_L + \text{h.c.}$$

- Decomposing the SU(2) indices and performing the standard basis transformation, one obtains:

$$\begin{aligned}\mathcal{L} \supset & -(U^T_d y^{LL}_3 U_\nu)_{ij} \bar{d}^{C\,i}_L \Delta^{1/3}_3 \nu^j_L - \sqrt{2}(U^T_d y^{LL}_3 U_l)_{ij} \bar{d}^{C\,i}_L \Delta^{4/3}_3 l^j_L + \\ & + \sqrt{2}(U^T_u y^{LL}_3 U_\nu)_{ij} \bar{u}^{C\,i}_L \Delta^{-2/3}_3 \nu^j_L - (U^T_u y^{LL}_3 U_l)_{ij} \bar{u}^{C\,i}_L \Delta^{1/3}_3 l^j_L + \\ & + \text{h.c.}\end{aligned}$$

- Given a particular coupling structure, the dominant decay modes for the different charge states are given by:

$$\Delta^{-2/3}_3 \rightarrow t\,\nu \qquad \Delta^{1/3}_3 \rightarrow b\,\nu,\ t\mu^- \qquad \Delta^{4/3}_3 \rightarrow b\,\mu^-$$

[Experimental constraints and signatures]

- Consider the d-l coupling relevant to B decay phenomenology, in the mass basis:

$$\mathcal{L}^{Y}_{LQ} \supset \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{d}^C_L\, \lambda_{dl}\, l_L\, \Delta_3^{4/3} + \text{h.c.} \qquad -\sqrt{2}\,(U_d^T\, y_3^{LL}\, U_l) \equiv \lambda_{dl} = \lambda_0 \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

- One can constrain the new leptoquark Yukawa couplings with the available B decay data. From $R_{K(*)}$ one finds e.g.:

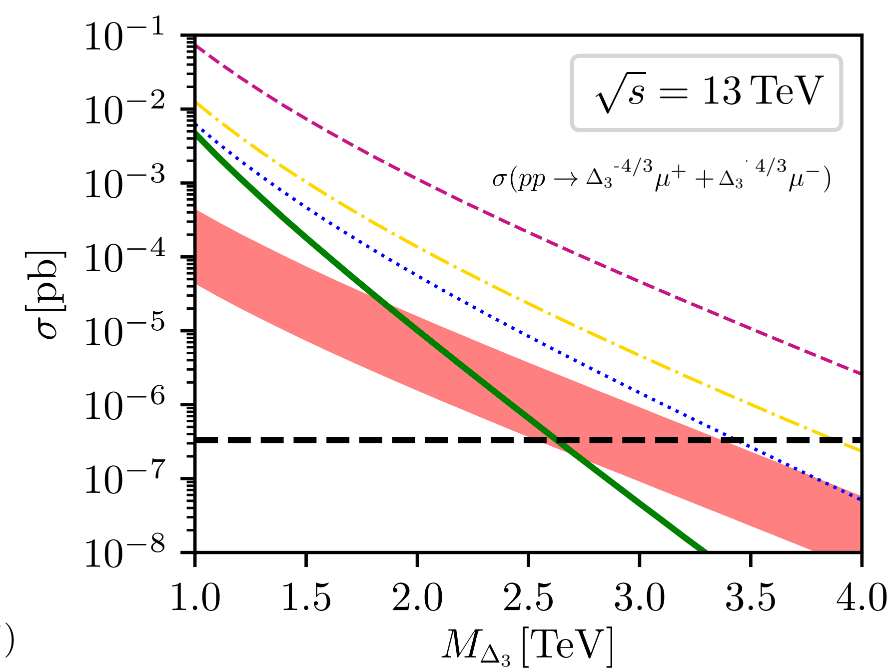
$$\lambda_{b\mu}\lambda^*_{s\mu} - \lambda_{be}\lambda^*_{se} \simeq 1.1 \frac{M^2}{(35\ \text{TeV})^2}$$

