Generating the SM fermion mass hierarchy by sequential loop suppression.

Antonio Enrique Cárcamo Hernández

Universidad Técnica Federico Santa María

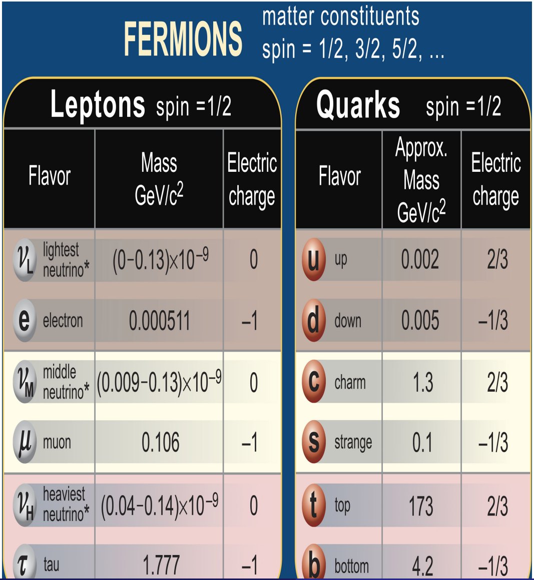
FLASY 2018 Conference, University of Basel, Switzerland, 3th July of 2018.

Based on: A. E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, arxiv:hep-ph/1611.09797, JHEP 1702 (2017) 125.

A. E. Cárcamo Hernández, S. Kovalenko, H. N. Long, I. Schmidt, arxiv:hep-ph/1705.09169

Overview

The origin of fermion masses and mixings is not explained by the SM.



FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c²	Electric charge
ν_L lightest neutrino*	(0-0.13)x10⁻⁹	0
e electron	0.000511	-1
ν_M middle neutrino*	(0.009-0.13)x10⁻⁹	0
μ muon	0.106	-1
ν_H heaviest neutrino*	(0.04-0.14)x10⁻⁹	0
τ tau	1.777	-1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

$$\sqrt{\left|\Delta m_{13}^2\right|} \sim \lambda^{20} m_t, \quad \sqrt{\Delta m_{12}^2} \sim \lambda^{21} m_t,$$

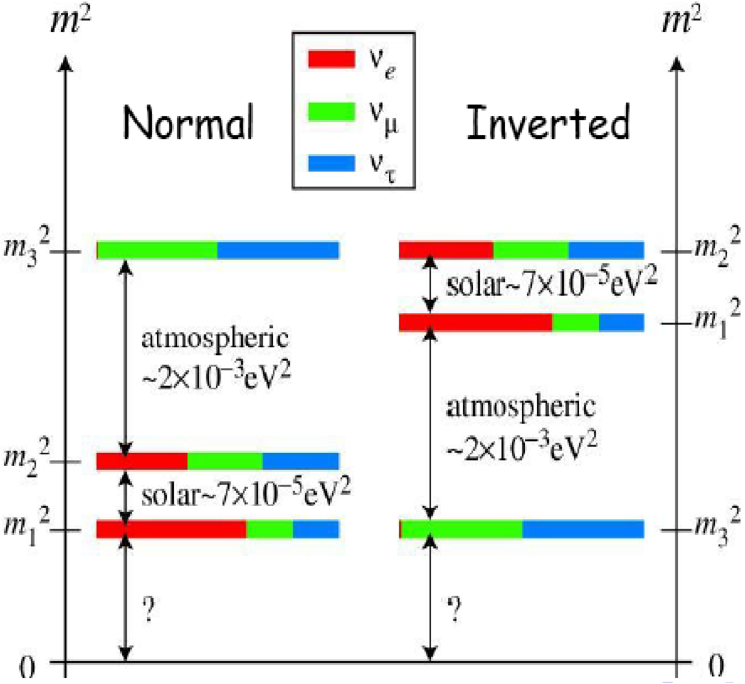
$$m_e \sim \lambda^9 m_t, \quad m_u \sim m_d \sim \lambda^8 m_t,$$

$$m_s \sim m_\mu \sim \lambda^5 m_t, \quad \lambda = 0.225,$$

$$m_c \sim \lambda^4 m_t, \quad m_b \sim m_\tau \sim \lambda^3 m_t,$$

$$\sin\theta_{12}^{(q)} \sim \lambda, \quad \sin\theta_{23}^{(q)} \sim \lambda^2, \quad \sin\theta_{13}^{(q)} \sim \lambda^4,$$

$$\sin\theta_{12}^{(l)} \sim \sqrt{\frac{1}{3}}, \quad \sin\theta_{23}^{(l)} \sim \sqrt{\frac{1}{2}}, \quad \sin\theta_{13}^{(l)} \sim \frac{\lambda}{\sqrt{2}}.$$


m^2
m^2
ν_e
ν_μ
ν_τ
Normal
Inverted
$m_3{}^2$
$m_2{}^2$
$m_1{}^2$
atmospheric
~2×10^{-3}eV^2
solar~7×10^{-5}eV^2
?
0
$m_2{}^2$
$m_1{}^2$
$m_3{}^2$
solar~7×10^{-5}eV^2
atmospheric
~2×10^{-3}eV^2
?
0

Some mechanisms to describe the SM charged fermion mass hierarchy are:

1. Spontaneously broken abelian symmetries as originally proposed by Froggatt and Nielsen in NPB, 1979.
2. Universal Seesaw mechanism as originally proposed by Davidson and Wali in PRL, 1987
3. Localization of the profiles of the fermionic zero modes in extradimensions as originally proposed by Dvali and Schifman in PLB, 2000.
4. Sequential loop supression mechanism as originally proposed by A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt in JHEP, 2017.

Several mechanisms to generate light active neutrino masses are: Weinberg Operator, type I seesaw, type II seesaw, type III seesaw, double seesaw, linear seesaw, inverse seesaw, radiative seesaw at one, two, three or four loop level.

