Three-loop neutrino masses

Systematic classification of three-loop realizations of the Weinberg operator

[1807.00629]

(work done in collaboration with R. Fonseca and M. Hirsch)

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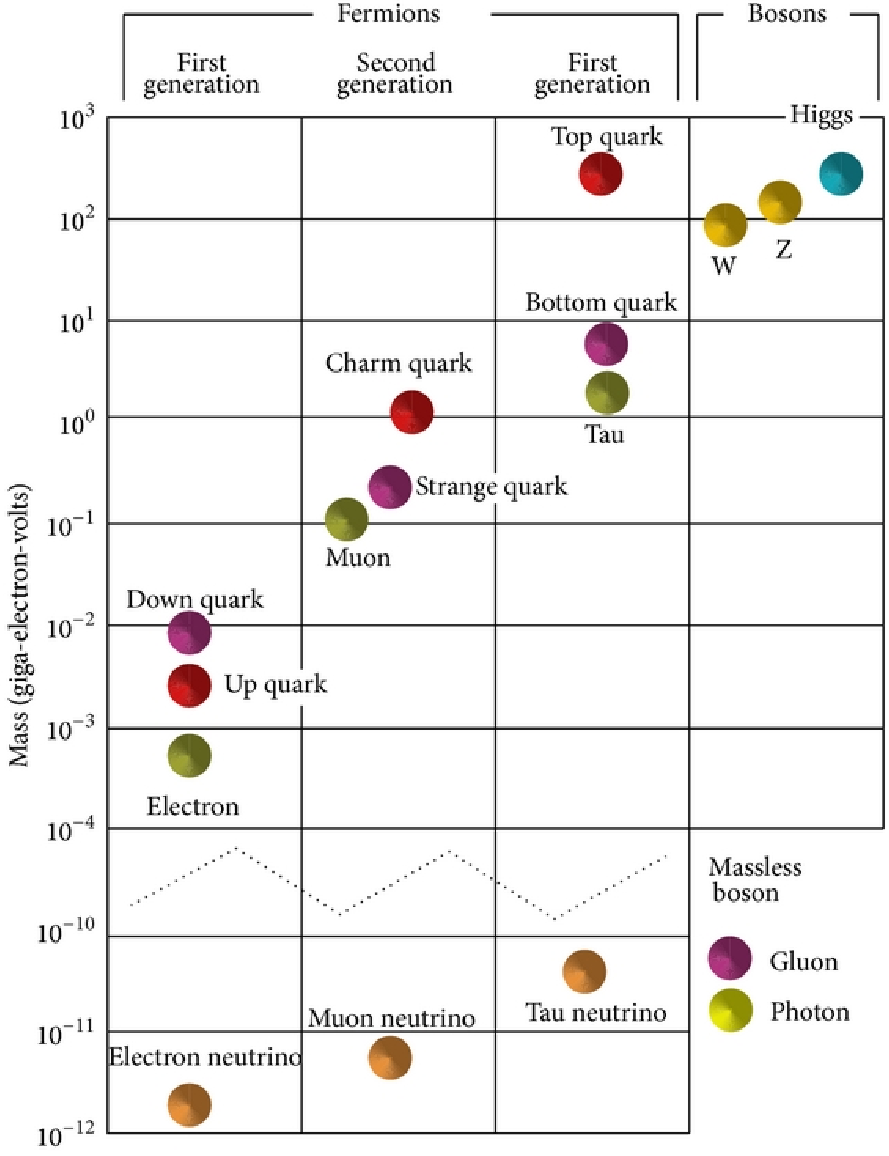
Contents

Introduction

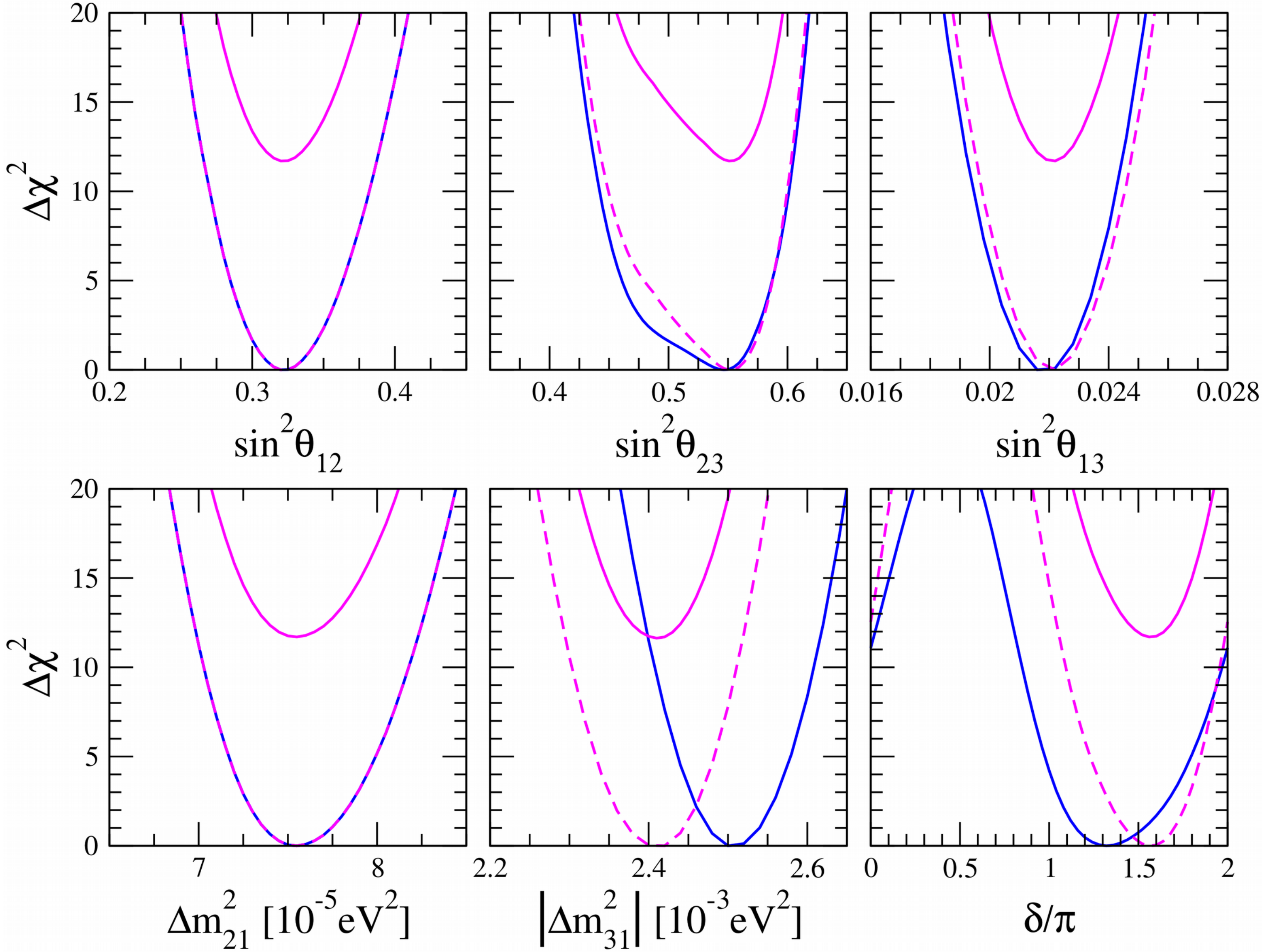


Neutrinos are much lighter than leptons by at least a factor $\mathbf{10^6}$

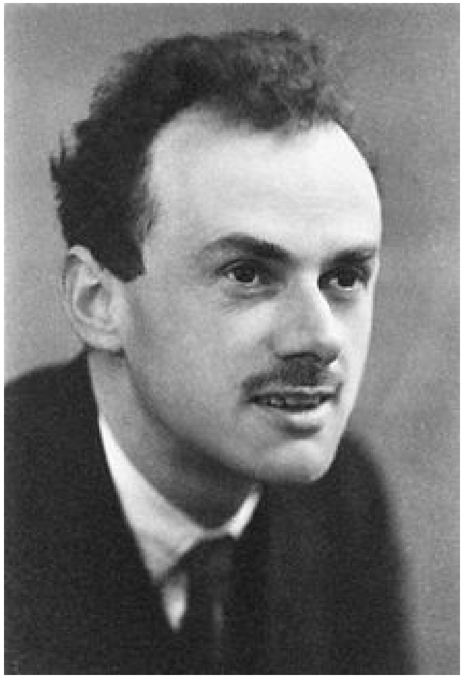
Introduction

[deSalas et al, Phys.Lett. B782 (2018) 633-640]

https://globalfit.astroparticles.es/



Introduction



νs

Introduction

νs

- Neutrinoless double beta decay
- Lepton Number Violation
- Testable at accelerators

Introduction

Weinberg operator:
(d=5)

$$\mathcal{O}_W = \frac{c_{\alpha\beta}}{\Lambda} L_\alpha L_\beta H H$$

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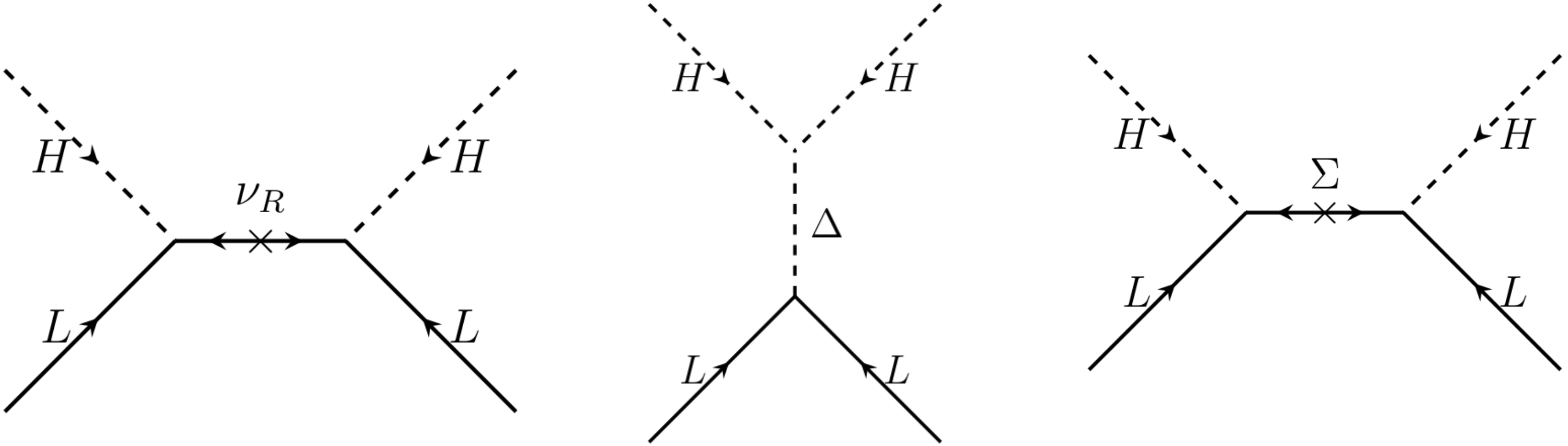


Seesaw type-I   Seesaw type-II   Seesaw type-III

$$c_{\alpha\beta} \simeq \mathcal{O}(1) \longrightarrow \Lambda \sim \mathcal{O}(10^{14})\ \mathrm{GeV}$$

Introduction

Weinberg operator: (d=5) $$\mathcal{O}_W = \frac{c_{\alpha\beta}}{\Lambda} L_\alpha L_\beta H H$$

$c_{\alpha\beta}$ could be naturally much smaller than one:

(i) Higher dimensional operators: $$\mathcal{O}_W \times (H^\dagger H)^{\frac{d-5}{2}}$$

[Anamiati et al; 1806.07264]

[Babu, Nandi, Tavartkiladze; Phys.Rev. D80 (2009) 071702]

[Bonnet et al; JHEP 0910 (2009) 076]

(ii) Radiative neutrino masses: $$c_{\alpha\beta} \propto 1/(16\pi^2)^n$$

[Review of Cai et al; Front.in Phys. 5 (2017) 63]

[RC, Helo, Hirsch; JHEP 1707 (2017) 079]

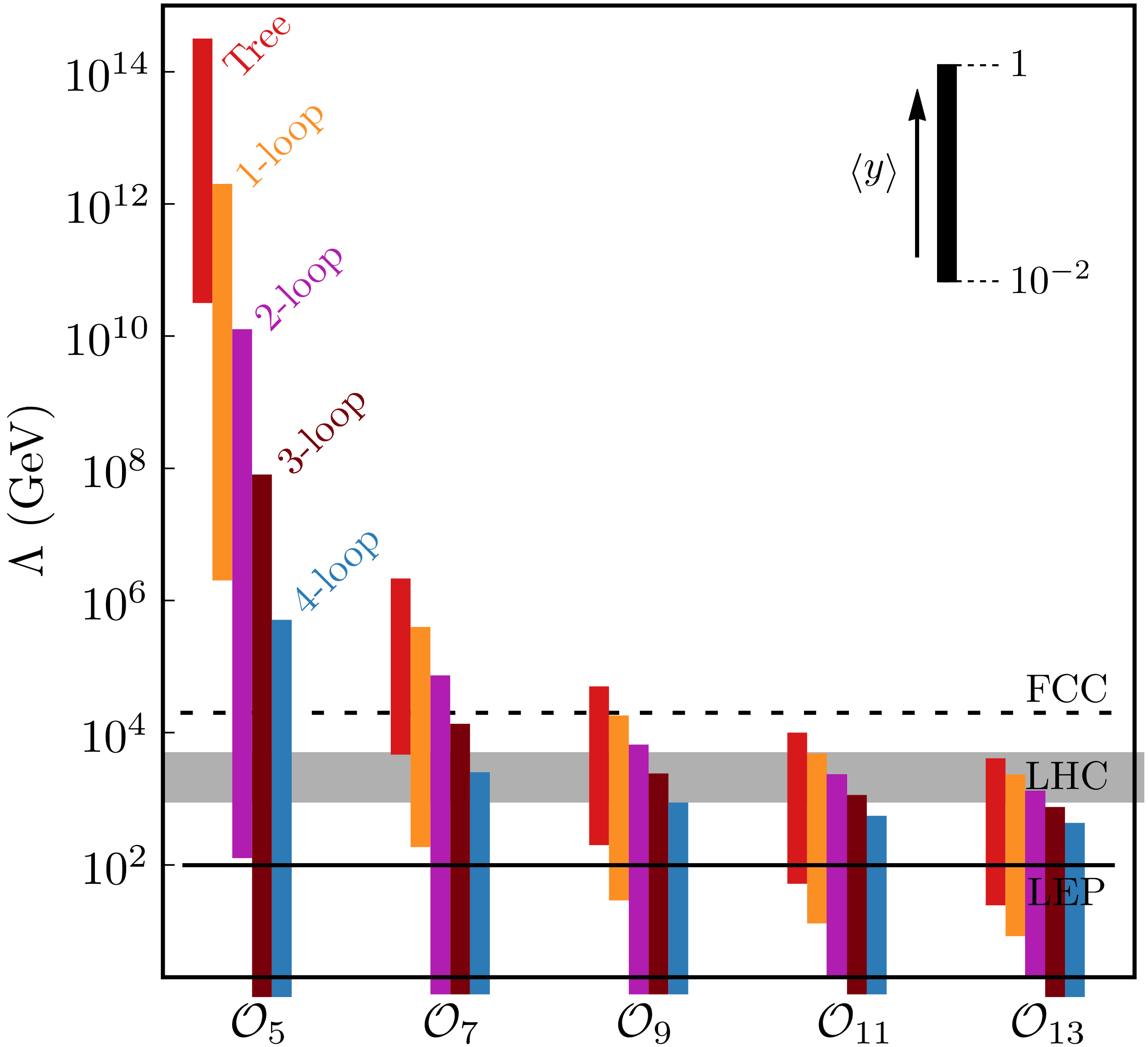
(iii) "Nearly" conserved Lepton Number: $$c_{\alpha\beta} < 1$$

[Mohapatra, Valle; Phys. Rev. D34, 1642 (1986)]

[Branco, Grimus, Lavoura; Nucl.Phys. B312 (1989) 492-508]

