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QCD Axion from **Flavour Symmetry** & **Flavourful Axion** Phenomenology



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Outline

- Accidental PQ symmetry from flavour symmetry
 - ✓ Basic issues
 - ✓ A Pati-Salam unified model
- Flavourful axion phenomenology
 - ✓ General axion couplings
 - ✓ Axion-meson mixing
 - ✓ Meson and lepton decays to axion
- Conclusion

Flavourful PQ symmetry

- PQ symmetry required to solve the strong CP problem.
- It may be identified with a flavour symmetry.

Wilczek, '82; ... ; Ema, et.al.; Calibbi, et.al., '16

- Planck scale effect may violate it, but it should be respected by operators up to $D=10$ to guarantee $\delta\theta < 10^{-10}$.

Holman, et.al.; Kamionkowski, et.al.; Barr-Seckel, '92

- An approximate PQ symmetry could arise accidentally from a discrete gauge (flavour) symmetry.

(*) $Z_{9,18}$ with 4 PQ singlets in SSM EJC, Lukas, '92

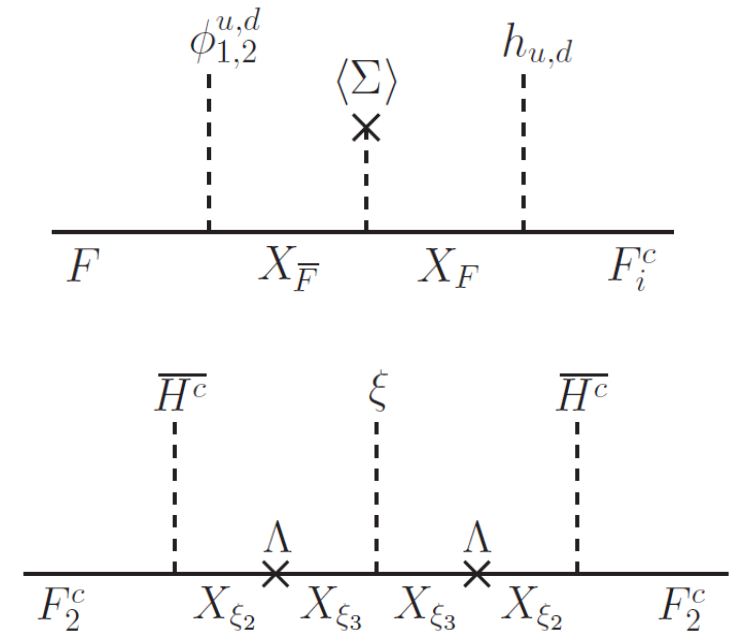
A-to-Z flavour Pati-Salam Model

King, '14

- A **Pati-Salam** unification model where **PQ symmetry**, protected up to D=10 operators, arises accidentally from discrete **flavour** symmetry $A_4 \times Z_5 \times Z_3 \times Z'_5$:

$$W_Y^{\text{eff}} = \lambda_3 (F \cdot h_3) F_3^c + \lambda_{1u} \frac{(F \cdot \phi_1^u) h_u F_1^c}{\langle \Sigma_u \rangle} + \lambda_{2u} \frac{(F \cdot \phi_2^u) h_u F_2^c}{\langle \Sigma_u \rangle} \\ + \lambda_{1d} \frac{(F \cdot \phi_1^d) h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \lambda_{2d} \frac{(F \cdot \phi_2^d) h_d F_2^c}{\langle \Sigma_d \rangle} + \lambda_{ud} \frac{(F \cdot \phi_1^u) h_d F_1^c}{\langle \Sigma_d \rangle}$$

$$W_{\text{Maj}}^{\text{eff}} = \frac{\overline{H^c} H^c}{\Lambda} \left(\frac{\xi^2}{\Lambda^2} F_1^c F_1^c + \frac{\xi}{\Lambda} F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda} F_1^c F_3^c \right)$$



$$x_{f_L} = (0,0,0)$$

$$x_{f_R} = (2,1,0)$$

$$N_{\text{DW}} = 6$$

"Family-dependent DFSZ"