The S_3 is the smallest non-abelian group having a doublet and two singlet irreducible representations. The S_3 group has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010):

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} &= (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} \\ &+ \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{2}}, \end{aligned} \quad (1)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_{\mathbf{2}}, \qquad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \quad (2)$$

Cárcamo Hernández-Kovalenko-Schmidt mechanism

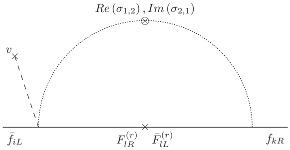
In the CKS mechanism the SM fermion mass hierarchy is explained by a sequential loop suppression, so that the masses are generated according to:

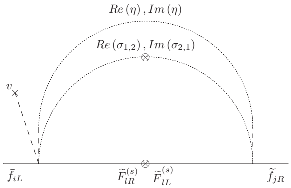
$$t\text{-quark} \rightarrow \textit{tree-level mass} \text{ from } \quad \overline{q}_{jL}\widetilde{\phi}u_{3R}, \tag{3}$$

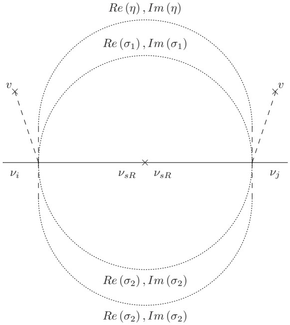
$$b, c, \ \tau, \mu \rightarrow \textit{1-loop mass;} \text{ tree-level forbidden by a } \textit{symmetry}. \tag{4}$$

$$s, u, d, \ e \rightarrow \textit{2-loop mass;} \text{ tree-level \& 1-loop forbidden by a } \textit{symmetry}. \tag{5}$$

$$\nu_i \rightarrow \textit{4-loop mass;} \text{ tree-level \& lower loops forbidden by a } \textit{symmetry}. \tag{6}$$


$Re(\sigma_{1,2}), Im(\sigma_{2,1})$
v
$\bar{f}_{iL}$
$F_{lR}^{(r)}$
$\bar{F}_{lL}^{(r)}$
f_{kR}


$Re(\eta), Im(\eta)$
$Re(\sigma_{1,2}), Im(\sigma_{2,1})$
v
$\bar{f}_{iL}$
$\widetilde{F}_{lR}^{(s)}$
$\bar{\widetilde{F}}_{lL}^{(s)}$
$\widetilde{f}_{jR}$


$Re(\eta), Im(\eta)$
$Re(\sigma_1), Im(\sigma_1)$
v
v
ν_i
ν_{sR}
ν_{sR}
ν_j
$Re(\sigma_2), Im(\sigma_2)$
$Re(\sigma_2), Im(\sigma_2)$

The mass matrices $M_{U,D}$ of up and down quarks, $M_{l,\nu}$, of charged leptons and light active neutrinos

$$M_U = \begin{pmatrix} \widetilde{\varepsilon}^{(u)}_{11} & \varepsilon^{(u)}_{12} & \kappa^{(u)}_{13} \\ \widetilde{\varepsilon}^{(u)}_{12} & \varepsilon^{(u)}_{22} & \kappa^{(u)}_{23} \\ \widetilde{\varepsilon}^{(u)}_{13} & \varepsilon^{(u)}_{32} & \kappa^{(u)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \widetilde{\varepsilon}^{(d)}_{11} & \widetilde{\varepsilon}^{(d)}_{12} & \varepsilon^{(d)}_{13} \\ \widetilde{\varepsilon}^{(d)}_{21} & \widetilde{\varepsilon}^{(d)}_{22} & \varepsilon^{(d)}_{23} \\ \widetilde{\varepsilon}^{(d)}_{31} & \widetilde{\varepsilon}^{(d)}_{32} & \varepsilon^{(d)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_l = \begin{pmatrix} \widetilde{\varepsilon}^{(l)}_{11} & \varepsilon^{(l)}_{12} & \varepsilon^{(l)}_{13} \\ \widetilde{\varepsilon}^{(l)}_{21} & \varepsilon^{(l)}_{22} & \varepsilon^{(l)}_{23} \\ \widetilde{\varepsilon}^{(l)}_{31} & \varepsilon^{(l)}_{32} & \varepsilon^{(l)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_\nu = \begin{pmatrix} \varepsilon^{(\nu)}_{11} & \varepsilon^{(\nu)}_{12} & \varepsilon^{(\nu)}_{13} \\ \varepsilon^{(\nu)}_{12} & \varepsilon^{(\nu)}_{22} & \varepsilon^{(\nu)}_{23} \\ \varepsilon^{(\nu)}_{13} & \varepsilon^{(\nu)}_{23} & \varepsilon^{(\nu)}_{33} \end{pmatrix} \frac{v^2}{\sqrt{2}\,\Lambda},$$

their entries are generated at different loop-levels:

$$\kappa^{(u)}_{j3} \quad \rightarrow \quad \text{tree-level} \tag{7}$$

$$\varepsilon^{(u)}_{j2}, \varepsilon^{(d)}_{j3}, \varepsilon^{(l)}_{j2}, \varepsilon^{(l)}_{j3} \quad \rightarrow \quad \text{1-loop-level} \tag{8}$$

$$\widetilde{\varepsilon}^{(u)}_{j1}, \widetilde{\varepsilon}^{(d)}_{j1}, \widetilde{\varepsilon}^{(d)}_{j2}, \widetilde{\varepsilon}^{(l)}_{j1} \quad \rightarrow \quad \text{2-loop-level} \tag{9}$$

$$\varepsilon^{(\nu)}_{jk} \quad \rightarrow \quad \text{4-loop-level,} \tag{10}$$

where $j, k = 1, 2, 3$.

