CP Phases and Symmetries

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Outline

- scenario with flavor and CP symmetry
- phenomenology for example case
  - CP violation at low energies
  - CP violation at high energies
- conclusions

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- generations of LH leptons are assigned to $\mathbf{3}$
- impose flavor and CP symmetry in fundamental theory

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- features: ***(Feruglio/H/Ziegler ('12,'13))***
  - prediction of Majorana phases becomes possible
  - at the same time lepton mixing angles are less constrained and non-trivial Dirac phase is possible
  - all angles and CP phases depend on one free parameter θ

($\mu - \tau$ reflection symmetry: ***Harrison/Scott ('02,'04), Grimus/Lavoura ('03)***)

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- features: ***(H/Molinaro ('16))***
  - correlation between low and high energy CP phases

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- features: *(H/Molinaro ('16))*
  - unflavored leptogenesis: $Y_B \propto \kappa^2$ and sign fixed
  - flavored leptogenesis: $Y_B \propto \kappa$
  - flavored leptogenesis: $Y_B \not\propto \kappa$

  (κ symmetry breaking parameter)

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- leptonic mixing arises from mismatch of residual symmetries G_e and CP $\subset G_\nu$
- high energy CP phases are constrained by residual symmetries as well
- in a theory with G_f and CP certain conditions have to be fulfilled

(Feruglio/H/Ziegler ('12,'13), Holthausen et al. ('12), Chen et al. ('14))

Phenomenology for example case

(H/Meroni/Molinaro ('14); see also Ding et al. ('14))

$\Delta(3\,n^2), \Delta(6\,n^2)$ and CP

$\swarrow$ $\searrow$

charged leptons | neutrinos

$G_e = Z_3$ | $G_\nu = Z_2 \times$ CP

U_e | $U_\nu = \Omega_\nu R(\theta) K_\nu$

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

four different types of mixing patterns with different characteristics

example: Case 2) with CP transformations $X(u, v)$

CP violation at low energies

Case 2) *(H/Meroni/Molinaro ('14))*

- one column is trimaximal, i.e. $\sin^2\theta_{12} \gtrsim \frac{1}{3}$
- smallness of θ_{13} constrains θ and u/n

CP violation at low energies

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- two sum rules for mixing angles

$$\sin^2\theta_{12} = \frac{1}{3\,\cos^2\theta_{13}}$$

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$$6\,\sin^2\theta_{23}\,\cos^2\theta_{13} = 3 \pm \sqrt{3}\,\tan\phi - 3\,\left(1 \pm \sqrt{3}\,\tan\phi\right)\,\sin^2\theta_{13}$$

with

$$\phi = \frac{\pi\,u}{n} + \sigma\,\frac{\pi}{3} \quad \text{and} \quad \sigma = 0,\,1,\,-1$$

(see also e.g. ***Ding/King ('14)***)

CP violation at low energies

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- one column is trimaximal, i.e. $\sin^2\theta_{12} \gtrsim \frac{1}{3}$
- smallness of θ_{13} constrains θ and u/n
- Dirac phase is function of u/n
- Majorana phase β depends on u/n as well
- value of Majorana phase α is (mainly) fixed by v/n

CP violation at low energies

Case 2) with $n = 10$ and $u = 4$ ***(H/Meroni/Molinaro ('14))***

- results for mixing angles (as example $\theta_{23} > \pi/4$)

$$0.340 \lesssim \sin^2\theta_{12} \lesssim 0.342$$

$$0.0187 \lesssim \sin^2\theta_{13} \lesssim 0.0250$$

$$0.558 \lesssim \sin^2\theta_{23} \lesssim 0.559$$

- Dirac phase and Majorana phase β are constrained as (one fixed combination of CP parities)

$$0.83 \lesssim \sin\beta \lesssim 0.94$$

$$-0.86 \lesssim \sin\delta \lesssim -0.80$$

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- Majorana phase α can take several values

v	0	$6, 24$	$12, 18$
$\sin\alpha$	$-0.035 \div -0.028$	$0.94 \div 0.96$	$-0.62 \div -0.56$