	Field	G_{PS}	A_4	Z_5	Z_3	Z'_5	R	$U(1)_{PQ}$
Fermions	F	(4, 2, 1)	3	1	1	1	1	0
	$F_{1,2,3}^c$	($\bar{4}$, 1, 2)	1	$\alpha, \alpha^3, 1$	$\beta, \beta^2, 1$	$\gamma^3, \gamma^4, 1$	1	-2, -1, 0
	\bar{H}^c	(4, 1, 2)	1	1	1	1	0	0
	H^c	($\bar{4}$, 1, 2)	1	1	1	1	0	0
Flavons	$\phi_{1,2}^u$	(1, 1, 1)	3	α^4, α^2	β^2, β	γ^2, γ	0	2, 1
	$\phi_{1,2}^d$	(1, 1, 1)	3	α^3, α	β^2, β	γ^2, γ	0	2, 1
Higgses	h_3	(1, 2, 2)	3	1	1	1	0	0
	h_u	(1, 2, 2)	1''	α	1	1	0	0
	h_{15}^u	(15, 2, 2)	1	α	1	1	0	0
	h_d	(1, 2, 2)	1'	α^3	1	1	0	0
	h_{15}^d	(15, 2, 2)	1'	α^4	1	1	0	0
	Σ_u	(1, 1, 1)	1''	α	1	1	0	0
	Σ_d	(1, 1, 1)	1'	α^3	1	1	0	0
	Σ_{15}^d	(15, 1, 1)	1'	α^2	1	1	0	0
	ξ	(1, 1, 1)	1	α^4	β^2	γ^2	0	2
Messengers	$X_{F''_{1,3}}$	(4, 2, 1)	1''	α, α^3	β^2, β	γ^2, γ	1	2, 1
	$X_{F'_{1,3}}$	(4, 2, 1)	1'	α, α^3	β, β^2	γ, γ^2	1	1, 2
	$X_{\bar{F}_i}$	($\bar{4}$, 2, 1)	1	α^i	$\beta, \beta, \beta^2, \beta^2$	$\gamma^3, \gamma^3, \gamma^4, \gamma^4$	1	-2, -2, -1, -1
	X_{ξ_i}	(1, 1, 1)	1	α^i	$\beta, \beta, \beta^2, \beta^2, 1$	$\gamma^3, \gamma, \gamma^4, \gamma^2, 1$	1	-2, 1, -1, 2, 0
	$\bar{\phi}_{1,2}^u$	(1, 1, 1)	3	α, α^3	β, β^2	γ^3, γ^4	0	-2, -1
	$\bar{\phi}_{1,2}^d$	(1, 1, 1)	3	α^2, α^4	β, β^2	γ^3, γ^4	0	-2, -1
	$\bar{\xi}$	(1, 1, 1)	1	α	β	γ^3	0	-2

Axion-dependent Yukawa

- Flavon fields $\phi_{1,2}^{u,d}$ and right-handed fermions $F_{1,2,3}^c$ are charged under the PQ symmetry:

