

# Theory of the Inclusive Decays $\bar{B} \rightarrow X_{s,d} \ell \ell$

TOBIAS HURTH    Johannes Gutenberg University Mainz



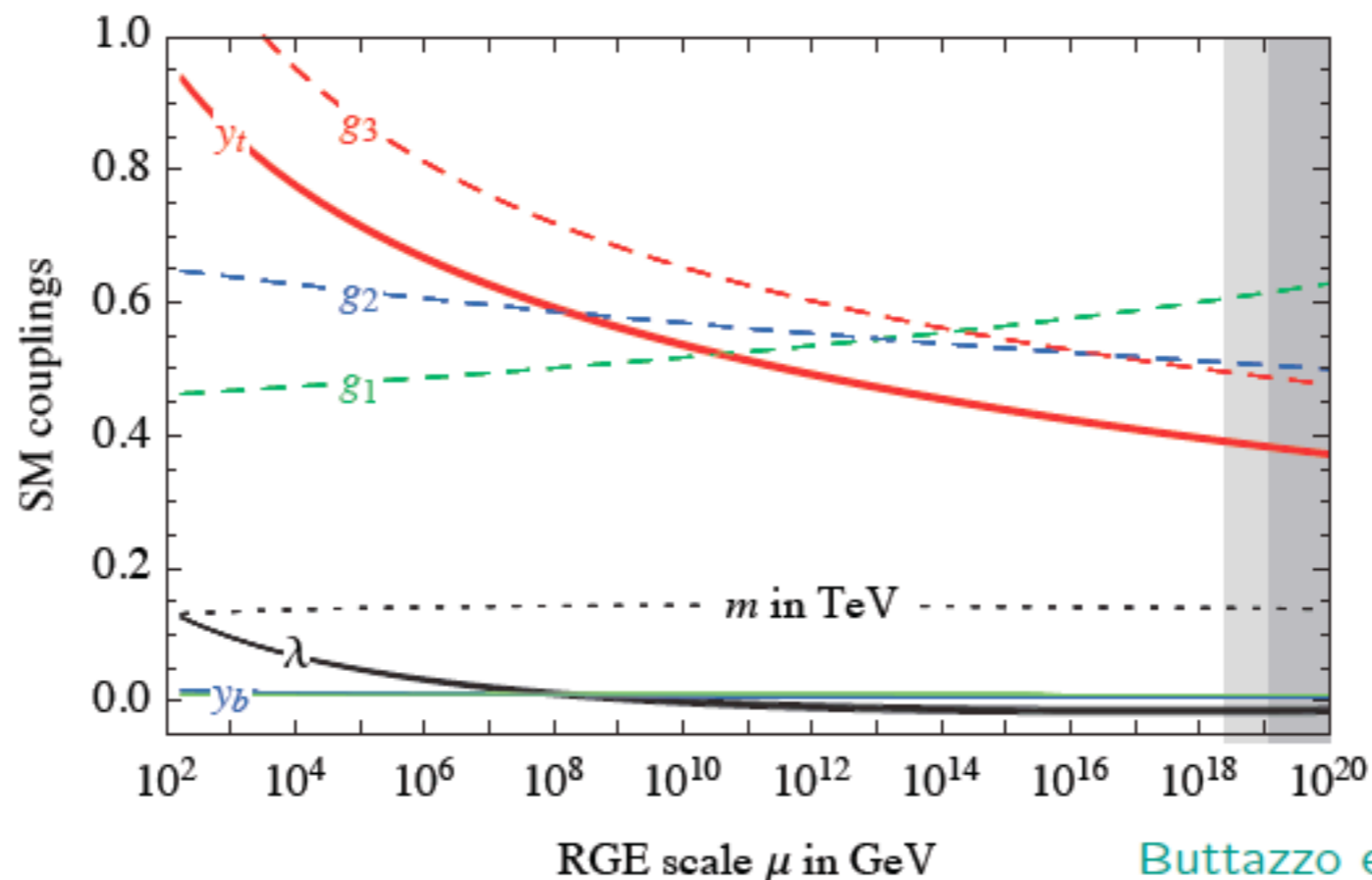
FLASY 2018: 7th workshop on flavour symmetries and consequences on accelerators and cosmology, Basel 2-5.7.

# Prologue

# Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the  $M_P$  as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles!).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

# Experimental evidence beyond SM

- **Dark matter** (visible matter accounts for only 4% of the Universe)
- **Neutrino masses** (Dirac or Majorana masses ?)
- **Baryon asymmetry of the Universe** (new sources of CP violation needed)

## Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

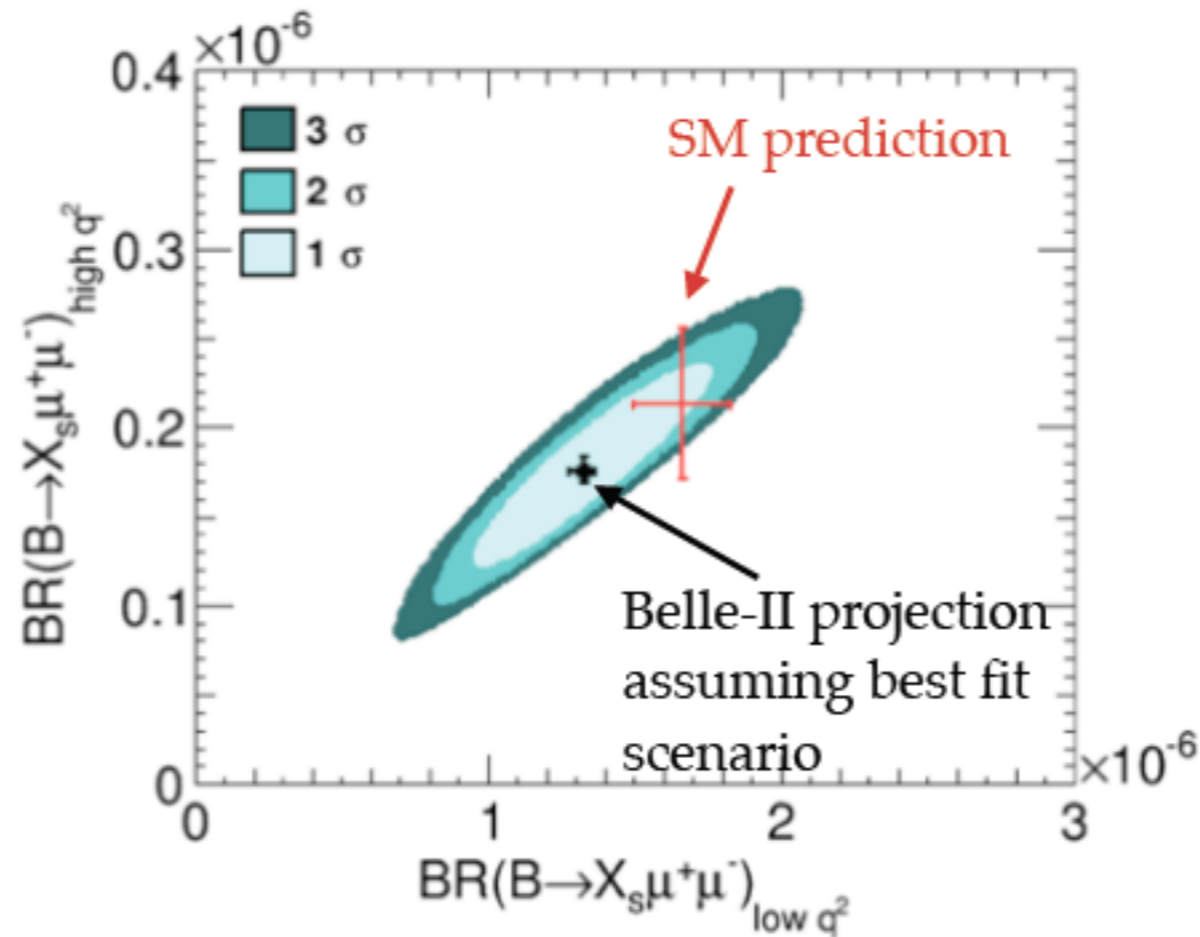
**Motivation: “From Flavour to New Physics”**



# Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545

if SM deviations in  $R_K$  and  $P'_5$  persist until Belle-II



If NP then the effect of  $C_9$  and  $C'_9$  are large enough to be checked at Belle-II with theoretically clean modes.

Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

# Global fit to 108 $b \rightarrow s$ observable with 20 operators

Arby, Hurth, Mahmoudi, Neshatpour arXiv: 1806.02791

20 Wilson coefficients sensitive to NP:  $C_7, C_8, C_9^\ell, C_{10}^\ell, C_{Q_1}^\ell, C_{Q_2}^\ell$

→ 10 independent WC (considering  $\ell = e, \mu$ ) + 10 primed

Set of WC	Nr. parameters	$\chi_{\min}^2$	Pull <sub>SM</sub>	Improvement
SM	0	118.8	-	-
$C_9^\mu$	1	85.1	$5.8\sigma$	$5.8\sigma$
$C_9^{(e,\mu)}$	2	83.9	$5.6\sigma$	$1.1\sigma$
$C_7, C_8, C_9^{(e,\mu)}, C_{10}^{(e,\mu)}$	6	81.2	$4.8\sigma$	$0.5\sigma$
All non-primed WC	10 (8)	81.0	$4.1 (4.5)\sigma$	$0.0 (0.1)\sigma$
All WC (incl. primed)	20 (16)	70.2	$3.6 (4.1)\sigma$	$0.9 (1.2)\sigma$

- No real improvement in the fits when going beyond the  $C_9^\mu$  case
- Pull with the SM decreases when all WC are varied

**NP significance of  $5.8\sigma$  in  $C_9^\mu$**

**16 effective degrees of freedom ( $C_{Q_{1/2}}^{e(\prime)}$  taken out)**

**NP significance of  $4.1\sigma$  in the global fit based on the assumption of 10% error for power corrections**

## Error of Branching ratio $\bar{B} \rightarrow X_s \ell^+ \ell^-$

$BF$ (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
$\geq 14.4$	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

## Error of Normalized Forward-Backward-Asymmetry

$AFB_n$ (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
$\geq 14.4$	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

$B \rightarrow (\pi, \rho) \ell^+ \ell^-$ , semi-inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  at 50/ab  
 (uncertainties like  $\bar{B} \rightarrow X_s \ell^+ \ell^-$  at 0.7/ab)

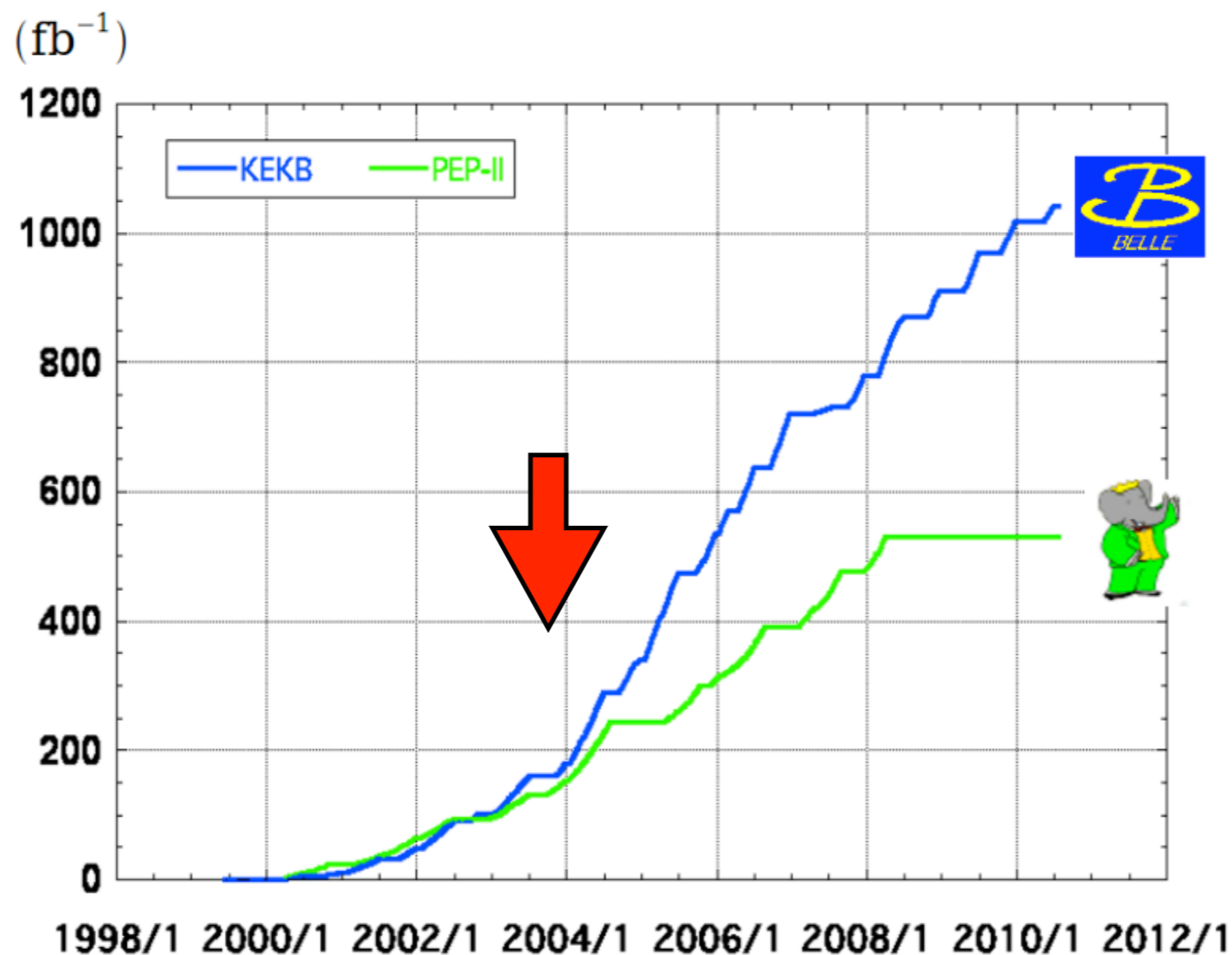


# Experiment

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based  $152 \times 10^6 B\bar{B}$  events)