Introduction

figure from Anamiati et al, 1806.07264



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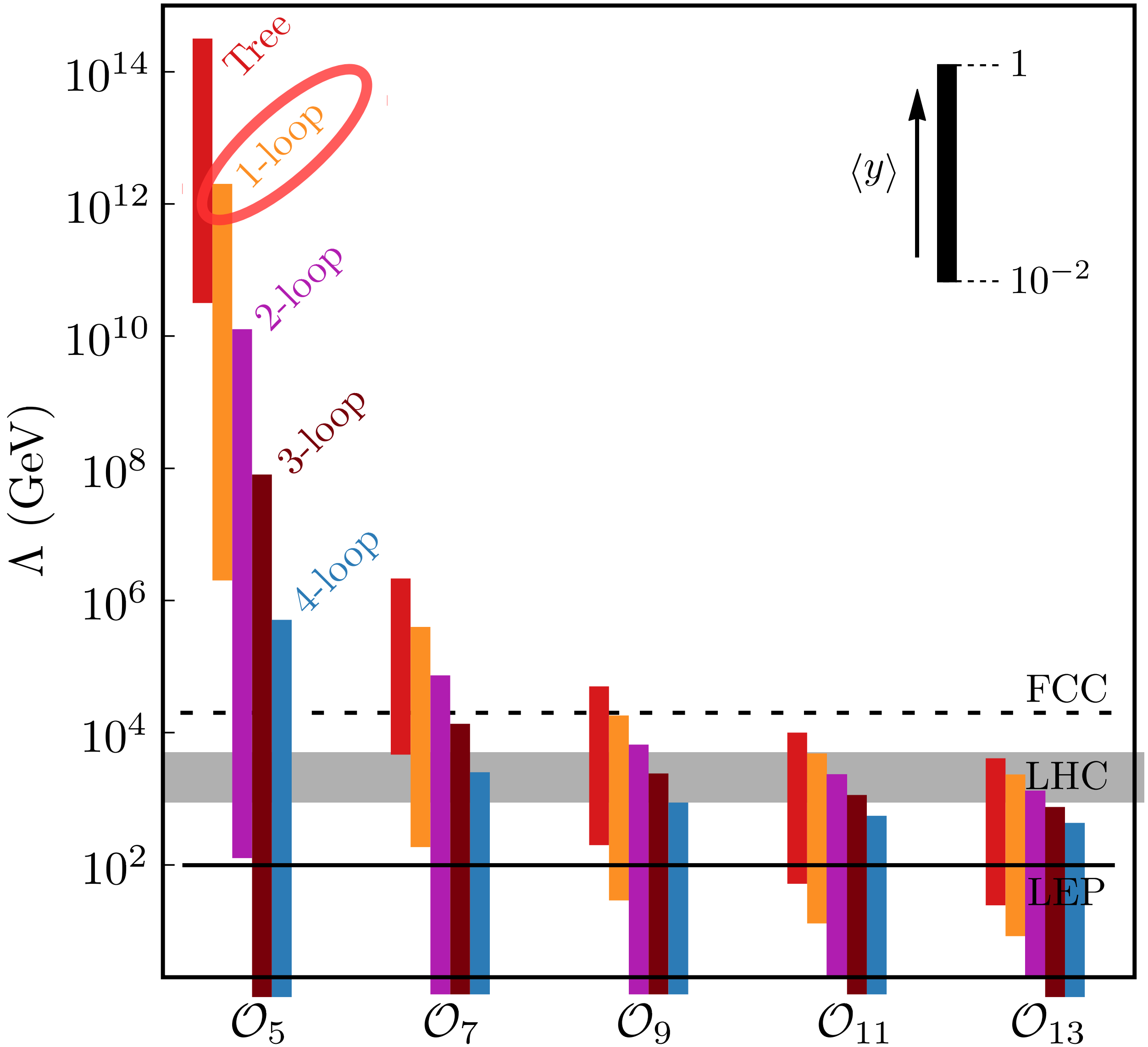

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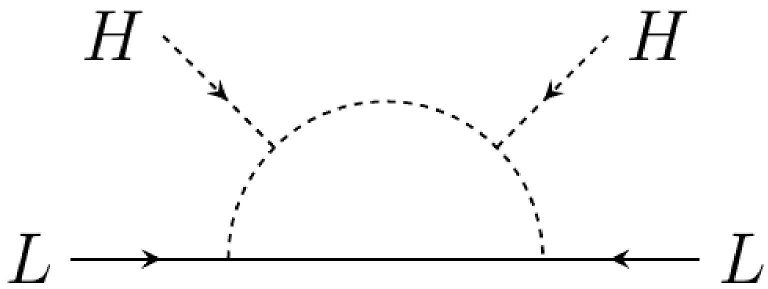

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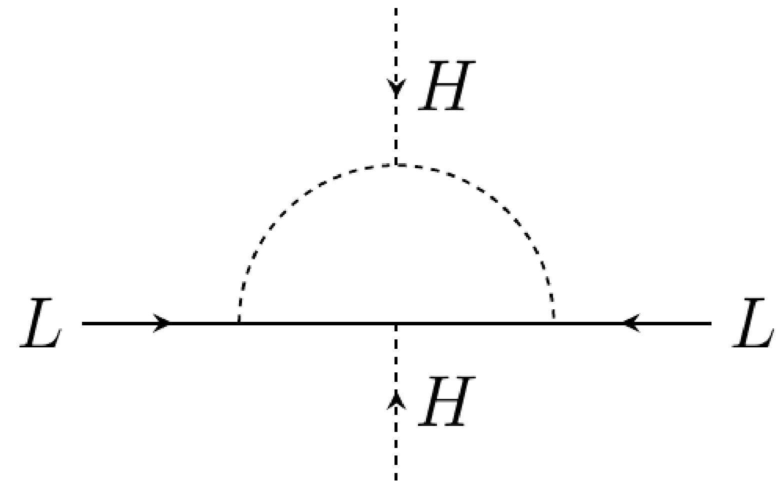

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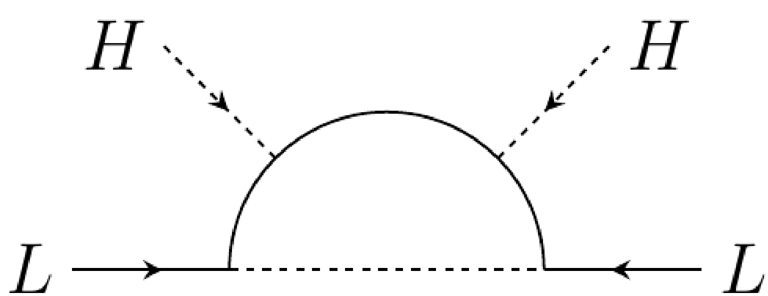
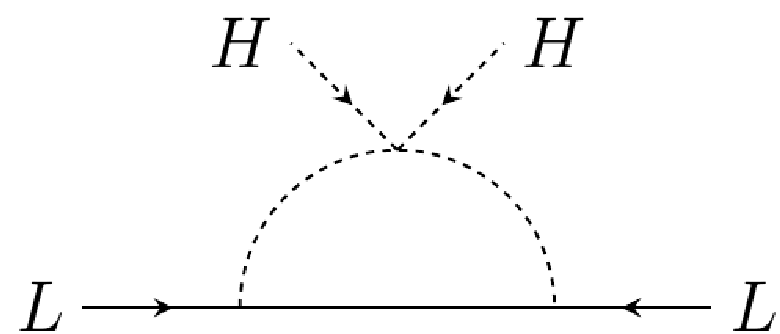


T-1-iii

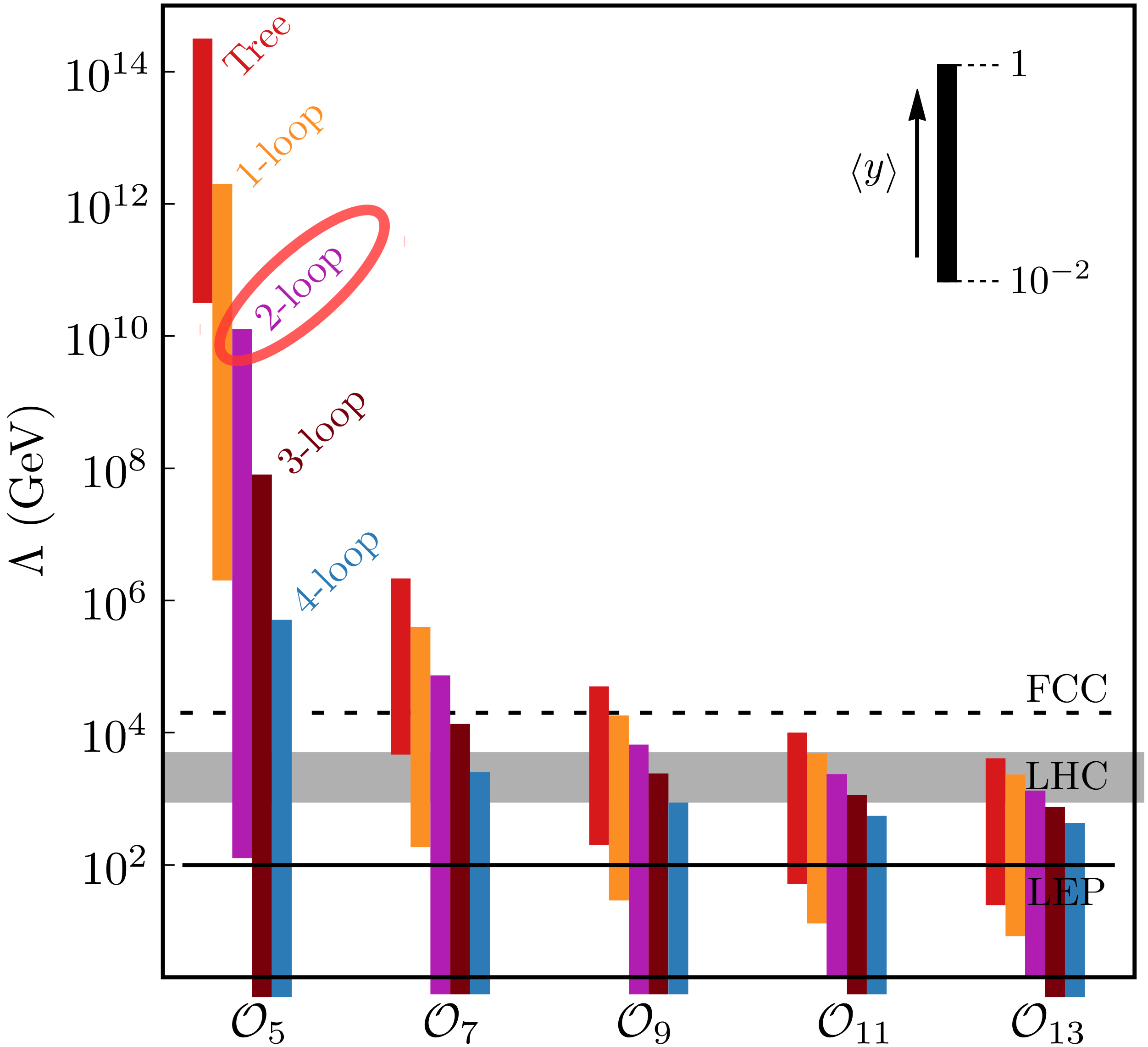


T-3

Systematic classification by Bonnet, Hirsch, Ota, Winter [JHEP 1207 (2012) 153]

Introduction

figure from Anamiati et al, 1806.07264

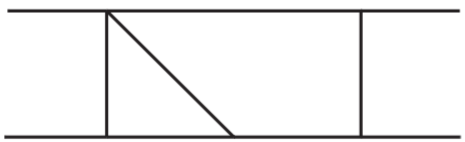


Introduction

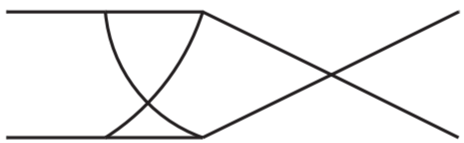
Systematic classification by Aristizabal, Degee, Dorame, Hirsch [JHEP 1503 (2015) 040]

$T2_1^{\mathrm{B}}$

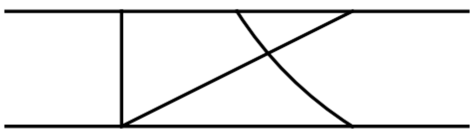
$T2_2^{\mathrm{B}}$



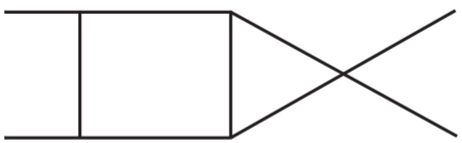
$T2_3^{\mathrm{B}}$



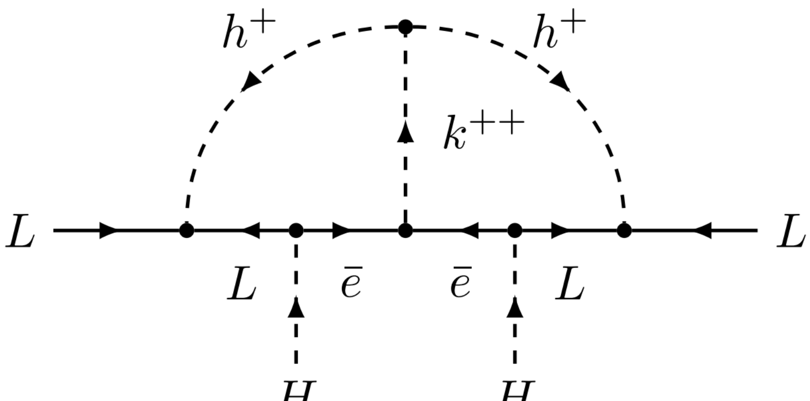
$T2_1^{\mathrm{T}}$



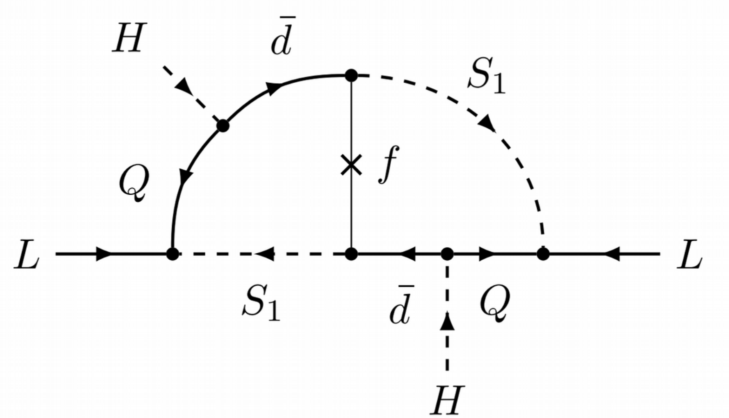
$T2_2^{\mathrm{T}}$



$T2_3^{\mathrm{T}}$



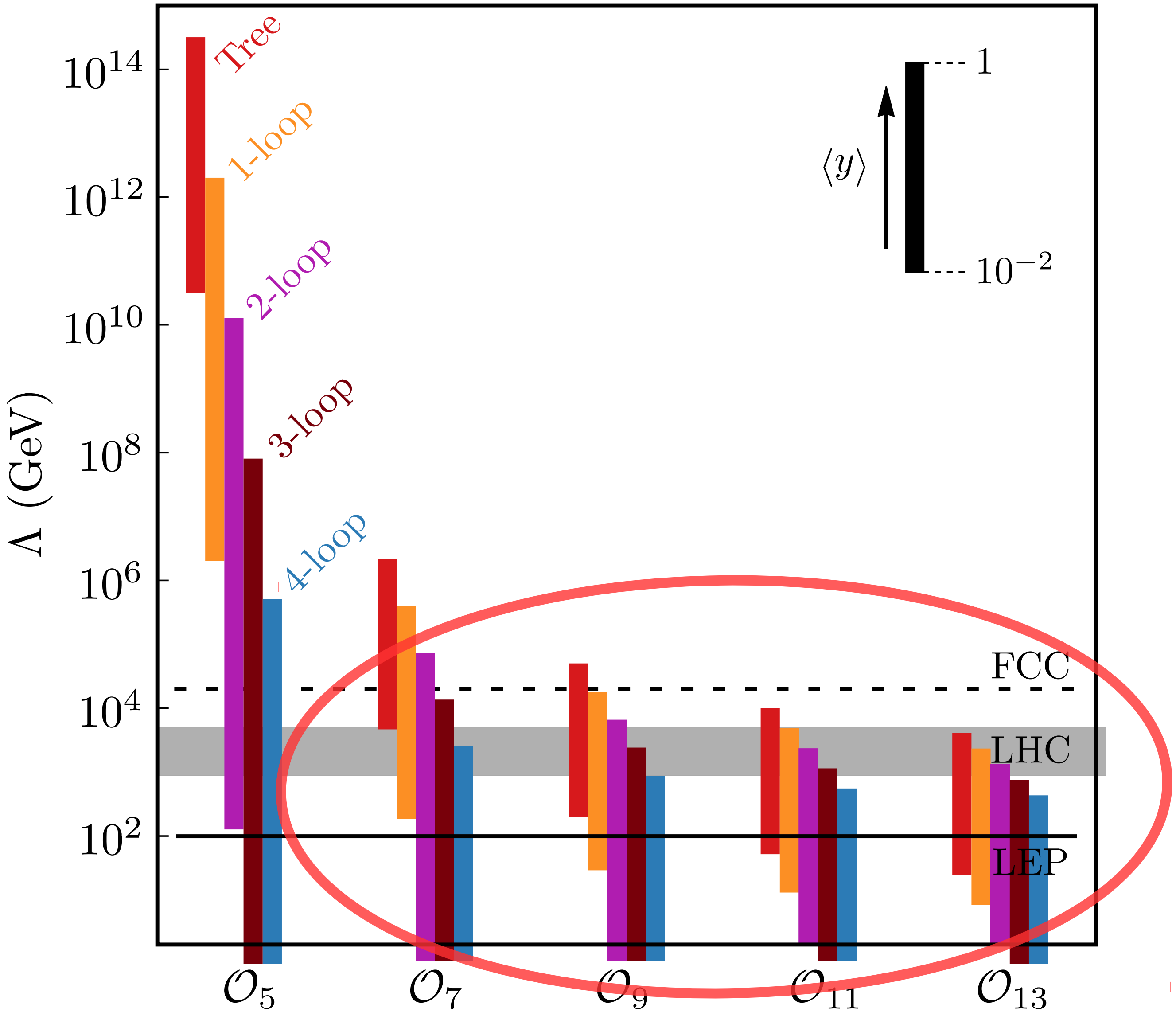
[Cheng, Li; Phys. Rev. D 22, 2860]