$$\begin{aligned}
 -\mathcal{L}_Y = & \lambda_3 (\bar{f} \cdot \langle h_3 \rangle^*) (f_{R3}) + \frac{\lambda_{1u} v_u}{\sqrt{2} v_{\Sigma_u}} (\bar{u}_L \cdot \langle \varphi_1^u \rangle^*) (u_{R1}) \exp \left[\frac{-ix_{\varphi_1^u} a}{v_{PQ}} \right] \quad x_{\varphi_1^{u,d}} = 2 \\
 & + \frac{\lambda_{2u} v_u}{\sqrt{2} v_{\Sigma_u}} (\bar{u}_L \cdot \langle \varphi_2^u \rangle^*) (u_{R2}) \exp \left[\frac{-ix_{\varphi_2^u} a}{v_{PQ}} \right] + \frac{\lambda_{1d} v_d}{\sqrt{2} v_{\Sigma_d}} (\bar{d}_L \cdot \langle \varphi_1^d \rangle^*) (d_{R1}) \exp \left[\frac{-ix_{\varphi_1^d} a}{v_{PQ}} \right] \\
 & + \frac{\lambda_{2d} v_d}{\sqrt{2} v_{\Sigma_d}} (\bar{d}_L \cdot \langle \varphi_2^d \rangle^*) (d_{R2}) \exp \left[\frac{-ix_{\varphi_2^d} a}{v_{PQ}} \right] + \frac{\lambda_{ud} v_d}{\sqrt{2} v_{\Sigma_d}} (\bar{d}_L \cdot \langle \varphi_1^u \rangle^*) (d_{R1}) \exp \left[\frac{-ix_{\varphi_1^u} a}{v_{PQ}} \right] \\
 & + \left\{ d_L \rightarrow e_L, d_R \rightarrow e_R, \lambda_{1d} \rightarrow \tilde{\lambda}_{1d}, \lambda_{2d} \rightarrow \tilde{\lambda}_{2d}, \lambda_{ud} \rightarrow \tilde{\lambda}_{ud} \right\} + \text{h.c.} \\
 & \quad \quad \quad x_{\varphi_2^{u,d}} = 1
 \end{aligned}$$

Fitting fermion masses and mixing

- Fermion mass matrices from 15 input parameters:

$$M^u = v_u \begin{pmatrix} 0 & b & \epsilon_{13}c \\ a & 4b & \epsilon_{23}c \\ a & 2b & c \end{pmatrix}, \quad M^d = v_d \begin{pmatrix} y_d^0 & 0 & 0 \\ By_d^0 & y_s^0 & 0 \\ By_d^0 & 0 & y_b^0 \end{pmatrix}, \quad M^e = v_d \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ By_d^0 & xy_s^0 & 0 \\ By_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

→ Specific flavour prediction for axion

- Best-fit:

Parameter	Value	Parameter	Value	Parameter	Value
$a/10^{-5}$	$1.246 e^{4.047i}$	$\epsilon_{13}/10^{-3}$	$6.215 e^{2.434i}$	m_a/meV	3.646
$b/10^{-3}$	$3.438 e^{2.080i}$	$\epsilon_{23}/10^{-2}$	$2.888 e^{3.867i}$	m_b/meV	1.935
c	-0.545	B	$10.20 e^{2.777i}$	m_c/meV	1.151
$y_d^0/10^{-5}$	$3.053 e^{4.816i}$	x	5.880	η	2.592
$y_s^0/10^{-4}$	$3.560 e^{2.097i}$			ξ	2.039
$y_b^0/10^{-2}$	3.607				

Flavourful axion phenomenology

Feng, et.al., '98

- An axion field residing in flavons:

$$\phi_i = \frac{v_i}{\sqrt{2}} e^{i \frac{x_i a}{v_{PQ}}} \quad a = \sum_i x_i \frac{v_i a_i}{v_{PQ}}; \quad v_{PQ}^2 = \sum_i x_i^2 v_i^2$$

- Fermion Yukawa couplings take a general form:

$$-\mathcal{L}_{\text{Yuk}} = e^{i \frac{a}{v_{PQ}} (x_{f_{L_i}} - x_{f_{R_j}})} M_{ij}^f \overline{f_{L_i}} f_{R_j}$$

- After the transformation: $f_{L/R_i} \rightarrow e^{i \frac{a}{v_{PQ}} x_{f_{L/R_i}}} f_{L/R_i}$

$$-\mathcal{L} = \frac{\partial_\mu a}{v_{PQ}} (x_{f_{L_i}} \overline{f_{L_i}} \gamma^\mu f_{L_i} + x_{f_{R_i}} \overline{f_{R_i}} \gamma^\mu f_{R_i}) - \mathcal{L}_{\text{anomaly}} + M_{ij}^f \overline{f_{L_i}} f_{R_j}$$