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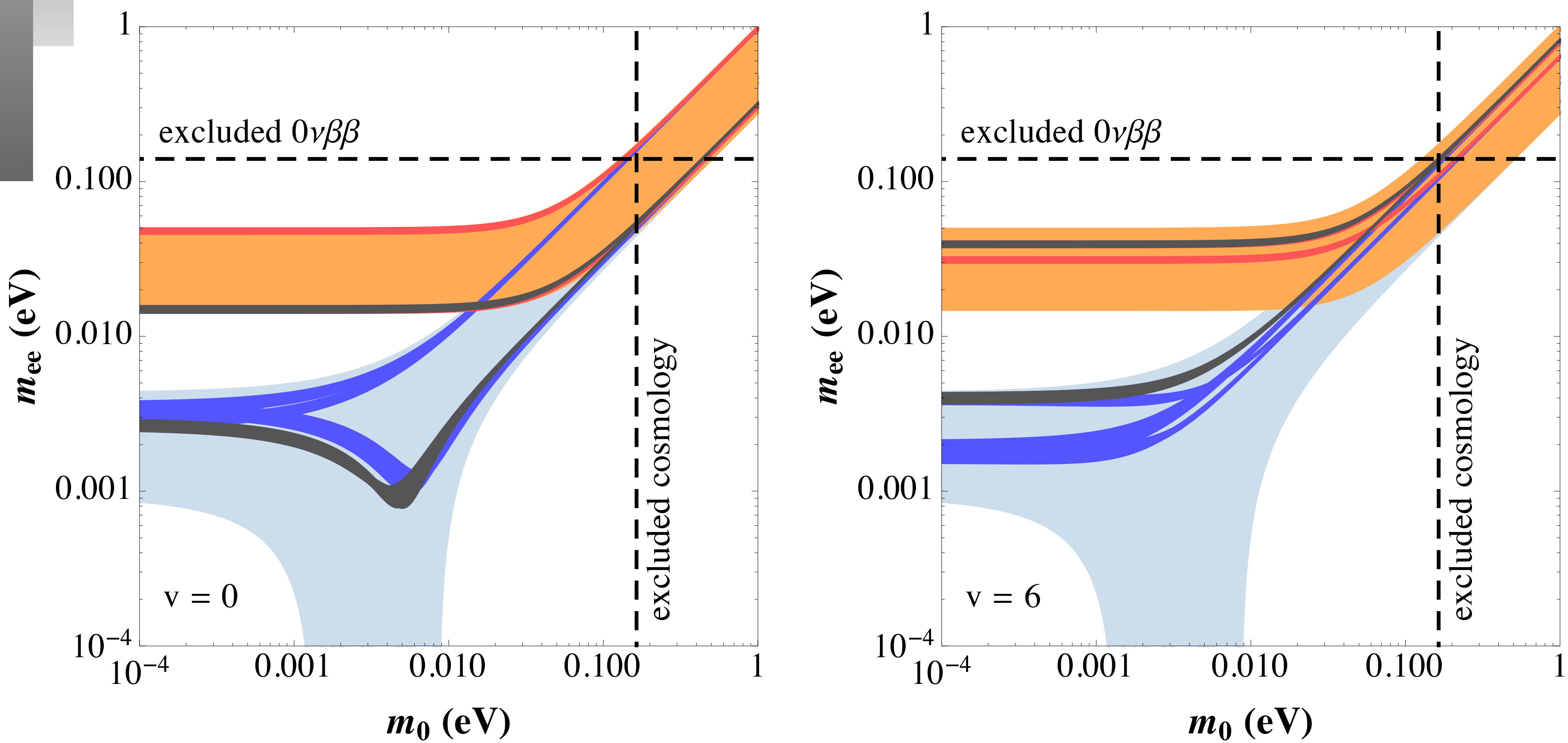

See also works by ***Ding, King, Neder***.

CP violation at high energies

- type-1 seesaw mechanism
- three RH neutrinos N_i forming $\mathbf{3}$
- baryogenesis through leptogenesis

$$Y_B \approx 10^{-3} \sum_{\alpha,i,j} \epsilon_i^\alpha \, \eta_{ij}^\alpha$$

ϵ_i^α: CP asymmetry from N_i decays to charged lepton flavor α
η_{ij}^α: efficiency factor

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- flavor and CP symmetries *(H/Molinaro ('16))*
  - $G_e = Z_3$ for charged leptons
  - $G_\nu = Z_2 \times$ CP for neutrinos

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  - $G_e = Z_3$ for charged leptons
  - $G_\nu = Z_2 \times$ CP for neutrinos
  - two realizations
    1. Y_D invariant under G_f and CP,
       M_R under CP $\subset G_\nu$
    2. Y_D and M_R both invariant under CP $\subset G_\nu$

Realization 1

(H/Molinaro ('16))

- charged lepton mass matrix m_e is invariant under $G_e = Z_3$
- light neutrino mass matrix is invariant under $G_\nu = Z_2 \times$ CP
  - Y_D is invariant under G_f and CP

$$Y_D = y_0 \, 1$$

  - lepton mixing encoded in RH neutrino mass matrix M_R, M_R is invariant under $G_\nu = Z_2 \times$ CP

$$U_R^T \, M_R \, U_R = \text{diag} \, (M_1, M_2, M_3) \quad \text{with} \quad U_R = \Omega_\nu \, R_{ij}(\theta) \, K_\nu$$