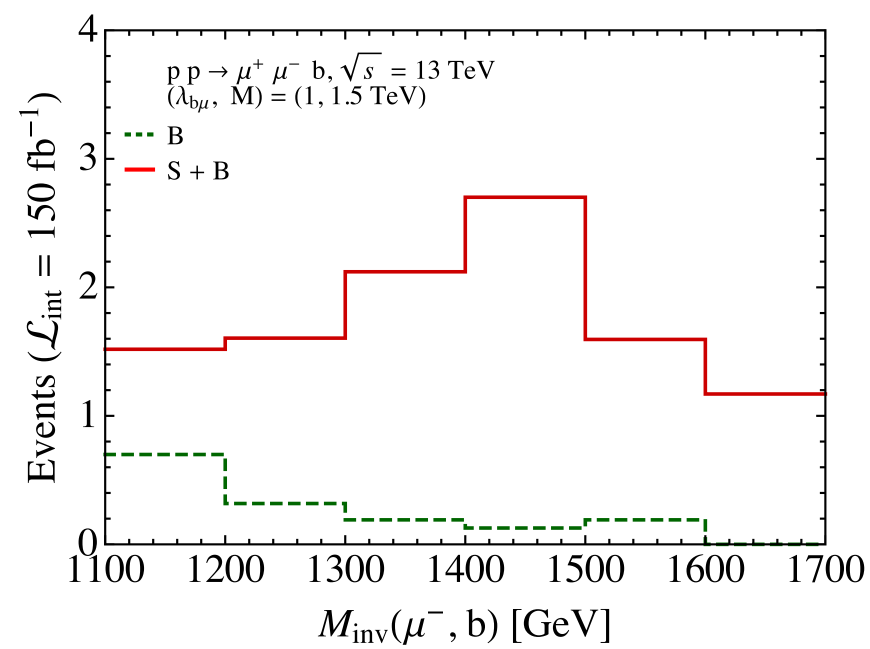
- Which then points to a rich collider phenomenology: GH, Loose, Nisandzic: hep-ph/1801.09399

Red Band: $\lambda_s \sim \lambda_0 \begin{pmatrix} 0 & 0 & 0 \\ * & \epsilon^2 & * \\ * & 1 & * \end{pmatrix}$

Thin curves: $\lambda_{d\mu}=1$, $\lambda_{s\mu}=1$, $\lambda_{b\mu}=1$

Green curve: $\sigma(pp \to \Delta_3^{-4/3}\Delta_3^{4/3})$





- However, **no theoretical origin for flavour structure of LQ Yukawa couplings**! Can this be constrained/modeled? (see e.g. IdMV, GH: 1503.01084 I GH, Loose, Schonwald: 1609.08895 I GH, Loose, Nisandzic: 1801.09399)

[2] [Residual flavour symmetries]

[Residual flavour symmetries]

- The SM (absent Yukawas) is invariant under a global U(3)^5 flavour symmetry (Chivukula/Georgi):

$$SM \;\sim\; q_L^i\,(3,2)_{+1/3}\,, \quad \overline{u}_L^i\,(\overline{3},1)_{-4/3}\,, \quad \overline{d}_L^i\,(\overline{3},1)_{+2/3}\,, \quad l_L^i\,(1,2)_{-1}\,, \quad \overline{e}_L^i\,(1,1)_{+2}$$

U(3)^5 ~ U(3)$_q$ x U(3)$_u$ x U(3)$_d$ x U(3)$_l$ x U(3)$_e$

- This symmetry can be preserved by promoting Yukawa couplings to spurions (cf. MFV), or by introducing new flavons…
- In broken phase, consider the Yukawa sector for SM leptons with Majorana neutrinos:

$$\mathcal{L}_{\text{mass}} = \frac{g}{\sqrt{2}}\bar{l}_L U_{PMNS}\,\gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_l l_L + \frac{1}{2}\bar{\nu}_L^c m_\nu \nu_L + ... + \text{h.c.}$$

- Neutrino mass term is *still* invariant under a Z_2 x Z_2 Klein symmetry (Lam):

$$\nu \to T_{\nu i}\nu, \qquad m_\nu \to T_{\nu i}^T m_\nu T_{\nu i} = m_\nu \qquad\qquad T_{\nu 1} = diag\,(1,-1,-1)\,, \qquad T_{\nu 2} = diag\,(-1,1,-1)$$

- While the charged leptons (and quarks) are *still* subject to the standard U(1)^3:

$$l_L \to T_l\, l_L, \qquad E_R \to T_l\, E_R, \qquad T_l = diag\left(e^{2\pi i \frac{a}{m}}, e^{2\pi i \frac{b}{m}}, e^{2\pi i \frac{c}{m}}\right)$$

- We interpret these *residual* flavour symmetries generated by T_i as remnant signatures of the parent flavour group. They are present regardless of the dynamics of the flavour model!