[Angel et al; JHEP 10 (2013) 118]

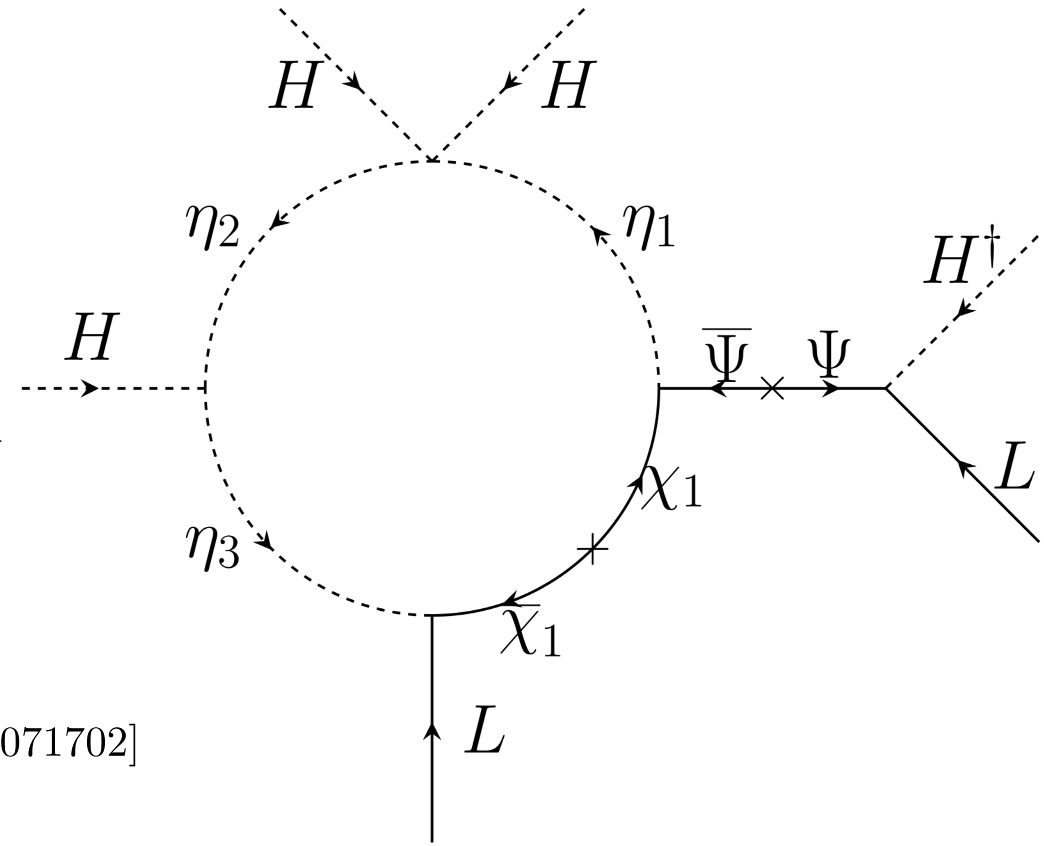
Introduction

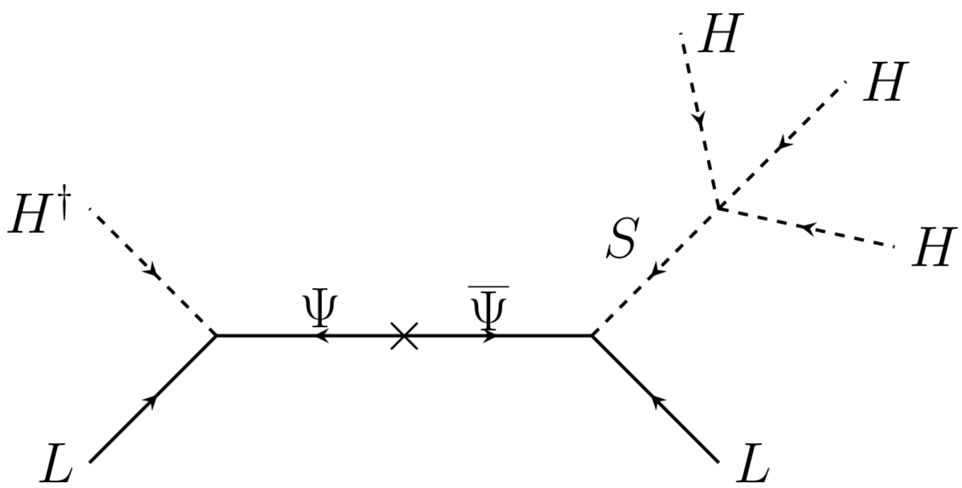
figure from Anamiati et al, 1806.07264



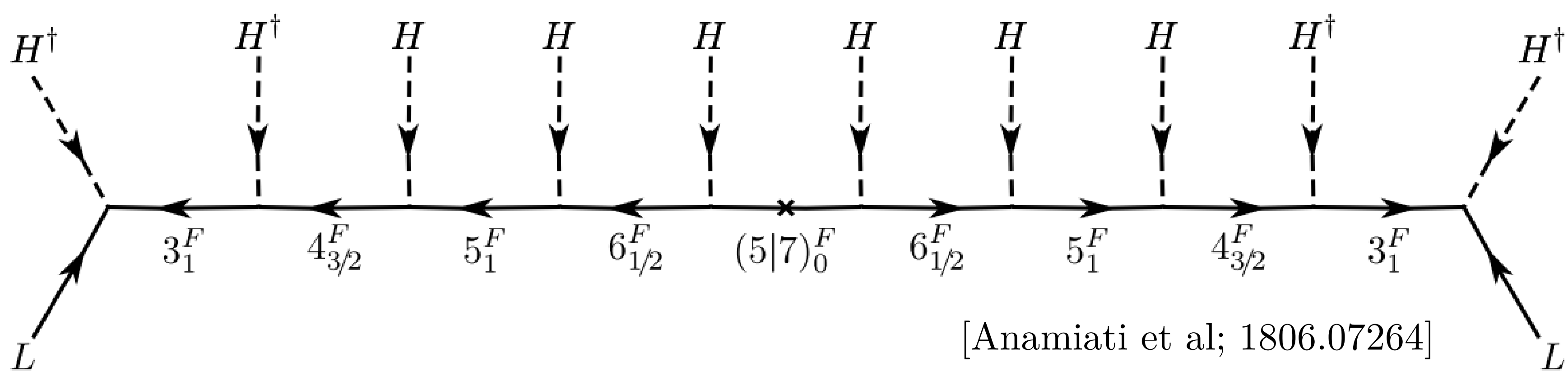
Introduction

[RC, Helo, Hirsch; JHEP 1801 (2018) 009]





[Babu, Nandi, Tavartkiladze; Phys.Rev. D80 (2009) 071702]



[Anamiati et al; 1806.07264]

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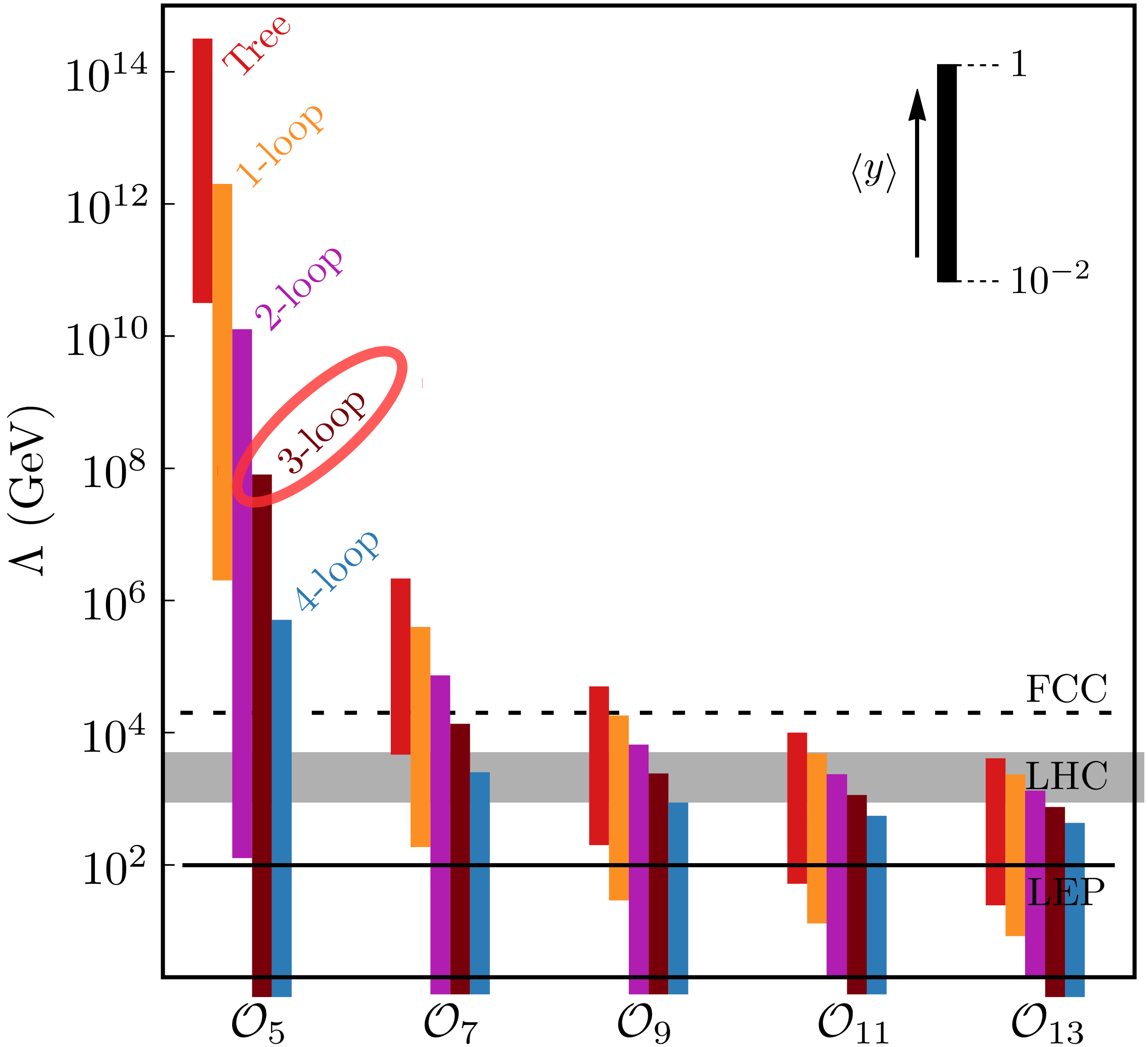
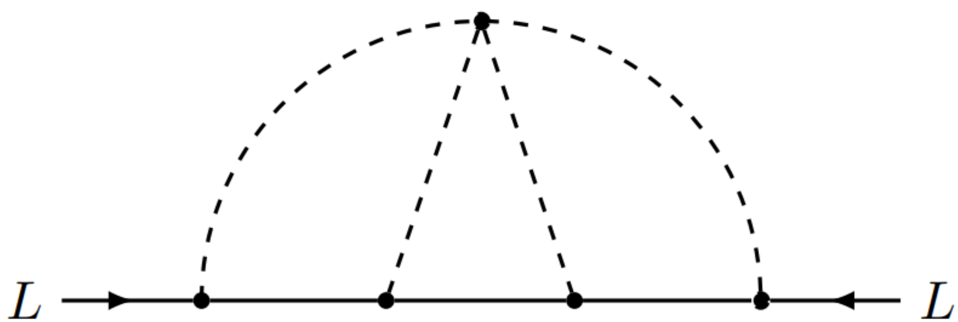
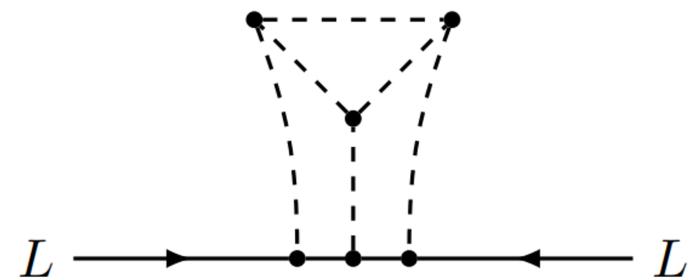
figure from Anamiati et al, 1806.07264



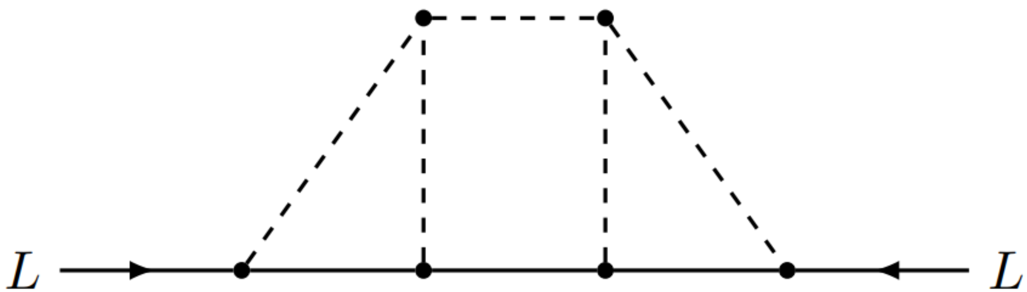
Introduction



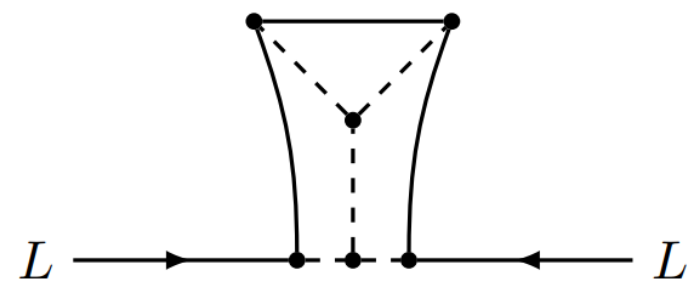
(a) KNT models.



(b) Cocktail models.



(c) AKS models. Cross diagrams may exist.



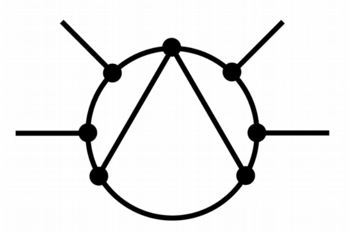
(d) Fermionic Cocktail models.

Figures from Cai et al; Front.in Phys. 5 (2017) 63

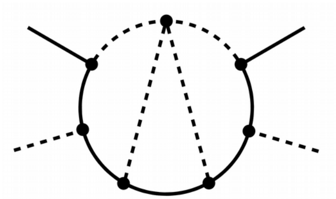
Topologies, Diagrams and Models

Some comments on nomenclature

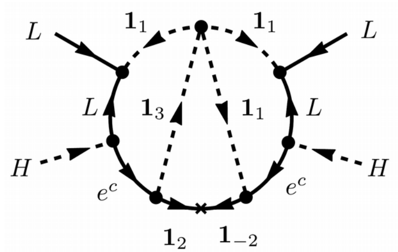
- **TOPOLOGIES:** Feynman diagrams where no property of the fields is considered.



- **DIAGRAMS:** scalars are differentiated from fermions.



- **MODEL-DIAGRAMS:** the quantum numbers for the internal particles are specified.



Topologies, Diagrams and Models

Some comments on nomenclature

- **GENUINENESS:**

  a **topology** is said to be genuine if it generates **at least one** model-diagram which fulfills the following three conditions:

  - It is renormalizable
  - The leading contribution to neutrino masses arises at 3-loops
  - No need for extra symmetries beyond those of the SM

Topologies, Diagrams and Models

Classification (or how to drive a student crazy)

(i) All connected topologies with 3- and 4- point vertices and 4 external legs.