General flavourful axion couplings

- In the mass basis: $f_{L/R} \rightarrow U_{fL/R} f_{L/R}$, $M^f \rightarrow U_{fL}^\dagger M^f U_{fR} = m^f$

$$-\mathcal{L} = \frac{\partial_\mu a}{v_{PQ}} \bar{f}_i \gamma^\mu \left(V_{ij}^f - A_{ij}^f \gamma_5 \right) f_j + \frac{a}{f_a} \left(\frac{\alpha_s}{8\pi} G \tilde{G} + c_{a\gamma} \frac{\alpha}{8\pi} F \tilde{F} \right) + m_i^f \bar{f}_{Li} f_{Rj}$$

$$V^f = \frac{1}{2} (U_{fL}^\dagger x_{fL} U_{fL} + U_{fR}^\dagger x_{fR} U_{fR})$$

$$A^f = \frac{1}{2} (U_{fL}^\dagger x_{fL} U_{fL} - U_{fR}^\dagger x_{fR} U_{fR})$$

$$f_a = v_{PQ} / N_{DW} \quad (N_{DW} = \text{QCD anomaly})$$

Axion-meson mixing

- After QCD condensation,

$$\frac{c_{ij}}{v_{PQ}} \partial_\mu a \bar{f}_i \gamma^\mu \gamma_5 f_j \Rightarrow c_P \frac{f_P}{f_a} \partial_\mu a \partial^\mu P$$

- Kinetic mixing diagonalization & mass re-diagonalization:

$$a \rightarrow a + c_P \frac{f_P}{f_a} \frac{m_P^2}{m_P^2 - m_a^2} P, \quad P \rightarrow P - c_P \frac{f_P}{f_a} \frac{m_a^2}{m_P^2 - m_a^2} a$$

mass-dependent mixing

Axion-pion mixing

- For QCD axion, the mixing is negligible ($f_P \ll f_a, m_a \ll m_\pi$).
- For ALP, it can lead to sizable contribution to $K^+ \rightarrow \pi^+ a$ & $a \rightarrow \gamma\gamma$ induced from $K^+ \rightarrow \pi^+ \pi^0$ & $\pi^0 \rightarrow \gamma\gamma$:

$$\Gamma(K^+ \rightarrow \pi^+ a) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2} \right)^2 \Gamma(K^+ \rightarrow \pi^+ \pi^0)$$

$$\Gamma(a \rightarrow \gamma\gamma) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2} \right)^2 \left(\frac{m_a}{m_\pi} \right)^3 \Gamma(\pi^0 \rightarrow \gamma\gamma) \Rightarrow (g_{a\gamma})_{mix} = \frac{\alpha}{\pi} \frac{c_\pi m_a^2}{m_\pi^2 - m_a^2} \frac{1}{f_a}$$

- $B(K^+ \rightarrow \pi^+ a) < 10^{-10}$ puts a limit:

$$f_a > 4 \left(\frac{c_\pi m_a^2}{m_\pi^2 - m_a^2} \right) TeV \Rightarrow (g_{a\gamma})_{mix} < 5.8 \times 10^{-7} GeV^{-1} \text{ for } m_a < 110 MeV$$

Impact on neutral meson mass splitting

- Flavourful axion couplings ($A_{12,23}^{u,d}$) induces mixing with heavy mesons (K, D, B) contributing to their mass splitting:

$$\Delta m_P \approx |\eta_P|^2 m_P = |c_P|^2 \frac{f_P^2}{v_{PQ}^2} m_P$$

System	$(\Delta m_P)_{\text{exp}}/\text{MeV}$	v_{PQ}/GeV
$K^0 - \bar{K}^0$	$(3.484 \pm 0.006) \times 10^{-12}$	$\gtrsim 4 \times 10^7 c_{K^0} $
$D^0 - \bar{D}^0$	$(6.25^{+2.70}_{-2.90}) \times 10^{-12}$	$\gtrsim 5 \times 10^6 c_{D^0} $
$B^0 - \bar{B}^0$	$(3.333 \pm 0.013) \times 10^{-10}$	$\gtrsim 1 \times 10^7 c_{B^0} $
$B_s^0 - \bar{B}_s^0$	$(1.1688 \pm 0.0014) \times 10^{-8}$	$\gtrsim 3 \times 10^6 c_{B_s^0} $