Realization 1

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- charged lepton mass matrix m_e is invariant under $G_e = Z_3$
- light neutrino mass matrix is invariant under $G_\nu = Z_2 \times$ CP
- we find
  - seesaw relation of light and heavy neutrino masses

$$m_i \propto \frac{1}{M_i}$$

  - PMNS mixing matrix is given by

$$U_{PMNS} = U_\nu = U_R = \Omega_\nu \, R_{ij}(\theta) \, K_\nu$$

Realization 1

- however, CP asymmetries for unflavored leptogenesis

$$\epsilon_i = -\frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im}\left((\hat{Y}_D^\dagger \hat{Y}_D)_{ij}^2 \right)}{(\hat{Y}_D^\dagger \hat{Y}_D)_{ii}} \, f(M_j/M_i)$$

vanish,
since

$$\hat{Y}_D^\dagger \, \hat{Y}_D \propto 1$$

known in flavor symmetry-only context

(Jenkins/Manohar ('08), Bertuzzo et al. ('09), H/Molinaro/Petcov ('09), Aristizabal Sierra et al. ('09))

Realization 1

- in order to achieve non-zero ϵ_i further symmetry breaking is needed,
  in particular
  $$\hat{Y}_D = Y_D\, U_R$$
  should get corrections,
  i.e.
  $$\hat{Y}_D = Y_D\, U_R = \left(Y_D^0 + \delta Y_D\right)\,\left(U_R^0\, \delta U_R\right)$$

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  $$\hat{Y}_D = Y_D\, U_R = \left(Y_D^0 + \delta Y_D\right)\ \left(U_R^0\, \delta U_R\right)$$
- notice δU_R can only be effective, if there is also δY_D
- we focus on $\delta Y_D \neq 0$

Realization 1

- we get then

$$\hat{Y}_D^\dagger \hat{Y}_D \approx U_R^\dagger \left((Y_D^0)^\dagger Y_D^0 + (\delta Y_D)^\dagger Y_D^0 + (Y_D^0)^\dagger \delta Y_D \right) U_R$$

- the first term is irrelevant, since $(Y_D^0)^\dagger Y_D^0 \propto 1$
- the second and third term depend in general not only on CP phases contained in U_R,
  but also on phases encoded in correction δY_D
- four instances in which phases of δY_D become irrelevant ...

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  i) $(Y_D^0)^\dagger \delta Y_D$ is real
  ii) $(Y_D^0)^\dagger \delta Y_D$ is imaginary
  iii) $(Y_D^0)^\dagger \delta Y_D$ is complex and symmetric
  iv) $(Y_D^0)^\dagger \delta Y_D$ is complex and antisymmetric

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- condition iii) can naturally be realized
  - $Y_D^0 \propto 1$ at leading order
  - δY_D flavor diagonal, since invariant under $G_e = Z_3$
    δY_D has two (in general complex) parameters
  - $\delta Y_D \propto \kappa$ (κ symmetry breaking parameter)

Realization 1

- size of ϵ_i:
  for $\delta Y_D \propto \kappa$ we find
  $$\epsilon_i \propto \kappa^2$$
  that leads for $\kappa \sim 10^{-(2\div3)}$ to correct size of Y_B,
  taking into account size of Yukawa coupling y_0 and RH neutrino masses M_i
  as well as efficiency factor
  known in flavor symmetry-only context
  (Jenkins/Manohar ('08), Bertuzzo et al. ('09), H/Molinaro/Petcov ('09), Aristizabal Sierra et al. ('09))

Realization 1

Results for Case 2)

- CP asymmetries ϵ_i read, e.g.

$$\epsilon_1 \approx \frac{\kappa^2}{6\,\pi}\left[(-1)^{k_1} f\left(\frac{m_1}{m_2}\right)\left([\cos(\phi+2\zeta)+\cos 2\theta]\sin\phi_v - \sin(\phi+2\zeta)\sin 2\theta\cos\phi_v\right)\right.$$

$$\left. +(-1)^{k_2+1} f\left(\frac{m_1}{m_3}\right)\sin 2\,(\phi-\zeta)\sin 2\theta\right]$$

$$= -\frac{3\,\kappa^2}{2\,\pi}\left[I_1\,(\phi\to\phi+2\,\zeta)\; f\left(\frac{m_1}{m_2}\right) + I_2\,(\phi\to\phi-\zeta)\; f\left(\frac{m_1}{m_3}\right)\right]$$