[Residual symmetries with a scalar leptoquark]

- These symmetries are well understood for the SM fermions. What happens when we include the d-l leptoquark coupling?

$$\mathcal{L}_{LQ}^{Y} \supset \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{d}_L^C\, \lambda_{dl}\, l_L\, \Delta_3^{4/3} + \text{h.c.} \qquad d_{L,R} \to T_d\, d_{L,R}, \qquad l_L \to T_l\, l_L, \qquad E_R \to T_l\, E_R$$

$$\begin{pmatrix} e^{i(\alpha_d+\alpha_l)}\,\lambda_{de} & e^{i(\alpha_d+\beta_l)}\,\lambda_{d\mu} & e^{i(\alpha_d+\gamma_l)}\,\lambda_{d\tau} \\ e^{i(\beta_d+\alpha_l)}\,\lambda_{se} & e^{i(\beta_d+\beta_l)}\,\lambda_{s\mu} & e^{i(\beta_d+\gamma_l)}\,\lambda_{s\tau} \\ e^{i(\gamma_d+\alpha_l)}\,\lambda_{be} & e^{i(\gamma_d+\beta_l)}\,\lambda_{b\mu} & e^{i(\gamma_d+\gamma_l)}\,\lambda_{b\tau} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

- There are very few interesting solutions to this equation!

'Isolation patterns'

$$\lambda_{dl}^{[e]} \sim \begin{pmatrix} \lambda_{de} & 0 & 0 \\ \lambda_{se} & 0 & 0 \\ \lambda_{be} & 0 & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\mu]} \sim \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ 0 & \lambda_{s\mu} & 0 \\ 0 & \lambda_{b\mu} & 0 \end{pmatrix}, \quad \lambda_{dl}^{[\tau]} \sim \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ 0 & 0 & \lambda_{s\tau} \\ 0 & 0 & \lambda_{b\tau} \end{pmatrix}$$

'Two column patterns'

$$\lambda_{dl}^{[e\mu]} \sim \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix}, \quad \lambda_{dl}^{[e\tau]} \sim \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix}, \quad \lambda_{dl}^{[\mu\tau]} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

'Three column patterns'

$$\lambda_{dl}^{[e\mu 1]} \sim \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix}, \quad \lambda_{dl}^{[e1\tau]} \sim \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix}, \quad \lambda_{dl}^{[1\mu\tau]} \sim \begin{pmatrix} \lambda_{de} & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$$

[Including the additional *q-l* couplings]

- But we must consider the Yukawa sector appended by all of the scalar states:

$$\mathcal{L}^{Y}_{LQ} \supset \frac{1}{2}\bar{\nu}^c_L\, m_\nu\, \nu_L + \bar{E}_R\, m_l\, l_L + \bar{d}_R\, m_d\, d_L + \bar{u}_R\, m_u\, u_L \\ + \bar{d}^C_L\, \lambda_{dl}\, l_L\, \Delta_3^{4/3} + \bar{d}^C_L\, \lambda_{d\nu}\, \nu_L\, \Delta_3^{1/3} + \bar{u}^C_L\, \lambda_{ul}\, l_L\, \Delta_3^{1/3} + \bar{u}^C_L\, \lambda_{u\nu}\, \nu_L\, \Delta_3^{-2/3} \\ + \text{h.c.}$$

- Other charged states are related to the d-l coupling via SU(2) transformations:

$$\lambda_{d\nu} = \frac{1}{\sqrt{2}}\lambda_{dl}\, U_{PMNS}, \quad \lambda_{ul} = \frac{1}{\sqrt{2}} U^\star_{CKM}\, \lambda_{dl}, \quad \lambda_{u\nu} = -U^\star_{CKM}\, \lambda_{dl}\, U_{PMNS}$$