Meson decay to axion

- Meson decays $P \rightarrow P' a$ from flavourful vector couplings:

$$B(P(q_i) \rightarrow P'(q_j) a) = \frac{1}{16\pi\Gamma(P)} |V_{ij}^q|^2 |f_+(0)|^2 \frac{m_P^3}{v_{PQ}^2} \left(1 - \frac{m_{P'}^2}{m_P^2}\right)^3$$
$$\equiv \tilde{c}_{P \rightarrow P'} |V_{ij}^q|^2 \left(\frac{10^{12} \text{ GeV}}{v_{PQ}}\right)^2$$

Decay	Branching ratio	Experiment	$\tilde{c}_{P \rightarrow P'}$	v_{PQ}/GeV
$K^+ \rightarrow \pi^+ a$	$< \mathbf{0.73} \times \mathbf{10}^{-10}$	E949 + E787 [56]	3.51×10^{-11}	$> 6.9 \times 10^{11} V_{21}^d $
	$< 0.01 \times 10^{-10*}$	NA62 (future) [59]		$> 5.9 \times 10^{12} V_{21}^d $
	$< 1.2 \times 10^{-10}$	E949 + E787 [55]		
	$< 0.59 \times 10^{-10}$	E787 [70]		
$K_L^0 \rightarrow \pi^0 a$	$< \mathbf{5} \times \mathbf{10}^{-8}$	KOTO [65]	3.67×10^{-11}	$> 2.7 \times 10^{10} V_{21}^d $
$(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$	$(< 2.6 \times 10^{-8})$	E391a [63]		
$B^\pm \rightarrow \pi^\pm a$	$< \mathbf{4.9} \times \mathbf{10}^{-5}$	CLEO [68]	5.30×10^{-13}	$> 1.0 \times 10^8 V_{31}^d $
$(B^\pm \rightarrow \pi^\pm \nu \bar{\nu})$	$(< 1.0 \times 10^{-4})$	BaBar [71]		
	$(< 1.4 \times 10^{-4})$	Belle [72]		
$B^\pm \rightarrow K^\pm a$	$< \mathbf{4.9} \times \mathbf{10}^{-5}$	CLEO [68]	7.26×10^{-13}	$> 1.2 \times 10^8 V_{32}^d $
$(B^\pm \rightarrow K^\pm \nu \bar{\nu})$	$(< 1.3 \times 10^{-5})$	BaBar [73]		
	$(< 1.9 \times 10^{-5})$	Belle [72]		
	$(< 1.5 \times 10^{-6})^*$	Belle-II (future) [74]		
$B^0 \rightarrow \pi^0 a$			4.92×10^{-13}	
$(B^0 \rightarrow \pi^0 \nu \bar{\nu})$	$(< 0.9 \times 10^{-5})$	Belle [72]		$\gtrsim 2.3 \times 10^8 V_{31}^d $
$B^0 \rightarrow K_{(S)}^0 a$	$< \mathbf{5.3} \times \mathbf{10}^{-5}$	CLEO [68]	6.74×10^{-13}	$> 1.1 \times 10^8 V_{32}^d $
$(B^0 \rightarrow K^0 \nu \bar{\nu})$	$(< 1.3 \times 10^{-5})$	Belle [72]		
$D^\pm \rightarrow \pi^\pm a$	< 1		1.11×10^{-13}	$> 3.3 \times 10^5 V_{21}^u $
$D^0 \rightarrow \pi^0 a$	< 1		4.33×10^{-14}	$> 2.1 \times 10^5 V_{21}^u $
$D_s^\pm \rightarrow K^\pm a$	< 1		4.38×10^{-14}	$> 2.1 \times 10^5 V_{21}^u $
$B_s^0 \rightarrow \bar{K}^0 a$	< 1		3.64×10^{-13}	$> 6.0 \times 10^5 V_{31}^d $