The $S_3 \times Z_2$ discrete group is assumed to be softly broken.

$$\phi \sim (\mathbf{1}, 1) , \qquad \sigma = (\sigma_1, \sigma_2) \sim (\mathbf{2}, 1) , \qquad \eta \sim (\mathbf{1}, -1) , \tag{11}$$

$$\begin{aligned}
u_{1R} &\sim (\mathbf{1}', -1) , \quad u_{2R} \sim (\mathbf{1}', 1) , \quad u_{3R} \sim (\mathbf{1}, 1) , \\
d_{1R} &\sim (\mathbf{1}', -1) , \quad d_{2R} \sim (\mathbf{1}', -1) , \quad d_{3R} \sim (\mathbf{1}', 1), \\
l_{1R} &\sim (\mathbf{1}', -1) , \quad l_{2R} \sim (\mathbf{1}', 1) , \quad l_{3R} \sim (\mathbf{1}', 1) , \\
q_{jL} &\sim (\mathbf{1}, 1) , \quad l_{jL} \sim (\mathbf{1}, 1) , \quad j = 1, 2, 3, \quad \nu_{sR} = (\mathbf{1}', -1) , \quad s = 1, 2, \\
T_L &= (T_{1L}, T_{2L}) \sim (\mathbf{2}, 1) , \quad T_R = (T_{1R}, T_{2R}) \sim (\mathbf{2}, 1) , \\
\widetilde{T}_L &= \left(\widetilde{T}_{1L}, \widetilde{T}_{2L}\right) \sim (\mathbf{2}, 1) , \quad \widetilde{T}_R = \left(\widetilde{T}_{1R}, \widetilde{T}_{2R}\right) \sim (\mathbf{2}, -1) , \\
B_L &= (B_{1L}, B_{2L}) \sim (\mathbf{2}, 1) , \quad B_R = (B_{1R}, B_{2R}) \sim (\mathbf{2}, 1) , \\
\widetilde{E}_L &= \left(\widetilde{E}_{1L}, \widetilde{E}_{2L}\right) \sim (\mathbf{2}, 1) , \quad \widetilde{E}_R = \left(\widetilde{E}_{1R}, \widetilde{E}_{2R}\right) \sim (\mathbf{2} - 1) , \\
\widetilde{B}_L^{(s)} &= \left(\widetilde{B}_{1L}^{(s)}, \widetilde{B}_{2L}^{(s)}\right) \sim (\mathbf{2}, 1) , \quad \widetilde{B}_R^{(s)} = \left(\widetilde{B}_{1R}^{(s)}, \widetilde{B}_{2R}^{(s)}\right) \sim (\mathbf{2}, -1) , \\
E_L^{(s)} &= \left(E_{1L}^{(s)}, E_{2L}^{(s)}\right) \sim (\mathbf{2}, 1) , \quad E_R^{(s)} = \left(E_{1R}^{(s)}, E_{2R}^{(s)}\right) \sim (\mathbf{2}, 1) . && (12)
\end{aligned}$$

(13)

$$
\begin{aligned}
-\mathcal{L}_Y = & \sum_{j=1}^{3} y_{j3}^{(u)} \overline{q}_{jL} \widetilde{\phi} u_{3R} + \sum_{r,F} m_F^{(r)} \overline{F}_L^{(r)} F_R^{(r)} + \sum_{r,\widetilde{F}} m_{\widetilde{F}}^{(r)} \overline{\widetilde{F}}_L^{(r)} \widetilde{F}_R^{(r)} \\
& + \sum_{j=1}^{3} \sum_{r,f,F} z_{jr}^{(f)} \overline{f}_{jL} H \left(F_R^{(r)} \sigma \right)_{\mathbf{1}} \frac{1}{\Lambda} + \sum_{r,k,f,F} w_{rk}^{(f)} \left(\overline{F}_L^{(r)} \sigma \right)_{\mathbf{1}'} f_{kR} \\
& + \sum_{j=1}^{3} \sum_{r,f,\widetilde{F}} y_{jr}^{(f)} \overline{f}_{jL} H \left(\widetilde{F}_R^{(r)} \sigma \right)_{\mathbf{1}} \frac{\eta}{\Lambda^2} + \sum_{r,k,f,\widetilde{F}} x_{rk}^{(f)} \left(\overline{\widetilde{F}}_L^{(r)} \sigma \right)_{\mathbf{1}'} \widetilde{f}_{kR} \frac{\eta}{\Lambda} \\
& + \sum_{j=1}^{3} \sum_{s=1}^{2} y_{js}^{(\nu)} \overline{l}_{jL} \widetilde{\phi} \nu_{sR} \frac{\left[\sigma \left(\sigma \sigma \right)_{\mathbf{2}} \right]_{\mathbf{1}'} \eta}{\Lambda^4} + \sum_{s=1}^{2} m_s \bar{\nu}_{sR} \nu_{sR}^C + h.c
\end{aligned}
\tag{14}
$$

$$
\mathcal{L}_{soft}^{\sigma} = \mu_{12}^2 \sigma_1 \sigma_2 \tag{15}
$$

$$m_b \sim \frac{y_b^2}{16\pi^2} f_1 \frac{v}{\Lambda} \frac{\mu_{12}}{M} \mu_{12}, \quad (16)$$

$$m_s \sim \frac{y_s^2}{\left(16\pi^2\right)^2} f_2 \frac{v}{M} \frac{\mu_{12}^3}{\Lambda^3} \mu_{12}, \quad (17)$$

Assuming $y_b^2 f_1 \sim y_s^2 f_2 \sim 1$ and $\mu_{12} \sim M$, we find a rough estimate

$$\Lambda \sim 10v \sim 2.5\mathrm{TeV} \quad (18)$$

for the correct order of magnitude of m_b and m_s.