- All of these terms are *simultaneously* subject to the residual symmetry constraints:

$$T_Q^T\, \lambda_{QL}\, T_l \overset{!}{=} \lambda_{QL} \quad \forall \quad \{Q, L\}$$

$$\lambda_{QL} \in \left\{ \begin{pmatrix} \lambda_{Q_1L_1} & 0 & 0 \\ \lambda_{Q_2L_1} & 0 & 0 \\ \lambda_{Q_3L_1} & 0 & 0 \end{pmatrix}, \begin{pmatrix} \lambda_{Q_1L_1} & \lambda_{Q_1L_2} & 0 \\ \lambda_{Q_2L_1} & \lambda_{Q_1L_2} & 0 \\ \lambda_{Q_3L_1} & \lambda_{Q_1L_2} & 0 \end{pmatrix}, \begin{pmatrix} \lambda_{Q_1L_1} & \lambda_{Q_1L_2} & \lambda_{Q_1L_3} \\ \lambda_{Q_2L_1} & \lambda_{Q_1L_2} & 0 \\ \lambda_{Q_3L_1} & \lambda_{Q_1L_2} & 0 \end{pmatrix} \right.$$

We focus here today!

[Model-independent conclusions]

- $SU(2)_L$ simultaneously constrains couplings between other quark and lepton species:

$$\lambda_{d\nu}^{[I]} = \frac{\lambda_0}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ U_{11}\lambda_{se} & U_{12}\lambda_{se} & U_{13}\lambda_{se} \\ U_{11}\lambda_{be} & U_{12}\lambda_{be} & U_{13}\lambda_{be} \end{pmatrix} \qquad \lambda_{ul} = \frac{\lambda_0}{\sqrt{2}} \begin{pmatrix} V_{13}\lambda_{be} + V_{12}\lambda_{se} & 0 & 0 \\ V_{23}\lambda_{be} + V_{22}\lambda_{se} & 0 & 0 \\ V_{33}\lambda_{be} + V_{32}\lambda_{se} & 0 & 0 \end{pmatrix}$$

$U_{PMNS}^{ij} = U_{ij}$ and $(U_{CKM}^{ij})^\star = V_{ij}$.

- Experimental constraints on one of the up-lepton elements permit two up-neutrino couplings

$$\lambda_{u\nu}^{[I_1]} = \lambda_0 \begin{pmatrix} 0 & 0 & 0 \\ U_{11}\left(\frac{V_{13}V_{22}}{V_{12}} - V_{23}\right)\lambda_{be} & U_{12}\left(\frac{V_{13}V_{22}}{V_{12}} - V_{23}\right)\lambda_{be} & 0 \\ U_{11}\left(\frac{V_{13}V_{32}}{V_{12}} - V_{33}\right)\lambda_{be} & U_{12}\left(\frac{V_{13}V_{32}}{V_{12}} - V_{33}\right)\lambda_{be} & 0 \end{pmatrix}$$

$$\lambda_{u\nu}^{[I_2]} = \lambda_0 \begin{pmatrix} U_{11}\left(\frac{V_{12}V_{23}}{V_{22}} - V_{13}\right)\lambda_{be} & U_{12}\left(\frac{V_{12}V_{23}}{V_{22}} - V_{13}\right)\lambda_{be} & 0 \\ 0 & 0 & 0 \\ U_{11}\left(\frac{V_{23}V_{32}}{V_{22}} - V_{33}\right)\lambda_{be} & U_{12}\left(\frac{V_{23}V_{32}}{V_{22}} - V_{33}\right)\lambda_{be} & 0 \end{pmatrix}$$

- That is, the residual flavour symmetries of the SM restrict us to **two possible patterns**!

$$\{\lambda_{dl}, \lambda_{d\nu}, \lambda_{ul}, \lambda_{u\nu}\} = \begin{cases} \{\lambda_{dl}^{[e]}, \lambda_{d\nu}^{[I]}, \lambda_{ul}^{[I_1]}, \lambda_{u\nu}^{[I_1]}\} \text{ with } \beta_d = \gamma_d = -\alpha_\nu = -\beta_\nu = -\alpha_l = \beta_u = \gamma_u \\ \\ \{\lambda_{dl}^{[e]}, \lambda_{d\nu}^{[I]}, \lambda_{ul}^{[I_2]}, \lambda_{u\nu}^{[I_2]}\} \text{ with } \beta_d = \gamma_d = -\alpha_\nu = -\beta_\nu = -\alpha_l = \alpha_u = \gamma_u \end{cases}$$