## Integrated luminosity of B factories



**> 1  $\text{ab}^{-1}$**

**On resonance:**

$\Upsilon(5S)$ : 121  $\text{fb}^{-1}$

$\Upsilon(4S)$ : 711  $\text{fb}^{-1}$

$\Upsilon(3S)$ : 3  $\text{fb}^{-1}$

$\Upsilon(2S)$ : 25  $\text{fb}^{-1}$

$\Upsilon(1S)$ : 6  $\text{fb}^{-1}$

**Off reson./scan:**

$\sim 100 \text{ fb}^{-1}$

**$\sim 550 \text{ fb}^{-1}$**

**On resonance:**

$\Upsilon(4S)$ : 433  $\text{fb}^{-1}$

$\Upsilon(3S)$ : 30  $\text{fb}^{-1}$

$\Upsilon(2S)$ : 14  $\text{fb}^{-1}$

**Off resonance:**

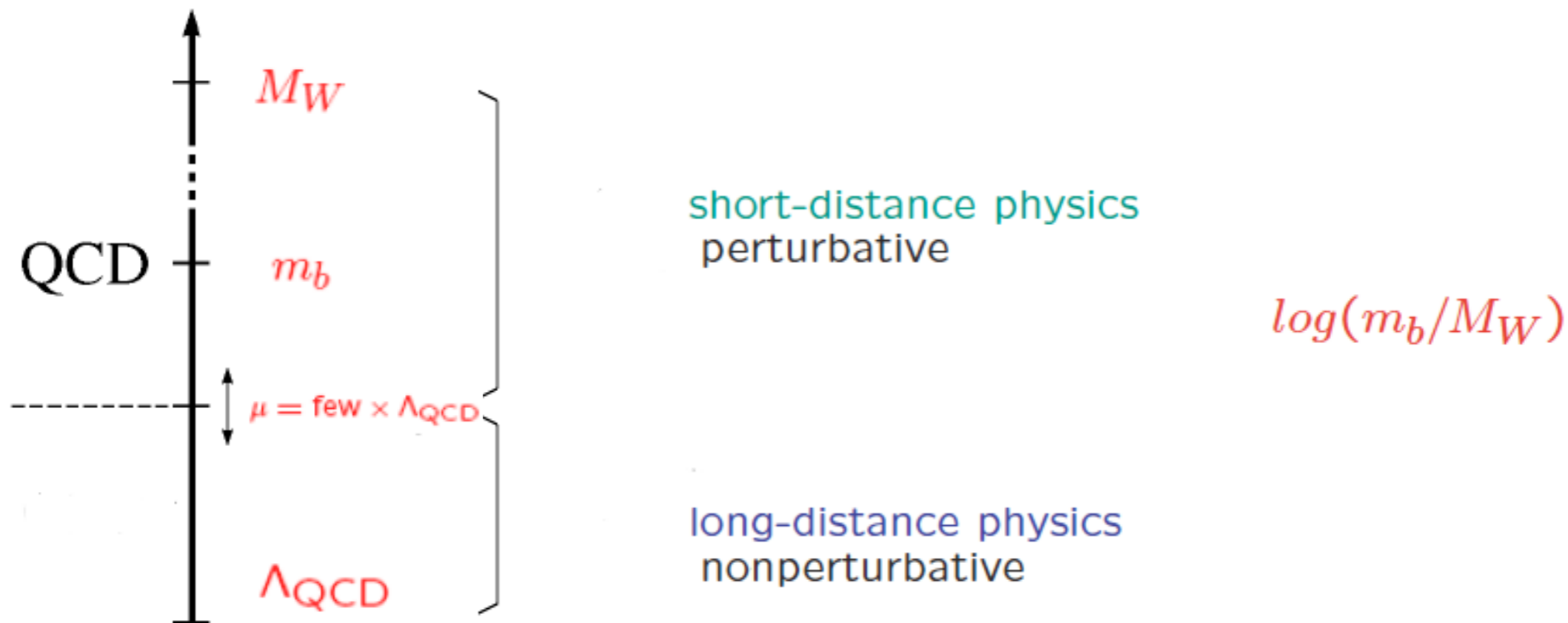
$\sim 54 \text{ fb}^{-1}$

New Babar analysis on dilepton spectrum arXiv:1312.3664

New Belle analysis on AFB arXiv:1402.7134

# Theoretical Tools

# Theoretical tools for flavour precision observables



## Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian:  $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 \gg M_W^2$ : 'new physics' effects:  $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements  $\mathcal{O}_i(\mu = m_b)$  ?

## Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$

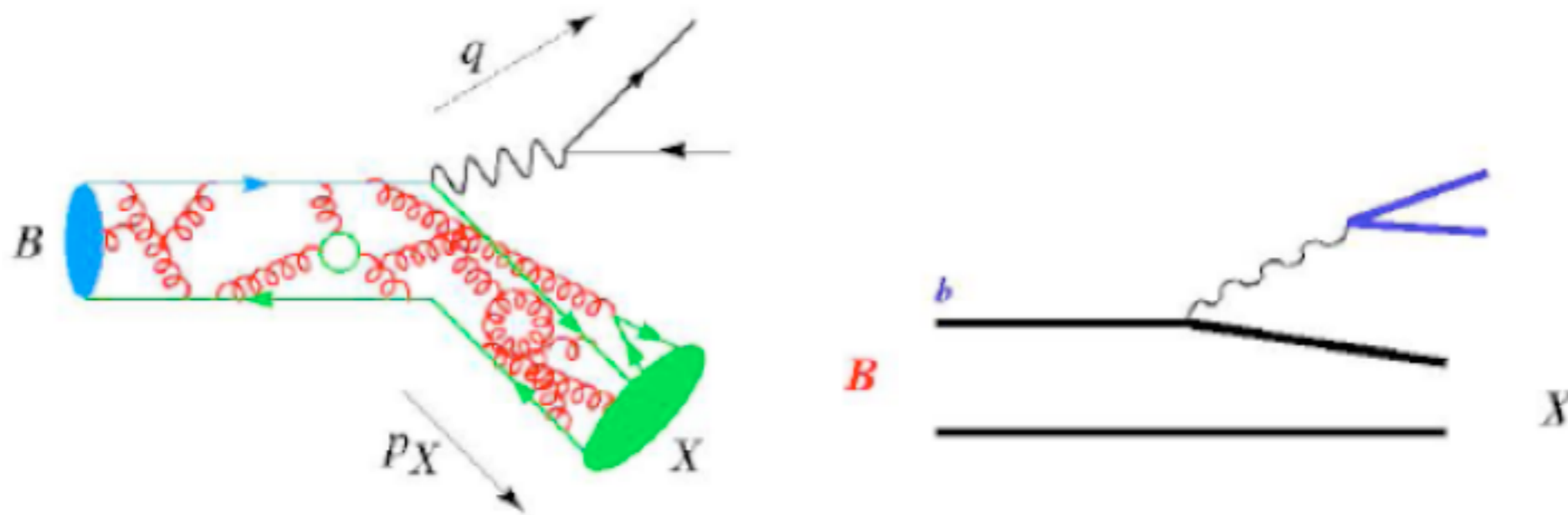
How to compute the hadronic matrix elements  $\mathcal{O}_i(\mu = m_b)$  ?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) \xrightarrow{m_b \rightarrow \infty} \Gamma(b \rightarrow X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \Lambda_{QCD}^2 / m_b^2$$

No linear term  $\Lambda_{QCD}/m_b$  (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



## Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$

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### An old story:

- If one goes beyond the leading operator ( $\mathcal{O}_7, \mathcal{O}_9$ ):  
breakdown of local expansion

### A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012](#)



Analysis in  $B \rightarrow X_s l l$  in this talk; [Benzke, Fickinger, Hurth, Turczyk](#)

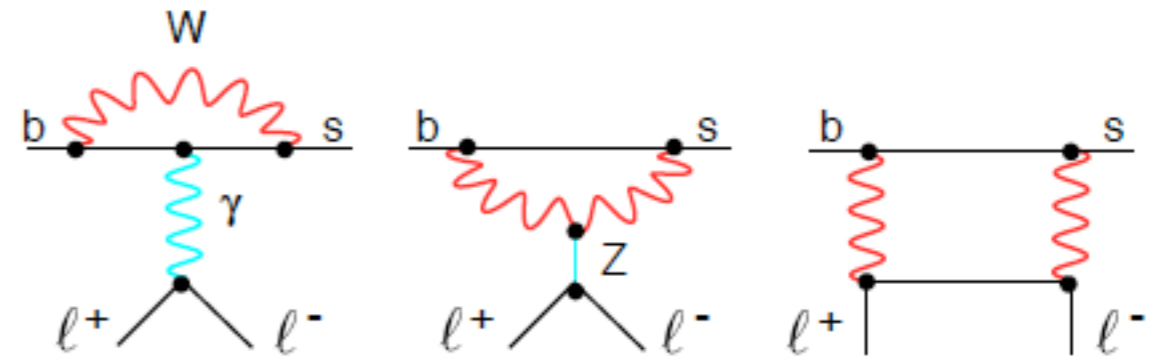
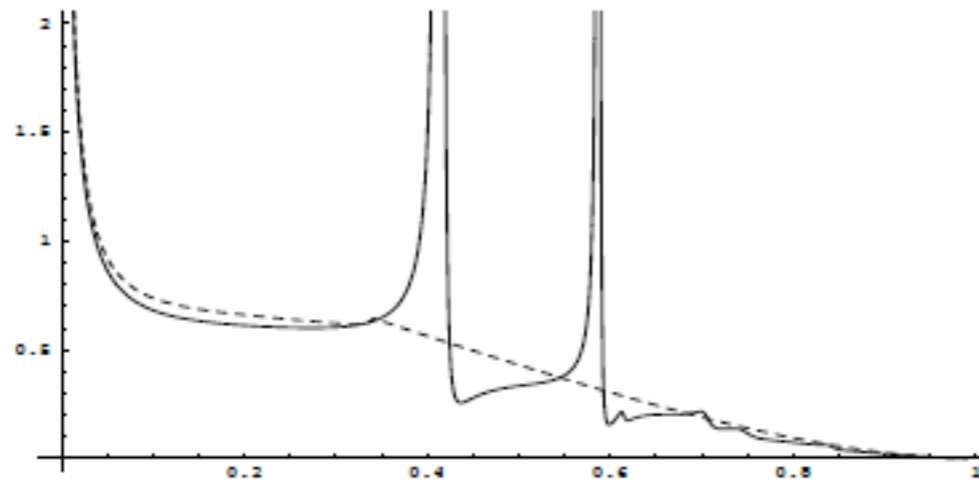


# Perturbative contributions

# Review of previous calculations for $B \rightarrow X_s ll$

- On-shell- $c\bar{c}$ -resonances  $\Rightarrow$  cuts in dilepton mass spectrum necessary :  
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$  and  $14.4\text{GeV}^2 < q^2 \Rightarrow$  perturbative contributions dominant

$$\frac{d}{d\hat{s}} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



$$\hat{s} = q^2/m_b^2$$

- NNLL prediction of  $\bar{B} \rightarrow X_s l^+ l^-$ : dilepton mass spectrum  
 Asatryan, Asatrian, Greub, Walker, hep-ph/0204341  
 Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \rightarrow X_s l^+ l^-)_{\text{Cut: } q^2 > 14.4\text{GeV}^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections  $q^2 \in [1\text{GeV}^2, 6\text{GeV}^2]$

central value:  $-14\%$ , perturbative error:  $13\% \rightarrow 6.5\%$

- Further refinements:
  - Completing NNLL QCD corrections:  
Mixing into  $\mathcal{O}_9$  (+1%), NNLL matrixelement of  $\mathcal{O}_9$  (-4%)
  - NLL QED two-loop corrections to Wilson coefficients  
-1.5% shift for  $\alpha_{em}(\mu = m_b)$ , -8.5% for  $\alpha_{em}(\mu = m_W)$   
Bobeth, Gambino, Gorbahn, Haisch, hep-ph/0312090
  - QED two-loop corrections to matrix elements  
Large collinear logarithm  $Log(m_b/m_\ell)$  which survive integration  
if a restricted part of the dilepton mass spectrum is considered  
+2% effect in the low- $q^2$  region for muons, for the electrons  
the effect depends on the experimental cut parameters  
Huber, Lunghi, Misiak, Wyler, hep-ph/0512066  
Huber, Hurth, Lunghi, Nucl.Phys.B802(2008)40
- NNLL prediction of  $\bar{B} \rightarrow X_s \ell^+ \ell^-$ : forward-backward-asymmetry (FBA)  
Asatrian, Bieri, Greub, Hovhannisyann, hep-ph/0209006  
Ghinculov, Hurth, Isidori, Yao, hep-ph/0208088, hep-ph/0312128 :

$$A_{FB} \equiv \frac{1}{\Gamma_{semilep}} \left( \int_0^1 d(\cos \theta) \frac{d^2\Gamma}{dq^2 d \cos \theta} - \int_{-1}^0 d(\cos \theta) \frac{d^2\Gamma}{dq^2 d \cos \theta} \right)$$

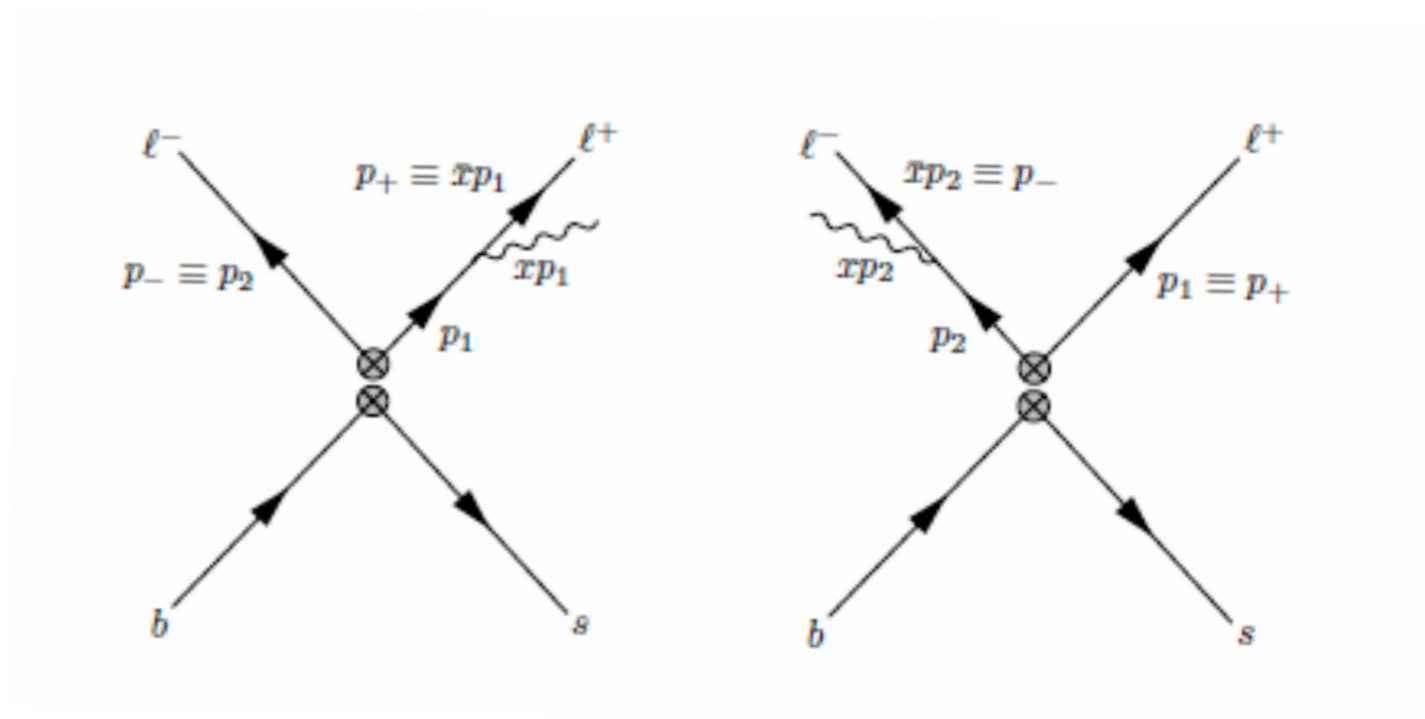
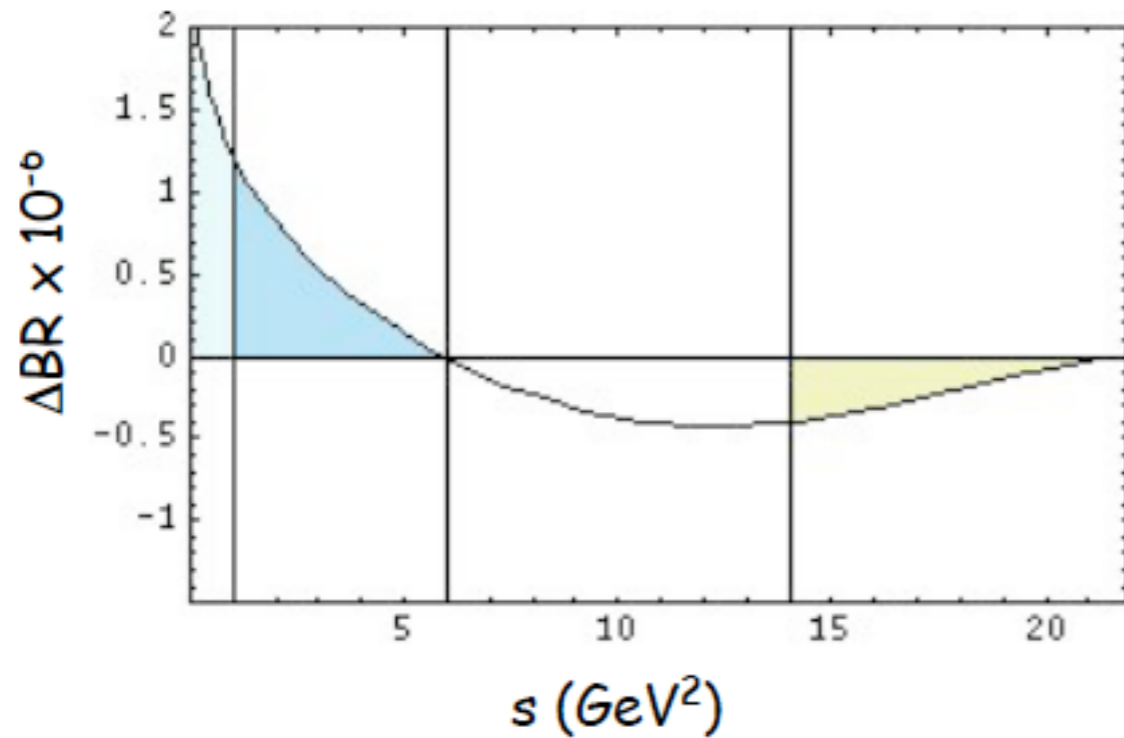
( $\theta$  angle between  $\ell^+$  and  $B$  momenta in dilepton CMS)