Model Phenomenology.

$$M_\nu = \frac{\mu_\eta^2\mu_\sigma^6 v}{(16\pi^2)^4\Lambda^8}\begin{pmatrix} \beta_1^2+\gamma_1^2 & \beta_1\beta_2+\gamma_1\gamma_2 & \beta_1\beta_3+\gamma_1\gamma_3 \\ \beta_1\beta_2+\gamma_1\gamma_2 & \beta_2^2+\gamma_2^2 & \beta_2\beta_3+\gamma_2\gamma_3 \\ \beta_1\beta_3+\gamma_1\gamma_3 & \beta_2\beta_3+\gamma_2\gamma_3 & \beta_3^2+\gamma_3^2 \end{pmatrix},$$

$$\beta_s = y_{s1}^{(\nu)}\frac{v}{m_1}f_1^{(\nu)}, \quad \gamma_s = y_{s2}^{(\nu)}\frac{v}{m_2}f_2^{(\nu)}, \quad s=1,2. \tag{19}$$

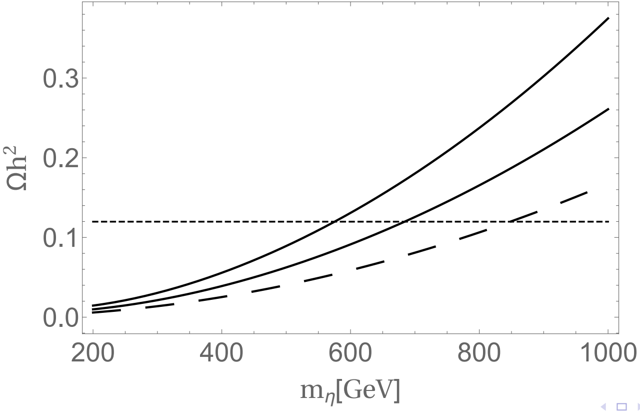
$$m_\nu \sim \frac{\left(y^{(\nu)}\right)^2}{(16\pi^2)^4}f^{(\nu)}\frac{v}{m_s}\frac{\mu_\eta^2\mu_\sigma^6}{\Lambda^8}v. \tag{20}$$

Assuming $\left(y^{(\nu)}\right)^2 \cdot f^{(\nu)} \sim 1$, $\mu_\eta \sim \mu_\sigma \sim m_s \sim \alpha\cdot\Lambda$ and taking $\Lambda = 2.5$TeV from the quark sector (18) we find for $\alpha \sim 1$ the light neutrino mass scale $m_\nu \sim 1$eV, which is too heavy. Assuming, for instance, $\alpha = 0.3$ we arrive at the correct neutrino mass scale $m_\nu \sim 50$ meV. We expect a typical mass scale for all the non-SM particles – the η-DM candidate, in particular, – to be $m_{\mathrm{non-SM}} \sim m_\eta \sim \alpha\cdot\Lambda \sim 750$ GeV.

The only possible decay modes of η are

$$\eta \rightarrow \sigma_{1,2}\widetilde{T}_{2L,1L}u_{1R},\ \sigma_{1,2}\widetilde{T}_{1R,2R}u_{iL},\ \sigma_{1,2}\widetilde{B}^{(s)}_{2L,1L}d_{kR},\ \sigma_{1,2}\widetilde{B}^{(s)}_{1R,2R}d_{iL},$$
$$\sigma_{1,2}\widetilde{E}_{2L,1L}l_{1R},\ \sigma_{1,2}\widetilde{E}_{1R,2R}e_{iL},\ \sigma_1 2\sigma_2\nu_{iL}\nu_{sR} \quad (21)$$

with $s, k = 1, 2$ and $i = 1, 2, 3$.
We assume that our DM candidate η annihilates mainly into WW, ZZ, $t\bar{t}$, $b\bar{b}$ and hh. We take $\lambda_{h^2\eta^2} = 1, 1.2, 1.5$ (from top to bottom, respectively).



The simplified 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$

We consider a $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with (Singer-Valle-Schechter, 1980):

$$Q = T_3 + \beta T_8 + XI, \qquad \beta = -\frac{1}{\sqrt{3}}, \tag{22}$$

331 Models ($SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ Models) are important because:

1. Can explain the origin of fermion generations (Frampton, 1992; Pisano-Pleitez, 1992).
2. These models can have DM candidates (J.K. Mizukoshi, et al, 2011).
3. Can explain the large mass splitting between the heaviest quark family and the two lighter ones (Frampton, 1995).
4. Allow the quantization of electric charge (Pires-Ravinez, 1998; Dong-Long, 2005).
5. Have several sources of CP violation (J.K. Mizukoshi, et al, 1998).
6. Can explain why the Weinberg mixing angle satisfies $\sin^2\theta_W < \frac{1}{4}$.

Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. Possible only with 3 generations!.

Quarks and leptons are unified in the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$Q_L^{1,2} = \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), \qquad Q_L^3 = \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), \quad (23)$$

$$\begin{array}{cc} D_R^{1,2,3} : (3, 1, -1/3), & U_R^{1,2,3} : (3, 1, 2/3), \\ J_R^{1,2} : (3, 1, -1/3), & T_R : (3, 1, 2/3). \end{array} \quad (24)$$

$$L_L^{1,2,3} = \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), \quad (25)$$

$$\begin{array}{ccc} e_R : (1, 1, -1), & \mu_R : (1, 1, -1), & \tau_R : (1, 1, -1), \\ N_R^1 : (1, 1, 0), & N_R^2 : (1, 1, 0), & N_R^3 : (1, 1, 0). \end{array} \quad (26)$$

6 $SU(3)_L$ triplets and 6 $SU(3)_L$ antitriplets → Gauge anomalies cancel

Scalars are grouped in the following $[SU(3)_L, U(1)_X]$ representations:

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3),$$

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3),$$

$$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3, -1/3). \tag{27}$$

The physical scalars are: 4 massive charged Higgs ($H_1^\pm$, $H_2^\pm$), one CP-odd Higgs (A_1^0), 3 neutral CP-even Higgs (h^0, H_1^0, H_3^0) and 2 neutral Higgs ($H_2^0, \overline{H}_2^0$) bosons.