Lepton decay to axion

- LFV decays $l_i \rightarrow l_j a$ with the couplings (V_{ij}^e, A_{ij}^e) :

$$B(l_i \rightarrow l_j a) \equiv \tilde{c}_{l_i \rightarrow l_j} |C_{ij}^e|^2 \left(\frac{10^{12} \text{GeV}}{v_{PQ}} \right)^2$$

$$\tilde{c}_{l_i \rightarrow l_j} \approx \frac{1}{16\pi\Gamma(l_i)} \frac{m_{l_i}^3}{(10^{12} \text{GeV})^2} \quad |C_{ij}^e|^2 = |V_{ij}^e|^2 + |A_{ij}^e|^2$$

- Angular distribution:

$$\frac{d\Gamma}{d\cos\theta} = \frac{|C_{ij}^e|^2}{32\pi} \frac{m_{l_i}^3}{v_{PQ}^2} (1 - A P_{l_i} \cos\theta) \quad A = \frac{2\Re(A_{ij}^e V_{ij}^{e*})}{|C_{ij}^e|^2} = \left. \begin{array}{l} A = 0 \text{ (isotropy)} \\ A = -1 \text{ (} V - A \text{: SM)} \\ A = +1 \text{ (} V + A \text{: our model)} \end{array} \right\}$$

Decay	Branching ratio	Experiment	$\tilde{c}_{l_1 \rightarrow l_2}$	v_{PQ}/GeV
$\mu^+ \rightarrow e^+ a$	$< 2.6 \times 10^{-6}$	($A = 0$) Jodidio <i>et al</i> [81]	7.82×10^{-11}	$> 5.5 \times 10^9 V_{21}^e $
	$< 2.1 \times 10^{-5}$	($A = 0$) TWIST [82]		$> 1.9 \times 10^9 C_{21}^e $
	$< 1.0 \times 10^{-5}$	($A = 1$) TWIST [82]		$> 2.8 \times 10^9 C_{21}^e $
	$< 5.8 \times 10^{-5}$	($A = -1$) TWIST [82]		$> 1.2 \times 10^9 C_{21}^e $
	$\lesssim 5 \times 10^{-9*}$	Mu3e (future) [83]		$\gtrsim 1 \times 10^{11} C_{21}^e $
$\tau^+ \rightarrow e^+ a$	$< 1.5 \times 10^{-2}$	ARGUS [84]	4.92×10^{-14}	$> 1.8 \times 10^6 C_{31}^e $
$\tau^+ \rightarrow \mu^+ a$	$< 2.6 \times 10^{-2}$	ARGUS [84]	4.87×10^{-14}	$> 1.4 \times 10^6 C_{32}^e $

MEG-II ??

Radiative LFV decay: $l_1 \rightarrow l_2 a \gamma$

- Cristal Box: $Br(\mu \rightarrow e a \gamma) < 1.1 \times 10^{-9} \Rightarrow v_{PQ} > 9.4 \times 10^8 |C_{21}^e| \text{ GeV}$
 $[Br(\mu \rightarrow e \gamma) < 4.9 \times 10^{-11}]$

$$\frac{d^2\Gamma}{dx dc_\theta} = \frac{\alpha |C_{l_1 l_2}^e|^2 m_{l_1}^3}{32\pi^2 v_{PQ}^2} f(x, c_\theta), \quad f(x, c_\theta) = \frac{1 - x(1 - c_\theta) + x^2}{(1 - x)(1 - c_\theta)}. \quad \begin{array}{l} x = 2E_{l_2}/m_{l_1} \\ c_\theta \equiv \cos \theta_{2\gamma} \end{array}$$

- Future search?

