

# HIERARCHICAL FERMIONS AND DETECTABLE $Z'$ FROM AN EFFECTIVE TWO-HIGGS-TRIPLET 3-3-1 MODEL

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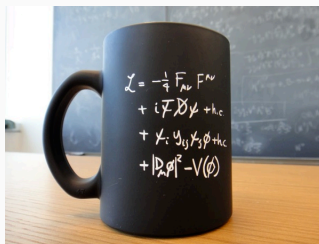
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## OPEN QUESTIONS AND THE STANDARD MODEL

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# SOME ISSUES WITH THE SM

- Based on the gauge group  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ , the SM is very successful but far from being complete.
- Open questions:
  - # fermion families;
  - fermion mass hierarchy;
  - neutrino masses and oscillations;
  - Strong CP problem;
  - dark matter;
  - etc;



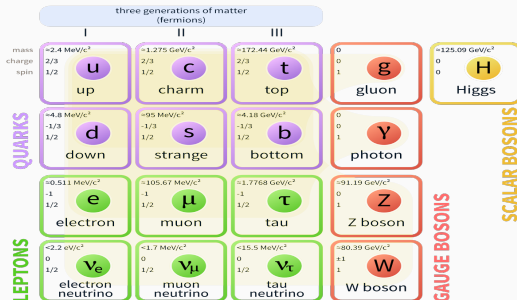
# FERMION MASS HIERARCHY

- 3 fermion generations are described in the SM.
- But why 3 exactly and not more? The SM does not provide an answer.
- In the SM, fermions couple to the Higgs doublet according to:

$$\lambda_{\alpha\beta}^f (\bar{F}_{\alpha L} \phi) f_{\beta R} \rightarrow m_{\alpha\beta}^f \bar{f}_{\alpha L} f_{\beta R}, \quad \mathbf{m}^f = \lambda^f \mathbf{v}. \quad (1)$$

- For up-type quarks, e.g., we have  $\lambda_t \sim \mathcal{O}(1)$  and  $\lambda_u/\lambda_t \sim \mathcal{O}(10^{-5})$ .
- Strong fermion mass hierarchy.

## Standard Model of Elementary Particles



- How to explain these hierarchies more naturally?
- The Froggatt-Nielsen mechanism makes use of a  $U(1)$  flavour symmetry.<sup>1</sup>
- In this case, fermion masses are given by

$$\lambda^f \left( \frac{S}{\Lambda} \right)^x (\bar{F}_L \phi) f_R \rightarrow m^f \bar{f}_L f_R, \quad m^f = \lambda_{eff}^f v, \quad (2)$$

so that  $\lambda_{eff}^f = \lambda^f \left( \frac{\langle S \rangle}{\Lambda} \right)^x$  and  $\lambda^f \sim \mathcal{O}(1)$ .

- The suitable exponent  $x$  is fixed by the flavour symmetry according to which the “Flavon”,  $S$ , and fermions get their charges.
- This symmetry can be identified with the Peccei-Quinn symmetry<sup>2</sup> that solves the strong CP problem and may explain DM abundance.

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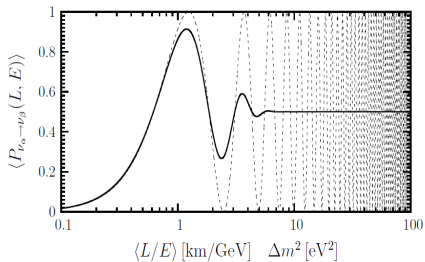
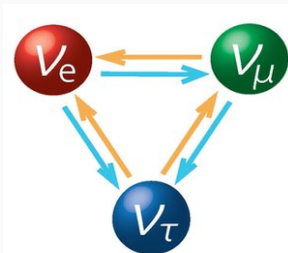
<sup>1</sup>C. D. Froggatt and H. B. Nielsen, (1979).

<sup>2</sup>R.D. Peccei and H R. Quinn, (1977)

# NEUTRINO MASSES

- Only LH neutrinos are present in the SM  $\Rightarrow$  Massless neutrinos.
- However, flavour oscillations have been observed, requiring neutrinos to be very light, but massive.