$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) GeV^2$$

- Electromagnetic corrections in high- $q^2$  and for  $A_{FB}$   
Huber, Hurth, Lunghi, Nucl. Phys. B802(2008)40

Corrections to matrix elements lead to large collinear log  $\text{Log}(m_b/m_\ell)$

$$\delta\text{BR}(B \rightarrow X_s \mu^+ \mu^-) = \begin{cases} (+2.0\%) & \text{low } q^2 \\ (-6.8\%) & \text{high } q^2 \end{cases} \quad \delta\text{BR}(B \rightarrow X_s e^+ e^-) = \begin{cases} (+5.2\%) & \text{low } q^2 \\ (-17.6\%) & \text{high } q^2 \end{cases}$$



⋮

# Complete angular analysis of inclusive $B \rightarrow X_s l l$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} [(1+z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2)] \quad (z = \cos \theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)$$

$$\frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

- Dependence on Wilson coefficients

$H_T$  suppressed in low- $q^2$  window

$$H_T(q^2) \propto 2s(1-s)^2 \left[ \left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$

$$H_A(q^2) \propto -4s(1-s)^2 \text{Re} \left[ C_{10} \left( C_9 + \frac{2}{s} C_7 \right) \right]$$

$$H_L(q^2) \propto (1-s)^2 \left[ |C_9 + 2 C_7|^2 + |C_{10}|^2 \right]$$

- Devide low- $q^2$  bin in two bins (zero of  $H_A$  in low- $q^2$ )

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156



# New physics sensitivity

Huber, Hurth, Lunghi, arXiv:1503.04849

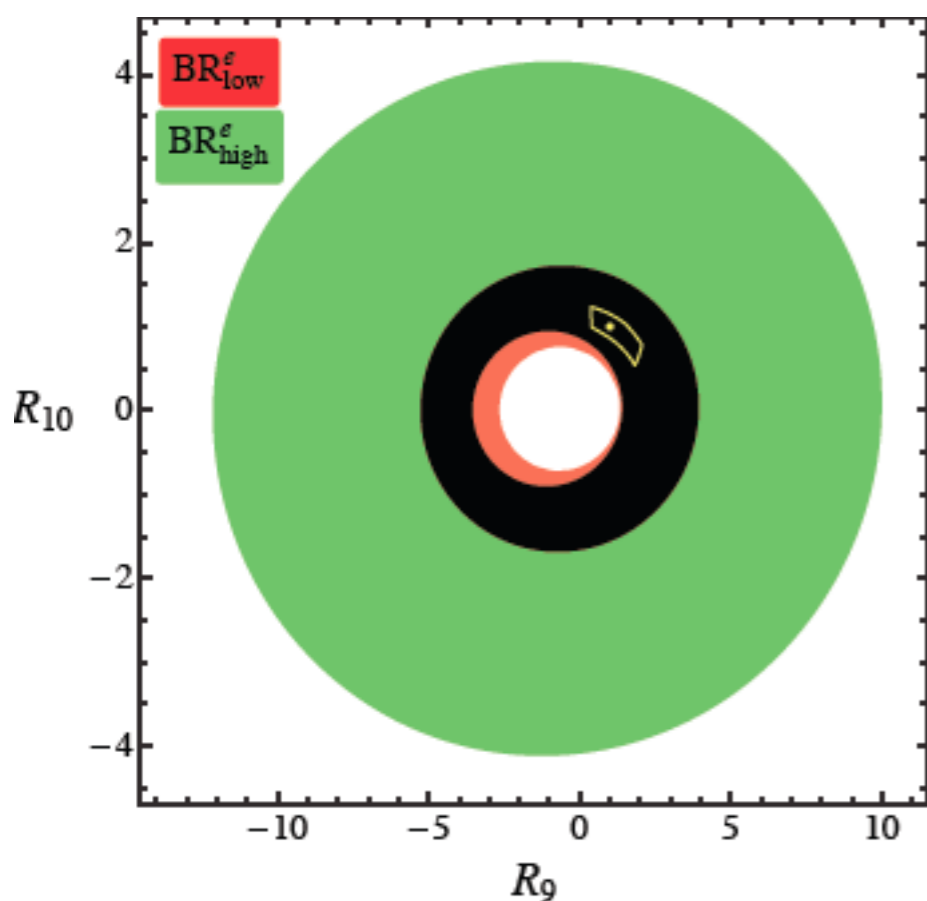
Constraints on Wilson coefficients  $C_9/C_9^{\text{SM}}$  and  $C_{10}/C_{10}^{\text{SM}}$

$$R_i = \frac{C_i(\mu_0)}{C_i^{\text{SM}}(\mu_0)}$$

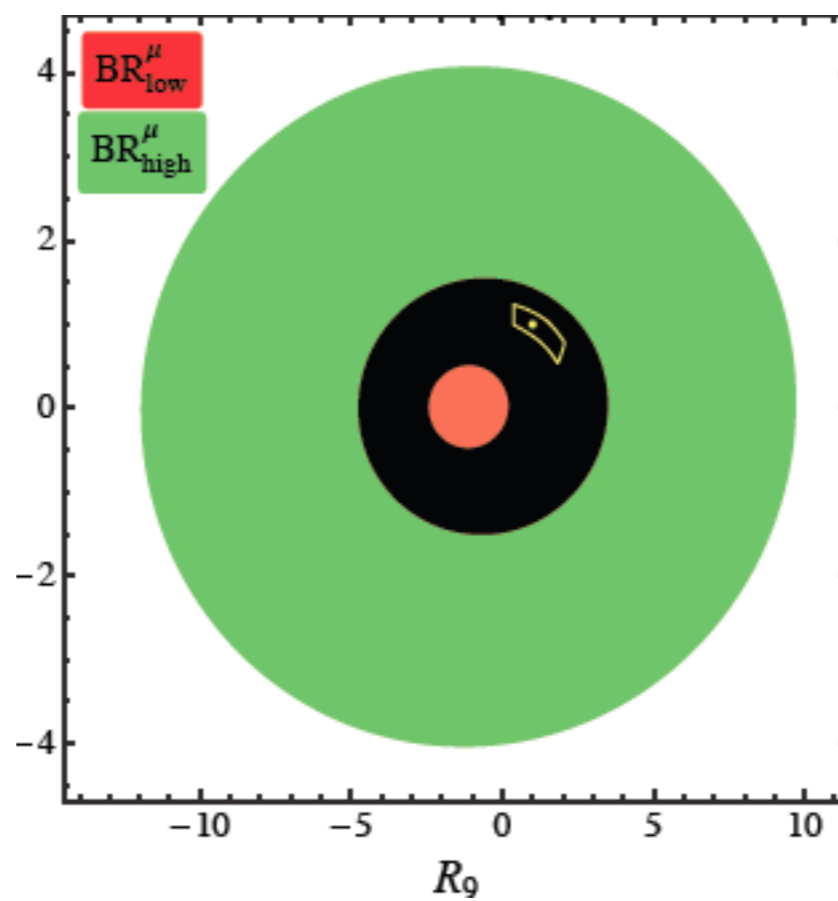
that we obtain at 95% C.L. from present experimental data  
(red low  $q^2$ , green high  $q^2$ )

that we will obtain at 95% C.L. from  $50ab^{-1}$  data at Belle-II  
(yellow)

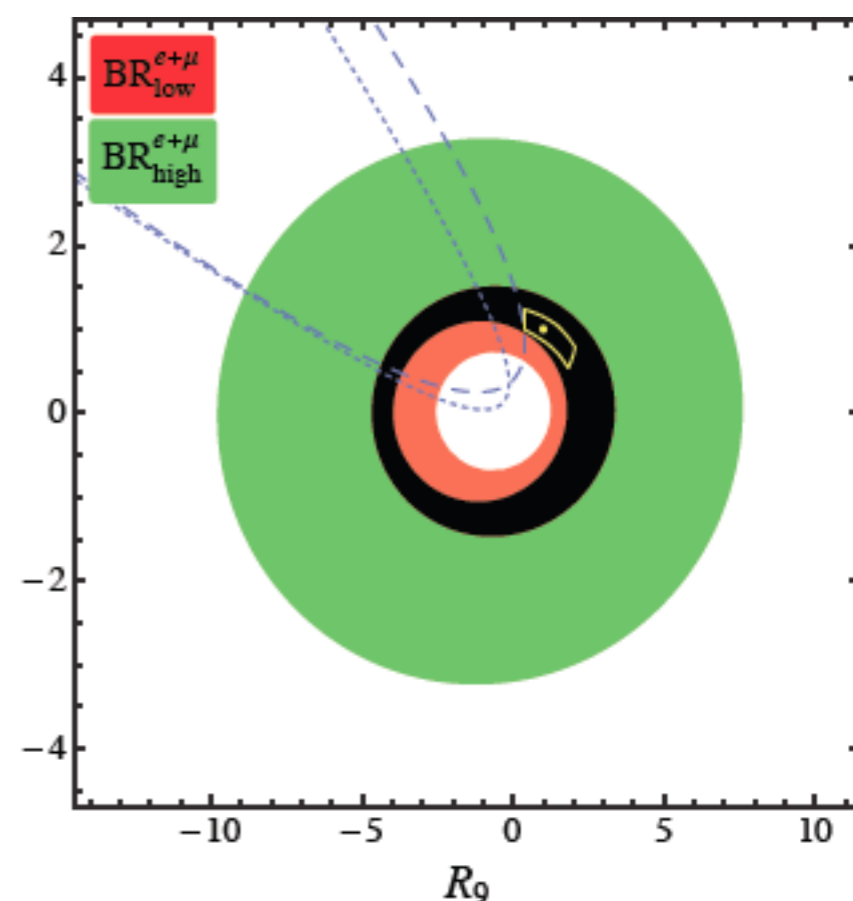
$B \rightarrow X_s e e$



$B \rightarrow X_s \mu \mu$



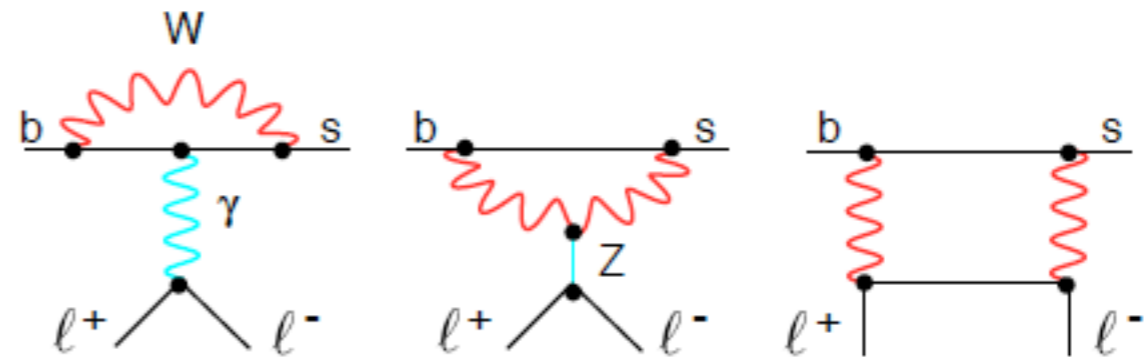
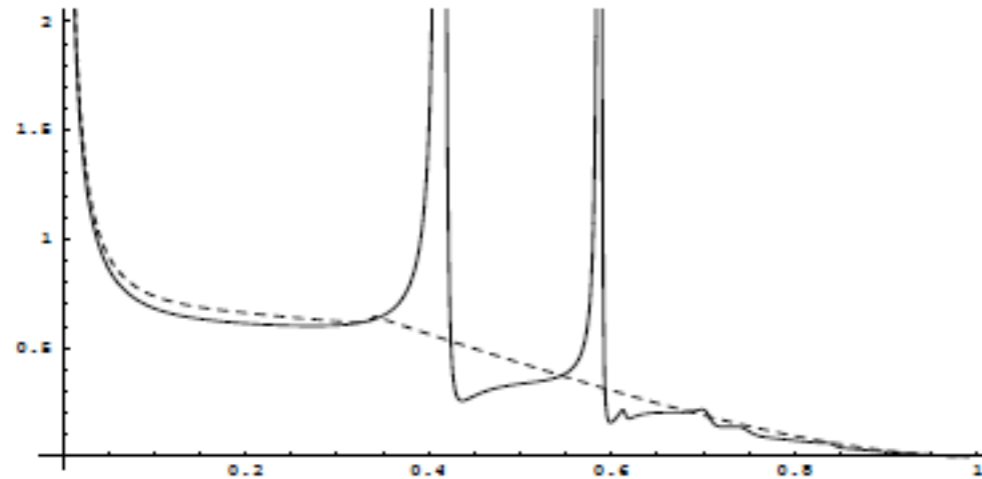
$B \rightarrow X_s l l$



# Cuts in the dilepton and hadronic mass spectra

- On-shell- $c\bar{c}$ -resonances  $\Rightarrow$  cuts in dilepton mass spectrum necessary :  
 $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$  and  $14.4\text{GeV}^2 < q^2 \Rightarrow$  perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s l^+ l^-) \times 10^{-5}$$