- Similar analyses can be performed for the other permissible down-lepton patterns…

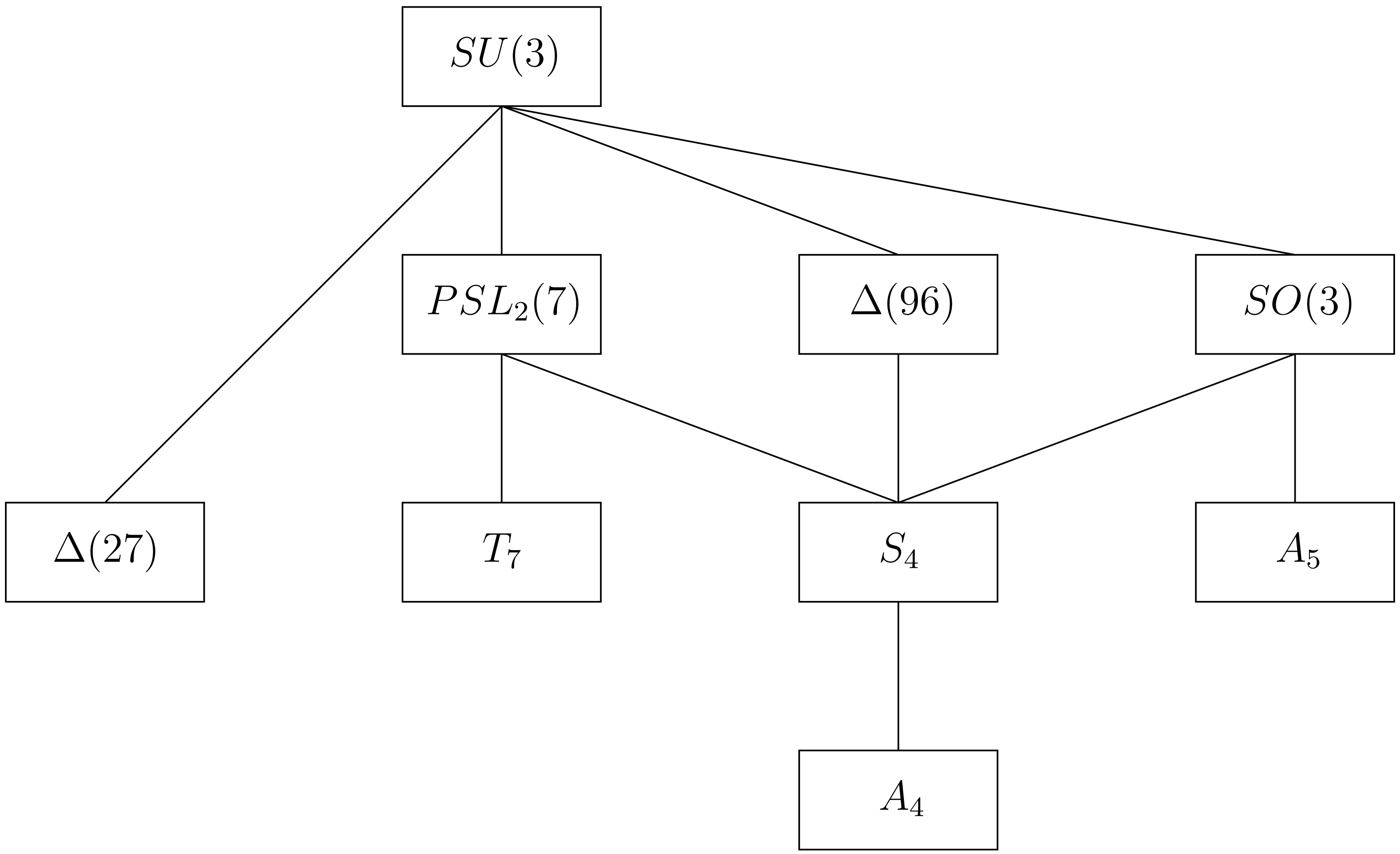
[3] [Guided model building]

[The discrete approach]

Reviews: King, Luhn: hep-ph/1301.1340, Grimus, Ludl: hep-ph/1110.6376, Altarelli, Feruglio: hep-ph/1002.0211
Encyclopedia: Ishimori et al.: hep-ph/1003.3552