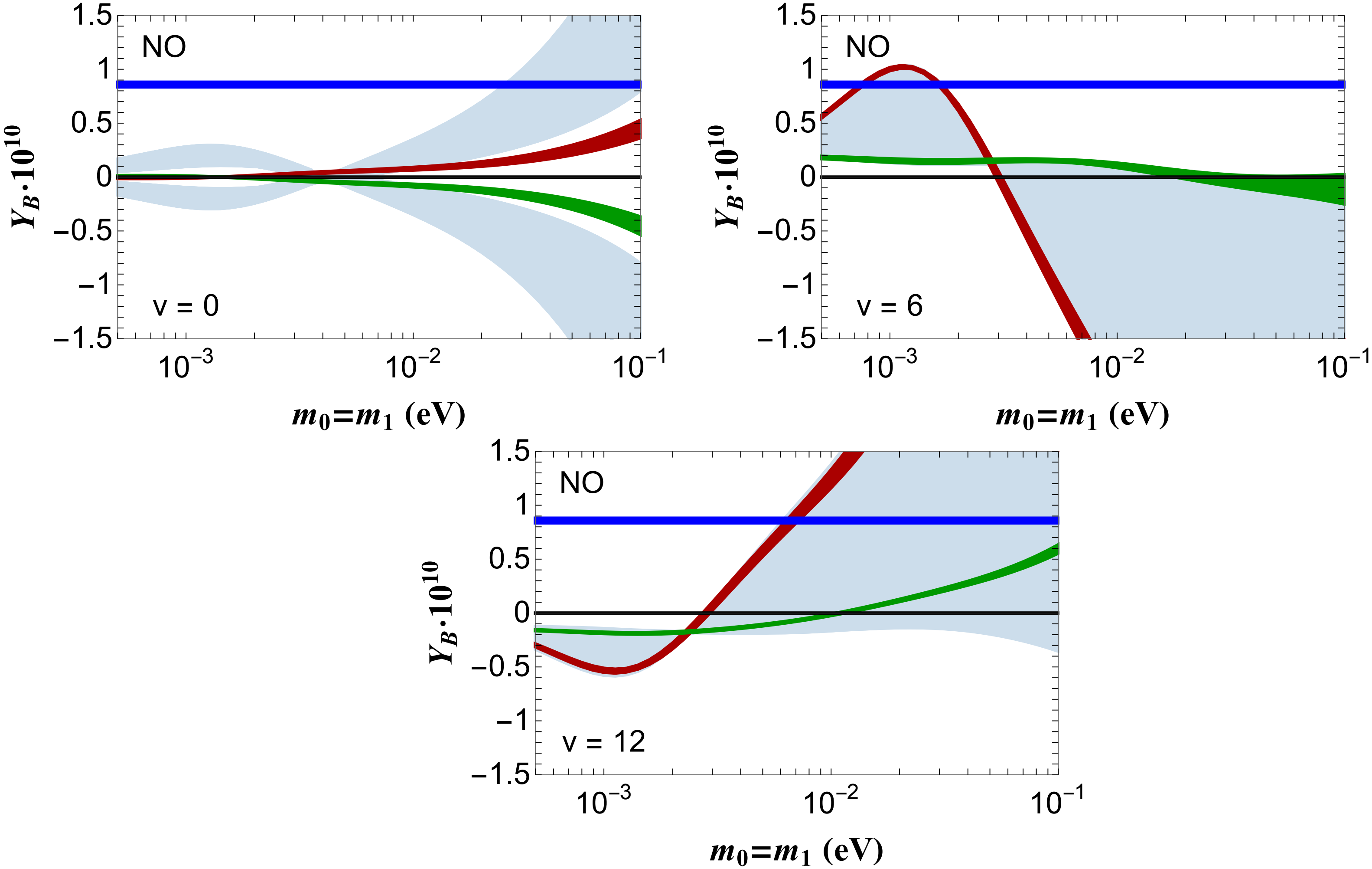
with

$$I_1 = \mathrm{Im}[U^2_{\mathrm{PMNS},12}(U^\star_{\mathrm{PMNS},11})^2] = \sin^2\theta_{12}\cos^2\theta_{12}\cos^4\theta_{13}\sin\alpha\,,$$

$$I_2 = \mathrm{Im}[U^2_{\mathrm{PMNS},13}(U^\star_{\mathrm{PMNS},11})^2] = \sin^2\theta_{13}\cos^2\theta_{12}\cos^2\theta_{13}\sin\beta$$