4367 topologies

Topologies, Diagrams and Models

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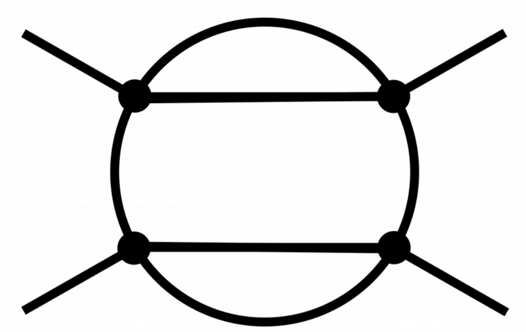
4367 topologies

(ii) Exclude **non-renormalizable** topologies. 3269 topologies



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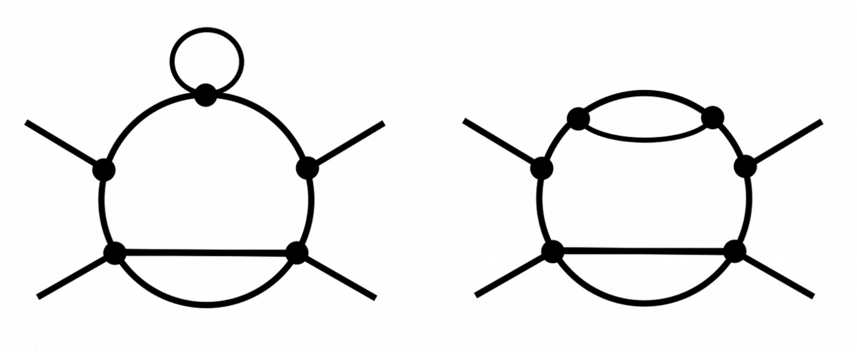

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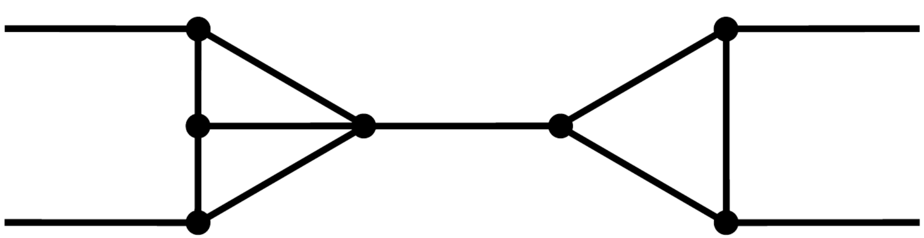

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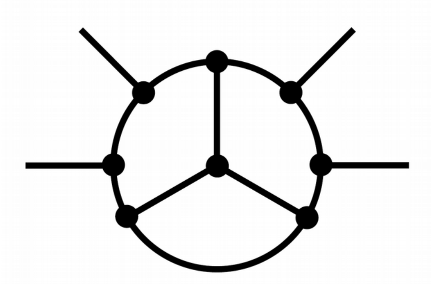

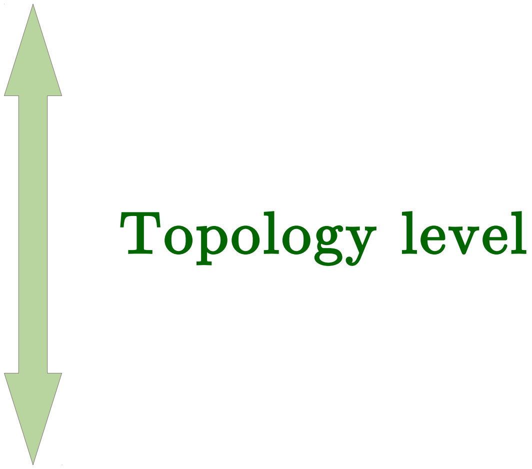

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Topology level

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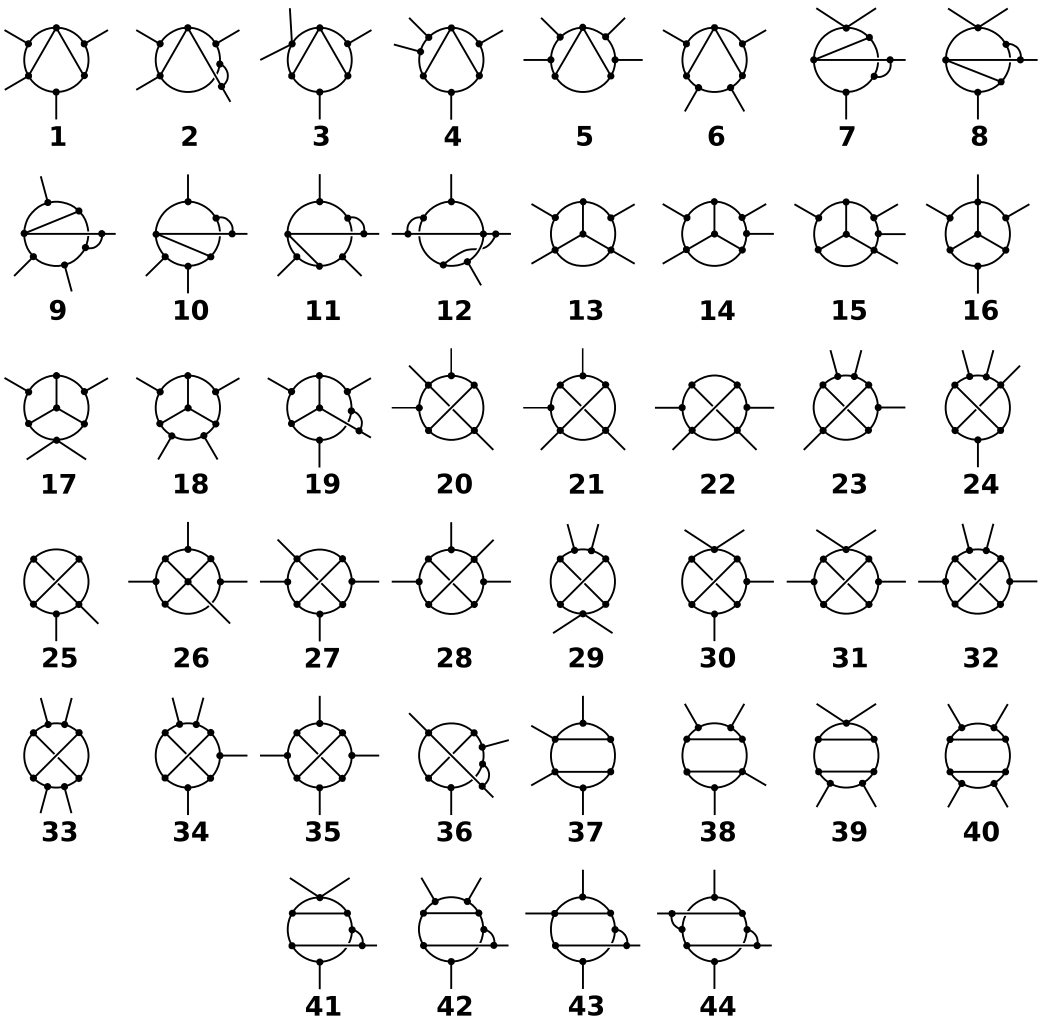
Diagram level

(vi) Remove compressible **4-point** loop vertices, i.e. 4 scalar vertices.

44 topologies

Topologies, Diagrams and Models

Normal genuine topologies (#44)



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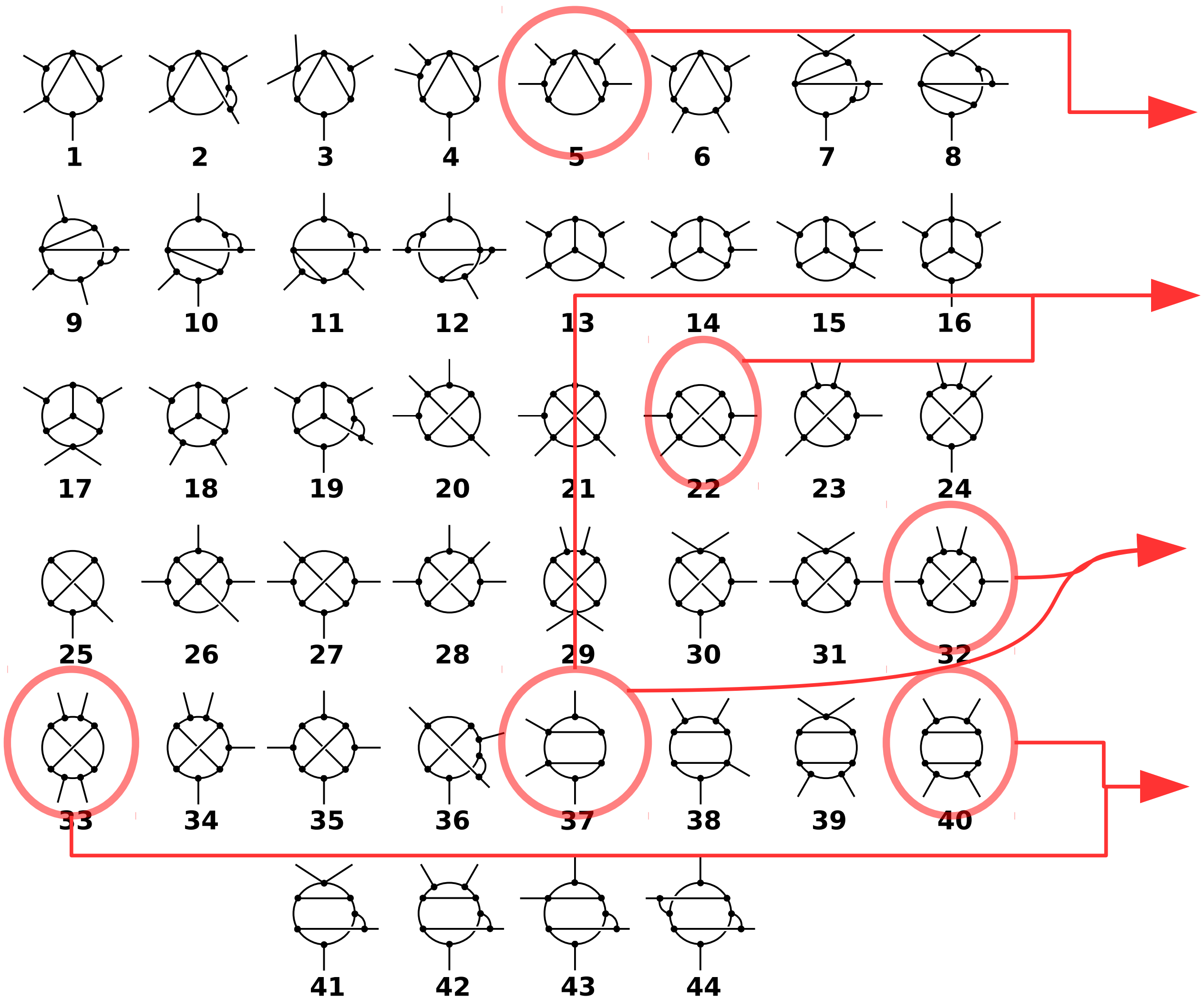

Krauss, Nasri, Trodden;
Phys. Rev. D67 (2003) 085002

Aoki, Kanemura, Seto;
Phys. Rev. Lett. 103 (2009) 051805

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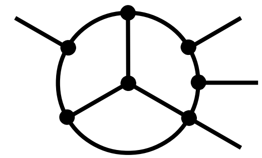
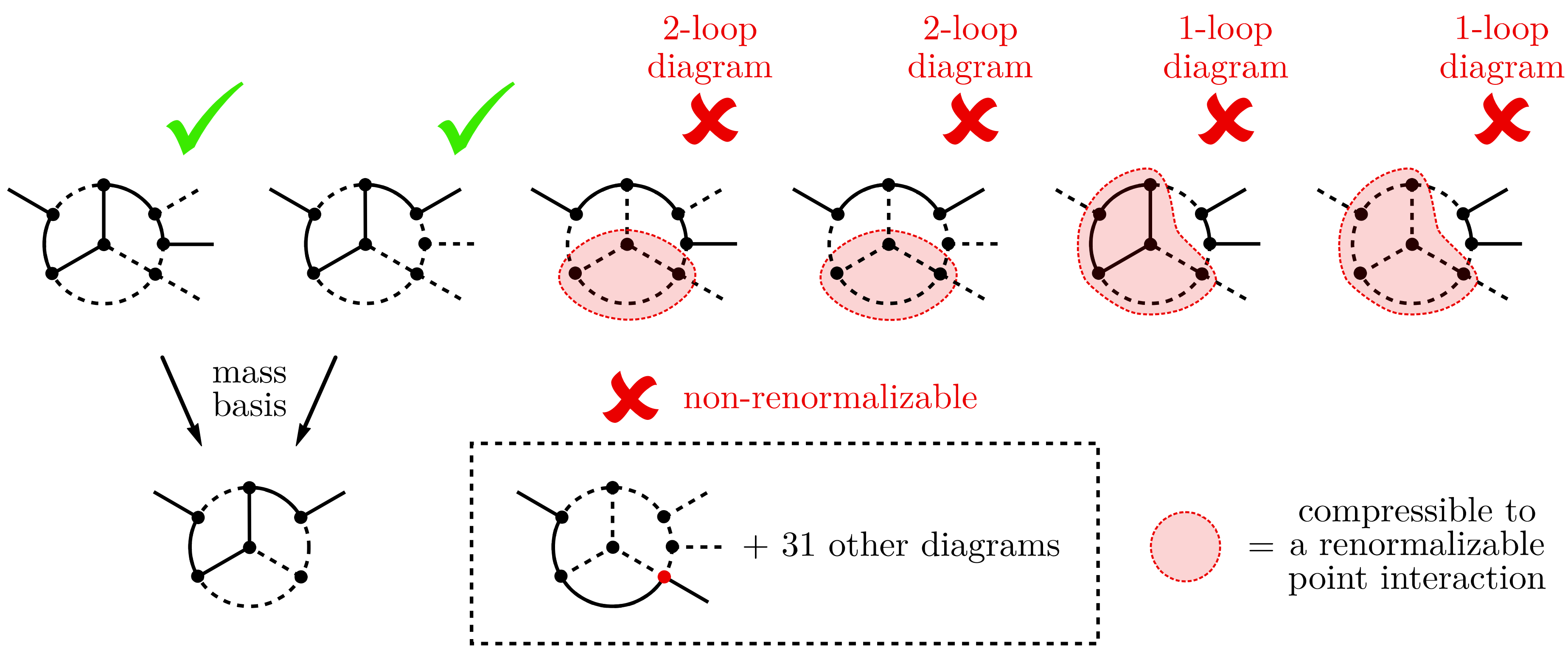
Topologies, Diagrams and Models