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \quad (3)$$



- Neutrino masses: SM  $\nu_L + N_R$  (singlets). In the flavour basis  $((\nu_L)^c, N_R)$

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \Rightarrow m_\nu \simeq \frac{(m_D)^2}{M_R} \text{ and } m_N \simeq M_R. \text{ (For } M_R \gg m_D \text{)} \quad (4)$$

- The heavier  $N_R$ , the lighter  $\nu_L \Rightarrow$  Seesaw Mechanism ( $M_R \sim 10^{14}$  GeV).
- Neutrino masses: SM  $\nu_L + N_R$  (singlets) +  $S_R$  (singlets).  $((\nu_L)^c, N_R, S_R)$

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & m_R \\ 0 & (m_R)^T & \mu \end{pmatrix} \Rightarrow m_\nu \simeq \mu \frac{(m_D)^2}{m_R^2}. \quad (5)$$

- $m_\nu \rightarrow 0$  when  $\mu \rightarrow 0 \Rightarrow$  Low-scale (inverse) seesaw mechanism<sup>3</sup>  
( $m_D \sim 10^2, m_R \sim 10^4, \mu \sim 10^{-5}$  in GeV).

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<sup>3</sup>R.N. Mohapatra and J.W.F. Valle (1986)



## EXTENDING THE SM

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- In view of the many open questions, several extensions of the SM have been proposed.
- Enlarging its gauge structure, so that the SM is seen as an effective model at low energies, is a common approach.
- In particular, we have extensions of the SM electroweak sector

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes SU(M)_L \otimes SU(N)_R \otimes U(1)_X . \quad (6)$$

- When  $M = N \geq 2$ , a left-right (LR) symmetry can be imposed.
- LR models can provide a dynamical origin for parity violation.
- For  $M = N = 2$ , we have the minimal LR SM extension<sup>4</sup>  $\Rightarrow$  for each LH fermion doublet in the SM, a RH copy is added, e.g.

$$L_L = (\nu_L \ l_L)^T \rightarrow L_R = (\nu_R \ l_R)^T . \quad (7)$$

- Therefore, RH neutrinos are essential in the minimal LR model, and the seesaw mechanism can be implemented.

<sup>4</sup>G. Senjanovic, R. N. Mohapatra (1975)

- The 3-3-1 model ( $M = 3, N = 1$ ), invariant under  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ , with the electric charge operator given by<sup>5</sup>

$$Q = T_3 + \beta T_8 + XI. \quad (8)$$

- Anomaly cancellation + asymptotic freedom  $\Rightarrow$  3 fermion families.
- For this to happen,  $\#3 = \#3^*$  of  $SU(3)_L$ ,

$$\begin{aligned} \psi_{iL} &= (\nu_i \ e_i^- \ k_i^q)_L^T \sim \left( \mathbf{1}, \mathbf{3}, \frac{q-1}{3} \right), \\ Q_{aL} &= (d_a \ -u_a \ J_a^{-q-1/3})_L^T \sim \left( \mathbf{3}, \mathbf{3}^*, -\frac{q}{3} \right), \\ Q_{3L} &= (u_3 \ d_3 \ J_3^{q+2/3})_L^T \sim \left( \mathbf{3}, \mathbf{3}, \frac{q+1}{3} \right), \end{aligned} \quad (9)$$

where  $i = 1, 2, 3, a = 1, 2$  and  $q = -1/2(1 + \sqrt{3}\beta)$ .

- Such a feature can generate different mass patterns among the quarks and lead to FCNC.

