University of Basel
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Towards minimal Flavor model via CP violation

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Kang, Simizu, Takagi, Takahashi, TM : arXiv: 1804.10468

Outline of my talk

1 Introduction

2 Neutrino mixing and Flavor symmetry

3 Minimal flavor model with A_4

4 Summary

1 Introduction

In the beginning of 21st century, neutrino data indicated $\sin^2\theta_{12} \sim 1/3$, $\sin^2\theta_{23} \sim 1/2$.

Harrison, Perkins, Scott (2002) proposed **Tri-bimaximal Mixing of Neutrino flavors.**

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Tri-bimaximal Mixing (TBM) is realized by the mass matrix

$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

in the diagonal basis of charged leptons.

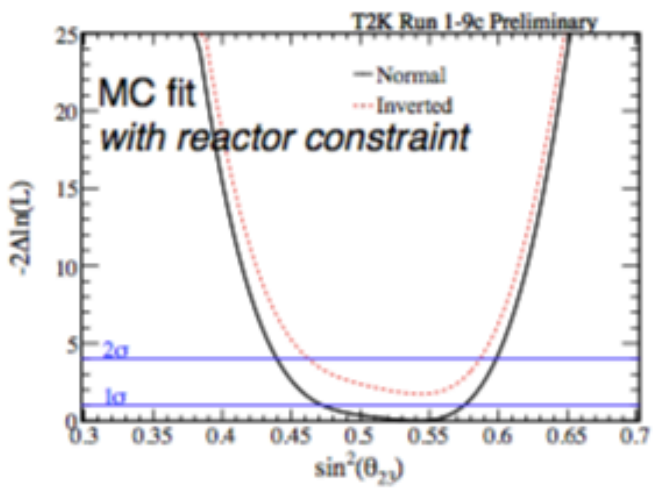
A_4 symmetric

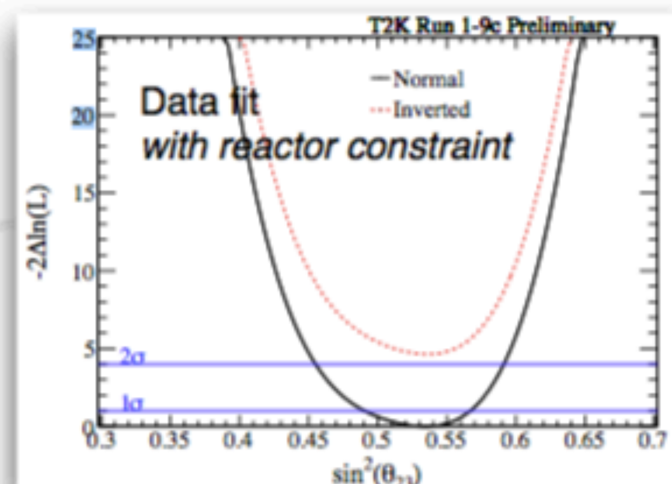
E. Ma, G. Rajasekaran (2001)

Integer (inter-family related) matrix elements suggest Non-Abelian Discrete Flavor Symmetry.