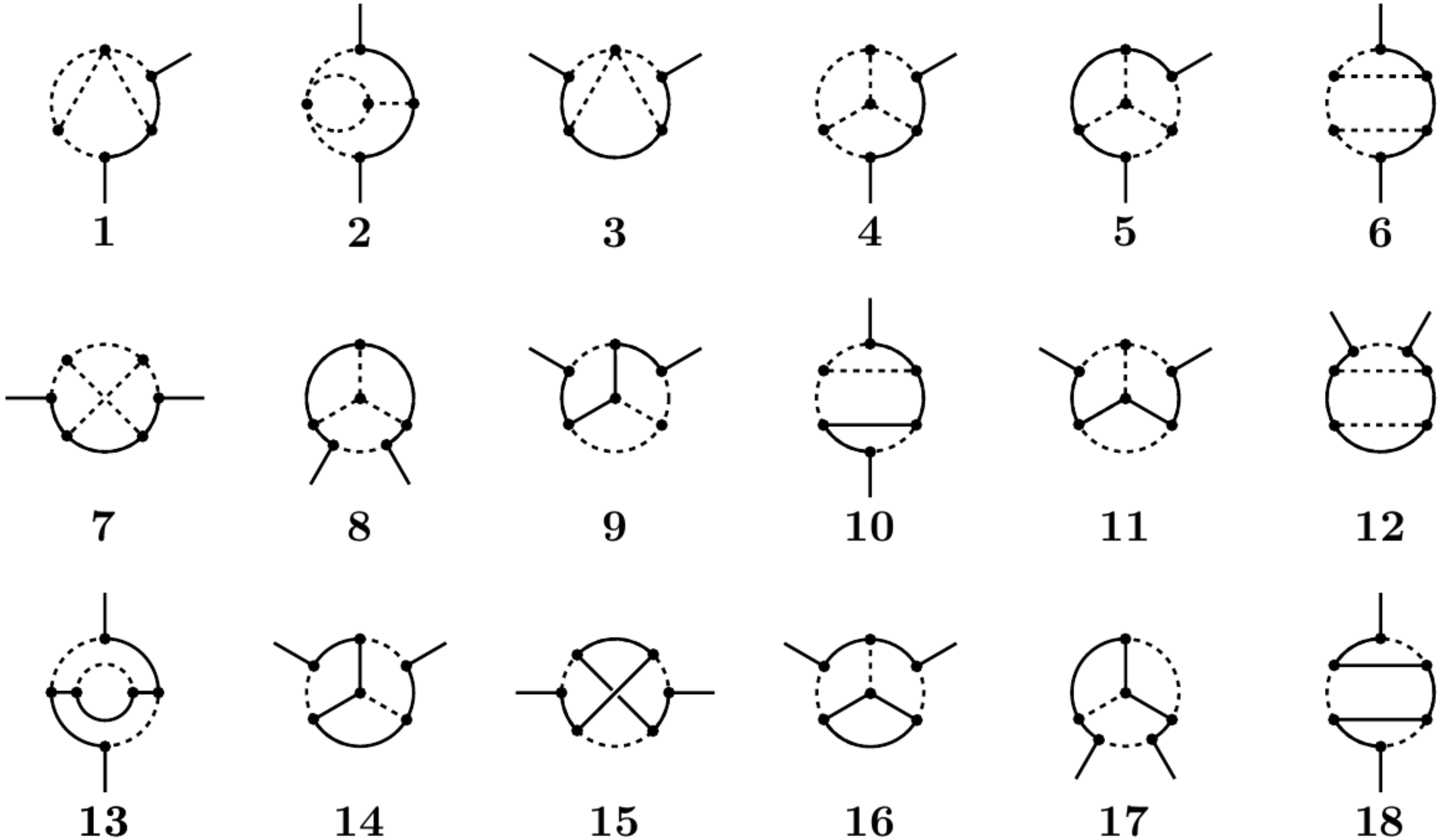
Genuine diagrams with this topology



Topologies, Diagrams and Models

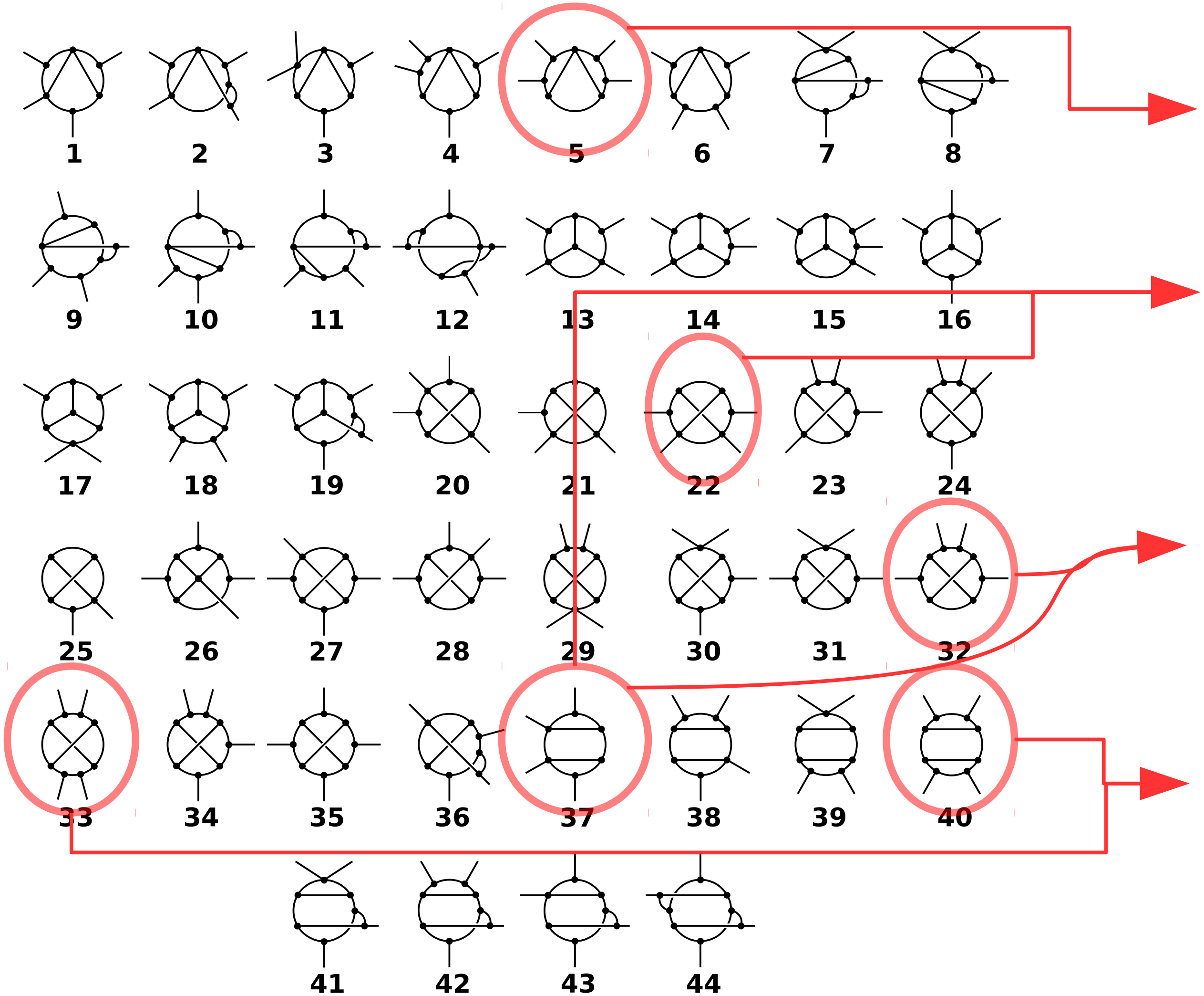
Mass diagrams (#18)

44 topologies ⟶ 228 diagrams ⟶ 18 diagrams in the mass basis



Topologies, Diagrams and Models

NORMAL genuine topologies (#44)



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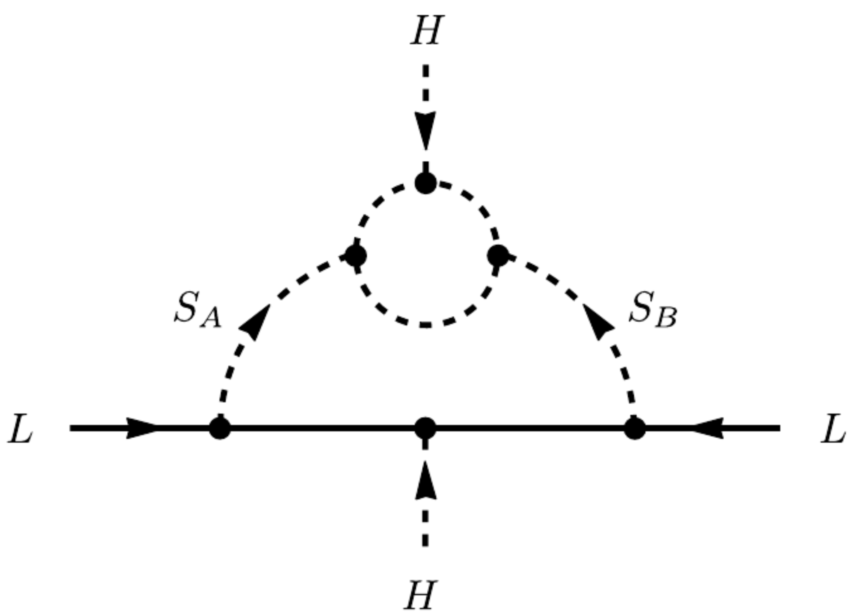
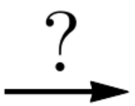
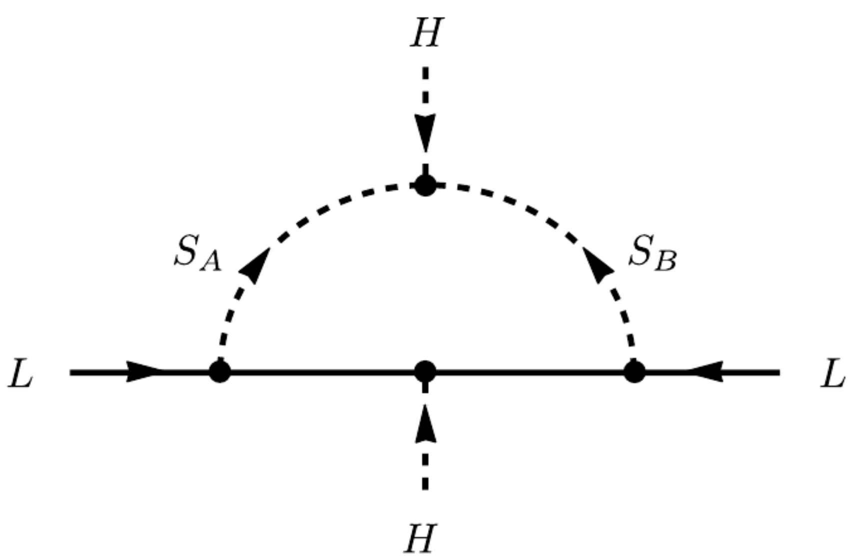
44 topologies

Topologies, Diagrams and Models

Loophole





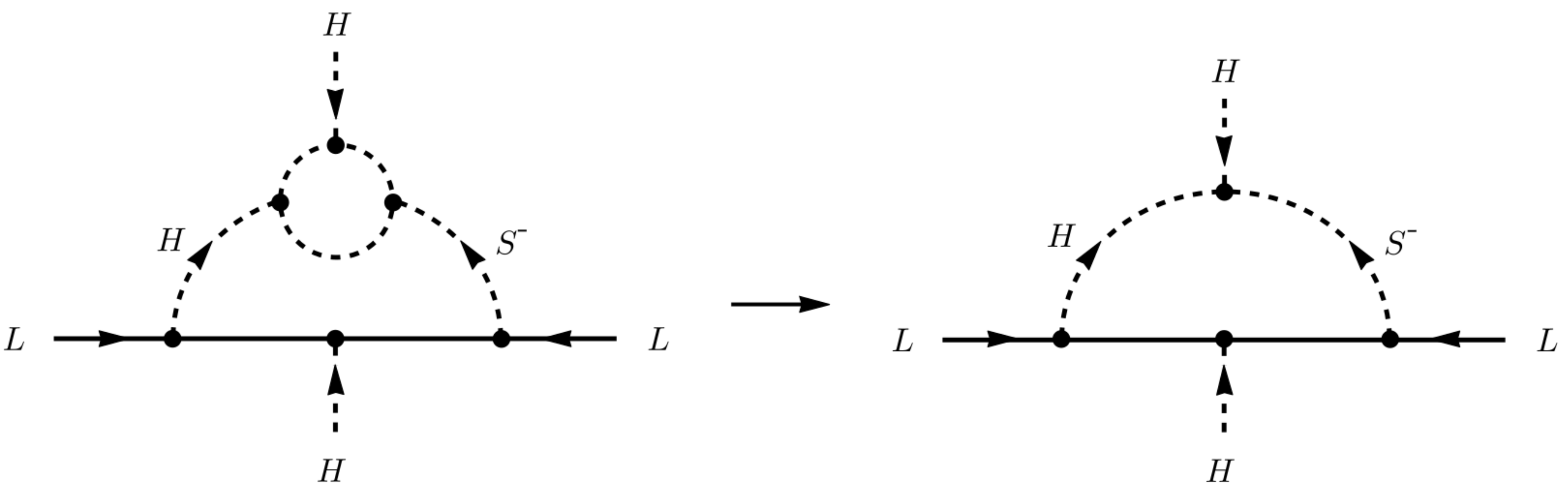


If the loop vertex H-S_A-S_B is allowed by symmetry,
so it is the tree level vertex H-S_A-S_B

BUT H-S_A-S_B can be identically zero

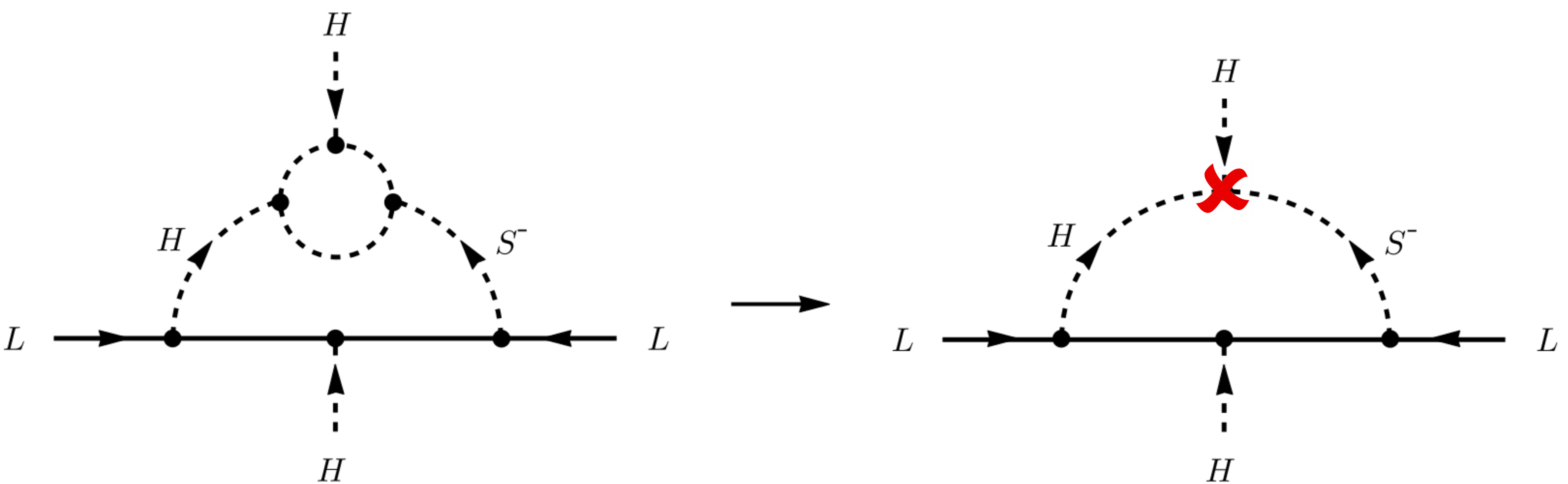
Topologies, Diagrams and Models

Loophole



Topologies, Diagrams and Models

Loophole



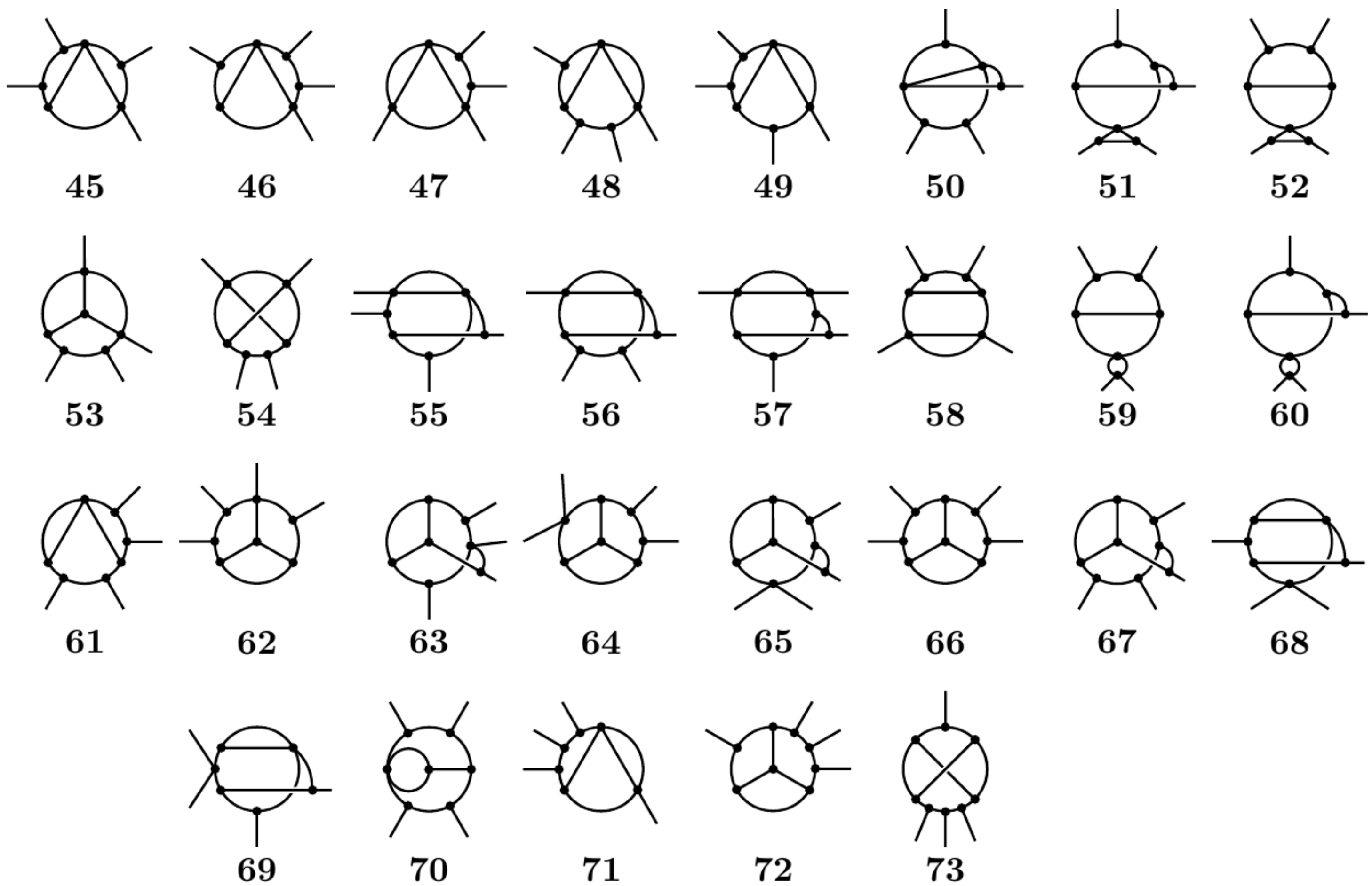
antisymmetric $SU(2)_L$ contraction $\longrightarrow H(x)H(x)S^-(x) = 0$

$$\boxed{H(x_1)H(x_2)S^-(x_3) \neq 0}$$

Topologies, Diagrams and Models

Special genuine topologies (#29)