- Hadronic invariant-mass cut is imposed in order to eliminate the background like  $b \rightarrow c (\rightarrow se^+\nu) e^-\bar{\nu} = b \rightarrow se^+e^- +$  missing energy
  - \* Babar, Belle:  $m_X < 1.8$  or  $2.0\text{GeV}$
  - \* high- $q^2$  region not affected by this cut
  - \* kinematics:  $X_s$  is jetlike and  $m_X^2 \leq m_b \Lambda_{QCD} \Rightarrow$  shape function region
  - \* SCET analysis: universality of jet and shape functions found:  
 the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the  $\bar{B} \rightarrow X_s \gamma$  shape function  
 5% additional uncertainty for  $2.0\text{GeV}$  cut due to subleading shape functions

Lee, Stewart hep-ph/0511334

Lee, Ligeti, Stewart, Tackmann hep-ph/0512191

Lee, Tackmann arXiv:0812.0001 (effect of subleading shape functions)

Bell, Beneke, Huber, Li arXiv:1007.3758 (NNLO matching QCD  $\rightarrow$  SCET)

# Nonlocal subleading contributions

# Subleading power factorization in $B \rightarrow X_s l^+ l^-$

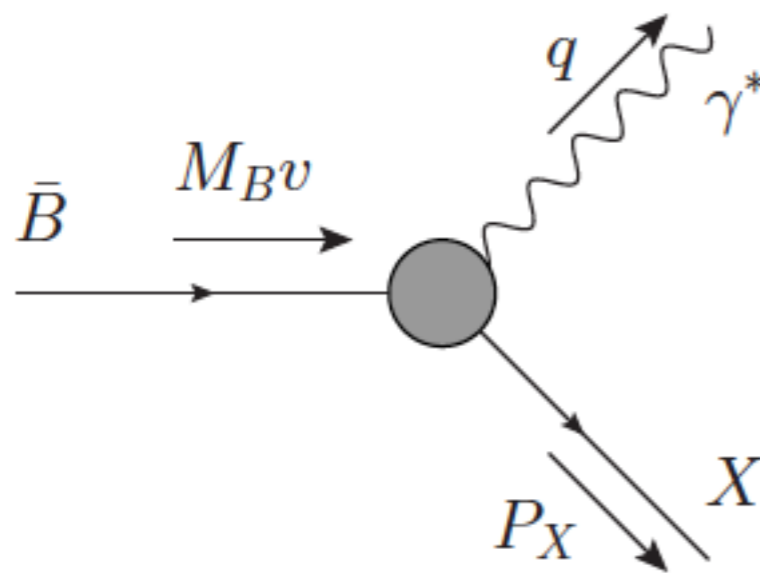
Benzke, Hurth, Turczyk, arXiv:1705.10366

## Hadronic cut

Additional cut in  $X_s$  necessary to reduce background affects only low- $q^2$  region.

Hadronic invariant  $m_X^2 < 1.8(2.0) \text{GeV}^2$

Multiscale problem  $\rightarrow$  SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\text{QCD}} m_b \gg \Lambda_{\text{QCD}}^2$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

**Scaling**

$$\lambda = \Lambda_{\text{QCD}}/m_b$$

## Kinematics

$B$  meson rest frame

$$q = p_B - p_X \quad 2 m_B E_X = m_B^2 + M_X^2 - q^2$$

$X_s$  system is jet-like with  $E_X \sim m_B$  and  $m_X^2 \ll E_X^2$

two light-cone components  $p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

$$q^+ = n q = m_B - p_X^+, \quad q^- = \bar{n} q = m_B - p_X^-$$



## Scaling

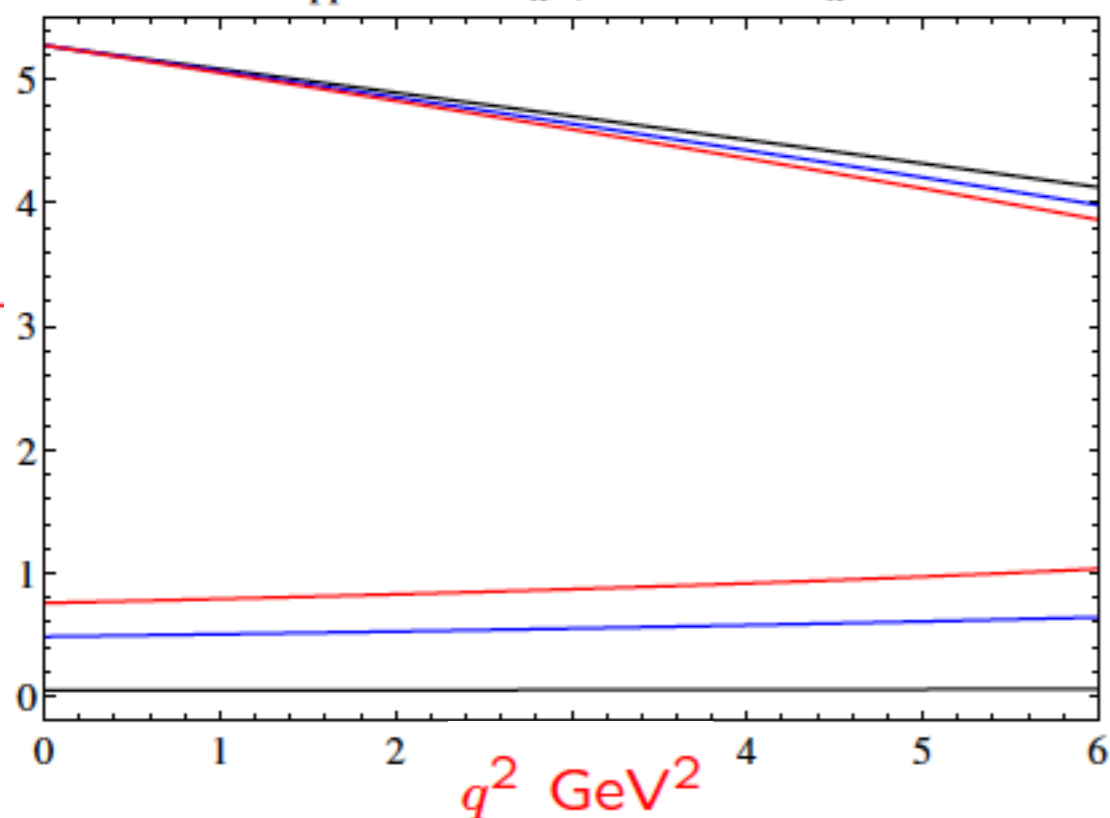
$$\lambda = \Lambda_{\text{QCD}}/m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

$M_X = [0.5, 1.6, 2]$  GeV [Black, Blue, Red]

Upper lines :  $P_X^-$ , lower lines :  $P_X^+$

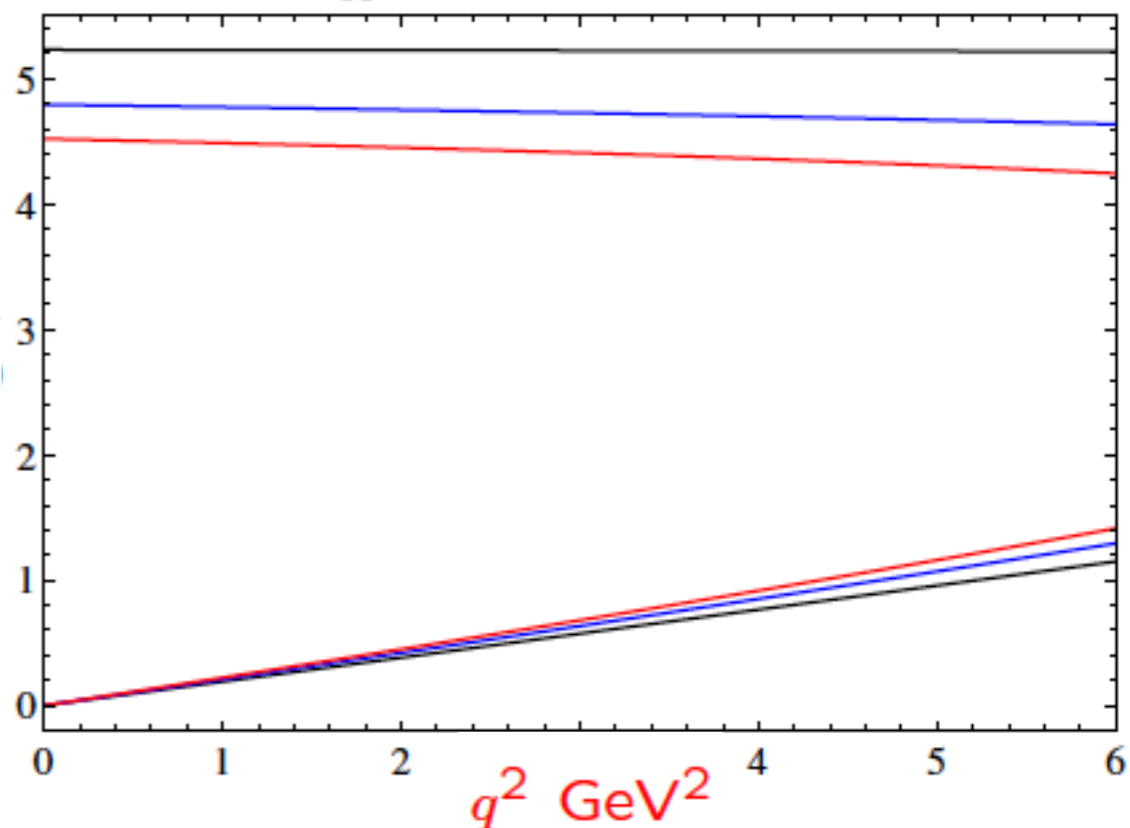
$P_X^-/+$   
GeV



$M_X = [0.5, 1.6, 2]$  GeV [Black, Blue, Red]

Upper lines :  $q^+$ , lower lines :  $q^-$

$q^+/-$   
GeV



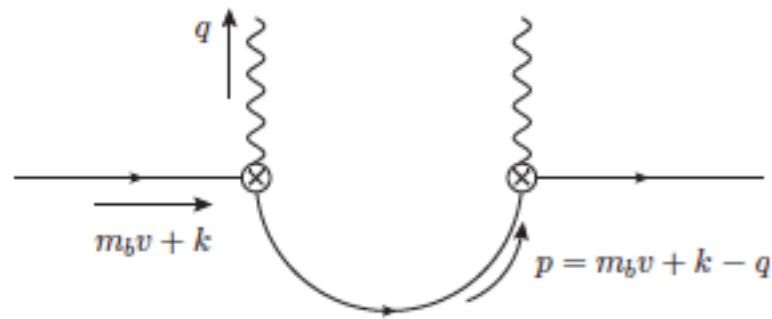
For  $q^2 < 6 \text{ GeV}^2$  the scaling of  $np_X$  and  $\bar{n}p_X$  implies  $\bar{n}q$  is of order  $\lambda$ , means  $q$  anti-hard-collinear (just kinematics).

Stewart and Lee assume  $\bar{n}q$  to be order 1, means  $q$  is hard.

This problematic assumption implies a different matching of SCET/QCD.

## Shapefunction region

Local OPE breaks down for  $m_X^2 \sim \lambda$ :



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left( 1 - \frac{n \cdot k}{m_b - n \cdot q} + \dots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of  $\bar{n}q$  does not matter here; zero in case of  $B \rightarrow X_s \gamma$ )

**Factorization theorem**  $d\Gamma \sim H \cdot J \otimes S$

The hard function  $H$  and the jet function  $J$  are perturbative quantities.

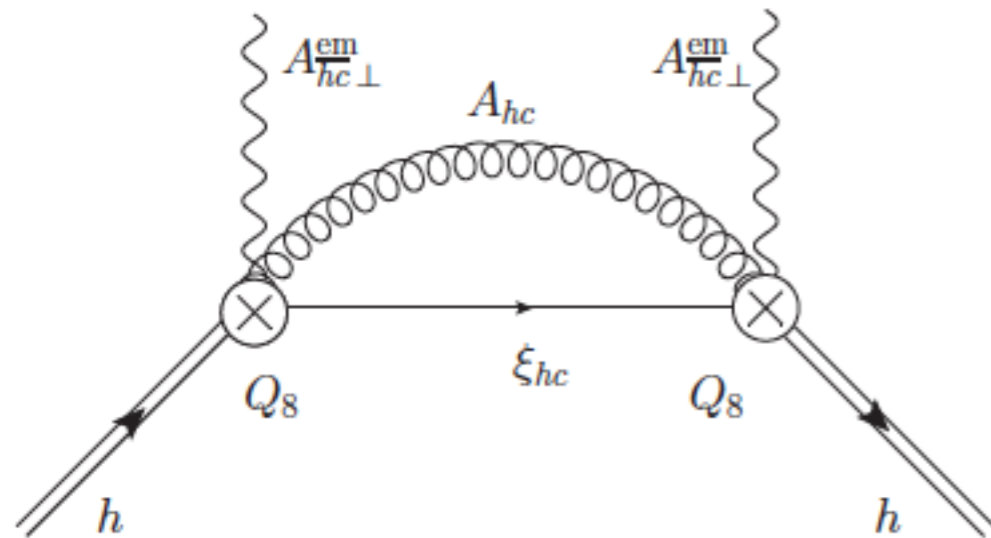
The shape function  $S$  is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