All of these symmetries have been explored in models…

Figure from King/Luhn



- $U(1)_{FN}$ symmetries difficult to reconcile with large neutrino mixing -> non-Abelian groups
- Discrete symmetries avoid Goldstone modes that could spoil phenomenology, easily embedded in SUSY GUTs, extra dimensional theories — naturally pumped out of orbifold compactifications!
- Easier facilitation of vacuum alignment than with continuous symmetries
- Huge literature: Pakvasa, Sugawara (1977) use S3 for Cabibbo angle. Deshpande uses S4 for full CKM and Pakvasa applies S4 to neutrino mass and mixing (1984). Early 90s discussion (Kaplan, Schmaltz; Frampton, Kephart), TBM and GUT models established early-mid 00s (Ma, Rajasekaran; Altarelli, Feruglio, de M. Varzielas, King, Ross +), new flood in 2012/13 after reactor angle…

[Building up non-Abelian discrete groups]

- How does the parent symmetry break to different families?

$$\mathcal{G}_{\mathcal{F}} \to \begin{cases} \mathcal{G}_{\mathcal{L}} \to \begin{cases} \mathcal{G}_{\nu} & \mathcal{G}_{\nu} = \mathbb{Z}_2 \times \mathbb{Z}_2 \\ \mathcal{G}_{\mathrm{e}} & \mathcal{G}_{e} = ? \end{cases} \\ \mathcal{G}_{\mathcal{Q}} \to \begin{cases} \mathcal{G}_{\mathrm{u}} \\ \mathcal{G}_{\mathrm{d}} \end{cases} \end{cases}$$

$$\mathcal{L}_{l,\mathrm{mass}} \sim \overline{E}_R m_l l_L + h.c.$$

Invariant under U(1)^3 rotations

$$m_l^\dagger m_l = T^\dagger m_l^\dagger m_l T \qquad T = diag\left(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}\right)$$

Discrete phases yield discrete symmetries: $\phi_i = 2\pi \frac{k_i}{m} \;\Rightarrow\; \mathcal{G}_e = \mathbb{Z}_m \quad m \geq 2, 3$

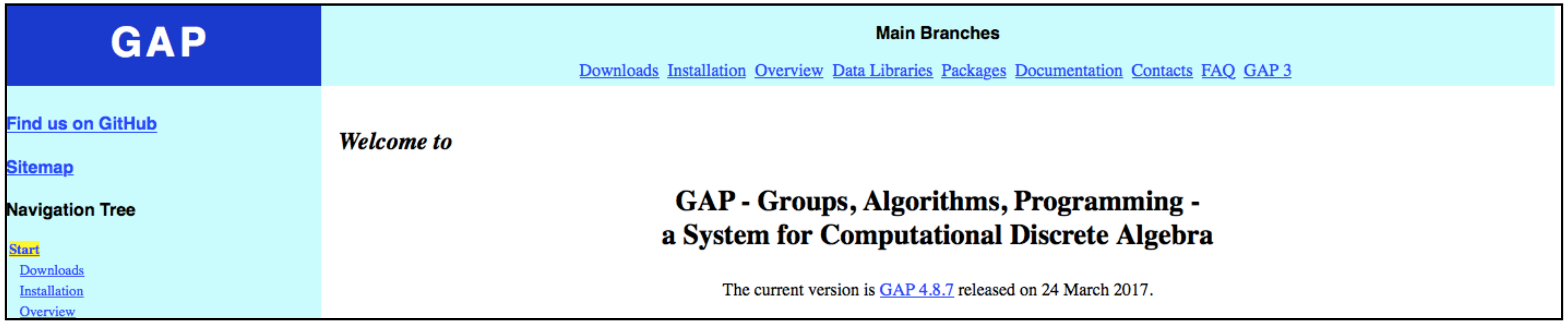
- The same is true for the up and down quark mass matrices.
- Hence the parent group *can* be understood as the closure of the residual generators:

$$\mathcal{G}_{\mathcal{F}} = \{S_{iU}, S_{jU}, T_k\}$$

- Multiple scans have been performed searching for symmetries capable of predicting SM quark and lepton mixing.