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<sup>5</sup>M. Singer, J. W. F. Valle, J. Schechter (1980)

- Arbitrary  $\beta$  versions are generally associated with exotic fermions and non-perturbativity at low scales.
- When  $\beta = \pm 1/\sqrt{3}$  though, such issues are avoided.
- Symmetry breaking happens in two stages:

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Q. \quad (10)$$

- This can be obtained with two scalar triplets  $\chi$  and  $\rho$ , with

$$\langle \chi_3^0 \rangle = w \quad \text{and} \quad \langle \rho_1^0 \rangle = v \sim 246 \text{ GeV} \ll w. \quad (11)$$

- The non-SM fields (new scalars, gauge bosons and fermions) are expected to get masses around the TeV scale ( $w$ ).

## 3-3-1 MODEL WITH TWO-HIGGS-TRIPLETS

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- We assume  $\beta = \frac{1}{\sqrt{3}}$  and choose a minimal scalar sector with:<sup>6</sup>

$$\rho \equiv (\rho_1^0 \ \rho_2^- \ \rho_3^-)^T, \quad \chi \equiv (\chi_1^+ \ \chi_2^0 \ \chi_3^0)^T. \quad (12)$$

- The third components of the fermion triplets are  $E_{iL}^-$ ,  $U_{aL}$  and  $D_L$ .
- We add a RH fermion singlet for each LH fermion in the triplets.
- The scalar potential and Yukawa Lagrangian follow

$$V = \mu_1^2(\rho^\dagger \rho) + \mu_2^2(\chi^\dagger \chi) + \lambda_1(\rho^\dagger \rho)^2 + \lambda_2(\chi^\dagger \chi)^2 \\ + \lambda_3(\rho^\dagger \rho)(\chi^\dagger \chi) + \lambda_4(\chi^\dagger \rho)(\rho^\dagger \chi), \quad (13)$$

$$-\mathcal{L}_Y = h_{is}^E \overline{\psi_{iL}} \chi e'_{sR} + h_{ij}^\nu \overline{\psi_{iL}} \rho \nu_{jR} + \frac{1}{2} m_{ij}^R \overline{(\nu_{iR})^c} \nu_{jR} + h_{am}^U \overline{Q_{aL}} \chi^* u'_{mR} \\ + h_{an}^d \overline{Q_{aL}} \rho^* d'_{nR} + h_m^u \overline{Q_{3L}} \rho u'_{mR} + h_n^D \overline{Q_{3L}} \chi d'_{nR} + h.c. \quad (14)$$

- With only **two triplets** some fermions remain massless as a result of a residual (Peccei-Quinn-like) symmetry in the model.

<sup>6</sup>E. R. Barreto, A. G. Dias, J. Leite, C. C. Nishi, R. L. N. Oliveira and W. C. Vieira, (2018).

- 12 d.o.f. (2 triplets) - 8 (GB) = 4 physical scalars:  $h$ ,  $H$  and  $\varphi_{\pm}$ .
- The spectrum is thus compact and does not contain CP-odd neutral scalars.
- $h$ , the SM Higgs, and  $H$ , the heavy Higgs, mix according to

$$\begin{aligned} m_h^2 &\simeq \left[ \lambda_1 - \lambda_3^2 / (4\lambda_2) \right] v^2 = (125 \text{ GeV})^2, \quad m_H^2 \simeq \lambda_2 w^2, \\ \tan(2\theta) &= \frac{\lambda_3 v w}{\lambda_2 w^2 - \lambda_1 v^2}. \end{aligned} \quad (15)$$

- $\varphi_{\pm}$  is also heavy

$$m_{\varphi_{\pm}}^2 = \frac{\lambda_4}{2} (v^2 + w^2). \quad (16)$$

- The symmetry gauge bosons are

$$P_\mu = \frac{g}{2} \begin{pmatrix} W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tB_\mu X & \sqrt{2}W_\mu^+ & \sqrt{2}V_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tB_\mu X & \sqrt{2}V_\mu^0 \\ \sqrt{2}V_\mu^- & \sqrt{2}V_\mu^{0\dagger} & -\frac{2W_{8\mu}}{\sqrt{3}} + 2tB_\mu X \end{pmatrix}, \quad (17)$$