# Calculation at subleading power

Example of **direct** photon contribution which factorizes

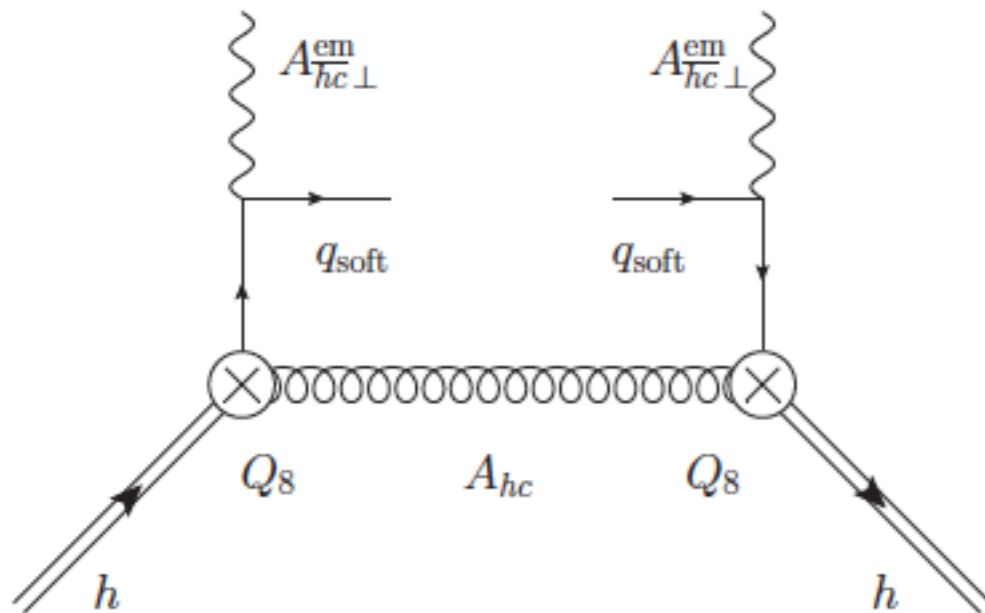
$$d\Gamma \sim H \cdot j \otimes S$$



$\rightarrow \frac{\alpha_s}{m_b}$  in low  $m_\chi^2$  region

Example of **resolved** photon contribution (double-resolved) which factorizes

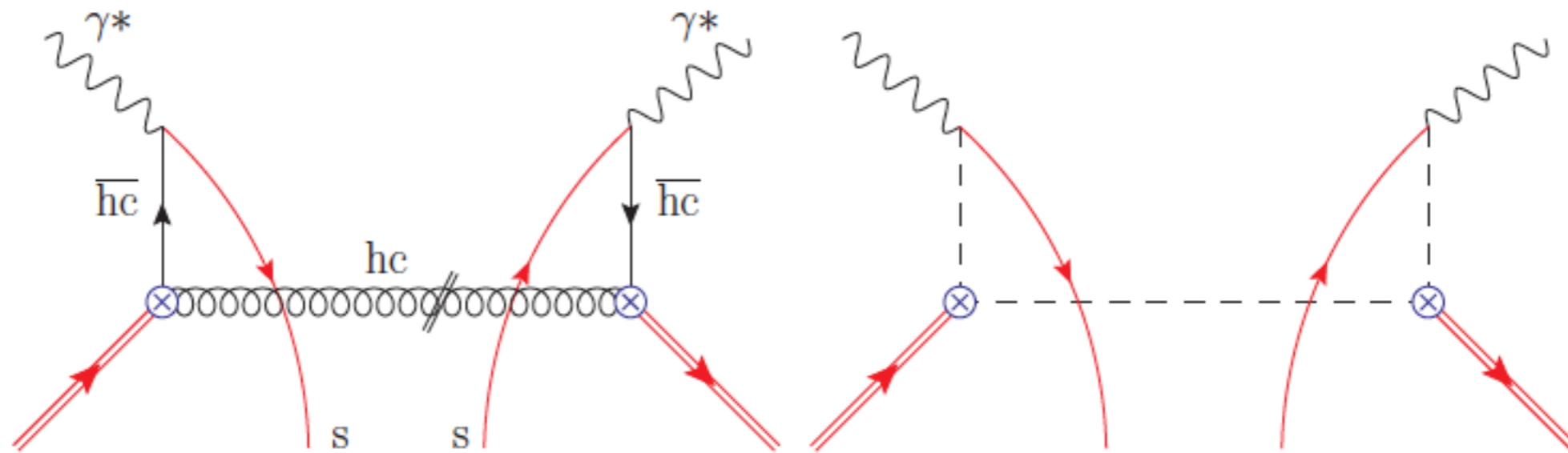
$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

## Interference of $Q_8$ and $\bar{Q}_8$



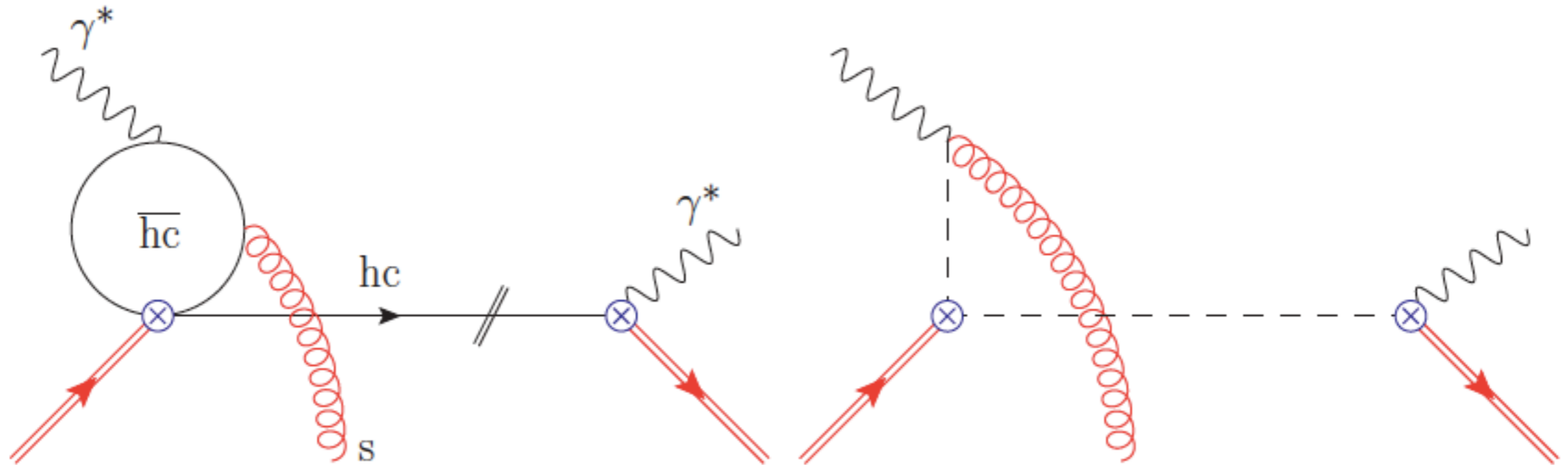
$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{n}) \bar{s}(\mathbf{r}\bar{n}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

Shape function is non-local in two light-cone directions.

It survives  $M_X \rightarrow 1$  limit (irreducible uncertainty).

## Interference of $Q_1$ and $Q_7$



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\epsilon}$$

$$\frac{1}{\omega_1} \left[ \bar{n} \cdot q \left( F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left( F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right.$$

$$\left. + \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) \right] g_{17}(\omega, \omega_1)$$

$$g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B} | \bar{h}(tn) \dots G_s^{\alpha\beta}(r\bar{n}) \dots h(0) | \bar{B} \rangle$$

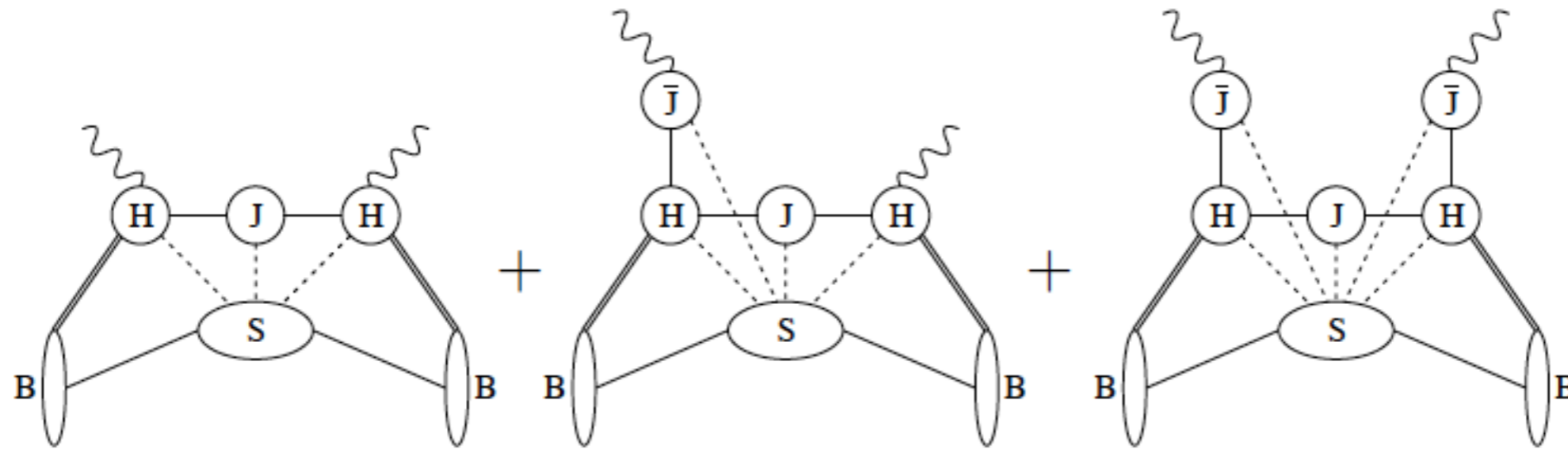
Expansion for  $m_c \sim m_b$  leads to Voloshin term in the total rate ( $-\lambda_2/m_c^2$ ), the terms stays non-local for  $m_c < m_b$ .



# Factorization formula

In the  $m_X^2 \sim \lambda$  and  $q^2 \sim \lambda$  region we have the following factorization formula

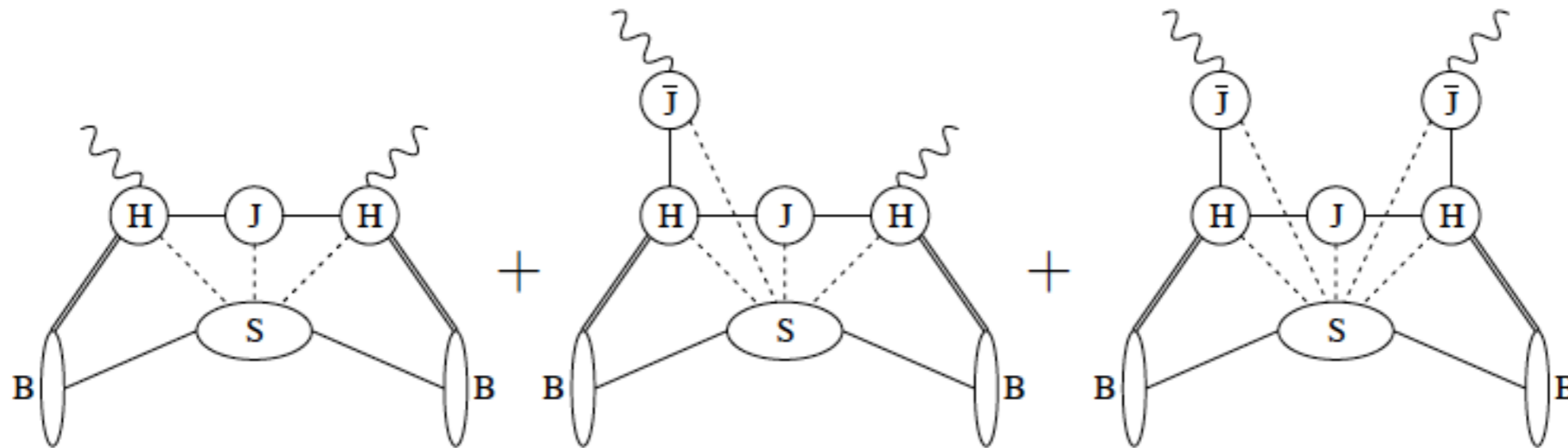
$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



## Factorization formula

In the  $m_X^2 \sim \lambda$  and  $q^2 \sim \lambda$  region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$



Subtlety in the  $Q_8$  and  $Q_8$  contribution: convolution integral is UV divergent

- This subtlety implies that there is no complete proof of the factorization formula.
- Nevertheless one shows that scale dependence of direct and resolved contribution cancel.
- No direct analogy to the problem of IR divergent convolution integrals in power-suppressed contributions to exclusive B decays.

## Angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \left[ (1 + z^2) H_T(q^2) + 2(1 - z^2) H_L(q^2) + 2z H_A(q^2) \right]$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

$$d\Gamma(\bar{B} \rightarrow X_s \ell^+ \ell^-) \equiv d\Lambda_{\alpha\beta} W^{\alpha\beta}(v, q),$$

$$d\Lambda_{\alpha\beta; 1/m_b} = dn \cdot q d\bar{n} \cdot q dz \frac{\alpha}{128\pi^3} (1 + z^2) \frac{n \cdot q}{\bar{n} \cdot q} g_{\perp, \alpha\beta}.$$

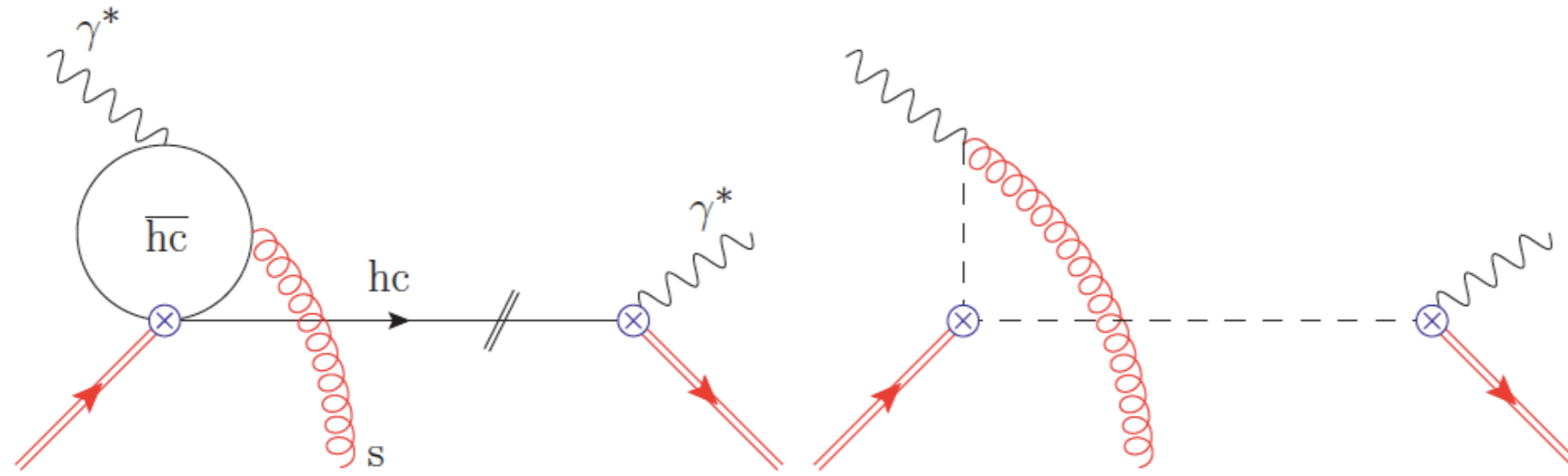
At  $O(1/m_b)$  nonlocal powercorrections only to  $H_T(q^2)$ .