The gauge symmetry in the 3-3-1 model is spontaneously broken in two steps as follows:

$$\mathcal{G} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{v_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v_\eta, v_\rho} SU(3)_C \otimes U(1)_Q , \quad (28)$$

where the hierarchy $v_\eta, v_\rho \ll v_\chi$ among the symmetry breaking scales is fullfilled. Here $v_\eta^2 + v_\rho^2 = v^2$, $v = 246$ GeV.
The quark Yukawa terms are:

$$\begin{aligned} -\mathcal{L}_Y^{(q)} &= \overline{Q}_L^3 \left(\eta h_{\eta 1j}^U + \chi h_{\chi 1j}^U\right) U_R^j + \overline{Q}_L^3 \rho h_{\rho 1j}^D D_R^j \\ &+ \overline{Q}_L^3 \left(\eta h_{\eta 11}^T + \chi h_{\chi 11}^T\right) T_R + \overline{Q}_L^3 \rho h_{\rho 1m}^J J_R^m \\ &+ \overline{Q}_L^n \rho^* h_{\rho nj}^U U_R^j + \overline{Q}_L^n \left(\eta^* h_{\eta nj}^D + \chi^* h_{\chi nj}^D\right) D_R^j \\ &+ \overline{Q}_L^n \rho^* h_{\rho n1}^T T_R^1 + \overline{Q}_L^n \left(\eta^* h_{\eta nm}^J + \chi^* h_{\chi nm}^J\right) J_R^m + h.c, \quad (29) \end{aligned}$$

where $n = 2, 3$ and $i, j = 1, 2, 3$.

The lepton Yukawa terms are:

$$-\mathcal{L}_Y^{(l)} = h_{\rho ij}^{(L)}\overline{L}_L^i \rho e_{jR} + \frac{1}{2}\left(h_\rho\right)_{ij}\varepsilon_{abc}\overline{L}_L^{ia}\left(L_L^{jC}\right)^b\left(\rho^*\right)^c + h_{\eta ij}^{(L)}\overline{L}_L^i \eta N_{jR} + h_{\eta ij}^{(L)}\overline{L}_L^i \chi N_{jR} + m_{Nij}\overline{N}_R^i N_R^{jC} + h.c \tag{30}$$

where $n = 2, 3$ and $i, j = 1, 2, 3$. The neutrino mass terms are:

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2}\begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + H.c, \tag{31}$$

where the neutrino mass matrix is:

$$M_\nu = \begin{pmatrix} 0_{3\times3} & M_1 & M_2 \\ M_1^T & 0_{3\times3} & M_3 \\ M_2^T & M_3^T & m_N \end{pmatrix} = \begin{pmatrix} 0_{3\times3} & \frac{v_\rho}{2\sqrt{2}}\left(h_\rho^\dagger - h_\rho^*\right) & \frac{v_\eta}{\sqrt{2}}h_\eta^* \\ \frac{v_\rho}{2\sqrt{2}}\left(h_\rho^\dagger - h_\rho^*\right)^T & 0_{3\times3} & \frac{v_\chi}{\sqrt{2}}h_\chi^* \\ \frac{v_\eta}{\sqrt{2}}h_\eta^\dagger & \frac{v_\chi}{\sqrt{2}}h_\chi^\dagger & m_N \end{pmatrix}, \tag{32}$$

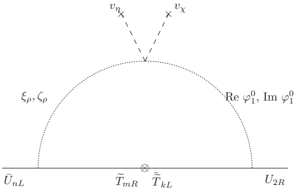
A 3-3-1 model with sequential loop suppression mechanism

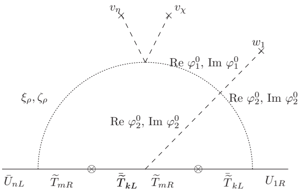
$$\begin{aligned} \mathcal{G} &= SU(3)_C \times SU(3)_L \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_g} \\ &\xrightarrow{v_\chi, v_\xi} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \times Z_2^{(L_g)} \\ &\xrightarrow{v_\eta} SU(3)_C \times U(1)_{em} \times Z_4 \times Z_2^{(L_g)}, \end{aligned} \tag{33}$$

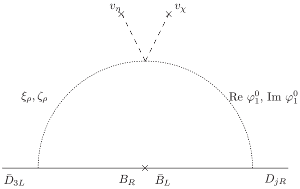
	χ	η	ρ	φ_1^0	φ_2^0	ϕ_1^+	ϕ_2^+	ϕ_3^+	ϕ_4^+	ξ^0
L_g	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	-2	-2	-2
Z_4	1	1	-1	-1	i	i	-1	-1	1	1
Z_2	-1	-1	1	1	1	1	1	-1	-1	1

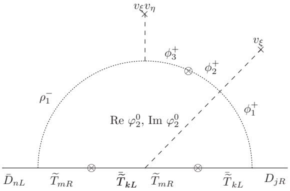
	Q_{1L}	Q_{2L}	Q_{3L}	U_{1R}	U_{2R}	U_{3R}	T_R	D_{1R}	D_{2R}	D_{3R}	J_{1R}	J_{2R}	$\widetilde{T}_{1L}$	$\widetilde{T}_{1R}$	$\widetilde{T}_{2L}$	$\widetilde{T}_{2R}$	B_L	B_R
L_g	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	0	0	0	2	2	0	0	0	0	0	0
Z_4	-1	-1	1	1	$-i$	1	1	1	1	1	-1	-1	i	1	i	1	-1	-1
Z_2	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	1	1	1

	L_{1L}	L_{2L}	L_{3L}	e_{1R}	e_{2R}	e_{3R}	E_{1L}	E_{2L}	E_{3L}	E_{1R}	E_{2R}	E_{3R}	N_{1R}	N_{2R}	N_{3R}	Ψ_R
L_g	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1	1	1	1	1	1	1	-1	-1	-1	1
Z_4	i	i	i	$-i$	$-i$	$-i$	1	i	i	$-i$	$-i$	$-i$	i	i	i	1
Z_2	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	-1

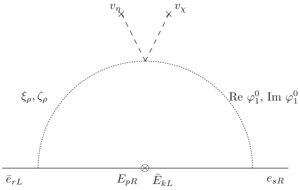

v_η
v_χ
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
$\bar{U}_{nL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
U_{2R}

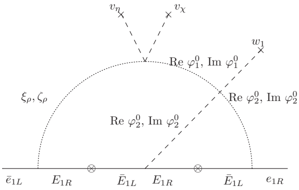

v_η
v_χ
w_1
Re φ_1^0, Im φ_1^0
ξ_ρ, ζ_ρ
Re φ_2^0, Im φ_2^0
Re φ_2^0, Im φ_2^0
$\bar{U}_{nL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
U_{1R}