They require a special choice of quantum numbers to be genuine



Topologies, Diagrams and Models

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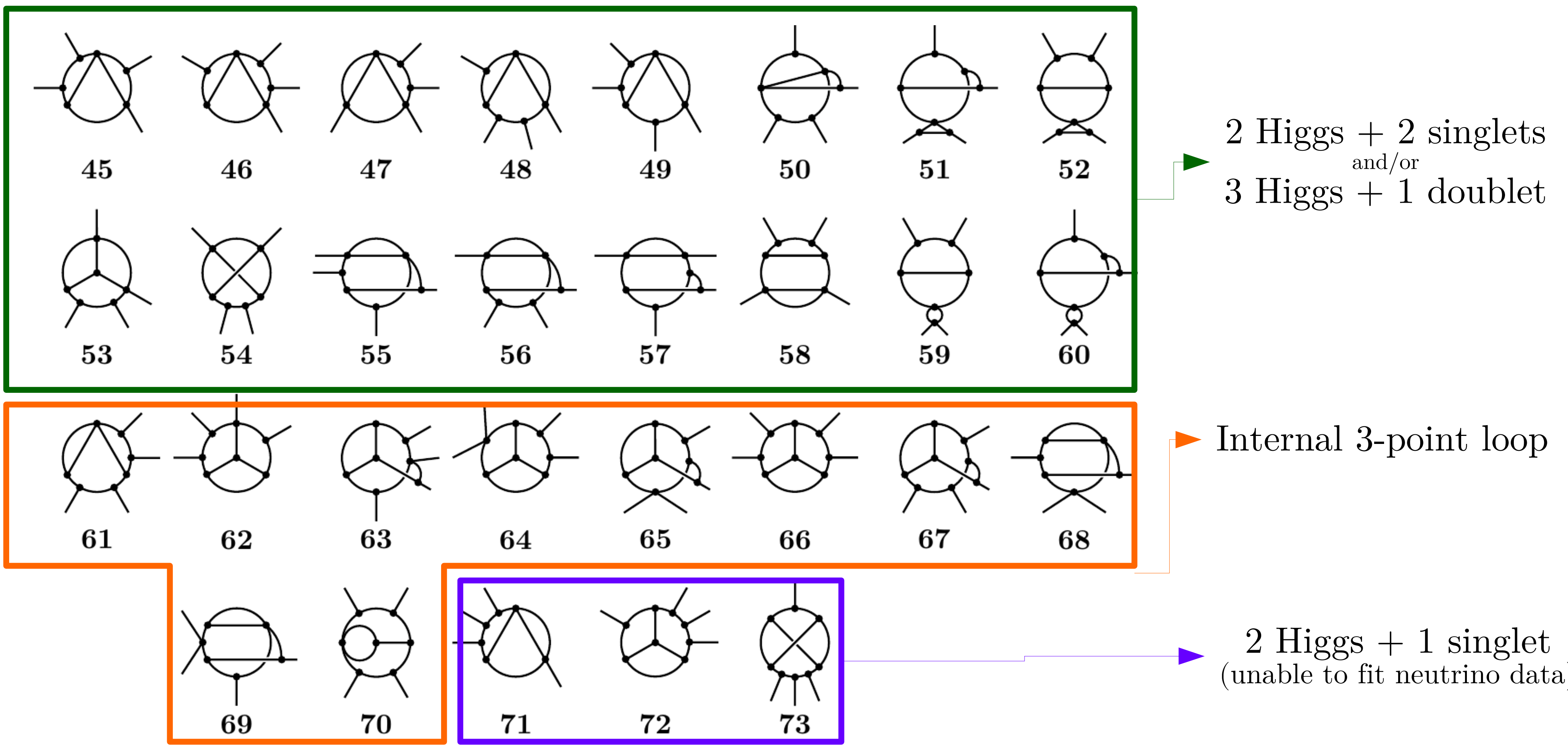
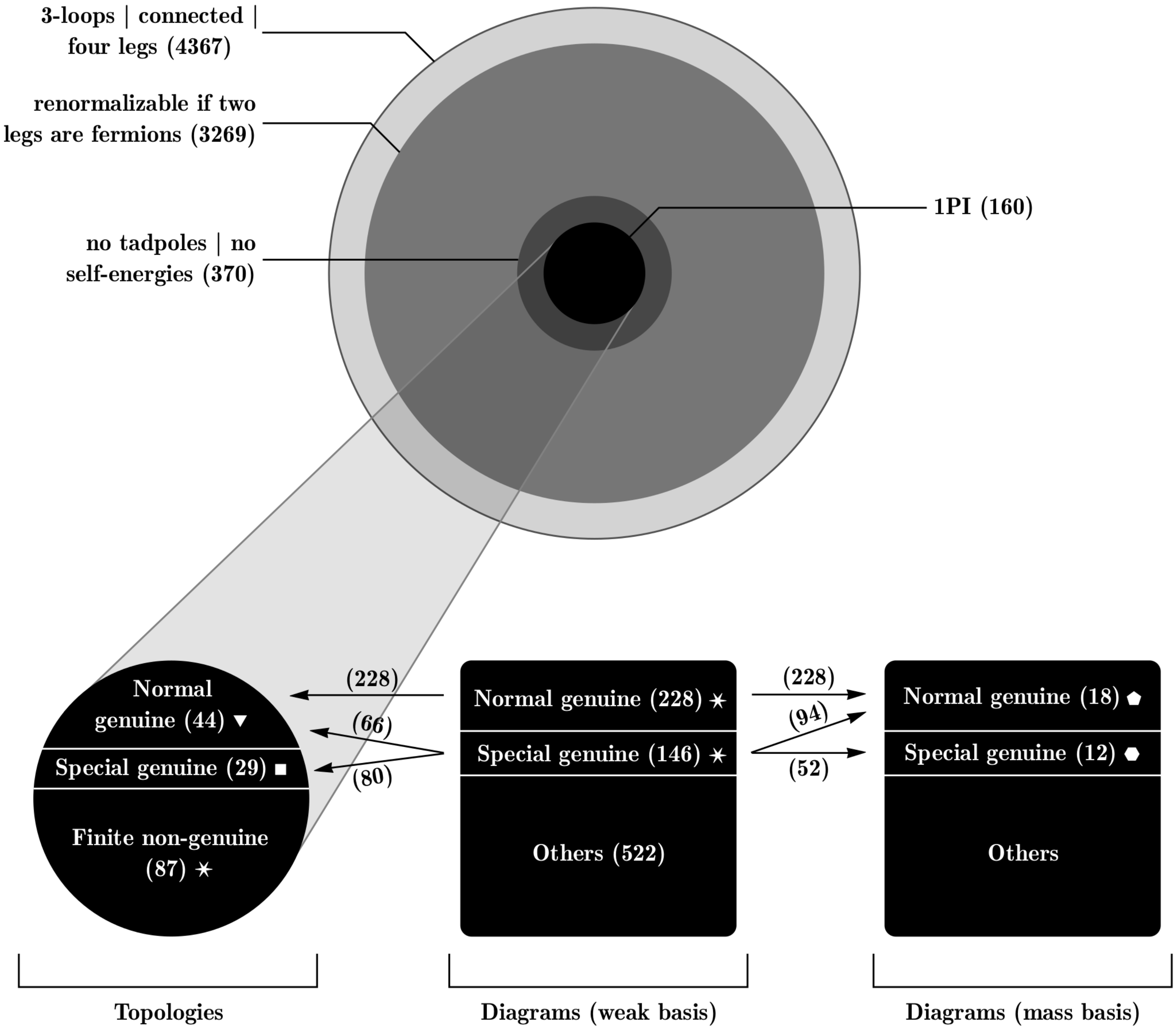
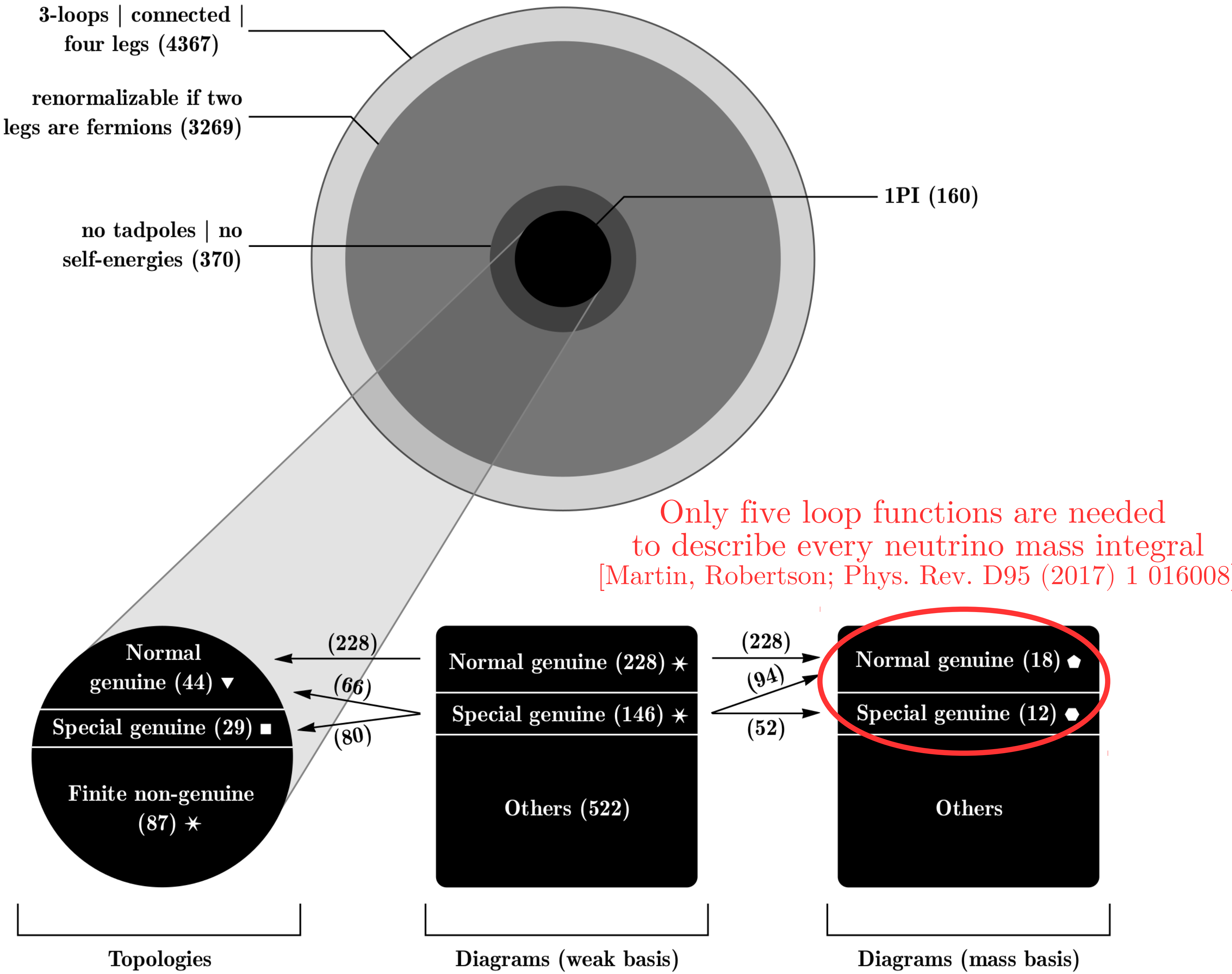




More diagrams on the web page http://renatofonseca.net/3loop-weinberg-operator.php



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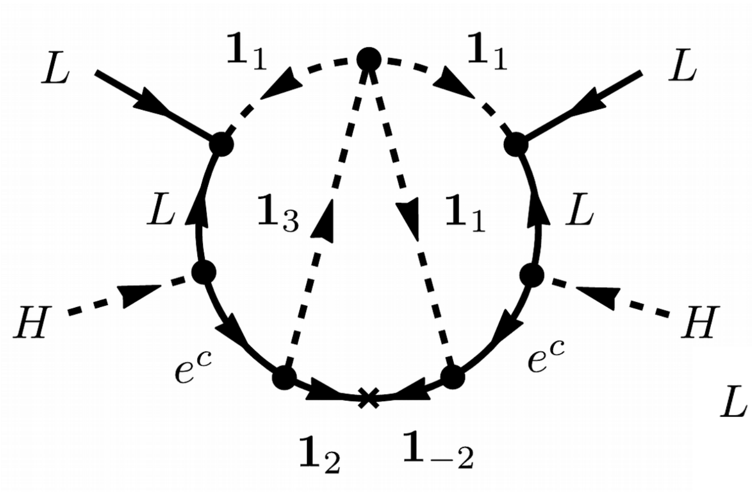
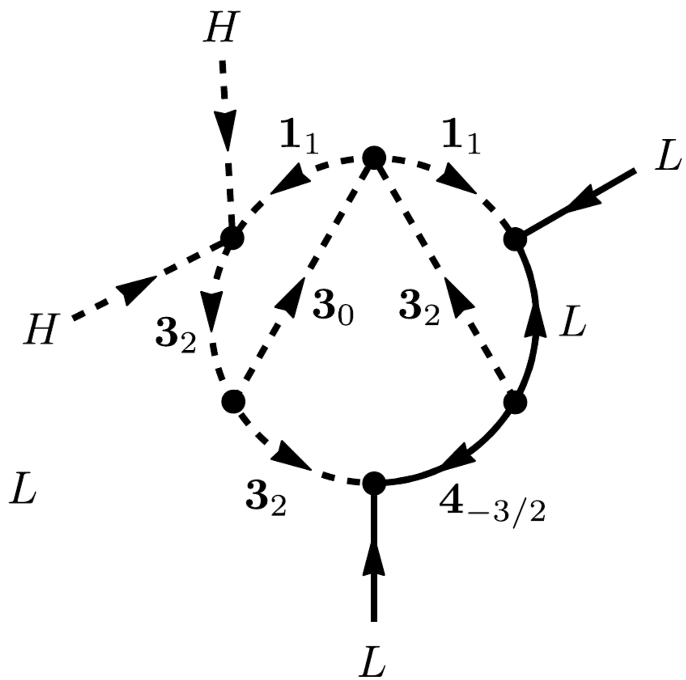
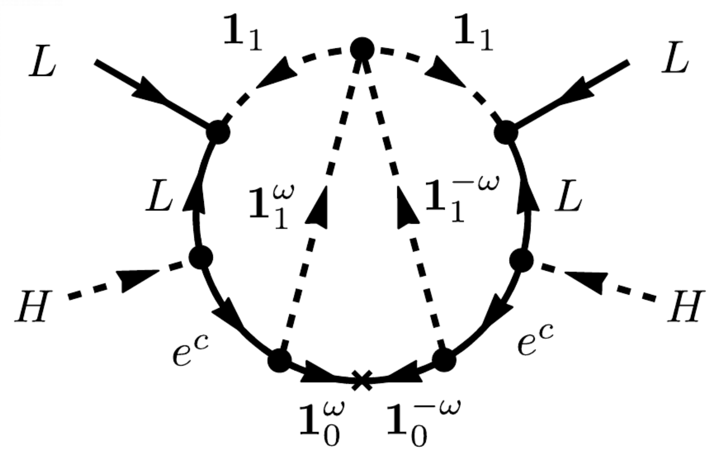
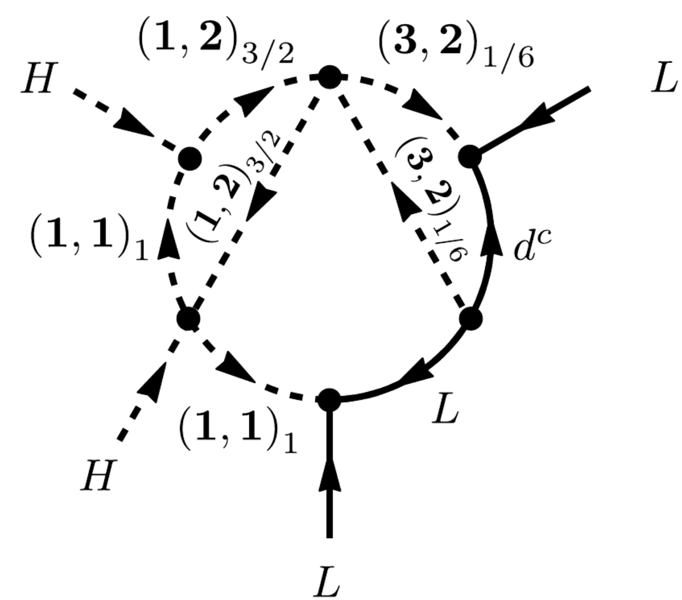
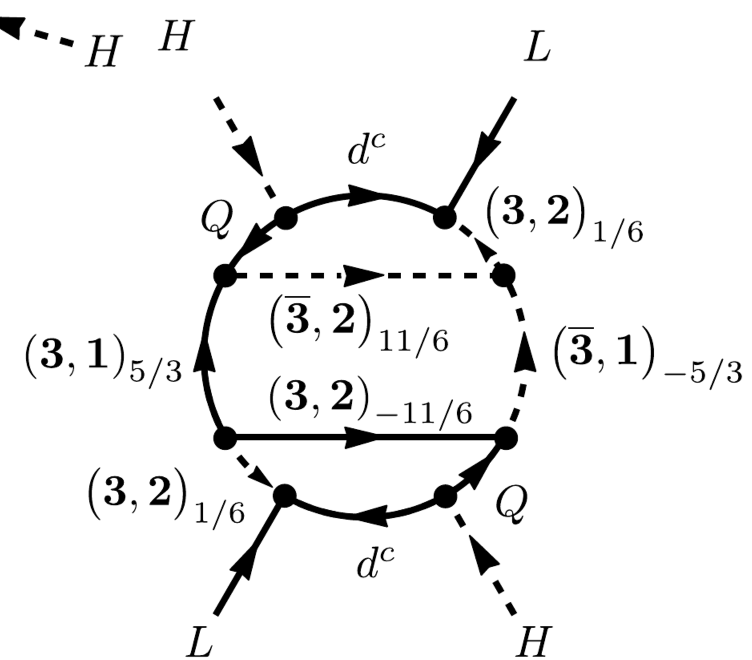
Some examples of models