## Numerical evaluation

- Subleading shape functions of resolved contributions similar to  $b \rightarrow s\gamma$
- Use explicit definition to determine properties:
  - \* PT invariance: soft functions are real
  - \* Moments of  $g_{17}$  related to HQET parameters
  - \* Vacuum insertion approximation relates  $g_{78}$  to the B meson LCDA
- Perform convolution integrals with model functions

# Numerical evaluation

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$\begin{aligned}
 d\Gamma_{17} = & \frac{1}{m_b} \operatorname{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q} \\
 & \times \operatorname{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon} \\
 & \times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\
 & \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu),
 \end{aligned}$$

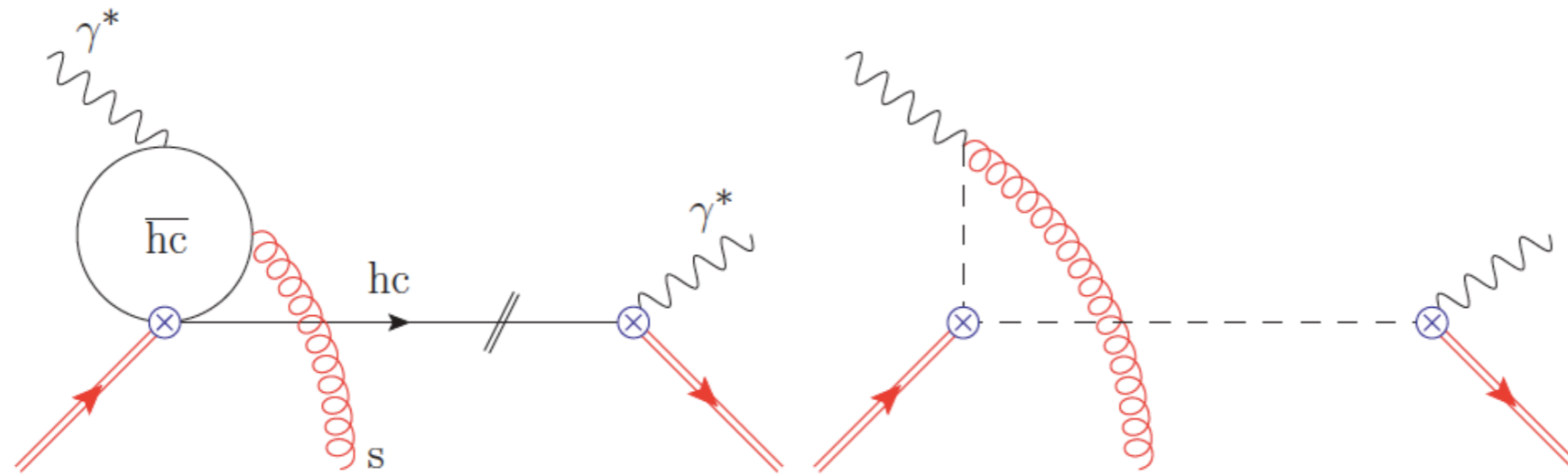
$$g_{17}(\omega, \omega_1, \mu) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t}$$

$$\times \frac{\langle \bar{B} | (\bar{h} S_n)(tn) \not{n} (1 + \gamma_5) (S_n^\dagger S_{\bar{n}})(0) i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n}) (S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle}{2M_B}$$



# Numerical evaluation

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$d\Gamma_{17} = \frac{1}{m_b} \text{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

$$\times \text{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$

$$\times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu),$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

• Limit  $m_c \rightarrow m_u = 0$

$$\times \frac{1}{\omega_1} [ \omega_1 ] g_{17}(\omega, \omega_1, \mu)$$

$$g_{17}(\omega, \omega_1, \mu) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t}$$

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## Numerical evaluation

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

- Trace formalism of HQET: 
$$\int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega (g_{17}(\omega, -\omega_1, \mu))^*$$

$$d\Gamma_{17} = \frac{1}{m_b} \text{Re} \left[ \hat{\Gamma}_{17} \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \frac{\alpha}{24\pi^3} dn \cdot q d\bar{n} \cdot q \frac{(n \cdot q)^3}{\bar{n} \cdot q}$$

$$\times \text{Re} \int d\omega \delta(\omega + m_b - n \cdot q) \int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$

$$\times \frac{1}{\omega_1} \left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right. \\ \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) \right) \right] g_{17}(\omega, \omega_1, \mu),$$

$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

- Limit  $m_c \rightarrow m_u = 0$

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$g_{17}$  is real

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$$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$$

- Limit  $m_c \rightarrow m_u = 0$

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- Integration of  $\omega_1$ :

Interference term  $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$  vanishes within the integrated rate

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}_{17}^q = \frac{1}{m_b} \frac{C_1(\mu)C_{7\gamma}(\mu)}{C_{\text{OPE}}} e_c \operatorname{Re} \left[ \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \int_{-\infty}^{+\infty} d\omega_1 J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) h_{17}(\omega_1, \mu)$$



## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

$$\mathcal{F}_{17}^q = \frac{1}{m_b} \frac{C_1(\mu)C_{7\gamma}(\mu)}{C_{\text{OPE}}} e_c \operatorname{Re} \left[ \frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \int_{-\infty}^{+\infty} d\omega_1 J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) h_{17}(\omega_1, \mu)$$

$$h(\omega_1, \mu) := \int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu)$$

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

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$$J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) = \operatorname{Re} \frac{1}{\omega_1 + i\epsilon} \int_{\frac{q_{\min}^2}{M_B}}^{\frac{q_{\max}^2}{M_B}} \frac{d\bar{n} \cdot q}{\bar{n} \cdot q} \frac{1}{\omega_1}$$

$$h(\omega_1, \mu) := \int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu)$$

$$\left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right. \\ \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right].$$

## Numerical evaluation

$$\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$$

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$$\left[ (\bar{n} \cdot q + \omega_1) \left( 1 - F \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) \right) - \bar{n} \cdot q \left( 1 - F \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right. \\ \left. - \bar{n} \cdot q \left( G \left( \frac{m_c^2}{m_b(\bar{n} \cdot q + \omega_1)} \right) - G \left( \frac{m_c^2}{m_b \bar{n} \cdot q} \right) \right) \right].$$

- One derives normalization of soft function:  $\int_{-\infty}^{\infty} d\omega_1 h_{17}(\omega_1, \mu) = 2 \lambda_2$
- $h_{17}$  should not have any significant structure (maxima or zeros) outside the hadronic range
- Values of  $h_{17}$  should be within the hadronic range

- First trial for a model function for  $h_{17}$ , a Gaussian, fulfills all needed properties.

$$h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

$\sigma = 0.5 \text{ GeV}$  as typical hadronic scale:  $\mathcal{F}_{17\text{exp}}^s \approx +1.6\%$

$\sigma = 0.1 \text{ GeV}$ :  $\mathcal{F}_{17\text{exp}}^s \approx +1.9\%$

However, convolution leads only to positive percentages !

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However, convolution leads only to positive percentages !

- More conservative estimate with 
$$h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

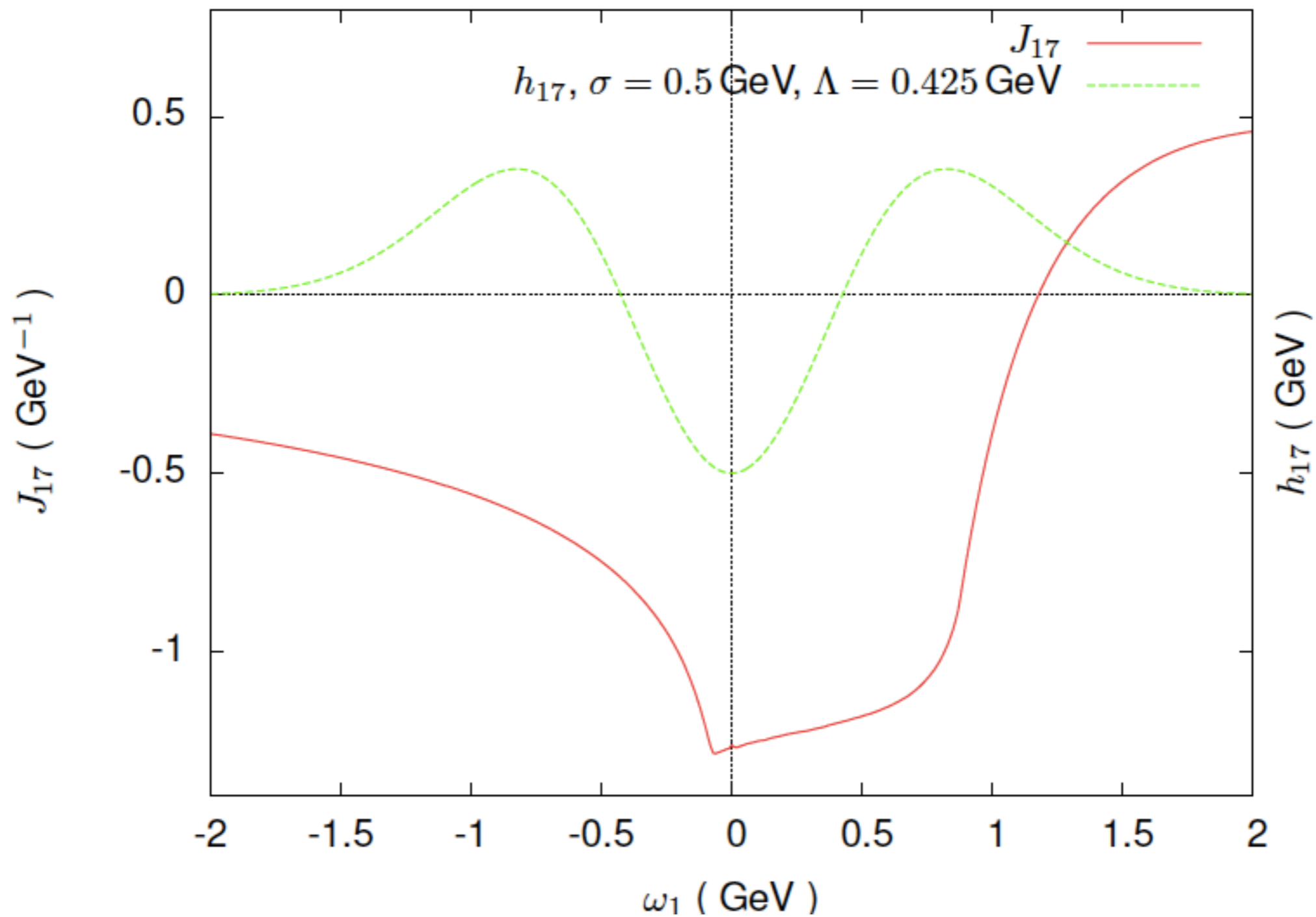
With  $\Lambda$  and  $\sigma$  of order  $\Lambda_{QCD}$  all general properties of  $h_{17}$  are fulfilled.

$$\sigma = 0.5 \text{ GeV; } \Lambda = 0.425 \text{ GeV: } \mathcal{F}_{17}^s = -0.5 \%$$

$$\Lambda = 0.575 \text{ GeV: } \mathcal{F}_{17}^s = +3.4 \%$$

$$\mathcal{F}_{17}^s \in [-0.5, +3.4] \%, \quad \mathcal{F}_{17}^d \in [-0.6, +4.1] \%$$





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- Relation to the Voloshin term: Voloshin 1997, Buchalla,Isidori,Rey 1997

We can rederive the leading Voloshin term under the following assumptions:

One starts with a narrow enough Gaussian as shape function, so that one can expand the jet function around  $\omega_1 = 0$  assuming  $\Lambda_{\text{QCD}} m_b / m_c^2$  to be small ( $(m_b \omega_1) / m_c^2$  corresponds to  $t = k \cdot q / m_c^2$  in Buchalla et al.):

$$\begin{aligned}
 [\dots] &= \omega_1^2 \bar{n} \cdot q \left[ \frac{1}{2\bar{n} \cdot q^2} - \frac{2m_c^2}{\bar{n} \cdot q^2} \frac{1}{4m_c^2 - m_b \bar{n} \cdot q} \sqrt{\frac{4m_c^2 - m_b \bar{n} \cdot q}{m_b \bar{n} \cdot q}} \arctan \frac{1}{\sqrt{\frac{4m_c^2 - m_b \bar{n} \cdot q}{m_b \bar{n} \cdot q}}} \right] \\
 &= -\frac{m_b \omega_1^2}{12m_c^2} F_V(r), \quad r = q^2 / (4m_c^2)
 \end{aligned}$$

However:

Voloshin term significantly underestimates the possible charm contributions.

Our final estimates of the resolved contributions to the leading order:  
(normalized to OPE result)

$$\mathcal{F}_{17}^s \in [-0.5, +3.4] \%, \quad \mathcal{F}_{17}^d \in [-0.6, +4.1] \%,$$

$$\mathcal{F}_{78}^{d,s} \in [-0.2, -0.1] \%, \quad \mathcal{F}_{88}^{d,s} \in [0, 0.5] \%$$

$$\mathcal{F}_{1/m_b}^d \in [-0.8, +4.5], \quad \mathcal{F}_{1/m_b}^s \in [-0.7, +3.8]$$

$\mathcal{F}_{19}$ :  $O(1/m_b^2)$  but  $|C_{9/10}| \sim 13|C_{7\gamma}|$   
(work in progress)

## Power corrections in the inclusive mode

- For  $q$  anti-hard-collinear we have identified a new type of subleading power corrections.
- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.
- They constitute an irreducible uncertainty because they survive the  $M_X \rightarrow 1$  limit.
- If  $q$  was hard then these resolved contributions would not exist

**Nonlocal power corrections of  $O(1/m_b^2)$  numerically relevant**

**$M_X$  cut effects in the low- $q^2$  region with  $q^2$  anti-hard-collinear**

(work in progress)

# Summary

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables available
- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events
- Nonlocal power corrections are under control and calculated to  $O(1/m_b)$
- Theory predictions for inclusive  $\bar{B} \rightarrow X_d \ell^+ \ell^-$  (including QED and power corrections) will be soon available (work in progress)
- Inclusive  $\bar{B} \rightarrow X_s \ell \ell$  has a complementary role in new physics search to  $\bar{B} \rightarrow X_s \gamma$  and  $B \rightarrow K^{(*)} \ell \ell$





# Mainz Institute for Theoretical Physics

## ACTIVITIES 2018

[www.mitp.uni-mainz.de](http://www.mitp.uni-mainz.de)

### SCIENTIFIC PROGRAMS

**Probing Physics Beyond the SM with Precision**  
Ansgar Denner U Würzburg, Stefan Dittmaier U Freiburg,  
Tilman Plehn Heidelberg U  
**February 26 - March 9, 2018**

**Bridging the Standard Model to New Physics with the Parity Violation Program at MESA**  
Jens Erler UNAM, Mikhail Gorshteyn, Hubert Spiesberger JGU  
**April 23 - May 4, 2018**

**Modern Techniques for CFT and AdS**  
Bartłomiej Czech IAS Princeton, Michal P. Heller  
MPI for Gravitational Physics, Alessandro Vichi EPFL  
**May 22 - 30, 2018**

**The Dawn of Gravitational Wave Science**  
Luis Lehner Perimeter Inst., Rafael A. Porto ICTP-SAIFR,  
Riccardo Sturani IIP Natal, Salvatore Vitale MIT  
**June 4 - 15, 2018**

**The Future of BSM Physics**  
Gian Giudice CERN, Giulia Ricciardi U Naples Federico II,  
Tobias Hurth, Joachim Kopp, Matthias Neubert JGU  
**June 4 - 15, 2018, Capri, Italy**

**Probing Baryogenesis via LHC and Gravitational Wave Signatures**  
Germano Nardini U Bern, Carlos E.M. Wagner  
U Chicago / Argonne Nat. Lab., Pedro Schwaller JGU  
**June 18 - 29, 2018**

**From Amplitudes to Phenomenology**  
Fabrizio Caola IPPP Durham,  
Bernhard Mistlberger, Giulia Zanderighi CERN  
**August 13 - 24, 2018**

**String Theory, Geometry and String Model Building**  
Philip Candelas, Xenia de la Ossa, Andre Lukas U Oxford,  
Daniel Waldram Imperial College London,  
Gabriele Honecker, Duco van Straten JGU  
**September 10 - 21, 2018**

### TOPICAL WORKSHOPS

**The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment**  
Carlo Carloni Calame INFN Pavia, Massimo Passera INFN Padua,  
Luca Trentadue U Parma, Graziano Venanzoni INFN Pisa  
**February 19 - 23, 2018**

**Applied Newton-Cartan Geometry**  
Eric Bergshoeff U Groningen, Niels Obers NBI Copenhagen,  
Dam Thanh Son U Chicago  
**March 12 - 16, 2018**

**Challenges in Semileptonic B Decays**  
Paolo Gambino U Turin, Andreas Kronfeld Fermilab,  
Marcello Rotondo INFN-LNF Frascati, Christoph Schwanda ÖAW Vienna  
**April 9 - 13, 2018**

**Tensions in the LCDM Paradigm**  
Cora Dvorkin Harvard, Silvia Galli IAP Paris,  
Fabio Iocco ICTP-SAIFR, Federico Marinacci MIT  
**May 14 - 18, 2018**