Lam: hep-ph/1208.5527
Holthausen, Lim, Linder: hep-ph/1212.2411
Holthausen, Lim: hep-ph/1306.4356
Lavoura, Ludl: hep-ph/1401.5036
JT: hep-ph/1409.7310
Yao, Ding: hep-ph/1505.03798
d.M.Varzielas, Rasmussen, JT: hep-ph/1605.03581
…

[Symmetry scans with GAP]



General Approach and Conclusions

- Assume a structure for residual subgroups and/or mixing
- Use Lagrange's Theorem to sift through finite groups up to a specified order
- Assume parent group a subgroup of SU(3), SU(2), etc.
- Assume certain types of irreducible representations

- For **quarks**, no group has been found to reproduce full CKM mixing exactly.
- For **quarks**, Cabibbo mixing can be (somewhat) realized with simple groups.
- For **leptons**, no group up to order (~10^3) can fully quantize within 1 sigma
- For **leptons**, groups order (~10^2) can quantize 1 column or full matrix within 3 sigma
- **HOWEVER, note that these scans assume a very specific type of symmetry breaking pattern…**
- See talks by Gui-Jun Ding later in the week!

Moving Forward

- What happens if an additional coupling is present, subject to the above flavour symmetries?

[Bottom-up scans: basic algorithm]

- Assign residual symmetries to each fermion sector: $G_a \sim Z_a^{n_a}$

- Discretize free parameters in mixing matrices:

$$\Theta_i \in \{\alpha_j^l, \alpha_j^d, ...\lambda_{de}, \lambda_{b\mu}, ...\}$$

$$\Theta_i \stackrel{!}{=} \frac{n}{m}, \quad \text{with} \quad \{n, m\} \in Integers$$

$$\Theta_i \stackrel{!}{=} \sqrt{\frac{n}{m}}$$

- Apply experimental constraints to discretized parameters:

$$\lambda_{b\mu}\lambda_{s\mu}^* - \lambda_{be}\lambda_{se}^* \simeq 1.1 \frac{M^2}{(35\ \mathrm{TeV})^2}$$

Matrices necessary to go to diagonal LQ basis...

- Form residual symmetry generators:

$$T_{\Delta d} = U_{\lambda\lambda^\dagger}\, T_d\, U_{\lambda\lambda^\dagger}^\dagger, \qquad T_{\Delta l} = U_{\lambda^\dagger\lambda}\, T_l\, U_{\lambda^\dagger\lambda}^\dagger$$

$$T_{\Delta u} = U_{CKM}^\star\, T_u\, U_{CKM}^T, \qquad T_{\Delta\nu} = U_{PMNS}\, T_\nu\, U_{PMNS}^\dagger$$

- Close groups with GAP and apply user-defined preferences:

$$\mathcal{G}_F \sim \{T_{\Delta d}, T_{\Delta l}, T_{\Delta u}, T_{\Delta\nu}\} \qquad \mathcal{G}_F \sim \{T_{\Delta d}, T_{\Delta l}, T_{\Delta\nu}\}$$

$$\mathcal{G}_F \sim \{T_{\Delta d}, T_{\Delta l}, T_{\Delta u}\} \qquad \mathcal{G}_F \sim \{T_{\Delta d}, T_{\Delta l}\}$$

[Isolation patterns for $R_{K(*)}$]

- Let us first consider the lepton isolation pattern allowed by the symmetries:

$$\lambda_{dl}^{[e]} \sim \begin{pmatrix} \lambda_{de} & 0 & 0 \\ \lambda_{se} & 0 & 0 \\ \lambda_{be} & 0 & 0 \end{pmatrix}, \qquad \lambda_{[\mu]} \sim \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ 0 & \lambda_{s\mu} & 0 \\ 0 & \lambda_{b\mu} & 0 \end{pmatrix}$$

- We can translate the experimental constraints from B physics to our parameterization:

$$\rho_d = \lambda_{dl}/\lambda_{bl}, \qquad \rho = \lambda_{sl}/\lambda_{bl} \qquad\qquad \rho_d \leq 0.02, \quad 10^{-4} \leq \rho \leq 1, \quad \rho_d/\rho \leq 1.6$$