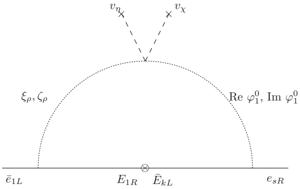

v_η
v_χ
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
$\bar{D}_{3L}$
B_R
$\bar{B}_L$
D_{jR}


$v_\xi v_\eta$
v_ξ
ϕ_3^+
ϕ_2^+
ρ_1^-
ϕ_1^+
Re φ_2^0, Im φ_2^0
$\bar{D}_{nL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
$\widetilde{T}_{mR}$
$\bar{\widetilde{T}}_{kL}$
D_{jR}

$$
\begin{aligned}
M_U &= \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} & 0 \\ \widetilde{\varepsilon}_{21}^{(u)} & \varepsilon_{22}^{(u)} & 0 \\ 0 & 0 & y \end{pmatrix} \frac{v}{\sqrt{2}}, \\
M_D &= \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(d)} & \widetilde{\varepsilon}_{12}^{(d)} & \widetilde{\varepsilon}_{13}^{(d)} \\ \widetilde{\varepsilon}_{21}^{(d)} & \widetilde{\varepsilon}_{22}^{(d)} & \widetilde{\varepsilon}_{23}^{(d)} \\ \varepsilon_{31}^{(d)} & \varepsilon_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}} \qquad (34)
\end{aligned}
$$


v_η
v_χ
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
$\bar{e}_{rL}$
E_{pR}
$\bar{E}_{kL}$
e_{sR}


v_η
v_χ
w_1
Re φ_1^0, Im φ_1^0
ξ_ρ, ζ_ρ
Re φ_2^0, Im φ_2^0
Re φ_2^0, Im φ_2^0
$\bar{e}_{1L}$
E_{1R}
$\bar{E}_{1L}$
E_{1R}
$\bar{E}_{1L}$
e_{1R}


v_η
v_χ
ξ_ρ, ζ_ρ
Re φ_1^0, Im φ_1^0
$\bar{e}_{1L}$
E_{1R}
$\bar{E}_{kL}$
e_{sR}

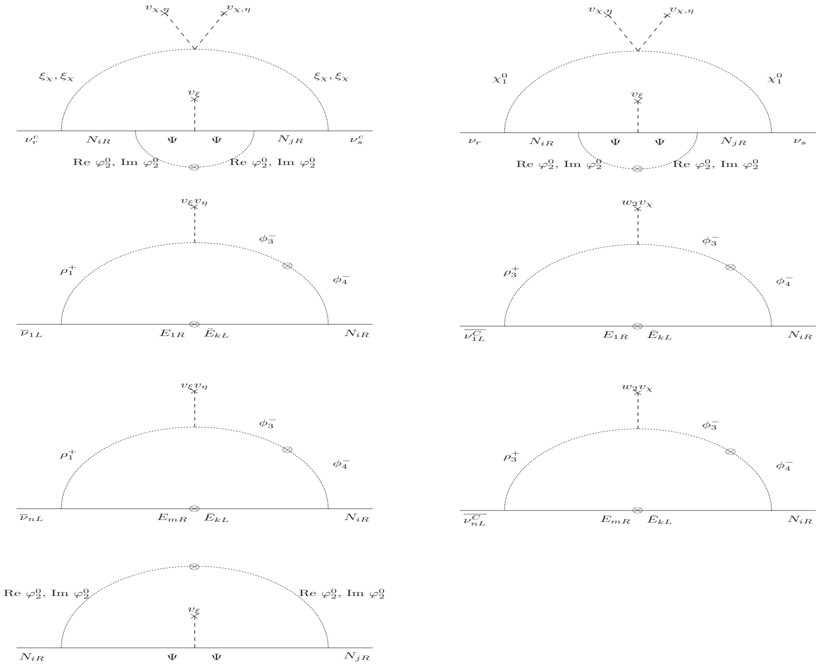
$$M_l = \begin{pmatrix} \widetilde{\varepsilon}^{(l)}_{11} & \varepsilon^{(l)}_{12} & \varepsilon^{(l)}_{13} \\ 0 & \varepsilon^{(l)}_{22} & \varepsilon^{(l)}_{23} \\ 0 & \varepsilon^{(l)}_{32} & \varepsilon^{(l)}_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{35}$$

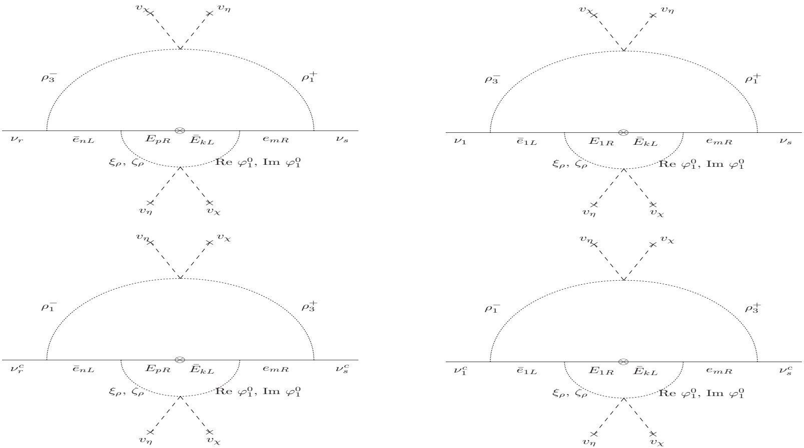
$$-\mathcal{L}^{(\nu)}_{mass} = \frac{1}{2} \begin{pmatrix} \overline{\nu^C_L} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu^C_R \\ N^C_R \end{pmatrix} + y_\Psi \overline{\Psi^c_R} \zeta^0 \Psi_R + h.c,$$

$$M_\nu = \begin{pmatrix} M_1 & 0_{3\times 3} & M_3 \\ 0_{3\times 3} & M_2 & M_4 \\ M_3 & M_4 & \mathcal{M} \end{pmatrix}, \qquad M_1 = \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}, \qquad M_3 = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_4 = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \\ d_1 & d_2 & d_3 \\ d_4 & d_5 & d_6 \end{pmatrix} \frac{v_\chi}{\sqrt{2}}, \qquad \mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} \\ \mathcal{M}_{12} & \mathcal{M}_{22} & \mathcal{M}_{23} \\ \mathcal{M}_{13} & \mathcal{M}_{23} & \mathcal{M}_{33} \end{pmatrix}$$