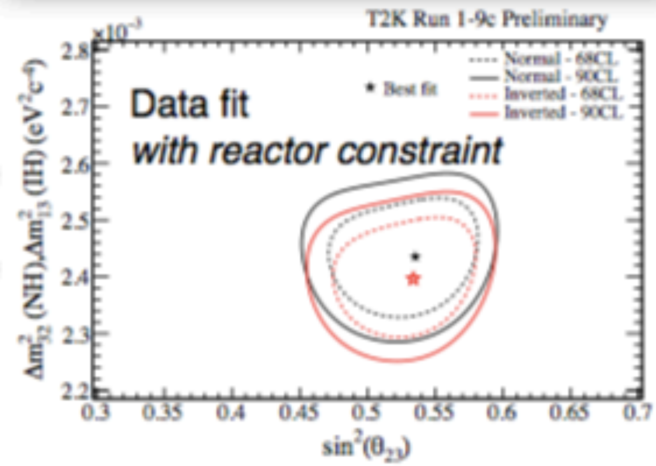
Neutrino2018 @ Heidelberg

Atmospheric sector: θ_{23}, $\Delta m^2_{32(1)}$

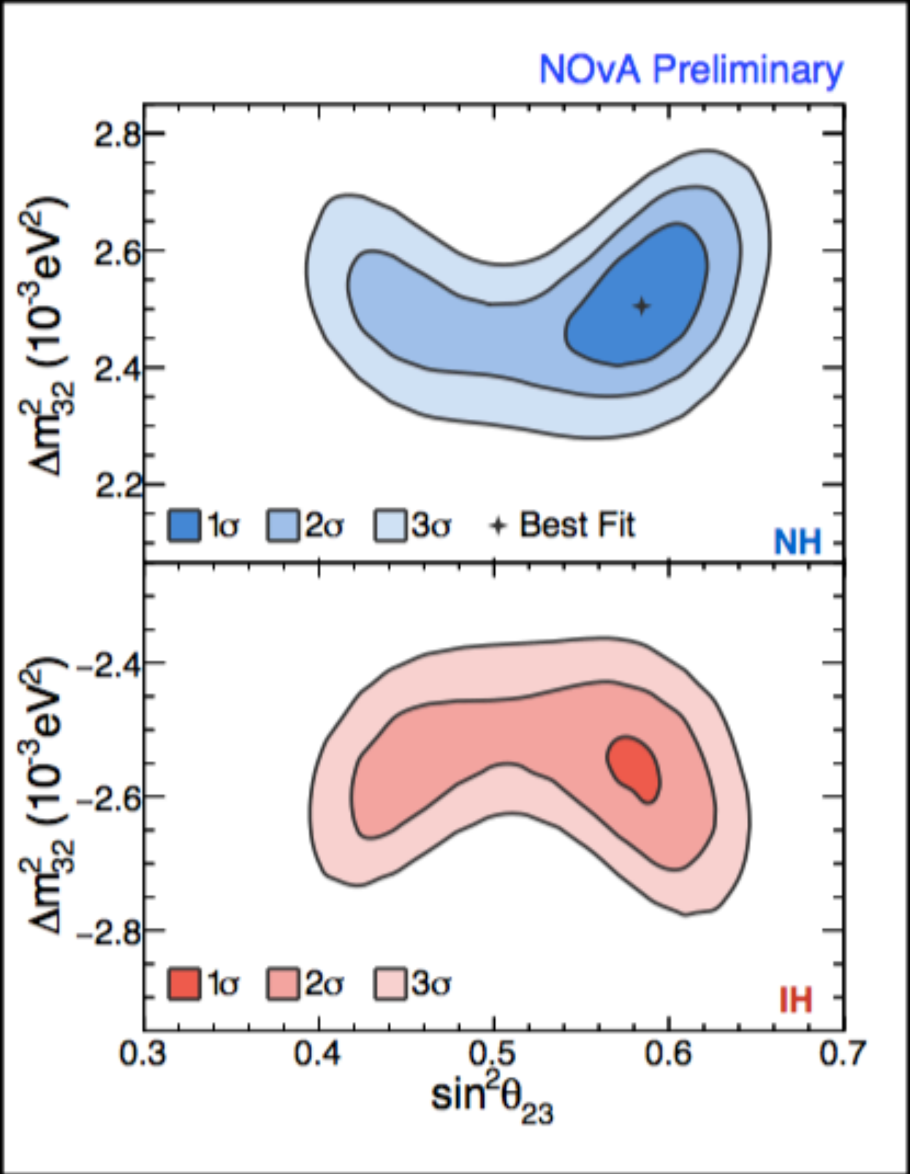


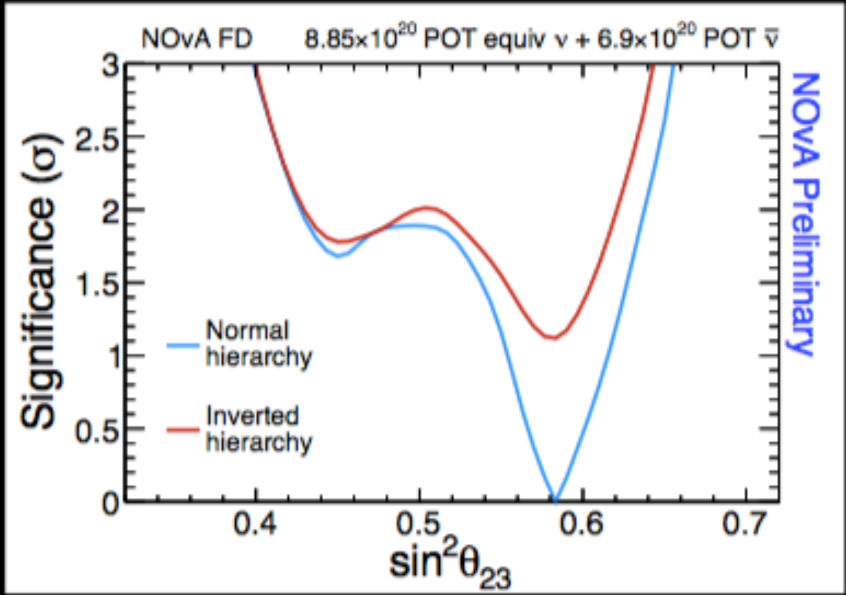


	NH	IH
$\sin^2\theta_{23}$	$0.536^{+0.031}_{-0.046}$	$0.536^{+0.031}_{-0.041}$
$\lvert\Delta m^2\rvert$	2.434 ± 0.064	$2.410^{+0.062}_{-0.063}$