**The Proton Radius Puzzle and Beyond**  
Richard Hill U Kentucky / Fermilab, Gil Paz Wayne State U, Randolph Pohl JGU  
**July 23 - 27, 2018**

**Scattering Amplitudes and Resonance Properties from Lattice QCD**  
Maxwell T. Hansen CERN, Sasa Prelovsek U Ljubljana / U Regensburg,  
Steve Sharpe U Washington, Georg von Hippel, Hartmut Wittig JGU  
**August 27 - 31, 2018**

**Quantum Fields – From Fundamental Concepts to Phenomenological Questions**  
Astrid Eichhorn Heidelberg U, Roberto Percacci SISSA Trieste,  
Frank Saueressig U Nijmegen  
**September 26 - 28, 2018**

### MITP SUMMER SCHOOL 2018

Johannes Henn, Matthias Neubert, Stefan Weinzierl, Felix Yu JGU  
**July 2018**





# Mainz Institute for Theoretical Physics

MEETINGS 2018

## SCIENTIFIC PROGRAMS

**Probing Physics Beyond the SM with Precision**  
 Ansgar Denner U Würzburg, Stefan Dittmaier U Freiburg,  
 Tilman Plehn Heidelberg U  
**February 26 - March 9, 2018**

**Bridging the Standard Model with the Parity Violation**  
 Jens Erler UNAM, Michigan State U  
**April 23 - May 4, 2018**

**Modern Topics in Particle Physics**  
 Giulia Ricciardi U Naples Federico II,  
 Joachim Kopp, Matthias Neubert JGU  
**June 4-15, 2018, Capri, Italy**

**Probing Baryogenesis via LHC and Gravitational Wave Signatures**  
 Germano Nardini U Bern, Carlos E.M. Wagner  
 U Chicago / Argonne Nat. Lab., Pedro Schwaller JGU  
**June 18-29, 2018**

**From Amplitudes to Phenomenology**  
 Fabrizio Caola IPPP Durham,  
 Bernhard Mistlberger, Giulia Zanderighi CERN  
**August 13-24, 2018**

**String Theory, Geometry and String Model Building**  
 Philip Candelas, Xenia de la Ossa, Andre Lukas U Oxford,  
 Daniel Waldram Imperial College London,  
 Gabriele Honecker, Duco van Straten JGU  
**September 10-21, 2018**

## WORKSHOPS

**Hadronic Contribution to the Anomalous Magnetic Moment**  
 Massimo Passera INFN Padua,  
 Roberto Zayadeh INFN Pisa

**Geometry and Cosmology**  
 Niels Obers NBI Copenhagen,  
 Chicago  
**May 16, 2018**

**Challenges in Semileptonic B Decays**  
 Paolo Gambino U Turin, Andreas Kronfeld Fermilab,  
 Marcello Rotondo INFN-LNF Frascati, Christoph Schwanda ÖAW Vienna  
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[www.mitp.uni-mainz.de](http://www.mitp.uni-mainz.de)

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**Extra**

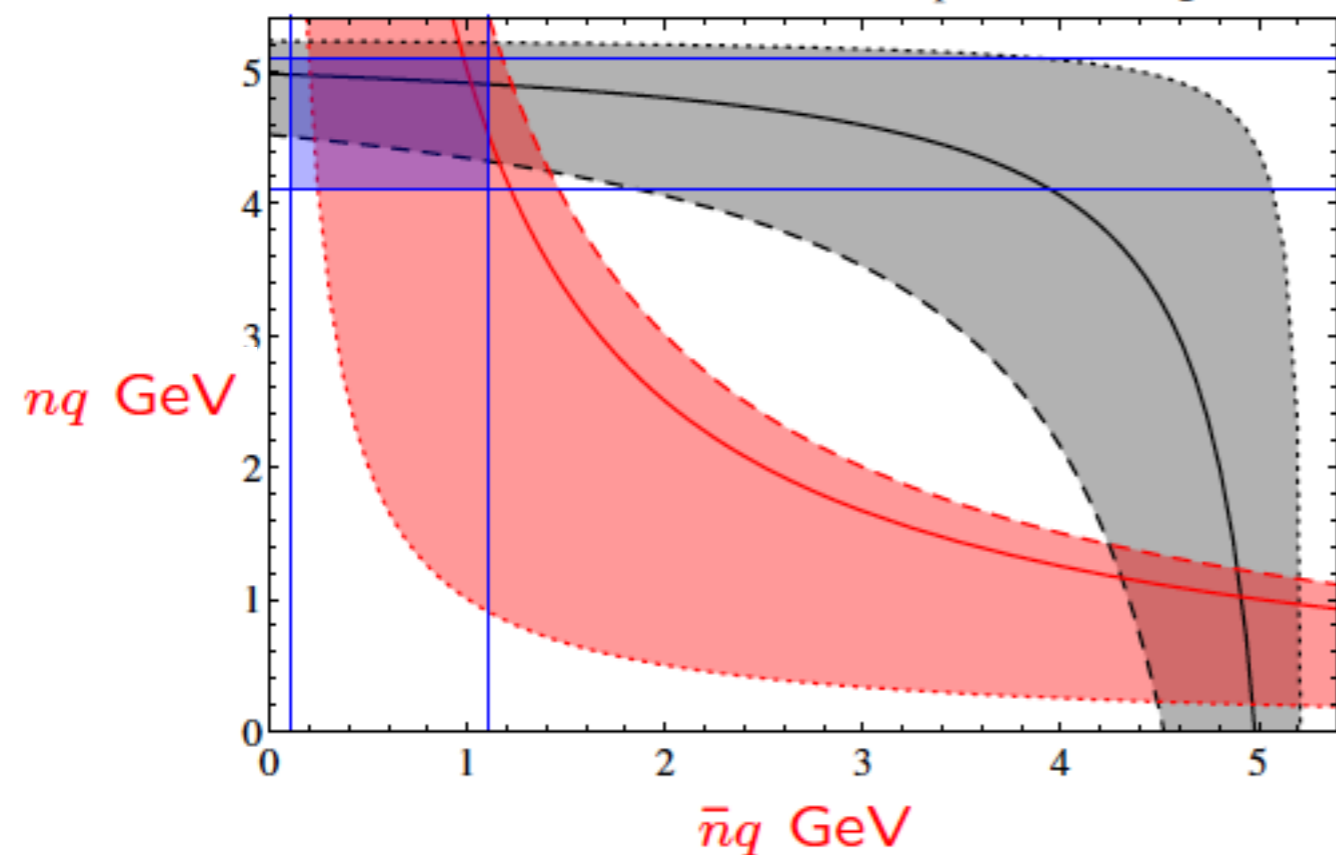
# Allowed regions

low- $q^2$

Red:  $q^2 = [1, 5, 6] \text{ GeV}^2$  [Dotted, Solid, Dashed]

Black:  $M_x = [0.495, 1.25, 2] \text{ GeV}$  [Dotted, Solid, Dashed]

Blue: anti-hard-collinear component scaling

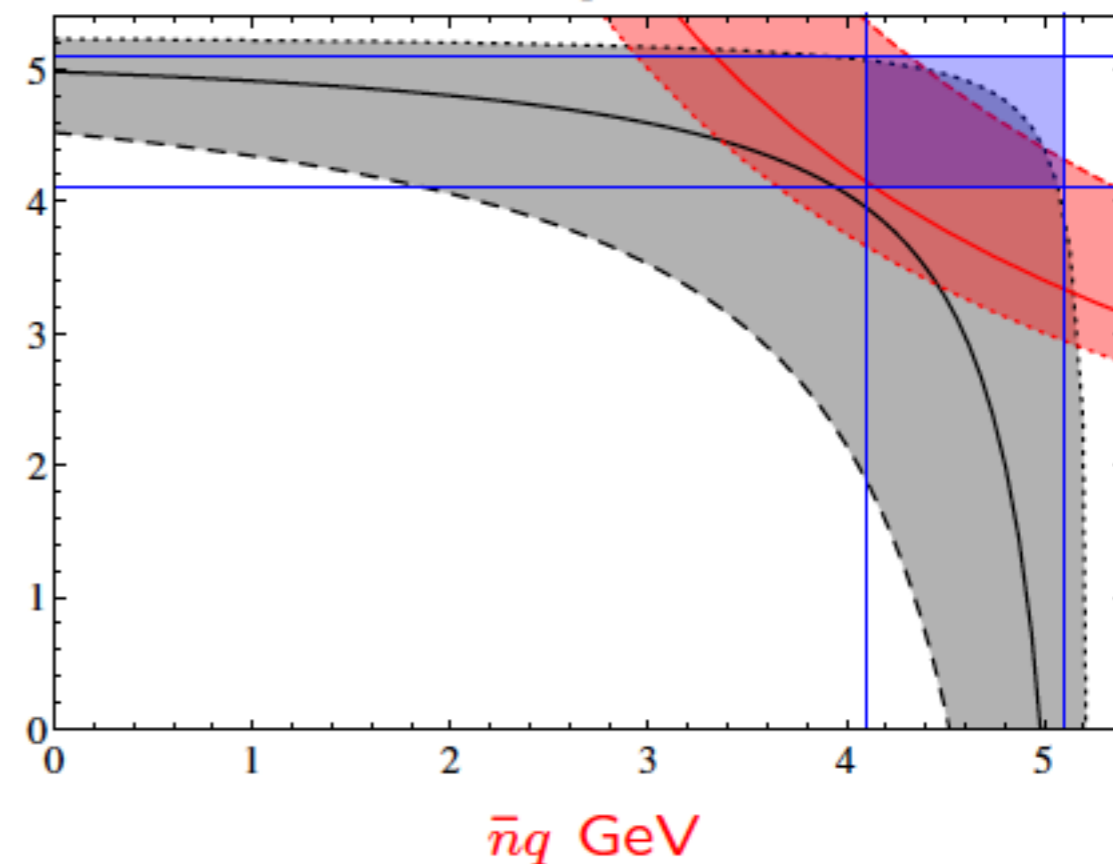


high- $q^2$

Red:  $q^2 = [15, 17, 22] \text{ GeV}^2$  [Dotted, Solid, Dashed]

Black:  $M_x = [0.495, 1.25, 2] \text{ GeV}$  [Dotted, Solid, Dashed]

Blue: hard component scaling



## Scaling

$$\lambda = \Lambda_{\text{QCD}}/m_b$$

$$m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

# Theory predictions

- Most important input parameters

$$m_b^{1S} = (4.691 \pm 0.037)\text{GeV}, \quad \bar{m}_c(\bar{m}_c) = (1.275 \pm 0.025)\text{GeV}$$

$$|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027, \quad BR_{b \rightarrow c e \nu}^{\text{exp.}} = (10.51 \pm 0.13)\%$$

- Perturbative expansion (NNLO QCD + NLO QED)  $\alpha_s$   $\kappa = \alpha_{\text{em}}/\alpha_s$

$$A = \kappa [A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3)]$$

$$+ \kappa^2 [A_{LO}^{\text{em}} + \alpha_s A_{NLO}^{\text{em}} + \alpha_s^2 A_{NNLO}^{\text{em}} + \mathcal{O}(\alpha_s^3)] + \mathcal{O}(\kappa^3)$$

$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$



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$$\text{LO} = \alpha_{\text{em}}/\alpha_s, \quad \text{NLO} = \alpha_{\text{em}}, \quad \text{NNLO} = \alpha_{\text{em}} \alpha_s$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Size of logs depend on experimental set-up

We assume no photons are included in the definition of  $q^2$  (di-muon channel at Babar/Belle, di-electron at Belle)

Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in  $q^2$

Monte Carlo techniques needed to estimate this effect !