- The non-diagonal fields get the following masses ( $M_{V^\pm}^2 - M_{V^0}^2 = M_{W^\pm}^2$ )

$$M_{W^\pm}^2 = \frac{g^2 v^2}{4}, \quad M_{V^\pm}^2 = \frac{g^2}{4}(v^2 + w^2), \quad M_{V^0}^2 = M_{V^{0\dagger}}^2 = \frac{g^2}{4}w^2. \quad (18)$$

- The diagonal fields mix and give rise to  $A$ ,  $Z_1$  and  $Z_2$  with masses

$$M_A^2 = 0, \quad M_{Z_1}^2 \simeq \frac{g^2 v^2}{4 \cos^2 \theta_W}, \quad M_{Z_2}^2 \simeq \frac{g^2 \cos^2 \theta_W w^2}{3 - 4 \sin^2 \theta_W}. \quad (19)$$

- And there is also the prediction, at tree level, that  $M_{Z_2}^2/M_{V^0}^2 \approx 1.48$ .

$$\Delta\rho_0 \equiv \frac{M_W^2}{\cos^2 \theta_W M_{Z_1}^2} - 1 \approx \frac{(v/w)^2}{4 \cos^4 \theta_W} \Rightarrow w \geq 6.5 \text{ TeV} \quad (20)$$



- Neutrino masses follow from the terms below

$$-\mathcal{L}_\nu = h_{ij}^\nu \overline{\psi_{iL}} \rho \nu_{jR} + \frac{1}{2} m_{ij}^R \overline{(\nu_{iR})^c} \nu_{jR} + h.c. \quad (21)$$

- With  $m_R \gg m_D = \frac{h^\nu v}{\sqrt{2}}$ , a type-I seesaw mechanism takes place, with sub-eV LH neutrinos ( $m_\nu \simeq m_D^2/m_R$ ) and very heavy RH ones ( $m_R \sim 10^{14}$  GeV).
- Other options, such as including a scalar sextet, could be explored but the scalar spectrum would possibly become less attractive.

- To break the residual PQL symmetry and generate mass to all charged fermions, we introduce

$$-\mathcal{L}_5 = \frac{y_{is}^e}{\Lambda} \overline{\psi_{iL}} \chi^* \rho^* e'_{sR} + \frac{y_{am}^u}{\Lambda} \overline{Q_{aL}} \chi \rho u'_{mR} + \frac{y_n^d}{\Lambda} \overline{Q_{3L}} \chi^* \rho^* d'_{nR} + h.c. , \quad (22)$$

- The second component of the effective scalar triplet develops a vev

$$\frac{\langle \chi^* \rho^* \rangle}{\Lambda} = \frac{vW}{2\Lambda} = \frac{\epsilon}{\sqrt{2}} , \quad (23)$$

with  $\Lambda \gg w, v$  so that  $\epsilon \ll v, w$ .

- These operators can emerge from a UV complete model, containing a heavy scalar triplet ( $\Lambda$ ) which is integrated out.
- When it comes to charged fermions, three energy scales are relevant  
 $v, w$  and  $\epsilon$  (or  $\Lambda$ ).
- Non-standard fermions obtain masses proportional to  $w$  (TeV scale).

- The  $6 \times 6$  charged lepton mass matrix, in the basis  $(e_i, E_i)_{L,R}$ , has the form

$$\mathcal{M}^l = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon \mathbf{y}^e \\ w \mathbf{h}^E \end{pmatrix} = \begin{pmatrix} M_e & M_{eE} \\ 0_{3 \times 3} & M_E \end{pmatrix}, \quad (24)$$