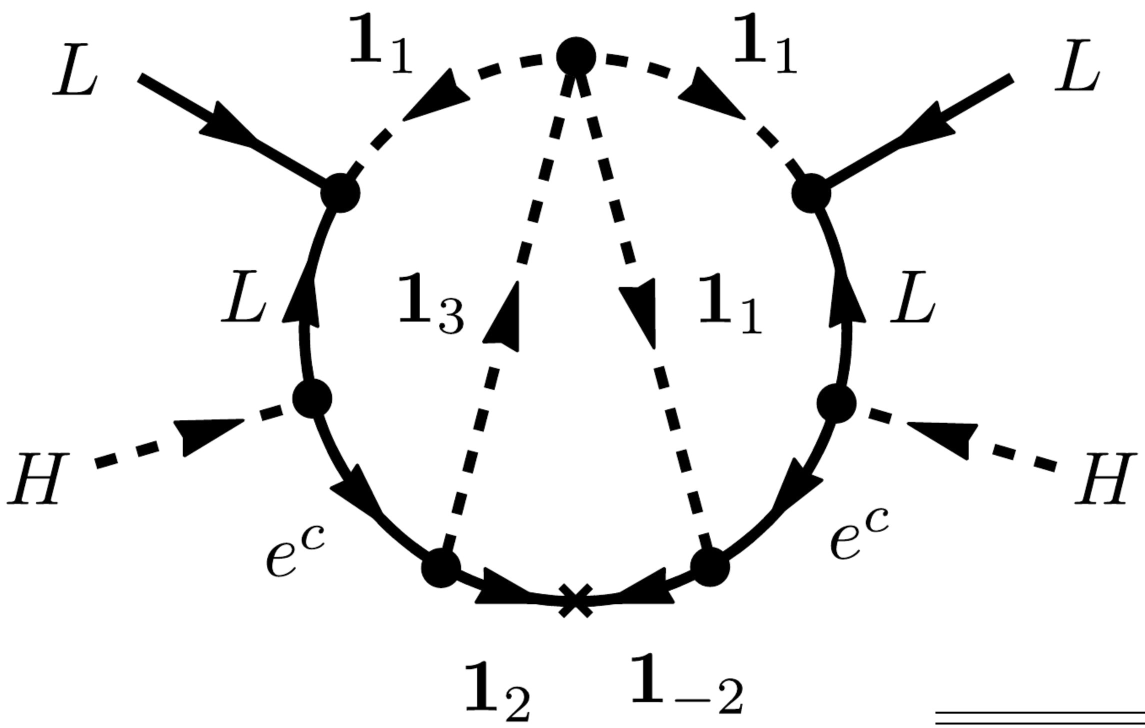






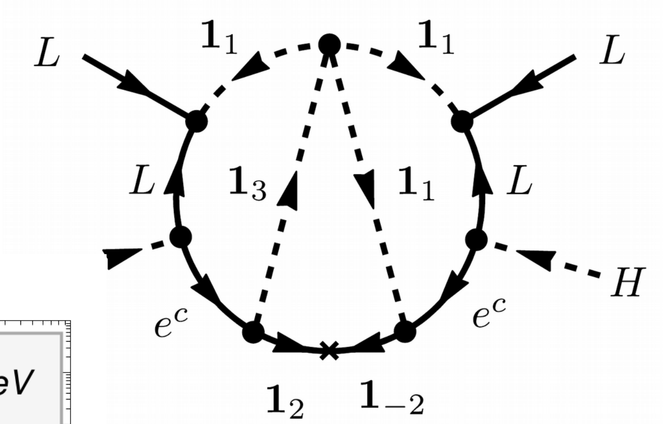


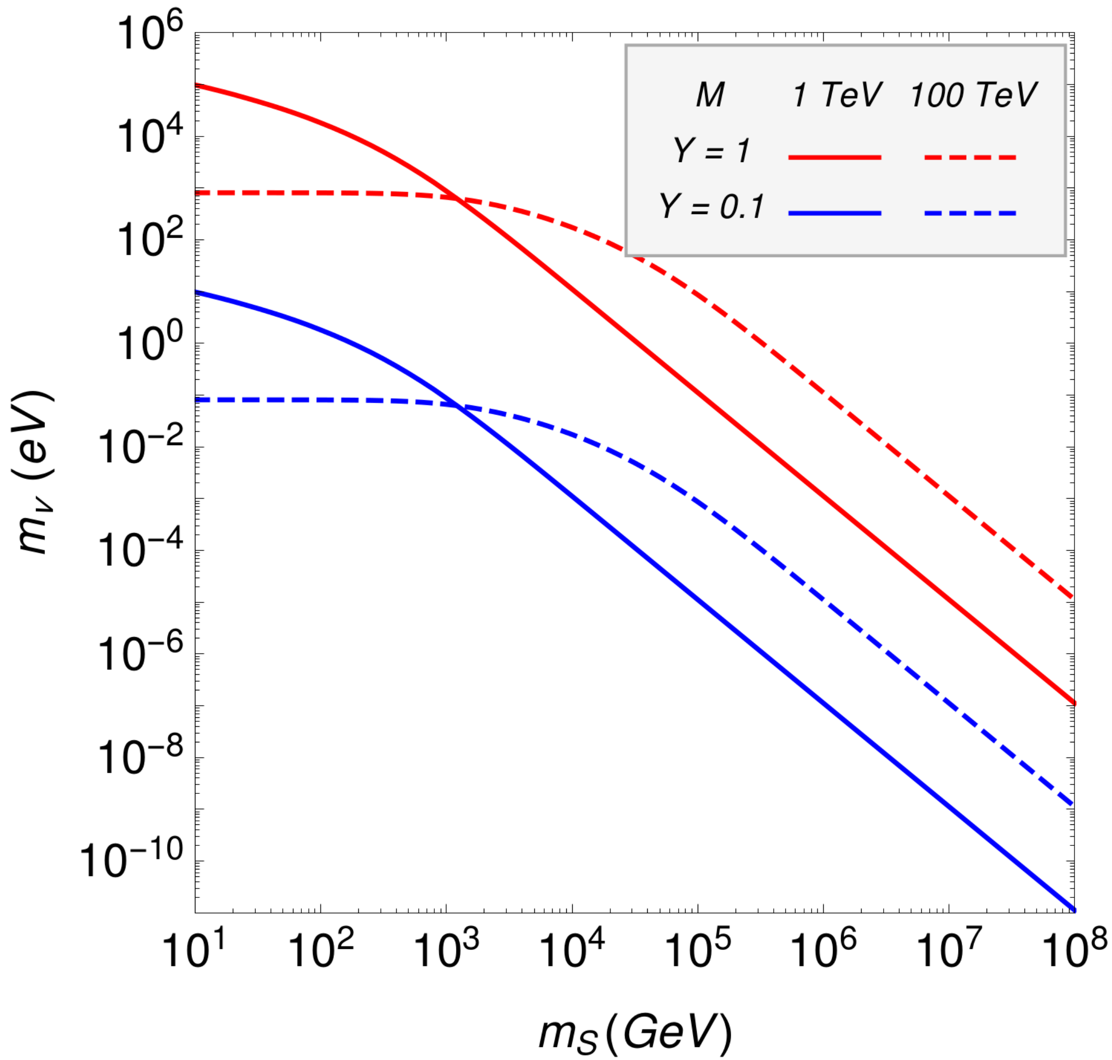
Some examples of models



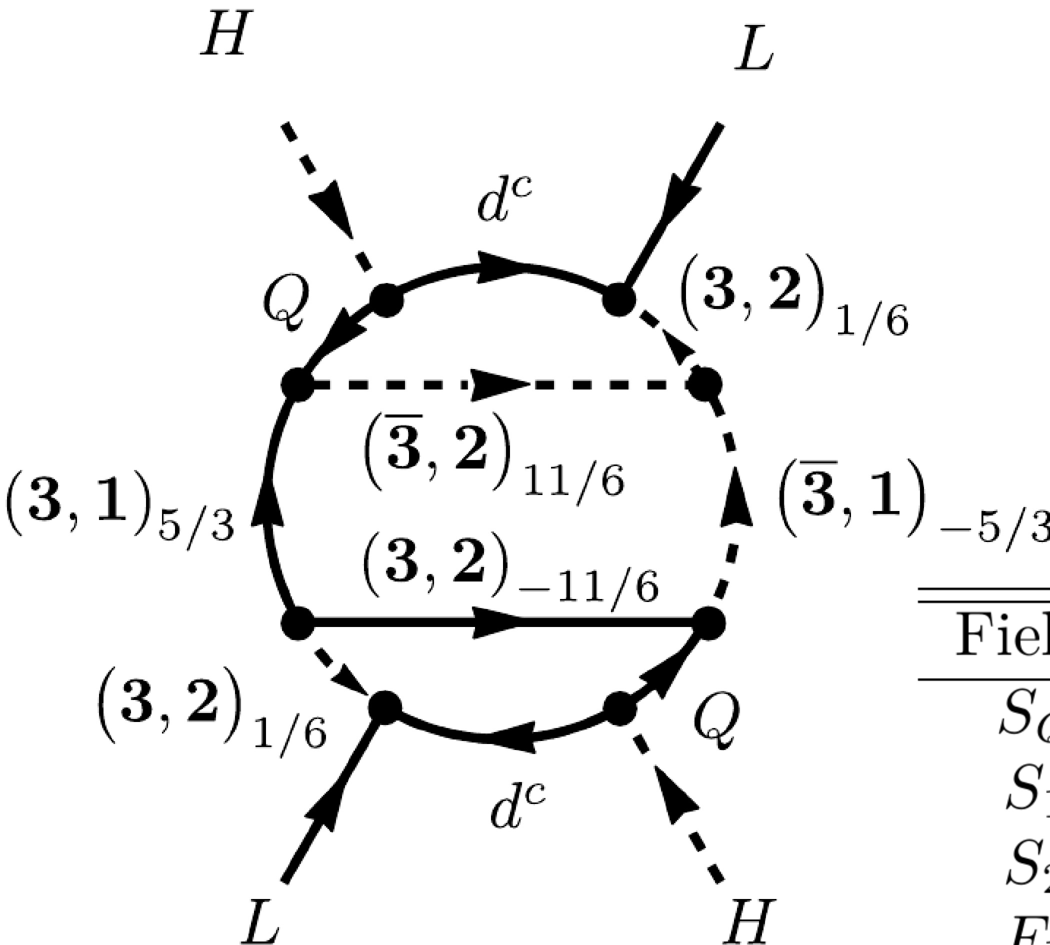
Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
S_1	1	1	1
S_2	1	1	3
F	1	1	2

Some examples of models



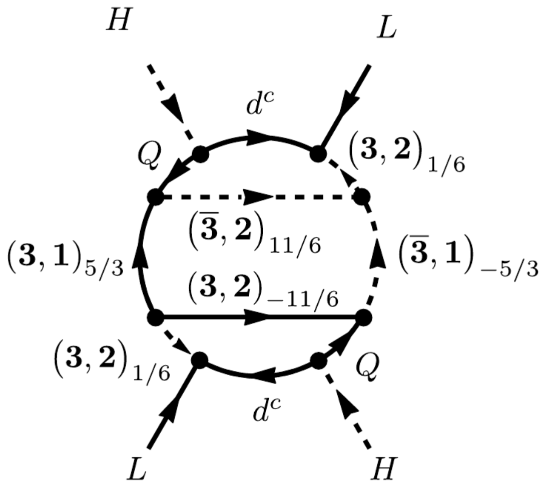


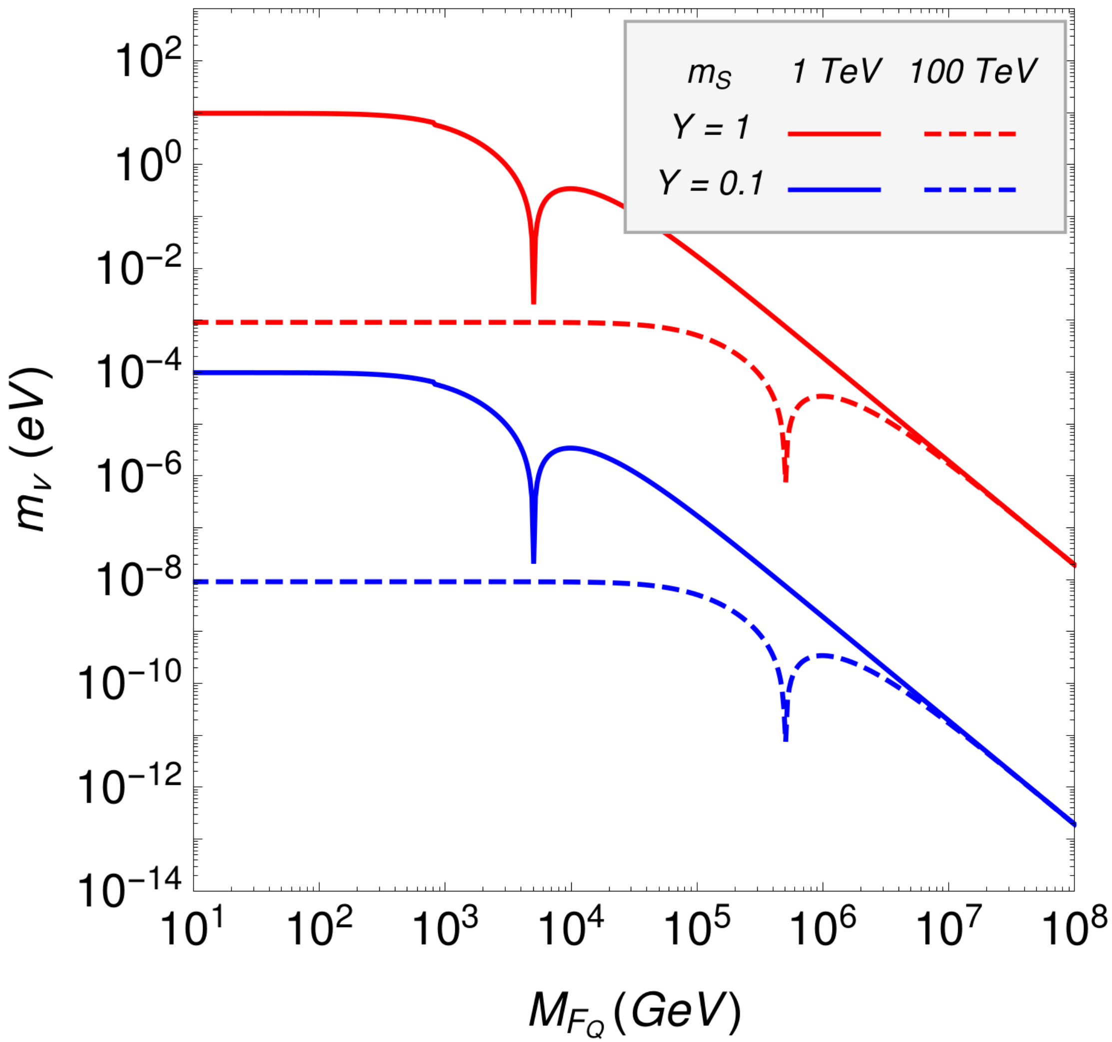
Some examples of models



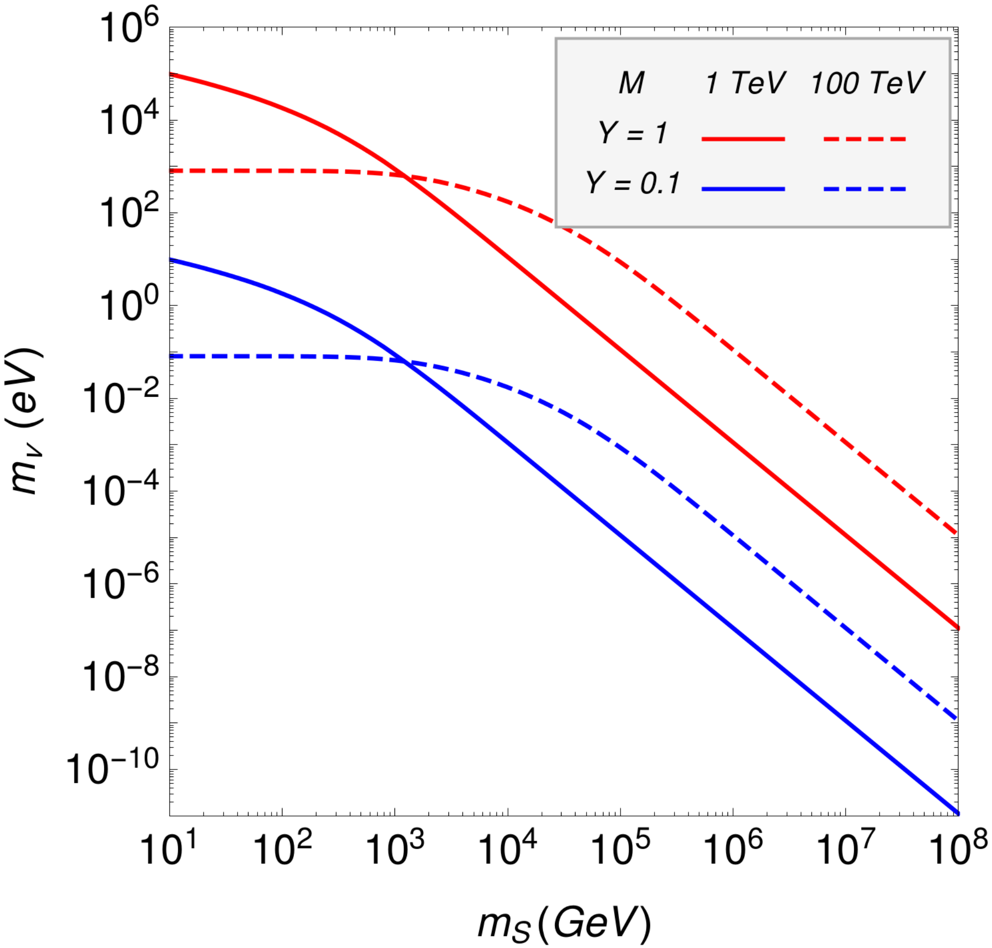
Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
S_Q	3	2	1/6
S_1	3	1	5/3
S_2	3	2	-11/6
F_1	3	1	5/3
F_2	3	2	-11/6