- We can also find the matrices acting on down quark and charged lepton fields such that these couplings are diagonalized:

$$U_{\lambda\lambda^\dagger} = \begin{pmatrix} -\frac{1}{\sqrt{1+\rho_d^2}} & \frac{\rho_d}{\sqrt{1+\rho^2+\rho_d^2}} & -\frac{\rho\,\rho_d}{\sqrt{1+\rho_d^2}\sqrt{1+\rho^2+\rho_d^2}} \\ 0 & \frac{\rho}{\sqrt{1+\rho^2+\rho_d^2}} & \frac{\sqrt{1+\rho_d^2}}{\sqrt{1+\rho^2+\rho_d^2}} \\ \frac{\rho_d}{\sqrt{1+\rho_d^2}} & \frac{1}{\sqrt{1+\rho^2+\rho_d^2}} & -\frac{\rho}{\sqrt{1+\rho_d^2}\sqrt{1+\rho^2+\rho_d^2}} \end{pmatrix}, \quad U_{\lambda^\dagger\lambda} = \begin{pmatrix} e^{2\pi i/a} & 0 & 0 \\ 0 & e^{2\pi i/b} & 0 \\ 0 & 0 & e^{2\pi i/c} \end{pmatrix}$$

- These then feed directly into our residual symmetry generators:

$$T_{\Delta d} = U_{\lambda\lambda^\dagger}\, T_d\, U_{\lambda\lambda^\dagger}^\dagger, \qquad\qquad T_{\Delta l} = U_{\lambda^\dagger\lambda}\, T_l\, U_{\lambda^\dagger\lambda}^\dagger$$

[Non-Abelian finite groups for $R_{K(*)}$]

- For illustration, we close groups capable of quantizing LQ couplings and TBM leptonic mixing:

$$U_{PMNS} \simeq U_{TBM} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Electron Isolation and TBM Lepton Mixing					
$\{y, x\}$	T_l^{ii}	T_d^{ii}	T_ν^{ii}	GAP-ID	Group Structure
$\{1, 1\}$	[-1, 1, -1]	[1, -1, -1]	[-1, -1, 1]	[24, 12]	S_4
$\{1, 1\}$	[-1, i, i]	[1, -1, -1]	[-1, -1, 1]	[96, 64]	$\Delta(96)$
$\{0, 1\}$	[-1, 1, -1]	[1, -1, -1]	[-1, -1, 1]	[8, 3]	D_8
$\{0, 2\}$	[-1, 1, i]	[1, -1, -1]	[-1, -1, 1]	[32, 11]	$\Sigma(32)$
...	...	...	...	...	...

- Similar results for LQ couplings and Cabibbo mixing:

Preliminary!!

$$U_{CKM} \simeq U_C \equiv \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electron Isolation and Cabibbo Mixing					
$\{y, x\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	GAP-ID	Group Structure
$\{1, 1\}$	[-1, 1, -1]	[1, -1, -1]	[1, -1, -1]	[56, 5]	D_{56}
$\{0, 1\}$	[-1, 1, -1]	[1, -1, -1]	[1, -1, -1]	[56, 12]	$Z_2 \times Z_2 \times D_{14}$
$\{0, 1\}$	[-1, 1, -1]	[1, -1, -1]	[1, -1, -1]	[28, 3]	D_{28}
$\{0, 1\}$	[i, -i, i]	[i, -i, -i]	[i, -i, -i]	[56, 6]	$Z_2 \times (Z_7 \rtimes Z_4)$
...	...	...	...	...	...

- Scans for *both* quark and lepton mixing also possible (work in progress…)

[Conclusions]

- Representing the action of global SM flavour symmetries via residual group generators, one constrains Yukawa couplings and thereby predicts flavored relationships.
- The presence of additional leptoquark couplings subject to these symmetries severely restricts the number of predictive patterns allowed.
- We have derived the allowed leptoquark Yukawa textures in a special case, which will have clear experimental signatures in other precision observables.
- Finally, we have shown how such residual symmetry constraints could originate from the breakdown of a non-Abelian discrete symmetry, and further discovered a host of predictive finite groups via a bottom-up numerical scan.

Moving Forward

- Derive full set of allowed leptoquark Yukawa textures for each permissible d-l pattern.
- Explore the phenomenological signatures of these constraints in other observables.
- Perform a more exhaustive numerical scan of predictive non-Abelian finite groups.

[Thanks!]