$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma, \text{coll}})^2$$

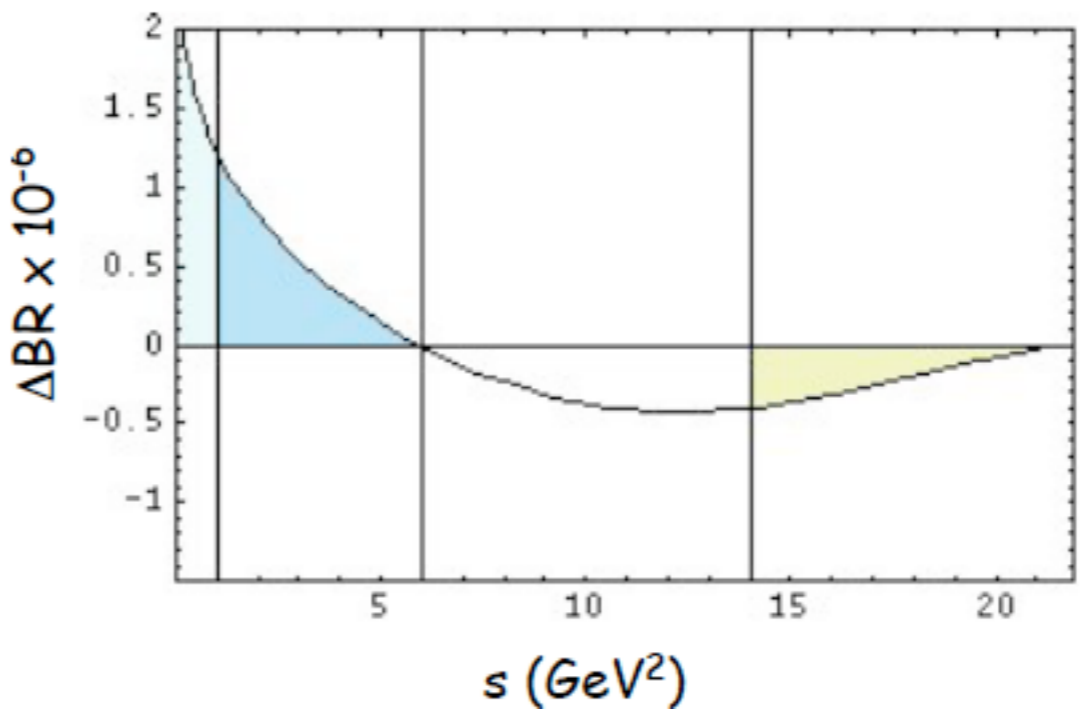
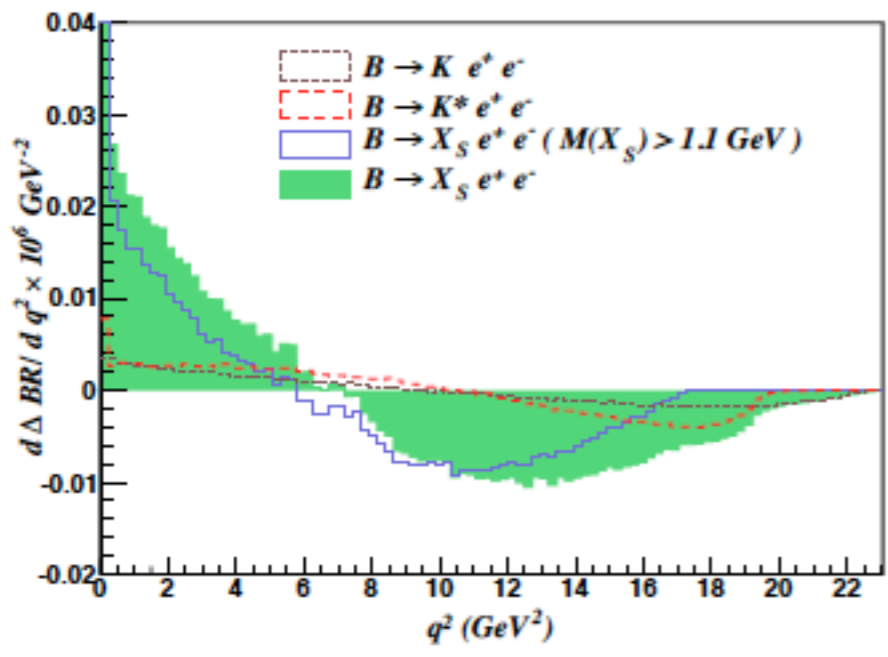
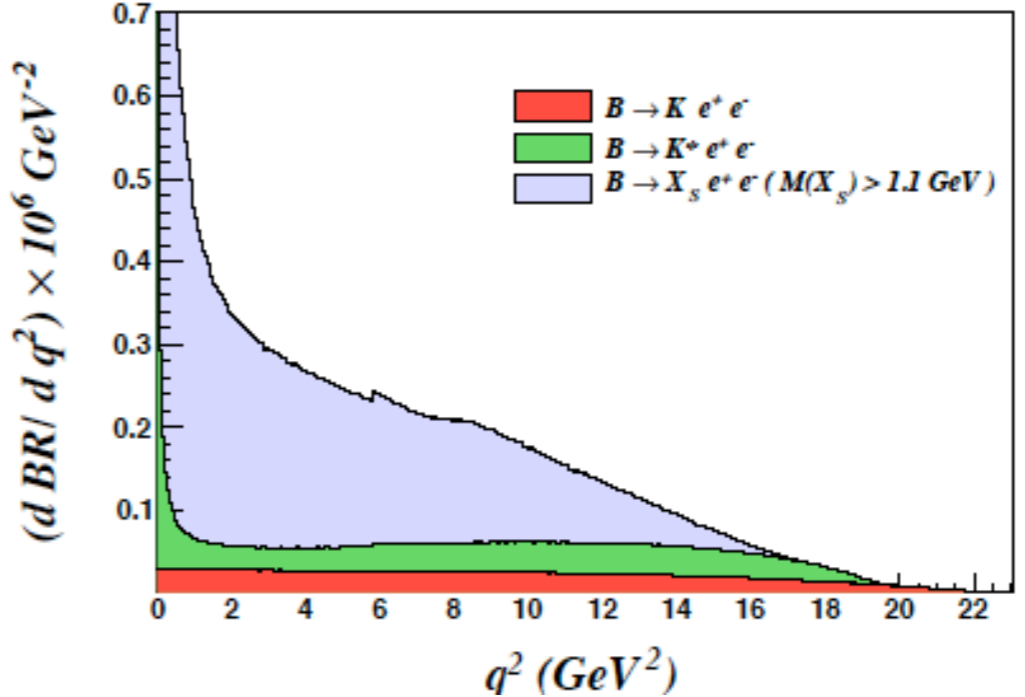
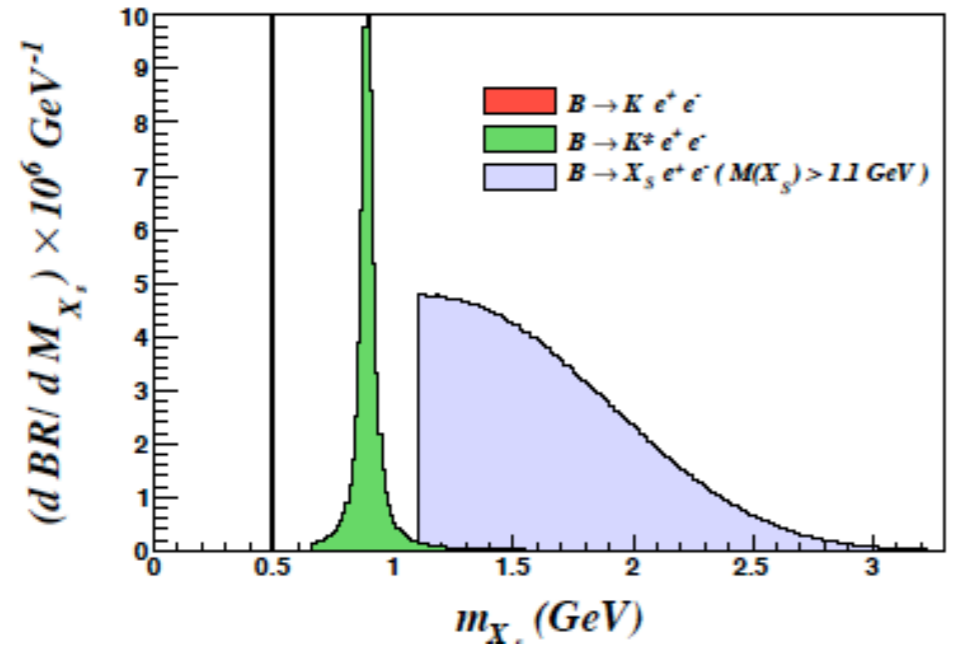


# Monte Carlo analysis

(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

$$\frac{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{[\mathcal{B}_{ee}^{\text{low}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{[\mathcal{B}_{ee}^{\text{high}}]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$



## Results

Low- $q^2$  ( $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ )

$$BR(B \rightarrow X_s ee) = (1.67 \pm 0.10) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (1.62 \pm 0.09) 10^{-6}$$

Babar:  $BR(B \rightarrow X_s ll) =$

$$= (1.60 (+0.41 - 0.39)_{stat} (+0.17 - 0.13)_{syst} (\pm 0.18)_{mod}) 10^{-6}$$

good agreement with SM

## Results

High- $q^2$ , Theory:  $q^2 > 14.4\text{GeV}^2$ , Babar:  $q^2 > 14.2\text{GeV}^2$

$$BR(B \rightarrow X_s ee) = (0.220 \pm 0.070) 10^{-6}$$

$$BR(B \rightarrow X_s \mu\mu) = (0.253 \pm 0.070) 10^{-6}$$

Babar:  $BR(B \rightarrow X_s ll) =$

$$(0.57 (+0.16 - 0.15)_{stat} (+0.03 - 0.02)_{syst}) 10^{-6}$$

$2\sigma$  higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077

(but perfect agreement if we use their prescriptions)

**Further results** in units of  $10^{-6}$ 

$$H_L[1, 3.5]_{ee} = 0.64 \pm 0.03$$

$$H_L[1, 3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5, 6]_{ee} = 0.50 \pm 0.03$$

$$H_L[3.5, 6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1, 6]_{ee} = 1.13 \pm 0.06$$

$$H_L[1, 6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1, 3.5]_{ee} = 0.29 \pm 0.02$$

$$H_T[1, 3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5, 6]_{ee} = 0.24 \pm 0.02$$

$$H_T[3.5, 6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1, 6]_{ee} = 0.53 \pm 0.04$$

$$H_T[1, 6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1, 3.5]_{ee} = -0.103 \pm 0.005$$

$$H_A[1, 3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5, 6]_{ee} = +0.073 \pm 0.012$$

$$H_A[3.5, 6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1, 6]_{ee} = -0.029 \pm 0.016$$

$$H_A[1, 6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error  $\mathcal{O}(5 - 8\%)$ . Still dominated by scale uncertainty.

## Further refinement

Normalization to semileptonic  $B \rightarrow X_u l \nu$  decay rate **with the same cut** reduces the impact of  $1/m_b$  corrections in the high- $q^2$  region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31) 10^{-3}$$

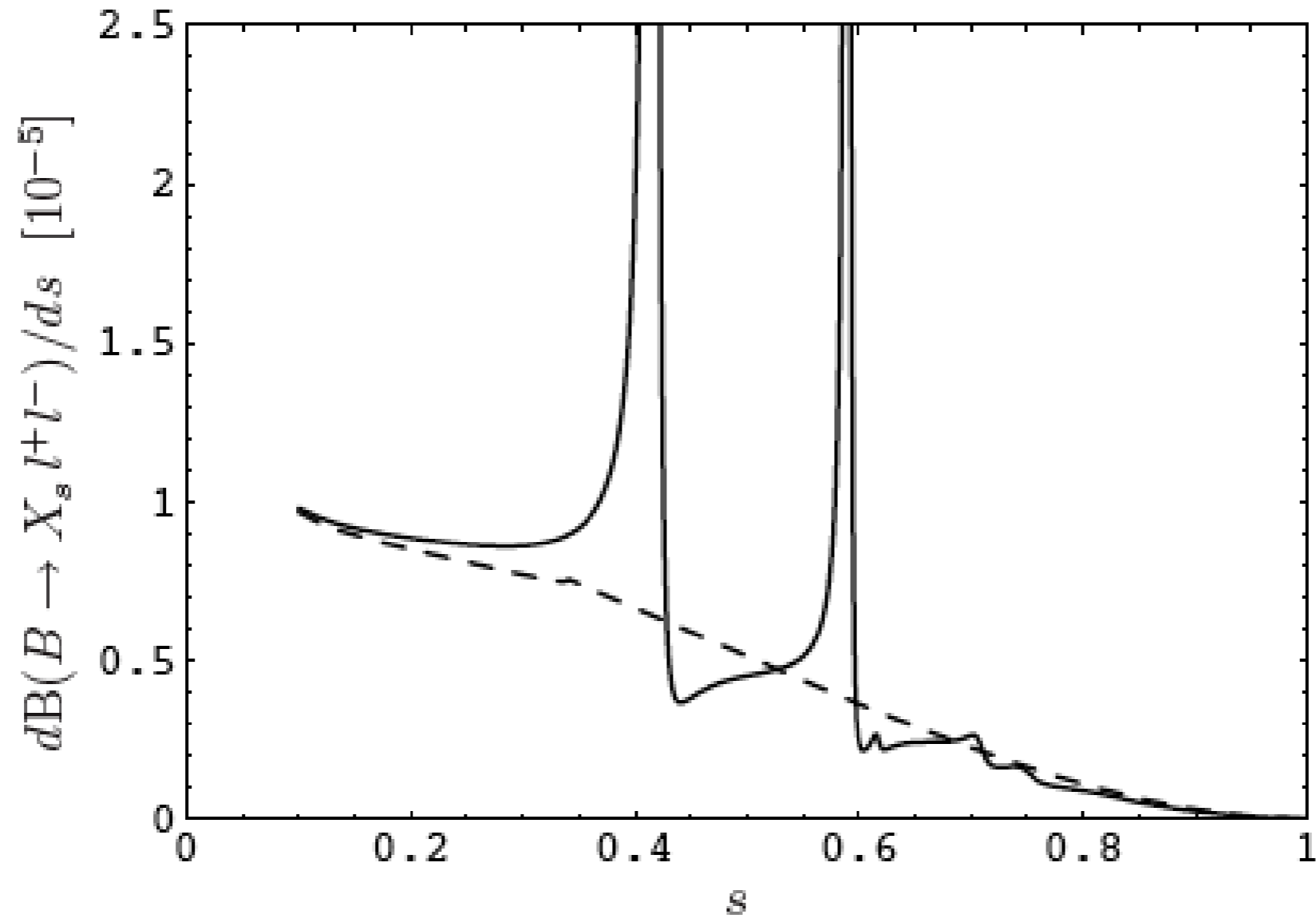
$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30) 10^{-3}$$

Largest source of error are CKM elements ( $V_{ub}$ )

Note: Additional  $O(5\%)$  uncertainty due to nonlocal power corrections  $O(\alpha_s \Lambda/m_b)$

# Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ ? BBNS, arXiv:0902.4446

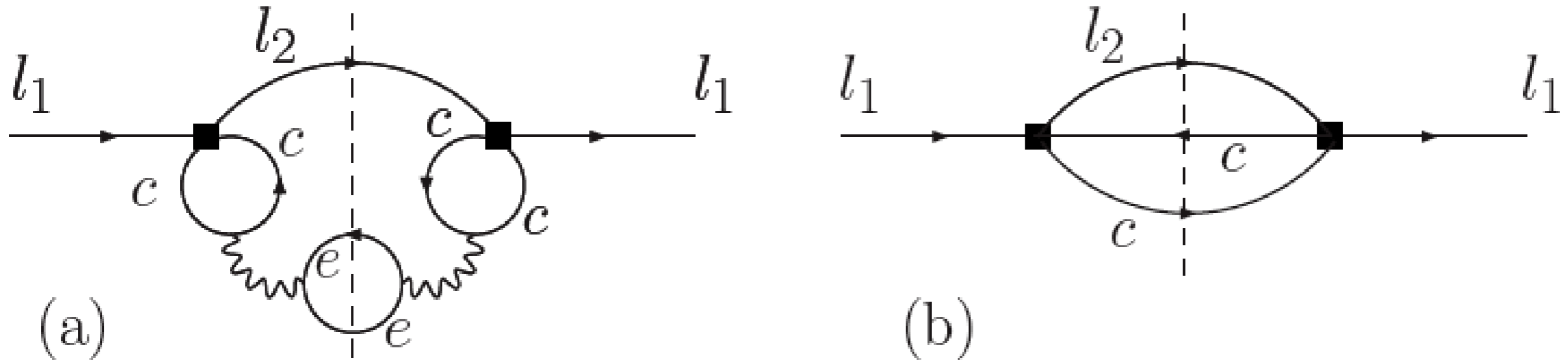
Within integrated branching ratio the resonances  $J/\psi$  and  $\psi'$  exceed the perturbative contributions **by two orders** of magnitude.





## Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+ \ell^-$ ? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances  $J/\psi$  and  $\psi'$  exceed the perturbative contributions **by two orders** of magnitude.



The rate  $l_1 \rightarrow l_2 e^+ e^-$  (a) is connected to the integral over  $|\Pi(q^2)|^2$  for which global duality is **NOT** expected to hold.

In contrast the inclusive hadronic rate  $l_1 \rightarrow l_2 X$  (b) corresponds to the imaginary part of the correlator  $\Pi(q^2)$ .

## Subtlety in the high- $q^2$ region

Locally: breakdown of OPE in  $\Lambda_{QCD}/m_b$  in the high- $s$  ( $q^2$ ) endpoint  
Partonic contribution vanishes in the limit  $s \rightarrow 1$ , while the  $1/m_b^2$   
corrections in  $R(s)$  tend towards a nonzero value.

Theoretically:  $s$ -quark propagator in the correlator of OPE:

$$S_s(k) = \frac{\not{k} + i \not{D}}{k^2 + 2ik \cdot D - \not{D} \not{D} + i\varepsilon}.$$

Endpoint region of the  $q^2$  spectrum in  $\bar{B} \rightarrow X_s l^+ l^-$  different from endpoint  
region of the photon spectrum of  $\bar{B} \rightarrow X_s \gamma$ :

$q^2 \approx m_b^2 \approx M_B^2 \Rightarrow k \sim \Lambda, \quad k^2 \sim \Lambda^2 \Rightarrow$  complete breakdown of OPE

no partial all-orders resummation possible, shape-function irrelevant

Buchalla, isidori

Practically: for integrated high- $s$  ( $q^2$ ) spectrum one finds an effective  
expansion ( $s_{\min} \approx 0.6$ ): Ghinculov, Hurth, Isidori, Yao, hep-ph/0312128

$$\int_{s_{\min}}^1 ds R(s) = \left[ 1 - \frac{1.6\lambda_2}{m_b^2(1 - \sqrt{s_{\min}})^2} + \frac{1.8\rho_1 + 1.7f_1}{m_b^3(1 - \sqrt{s_{\min}})^3} \right] \times \int_{s_{\min}}^1 ds R(s)|_{m_b \rightarrow \infty}$$