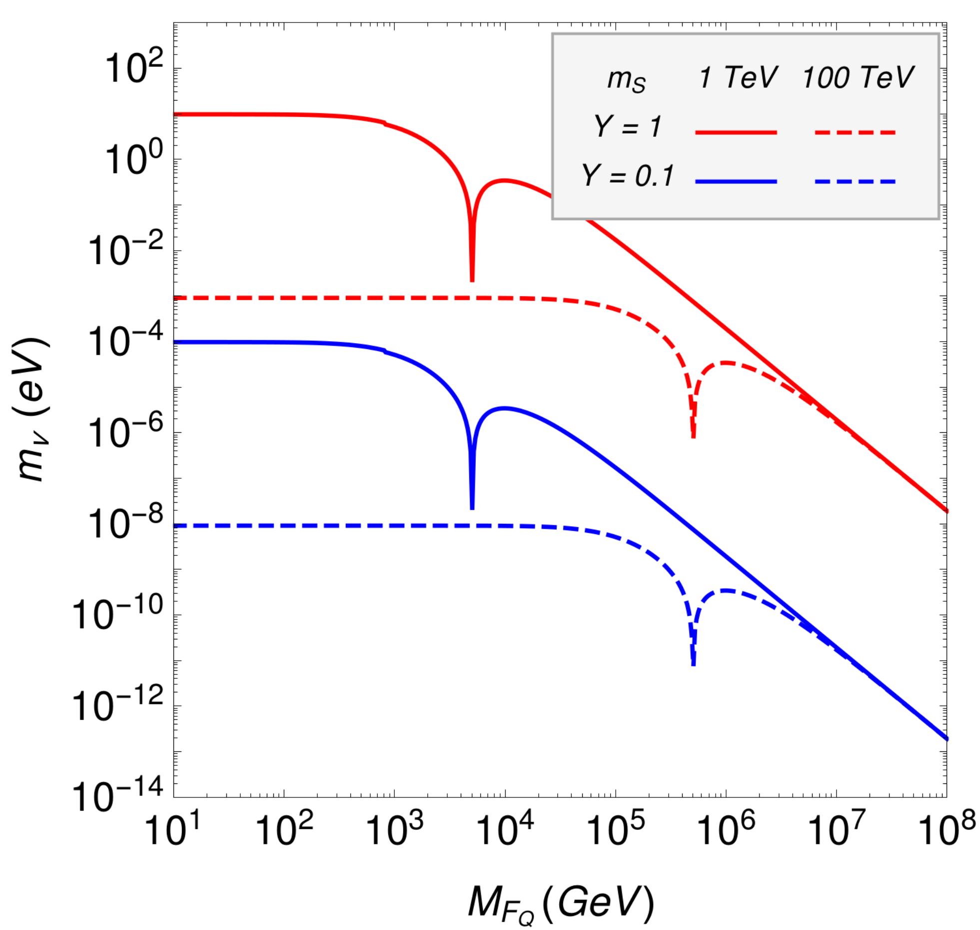
Some examples of models





Some examples of models





Conclusions

- We have found **73 topologies** that can generate genuine models in two classes, depending if they need a special combination of fields to be genuine.
    - 44 normal genuine topologies (228 diagrams + 66 diagrams)
    - 29 special genuine topologies (80 diagrams)
- We have found that every diagram fall into a set of **30 mass diagrams** after EWSB that they depend on five master integral.
- We have have computed two concrete examples to show the typical parameter range of these models.
    - For order (1 - 10^3) TeV d=5 3-loop models can give a **good fit to data**.
    - Interesting and partially **testable** in future colliders and LNV searches.

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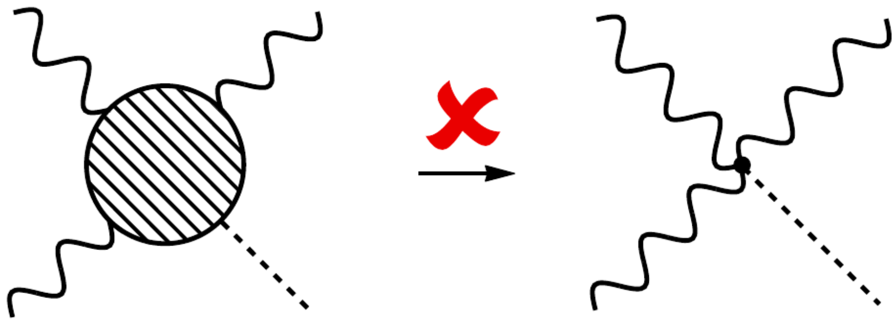
THANK YOU!

More?

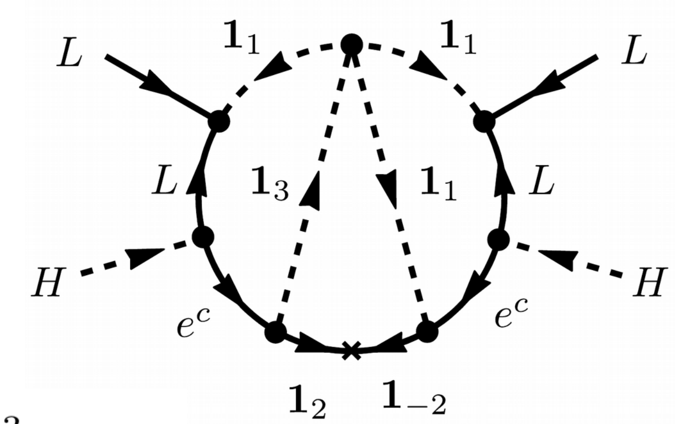
Topologies, Diagrams and Models

- We are considering only diagrams with **scalars** and **fermions**. Adding vectors:



(non-renormalizable)

Some examples of models

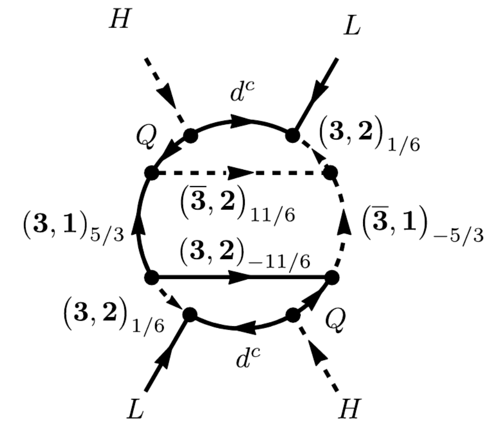


$$\begin{aligned}\mathscr{L} = \mathscr{L}_{SM} \quad &+ \quad Y_1 \overline{L^c} L S_1 + Y_2 \overline{F} e^c S_1 + Y_3 F e^c S_2^\dagger + \lambda_S S_2 (S_1^\dagger)^3 + h.c.\\ &+ \quad m_1^2 S_1^\dagger S_1 + m_2^2 S_2^\dagger S_2 + M \overline{F} F.\end{aligned}$$

$$(M_\nu)_{\alpha\beta} = -\frac{3!}{(16\pi^2)^3} \lambda_S \frac{m_\tau^2}{M} \left[(Y_1)_{\alpha\tau}(Y_2)_\tau (Y_3)_\tau (Y_1)_{\tau\beta} \; + \; (\alpha \leftrightarrow \beta)\right] \; F_{loop}(x_1, x_2)\,.$$

$$F_{loop}(x_1,x_2) = \iiint\limits_{(k_1,k_2,k_3)} \frac{1}{[k_1^2][k_1^2 - x_1][k_2^2][k_2^2 - x_1][k_3^2 - 1][(k_2-k_3)^2 - x_1][(k_3-k_1)^2 - x_2]}.$$

Some examples of models



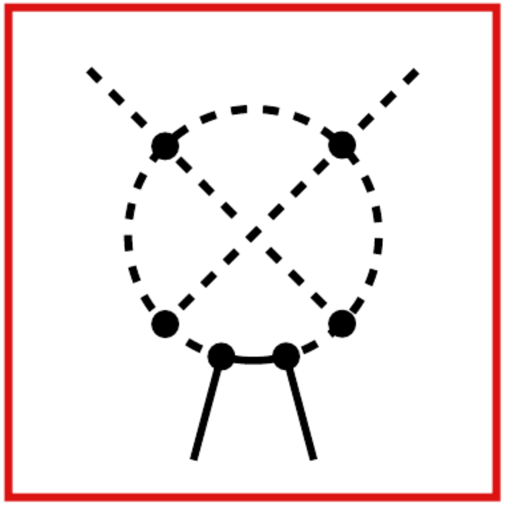
$$
\begin{aligned}
\mathscr{L} = \mathscr{L}_{SM} \quad &+ \quad Y_1 L d^c S_Q + Y_2 Q F_1 S_2 + Y_3 Q F_2 S_1 + Y_4 \overline{F_1}\,\overline{F_2} S_Q^\dagger + \mu_S S_Q^\dagger S_1^\dagger S_2^\dagger + h.c. \\
&+ \quad M_{F_1} \overline{F_1} F_1 + M_{F_2} \overline{F_2} F_2 + m_{S_Q}^2 S_Q^\dagger S_Q + m_{S_1}^2 S_1^\dagger S_1 + m_{S_2}^2 S_2^\dagger S_2 .
\end{aligned}
$$

$$
\begin{aligned}
(M_\nu)_{\alpha\beta} = \quad & - \quad \frac{12\mu_S}{(16\pi^2)^3} \frac{m_b^2}{m_{S_Q}^2} \left[(Y_1)_{\alpha b} (Y_2)_b (Y_4) (Y_3)_b (Y_1)_{b\beta} \; + \; (\alpha \leftrightarrow \beta) \right] \\
& \times \quad \left[F_L(x_1, x_2, x_3, x_4) + F_R(x_1, x_2, x_3, x_4) \right]
\end{aligned}
$$

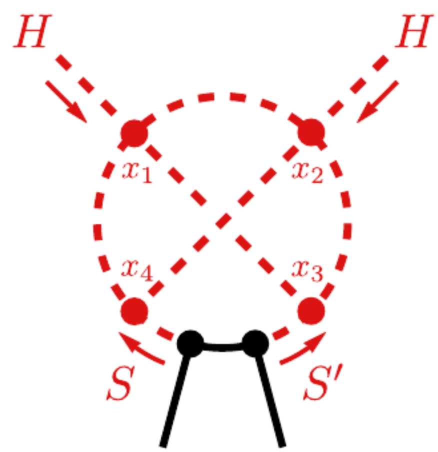
$$
\begin{aligned}
F_L(x_1,x_2,x_3,x_4) \quad &= \iiint\limits_{(k_1,k_2,k_3)} \frac{x_1 x_3}{[k_1^2][k_1^2-1][k_2^2][k_2^2-1][k_3^2-x_1][k_3^2-x_2][(k_2-k_3)^2-x_3][(k_3-k_1)^2-x_4]}, \\
F_R(x_1,x_2,x_3,x_4) \quad &= \iiint\limits_{(k_1,k_2,k_3)} \frac{\not{k}_3(\not{k}_2+\not{k}_3)}{[k_1^2][k_1^2-1][k_2^2][k_2^2-1][k_3^2-x_1][k_3^2-x_2][(k_2-k_3)^2-x_3][(k_3-k_1)^2-x_4]},
\end{aligned}
$$

Topologies, Diagrams and Models

Loophole

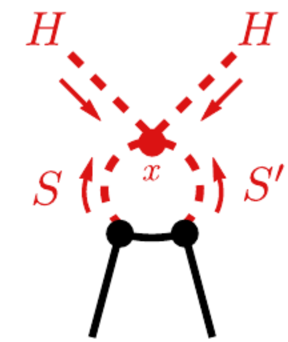


Genuine diagram
under special conditions



$$H(x_1)H(x_2)S(x_3)S'(x_4) \neq 0$$
$$H(x)H(x)S(x)S'(x) = 0$$

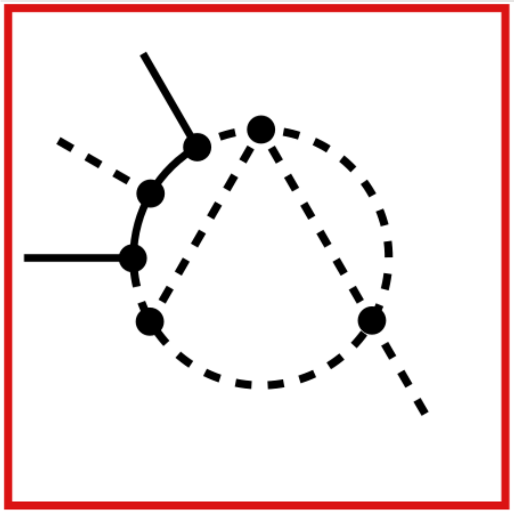
otherwise ...



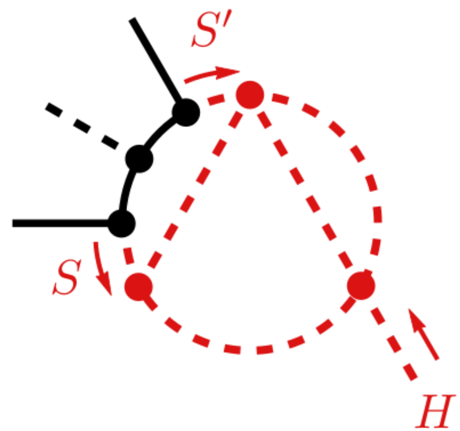
S has to be the SM Higgs, while S' is a doublet with hypercharge -2/3, or viceversa.

Topologies, Diagrams and Models

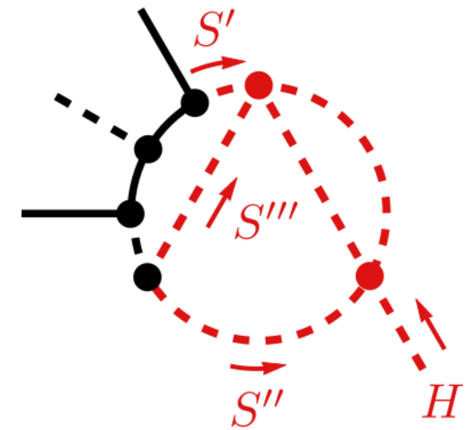
Loophole



Genuine diagram
under special conditions



$$S(x_1)S'(x_2)H(x_3) \neq 0$$
$$S(x)S'(x)H(x) = 0$$



$$S'(x_1)S''(x_2)S'''(x_1)H(x_2) \neq 0$$
$$S'(x)S''(x)S'''(x)H(x) = 0$$

The scalars should be:

- S = (1,1,-1)
- S’ = H
- S’’ = (1,1,y)
- S’’’ = (1,1,-1-y)