Decay	Branching ratio	Experiment
$\mu^+ \rightarrow e^+ \gamma$	$< 4.2 \times 10^{-13}$	MEG [87]
	$\lesssim 6 \times 10^{-14}^*$	MEG-II (future) [88] ??
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	BaBar [90]
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	BaBar [90]

Axion-mediated LFV processes

- Mu to 3e: $Br(\mu \rightarrow eee) < 10^{-12}$ (SINDRUM); 10^{-16} (Mu3e)

$$Br(\mu^+ \rightarrow e^+e^-e^+) \approx \frac{m_e^2 m_\mu^3}{16\pi^3 \Gamma(\mu)} \frac{|A_{11}^e|^2 |C_{21}^e|^2}{v_{PQ}^4} \left(\ln \frac{m_\mu^2}{m_e^2} - \frac{15}{4} \right),$$

$$\approx 1.43 \times 10^{-41} |A_{11}^e|^2 |C_{21}^e|^2 \left(\frac{10^{12} \text{ GeV}}{v_{PQ}} \right)^4.$$

- Mu-to-e conversion: $R_{\mu e} < 7 \times 10^{-13}$ (SINDRUM II); 6×10^{-17} (Mu2e)

$$R_{\mu e}^{(A,Z)} \equiv \frac{\Gamma(\mu^- \rightarrow e^- (A, Z))}{\Gamma_{\mu^- \text{ cap}}^{(A,Z)}} \sim \frac{m_\mu^5}{(q^2 - m_a^2)^2} \frac{(\alpha Z)^3}{\pi^2 \Gamma_{\mu^- \text{ cap}}^{(A,Z)}} \frac{m_\mu^2 m_N^2}{v_{PQ}^4} |C_{21}^e|^2 |S_N^{(A,Z)} C_{aN}|^2$$

↑ Nucleon spin-dependent

Cirigliano, Davidson, Kuno, '17

- Both put bounds around $v_{PQ} \gtrsim 10^6 \text{ GeV}$

Our model Predictions

- Best-fit values for axion-fermion couplings:

$$V^u = -A^u \simeq \begin{pmatrix} 1.0 & 4.3 \times 10^{-3} e^{-0.05i} & -1.7 \times 10^{-5} e^{-0.015i} \\ 4.3 \times 10^{-3} e^{0.05i} & -0.5 & -6.0 \times 10^{-4} \\ -1.7 \times 10^{-5} e^{0.015i} & -6.0 \times 10^{-4} & 7.3 \times 10^{-7} \end{pmatrix}$$

$$V^d = -A^d \simeq \begin{pmatrix} 0.78 & 0.25 & -0.0065 \\ 0.25 & 0.72 & -0.0057 \\ -0.0065 & -0.0057 & 7.5 \times 10^{-5} \end{pmatrix}, \quad V^e = -A^e \simeq \begin{pmatrix} 0.99 & 0.073 & -0.0085 \\ 0.073 & 0.51 & -0.0013 \\ -0.0085 & -0.0013 & 7.5 \times 10^{-5} \end{pmatrix}$$

- Rare decays to axion:

Process	Branching ratio ($v_{PQ} = 10^{12}$ GeV)	Experimental sensitivity
$K^+ \rightarrow \pi^+ a$	2.19×10^{-12}	$\lesssim 1 \times 10^{-12}$ (NA62 future)
$K_L^0 \rightarrow \pi^0 a$	2.29×10^{-12}	$< 5 \times 10^{-8}$ (KOTO)
$\mu^+ \rightarrow e^+ a$	8.3×10^{-13}	$\lesssim 5 \times 10^{-9}$ (Mu3e future)

- Correlated probes:

$$R_{\mu/K} \equiv \frac{\text{Br}(\mu^+ \rightarrow e^+ a)}{\text{Br}(K^+ \rightarrow \pi^+ a)} \simeq 4.45 \frac{|V_{21}^e|^2}{|V_{21}^d|^2} \approx 31 e^{-1.8\sqrt{x}} \approx 0.38.$$

Conclusion

- Approximate PQ symmetric may arise accidentally from discrete family symmetry.
- A specific Pati-Salam unified model is worked out.
- It leads to interesting flavourful axion phenomenology.
- Rare Kaon and muon decays may provide a sensitive probe for QCD axion.
- There appear numerous flavour observables interesting for ALP.