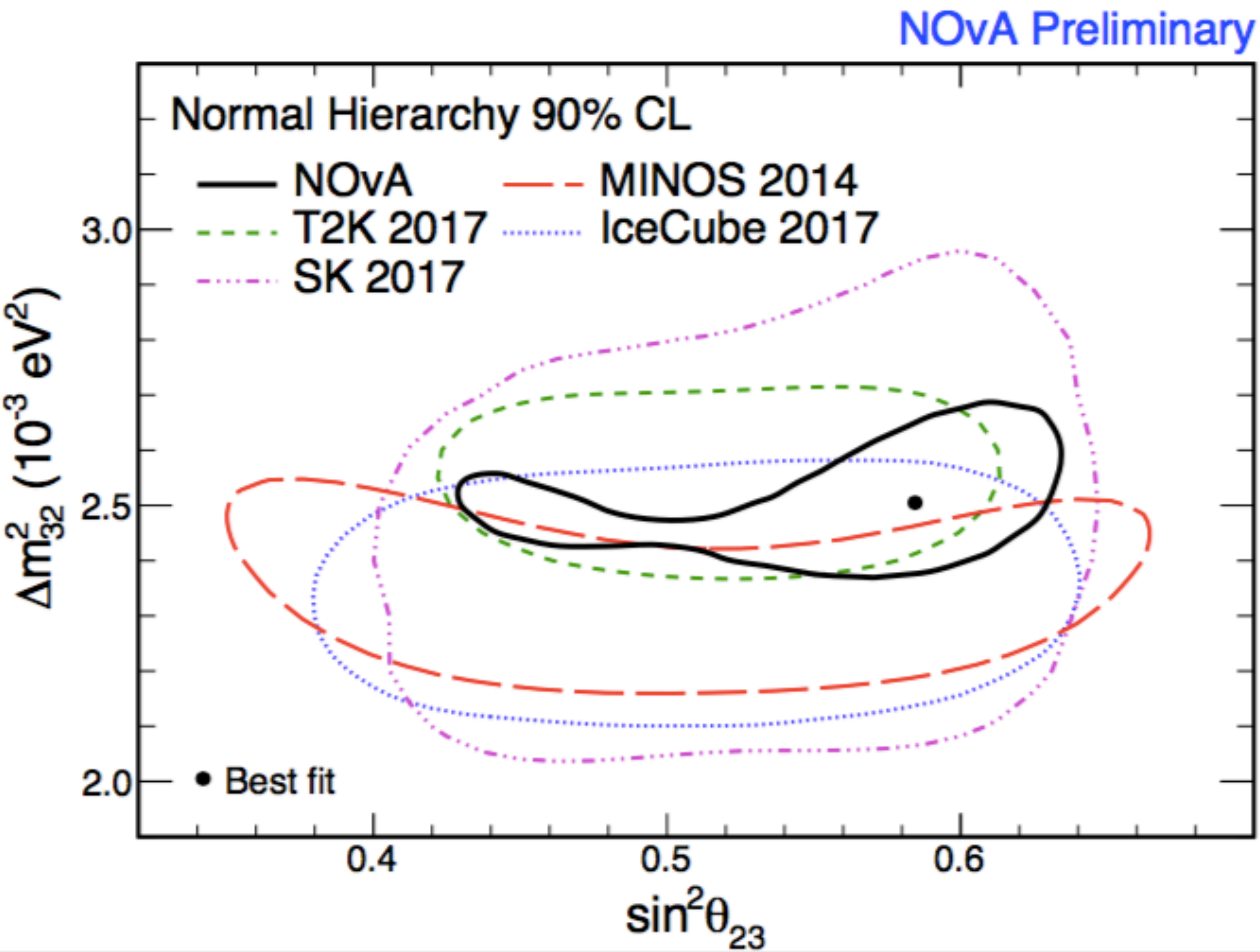
ALLOWED OSCILLATION PARAMETERS





- Best fit:
  Normal Hierarchy
  $\sin^2\theta_{23} = 0.58 \pm 0.03$ (UO)
  $\Delta m^2_{32} = (2.51^{+0.12}_{-0.08}) \cdot 10^{-3}\ eV^2$

Prefer non-maximal at 1.8σ
Exclude LO at similar level



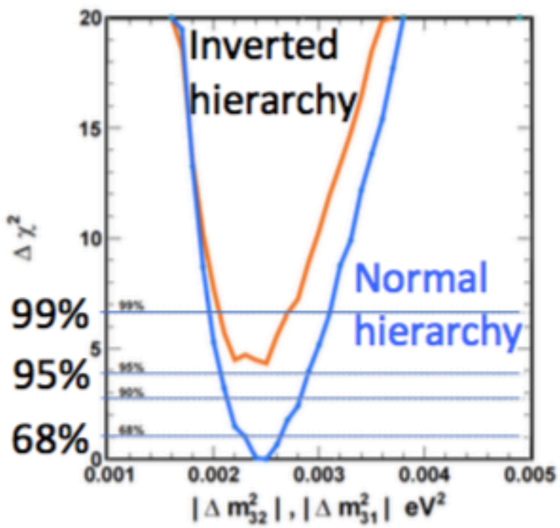