- The SM charged leptons obtain masses proportional to  $\epsilon$  so that we take  $\epsilon \sim m_\tau \sim 1 \text{ GeV}$  ( $\Lambda \sim 10^3 \text{ TeV}$ ).
- To obtain  $m_e$ , we need  $\lambda \sim 10^{-3}$ ; in the SM  $10^{-5}$  is required.
- For the up-type quarks, in the basis  $(u_i, U_a)_{L,R}^T$ ,

$$\mathcal{M}^u = \frac{1}{\sqrt{2}} \begin{pmatrix} -\epsilon \mathbf{y}^u \\ v \mathbf{f}^u \\ w \mathbf{h}^u \end{pmatrix} = \begin{pmatrix} M_u & M_{uU} \\ 0_{2 \times 3} & M_U \end{pmatrix}, \quad M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon \mathbf{y}_i^u \\ v \mathbf{f}_i^u \end{pmatrix} = \begin{pmatrix} \tilde{M}_u & \tilde{M}_{ut} \\ 0_{1 \times 2} & m_t \end{pmatrix}, \quad (25)$$

- The mixing angles with the new quarks are at most of order  $10^{-2}$  and  $10^{-4}$  for  $t_L$  and  $(u_L, c_L)$ , respectively.
- The mixing between  $t_L$  and  $(u_L, c_L)$  has a natural hierarchy of  $\epsilon/v \sim 10^{-2}$ .

- For the down-type quarks, in the basis  $(d_i, D)_{L,R}^T$ , we have

$$\mathcal{M}^d = \frac{1}{\sqrt{2}} \begin{pmatrix} v\mathbf{h}^d \\ \epsilon\mathbf{y}^d \\ w\mathbf{f}^D \end{pmatrix} = \begin{pmatrix} M_d & M_{dD} \\ 0_{1 \times 3} & M_D \end{pmatrix}, \quad (26)$$

Not a natural hierarchy between the first two and the third families.

- However, if  $h_{an}^d$  is an effective coupling, we have

$$vh_{an}^d = \epsilon' \bar{h}_{an}^d \sim 6 \times 10^{-4} v \bar{h}_{an}^d, \quad (27)$$

with  $\bar{h}_{an}^d \sim 1$  and  $\epsilon' \sim m_s \sim 0.1$  GeV, then  $m_b \sim \epsilon \sim 1$  GeV and  $m_s \sim \epsilon'$ .

- This can be arranged by introducing a scalar singlet  $\varphi$  and a  $Z_2$  symmetry under which only  $\varphi, Q_{aL}, U_{aR}, U_{aR}$  are odd.
- Instead of  $h_{an}^d \overline{Q}_{aL} \rho^* d'_{nR}$ , we would have  $\bar{h}_{an}^d (\varphi/\Lambda') \overline{Q}_{aL} \rho^* d'_{nR}$ .
- Could  $\varphi$  be introduced with a  $U(1)$  chiral symmetry and give rise to a “flaxion”?

- A UV completion is achieved with a heavy scalar triplet  $\eta \sim (\mathbf{1}, \mathbf{3}, 1/3)$ .
- $\eta$  couples to fermions according to

$$-\mathcal{L}_5 = y_{is}^e \bar{\psi}_{iL} \eta e'_{sR} + y_{am}^u \bar{Q}_{aL} \eta^* u'_{mR} + y_n^d \bar{Q}_{3L} \eta d'_{nR} + h.c. , \quad (28)$$

- and the potential contains new terms, in special,

$$V_\eta = M^2 \eta^\dagger \eta - (f \eta \rho \chi + h.c.) + \dots , \quad (29)$$

where  $M^2 > 0$  and  $[f] = [M]$ .

- When  $M \gg w, v$ , the new scalar can be integrated out, so that at low energies it is effectively given by

$$\eta \approx \frac{\rho^* \chi^*}{\Lambda} + \dots , \quad \text{with } \Lambda = \frac{M^2}{f}. \quad (30)$$

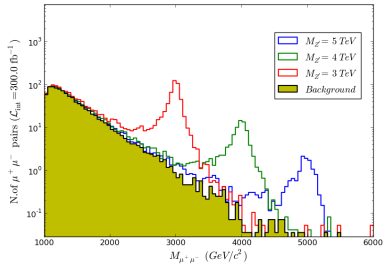
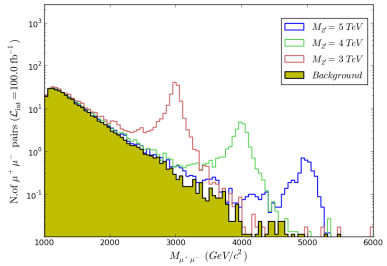
Upon replacing (30) into (28), we obtain the effective operators needed.