Realization 1

Case 2) with $n = 10$ and $u = 4$ *(H/Molinaro ('16))*



Realization 1

- however, CP asymmetries for flavored leptogenesis

$$\epsilon_i^\alpha = -\frac{1}{8\pi\,(\hat{Y}_D^\dagger \hat{Y}_D)_{ii}} \sum_{j\neq i} \Big\{ \mathrm{Im}\left((\hat{Y}_D^\dagger \hat{Y}_D)_{ij} (\hat{Y}_D)^\star_{\alpha i} (\hat{Y}_D)_{\alpha j} \right) f(M_j/M_i) + \mathrm{Im}\left((\hat{Y}_D^\dagger \hat{Y}_D)_{ji} (\hat{Y}_D)^\star_{\alpha i} (\hat{Y}_D)_{\alpha j} \right) g\,(M_j/M_i) \Big\},$$

  vanish,
  since

$$\hat{Y}_D^\dagger\, \hat{Y}_D \propto 1$$

  known in flavor symmetry-only context ***(Bertuzzo et al. ('09))***

- size of ϵ_i^α:
  for $\delta Y_D \propto \kappa$ we find

$$\epsilon_i^\alpha \propto \kappa$$

  so that correct size of Y_B is combination of size of κ, Yukawa coupling y_0 and efficiency factor

Realization 1

Results for Case 2)

- CP asymmetries ϵ_i^α read, e.g.

$$\epsilon_1^e \approx \frac{y_0\, \kappa}{12\sqrt{3}\,\pi}\left[f\left(\frac{m_1}{m_2}\right)\Big([\cos(\phi+\zeta)+\cos\zeta\ \cos 2\theta]\sin\phi_v - \sin(\phi+\zeta)\sin 2\theta\cos\phi_v\Big)\right.$$

$$\left. - f\left(\frac{m_1}{m_3}\right)\sin(2\phi-\zeta)\sin 2\theta\right]$$

$$+\frac{y_0\, \kappa}{12\sqrt{3}\,\pi}\left[g\left(\frac{m_1}{m_2}\right)\ \sin\zeta\ \sin 2\theta - g\left(\frac{m_1}{m_3}\right)\sin\zeta\sin 2\theta\right]$$

- sign of ϵ_i^α depends in general on ζ as well

Realization 2

(H/Molinaro ('16))

- charged lepton mass matrix m_e is invariant under $G_e = Z_3$
- light neutrino mass matrix is invariant under $G_\nu = Z_2 \times$ CP
  - Y_D is invariant under G_ν

    $$Z^\dagger \, Y_D \, Z \; = \; Y_D \quad \text{and} \quad X^\star \, Y_D \, X \; = \; Y_D^\star$$

    such that

    $$Y_D = \Omega_\nu \, R_{ij}(\theta_L) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} R_{ij}(-\theta_R) \, \Omega_\nu^\dagger$$

  - RH neutrino mass matrix M_R also

    $$U_R^T \, M_R \, U_R = \text{diag} \; (M_1, M_2, M_3) \quad \text{with} \quad U_R = \Omega_\nu \, R_{ij}(\theta) \, K_\nu$$

Realization 2

- CP asymmetries ϵ_i^α do not vanish – even without δY_D
- however, there is in general no one-to-one correspondence between α, β, δ and sign of Y_B, e.g. we find for Case 2)

$$\epsilon_{1\,(3)}^{\alpha} \quad = \quad \frac{y_1\, y_3\, (y_{1\,(3)}^2 - y_{3\,(1)}^2)\, \sin 2\,(\theta - \theta_R)\, \sin\left(\phi - \rho_\alpha\, \frac{\pi}{3}\right)}{48\,\pi\left(y_{1\,(3)}^2\, \cos^2(\theta - \theta_R) + y_{3\,(1)}^2\, \sin^2(\theta - \theta_R)\right)} \times \left[(-1)^{k_2}\, f\left(\frac{M_{3\,(1)}}{M_{1\,(3)}}\right) + g\left(\frac{M_{3\,(1)}}{M_{1\,(3)}}\right)\right]$$

and ϵ_2^α vanish

For further studies of flavored leptogenesis in theories with flavor & CP symmetry

see *Mohapatra/Nishi ('15), Chen et al. ('16), Yao/Ding ('16)*

Conclusions

- flavor and CP symmetries can be powerful in constraining lepton mixing parameters, in particular leptonic CP phases
- series of $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$, $n \geq 2$, and CP are very interesting
- strong correlation of α, β, δ and Y_B possible in unflavored leptogenesis framework
- in spite of less correlations also flavored leptogenesis can be interesting

Thank you for your attention.