$v_{\chi,\eta}$ $v_{\chi,\eta}$
ξ_χ, ξ_χ ξ_χ, ξ_χ
v_ξ
ν_r^c N_{iR} Ψ Ψ N_{jR} ν_s^c
Re φ_2^0, Im φ_2^0 Re φ_2^0, Im φ_2^0
$v_{\chi,\eta}$ $v_{\chi,\eta}$
χ_1^0 χ_1^0
v_ξ
ν_r N_{iR} Ψ Ψ N_{jR} ν_s
Re φ_2^0, Im φ_2^0 Re φ_2^0, Im φ_2^0
$v_\xi v_\eta$
ϕ_3^- ρ_1^+ ϕ_4^-
$\overline{\nu}_{1L}$ E_{1R} $\bar{E}_{kL}$ N_{iR}
$w_2 v_\chi$
ϕ_3^- ρ_3^+ ϕ_4^-
$\overline{\nu_{1L}^C}$ E_{1R} $\bar{E}_{kL}$ N_{iR}
$v_\xi v_\eta$
ϕ_3^- ρ_1^+ ϕ_4^-
$\overline{\nu}_{nL}$ E_{mR} $\bar{E}_{kL}$ N_{iR}
$w_2 v_\chi$
ϕ_3^- ρ_3^+ ϕ_4^-
$\overline{\nu_{nL}^C}$ E_{mR} $\bar{E}_{kL}$ N_{iR}
Re φ_2^0, Im φ_2^0 Re φ_2^0, Im φ_2^0
v_ξ
N_{iR} Ψ Ψ N_{jR}


v_χ v_η
ρ_3^- ρ_1^+
ν_r $\bar{e}_{nL}$ E_{pR} $\bar{E}_{kL}$ e_{mR} ν_s
ξ_ρ, ζ_ρ Re φ_1^0, Im φ_1^0
v_η v_χ
v_χ v_η
ρ_3^- ρ_1^+
ν_1 $\bar{e}_{1L}$ E_{1R} $\bar{E}_{kL}$ e_{mR} ν_s
ξ_ρ, ζ_ρ Re φ_1^0, Im φ_1^0
v_η v_χ
v_η v_χ
ρ_1^- ρ_3^+
ν_r^c $\bar{e}_{nL}$ E_{pR} $\bar{E}_{kL}$ e_{mR} ν_s^c
ξ_ρ, ζ_ρ Re φ_1^0, Im φ_1^0
v_η v_χ
v_η v_χ
ρ_1^- ρ_3^+
ν_1^c $\bar{e}_{1L}$ E_{1R} $\bar{E}_{kL}$ e_{mR} ν_s^c
ξ_ρ, ζ_ρ Re φ_1^0, Im φ_1^0
v_η v_χ

The light active neutrino mass matrix has the form:

$$M_{1\nu} = M_1 + \frac{1}{16} M_3 (M_4)^{-2} M_3^T M_3 (M_4)^{-1} M_3^T M_3 (M_4)^{-2} M_3^T + \frac{1}{8} M_3 (M_4)^{-1} \left(\mathcal{M} - M_3^T M_3 (M_4)^{-1} \right) (M_4)^{-1} \times \left(\mathcal{M} - (M_4)^{-1} M_3^T M_3 \right) (M_4)^{-1} M_3^T \quad (36)$$

whereas the sterile neutrino mass matrices are given by:

$$M_{2\nu} = -M_4$$
$$M_{3\nu} = M_4 + \frac{1}{\sqrt{2}} \left(M_3^T M_3 (M_4)^{-1} + (M_4)^{-1} M_3^T M_3 \right). \quad (37)$$

For the benchmark region where $m_\Psi \gg m_{\phi_2^0}, \mu_1$ their contribution is

$$a_{ij} \sim \alpha_1 \left(\frac{1}{4\pi} \right)^2 \left(\frac{v_\chi}{v} \right)^2 \frac{\mu_1^2}{m_\Psi} \log \left(m_\Psi / m_{\phi_2^0} \right). \quad (38)$$

$a_{ij} \to 0$ in the limit $m_\Psi \to 0$, since m_Ψ is source of LNV

(2nd term in Eq. (36)) $\sim \left(\frac{v}{v_\chi} \right)^5 v$, $(M_{1\nu})_{ij} \lesssim m_\nu \sim 50 meV$, $v_\chi \gtrsim 90$ TeV

$$\frac{1}{\Lambda^3}\left(\bar{L}\Psi_R\right)\left(\bar{e}_R L\right)\varphi_2^0 \tag{39}$$

For $m_\Psi < m_2^R$, Ψ_R is a DM candidate.

$$\frac{1}{\Lambda^2}\epsilon_{abc}\left(\eta^\dagger\right)^a\left(\chi^\dagger\right)^b\varphi_1^0\overline{L}_1^c e_{kR} \quad \text{for} \quad k = 2,3 \tag{40}$$

$$\Gamma(\varphi^0 \to Z e_1^+ e_{2,3}^-) \simeq \Gamma(\varphi^0 \to \zeta_\eta e_1^+ e_{2,3}^-) \sim \Gamma(\varphi^0 \to h e_1^+ e_{2,3}^-) \sim \frac{m_{\varphi^0}^3 v_\chi^2}{\Lambda^4},$$
$$\Gamma(\varphi^0 \to e_1^+ e_{2,3}^-) \sim m_{\varphi^0}\left(\frac{v_\chi v_\eta}{\Lambda^2}\right)^2. \tag{41}$$

Requiring that the DM candidate φ^0 lifetime be greater than the universe lifetime $\tau_u \approx 13.8$ Gyr, taking into account the limit $v_\chi \gtrsim 90\,\text{TeV}$ and assuming $m_{\varphi^0} \sim 1$ TeV, we estimate the cutoff scale of our model

$$\Lambda > 3 \times 10^{10}\text{GeV} \tag{42}$$

Conclusions

For the $S_3 \times Z_2$ flavor model:

- The SM fermion mass hierarchy is generated by the loops.
- The cutoff scale is $\Lambda \sim 2.5$ TeV.
- The model predicts one massless and two non-zero mass neutrinos.
- The mass scale of the non-SM particles are of the order of 1 TeV.
- The model possesses DM particle candidates.