- Two main types of interactions involving FCNC:
  - i) due to the mixing between the third family of quarks and the other two;
  - ii) due to the mixing between the heavy and the SM fermions;
- For  $Z'$ , for example, the flavour changing piece is

$$g_{Z'}^{-1} J_{Z'}^\mu |_{\text{FCNC}} = (-c_W^2) \bar{u}_{3L} \gamma^\mu u_{3L} + (-c_W^2) \bar{d}_{3L} \gamma^\mu d_{3L} + \left(-\frac{3}{2} + 2s_W^2\right) \bar{U}_{aL} \gamma^\mu U_{aL} \\ + \left(\frac{1}{2} - s_W^2\right) \bar{D}_L \gamma^\mu D_L + \left(c_{2W} + \frac{1}{2}\right) \bar{E}_{iL} \gamma^\mu E_{iL}, \quad (31)$$

- For type (i), the mixing is naturally suppressed by  $\epsilon/v \sim m_s/m_b \sim 10^{-2}$  which is typically below the current limits coming from meson mixing.
- For type (ii), FCNC are constrained by the search for singlet VLQs at the LHC and by indirect constraints coming from precision electroweak observables and from LEP.
- According to these, our heavy quarks of masses with mixing angle of less than  $10^{-2}$  are not currently observable.
- FCNC mediated by the remaining gauge bosons and scalars are similarly suppressed.

- The  $Z'$  branching ratios, for exotic masses just above 1 TeV, are  $\text{Br}[Z' \rightarrow \nu\bar{\nu}] \simeq 45\%$ ,  $\text{Br}[Z' \rightarrow \ell\bar{\ell}] \simeq 13\%$  and  $\text{Br}[Z' \rightarrow q\bar{q}] \simeq 42\%$ .
- Since the SM  $\text{Br}[Z \rightarrow \ell\bar{\ell}] \simeq 3\%$ , we consider the  $Z'$  search via a clean dilepton signal at the LHC.
- We focus on the invariant mass distribution of the emerging leptons at the LHC:  $p + p \rightarrow \mu^+ + \mu^- + X$  at 14 TeV, with  $\mathcal{L} = 100, 300 \text{ fb}^{-1}$ .
- In order to suppress the SM background, we look at  $M_{\mu\mu} > 1 \text{ TeV}$ .
- We assume three values for  $M_{Z'}$ , *i.e.* 3, 4, 5 TeV.
- A simple analysis is also performed considering the transverse momentum of the final muons.
- A considerable number of events is then predicted.





## CONCLUSIONS

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- We study a 3-3-1 model with  $\beta = 1/\sqrt{3}$ . (3-3-1  $\rightarrow$  SM at the TeV scale).
- 2 scalar triplets at low energies  $\Rightarrow h, H, \varphi^\pm$  (no CP-odd neutral scalar).
- A residual symmetry is explicitly broken with effective operators so that all fermions become massive.
- UV completion is obtained with a heavy scalar triplet ( $M \gg \mathcal{O}(\text{TeV})$ ).
- Neutrino masses via the type-I seesaw mechanism.
  - $m_{E_i^-}, m_{D_a}, m_U \sim w \sim 10 \text{ TeV}$ ;
  - $m_e, m_\mu, m_\tau, \sim \epsilon \sim 1 \text{ GeV}$ ;
  - $m_U, m_c \sim \epsilon$  and  $m_t \sim v \sim 10^2 \text{ GeV}$ ;
  - $m_d, m_s \sim v$  and  $m_b \sim \epsilon$ ;
- Natural mass hierarchies lead to small mixing and suppressed FCNC.
- By studying the decay of  $Z'$  into muons at the LHC, through the invariant mass and  $p_T$ , the existence of  $Z'$  can be confirmed.

THANKS!