For the 3-3-1 model with sequential loop suppression mechanism

- The SM fermion mass hierarchy arises from a sequential loop suppression.
- Light active neutrinos acquire their masses from a combination of linear and inverse seesaw mechanisms at two loop level.
- At tree level there is no quark mixing, the mixing angles in the quark sector are generated from a combination of one and two loop level radiative seesaw mechanisms.
- The contribution to the leptonic mixing angles coming from the charged leptons arise at one loop level, whereas the mixings in the light active neutrino sector are generated from a two loop level.
- The model possesses DM particle candidates.

Acknowledgements

Thank you very much to all of you for the attention.

A.E.C.H was supported by Fondecyt (Chile), Grant No. 1170803.

Extra Slides

$$
\begin{aligned}
-L_{gY}^{(q)} &= h_{\chi}^{(T)}\overline{Q}_{3L}\chi T_R + h_{\eta}^{(U)}\overline{Q}_{3L}\eta U_{3R} \\
&+ \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\rho nm}^{(\widetilde{T})}\overline{Q}_{nL}\rho^{*}\widetilde{T}_{mR} + \sum_{n=1}^{2} h_{\varphi_1^0 n2}^{(U)}\overline{\widetilde{T}}_{nL}\varphi_1^0 U_{2R} + \sum_{n=1}^{2} h_{\varphi_2^0 n1}^{(U)}\overline{\widetilde{T}}_{nL}\varphi_2^0 U_{1R} \\
&+ \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\chi nm}^{(J)}\overline{Q}_{nL}\chi^{*} J_{mR} + h_{\rho}^{(B)}\overline{Q}_{3L}\rho B_R + \sum_{j=1}^{3} h_{\varphi_1^0 j}^{(D)}\overline{B}_L\varphi_1^0 D_{jR} \qquad (1) \\
&+ \sum_{n=1}^{2}\sum_{j=1}^{3} h_{\phi_1^+ nj}^{(D)}\overline{\widetilde{T}}_{nL}\phi_1^{+} D_{jR} + \sum_{n=1}^{2}\sum_{m=1}^{2} h_{\varphi_2^0 nm}^{(\widetilde{T})}\overline{\widetilde{T}}_{nL}\varphi_2^0\widetilde{T}_{mR} + m_B\bar{B}_L B_R + h.c,
\end{aligned}
$$

$$
\begin{aligned}
-L_{gY}^{(l)} &= h_{\rho}^{(E)}\overline{L}_{1L}\rho E_{1R} + h_{\varphi_2^0}^{(E)}\overline{E}_{1L}\varphi_2^0 E_{1R} + h_{\varphi_2^0}^{(e)}\overline{E}_{1L}\varphi_2^0 e_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(E)}\overline{L}_{nL}\rho E_{mR} \\
&+ h_{\rho}^{(e)}\overline{L}_{1L}\rho e_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(e)}\overline{L}_{nL}\rho e_{mR} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\varphi_1^0 nm}^{(E)}\overline{E}_{nL}\varphi_1^0 E_{mR} \\
&+ \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\varphi_1^0 nm}^{(e)}\overline{E}_{nL}\varphi_1^0 e_{mR} + \sum_{n=2}^{3}\sum_{j=1}^{3} h_{\chi nj}^{(L)}\overline{L}_{nL}\chi N_{jR} \\
&+ \sum_{j=1}^{3}\sum_{n=2}^{3} h_{\phi_4^- nj}^{(e)}\overline{E}_{nL}\phi_4^{-} N_{jR} + \sum_{j=1}^{3} h_{\varphi_2^0}^{(N)}\overline{\Psi_R^c}\left(\varphi_2^0\right)^{*} N_{jR} + y_{\Psi}\overline{\Psi_R^c}\Psi_R\xi^0 \\
&+ h_{\rho 11}^{(L)}\varepsilon_{abc}\overline{L}_{1L}^{a}\left(L_{1L}^{C}\right)^{b}\left(\rho^{*}\right)^{c} + \sum_{n=2}^{3}\sum_{m=2}^{3} h_{\rho nm}^{(L)}\varepsilon_{abc}\overline{L}_{nL}^{a}\left(L_{mL}^{C}\right)^{b}\left(\rho^{*}\right)^{c} + h.c. \qquad (2)
\end{aligned}
$$

$$
V \supset \lambda_1\eta\chi\rho\varphi_1^0 + \lambda_2\eta\chi\rho\left(\varphi_1^0\right)^{*} + \lambda_3\phi_3^{-}\rho\eta^{\dagger}\xi^0 + \lambda_4\phi_1^{-}\phi_2^{+}\left(\varphi_2^0\right)^{*}\left(\xi^0\right)^{*} + w_1\left(\varphi_2^0\right)^2\varphi_1^0 + w_2\phi_3^{-}\rho\chi^{\dagger} + h.c.. \qquad (3)
$$

$$
\begin{aligned}
L_{gsoft}^{F} &= \sum_{n=1}^{2}\sum_{m=1}^{2}\left(m_{\widetilde{T}}\right)_{nm}\overline{\widetilde{T}}_{nL}\widetilde{T}_{mR} + m_{E_1}\overline{E}_{1L}E_{1R} + \sum_{n=2}^{3}\sum_{m=2}^{3}\left(m_E\right)_{nm}\overline{E}_{nL}E_{mR} \\
&+ \sum_{n=2}^{3}\left(m_E\right)_{n1}\overline{E}_{nL}E_{1R} + h.c., \qquad (4)
\end{aligned}
$$