NOvA Preliminary
Normal Hierarchy 90% CL
NOvA
MINOS 2014
T2K 2017
IceCube 2017
SK 2017
Best fit
3.0
2.5
2.0
0.4
0.5
0.6
Δm^2_{32} (10^{-3} eV^2)
$\sin^2\theta_{23}$

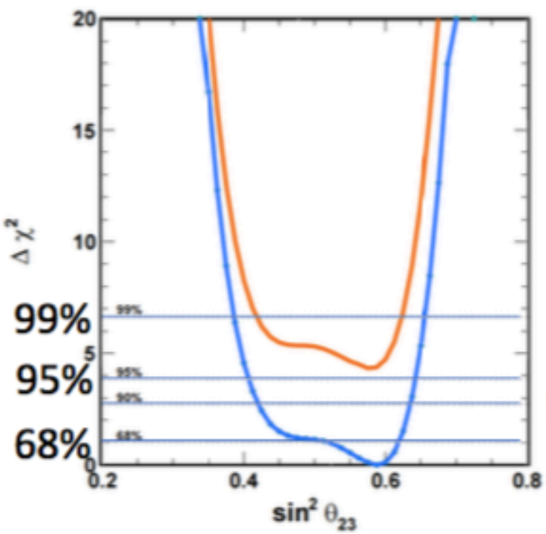
Determination of ν oscillation parameters

SK-I to SK-IV, 5326 days (2519 days from SK-IV), 328 kt·yr

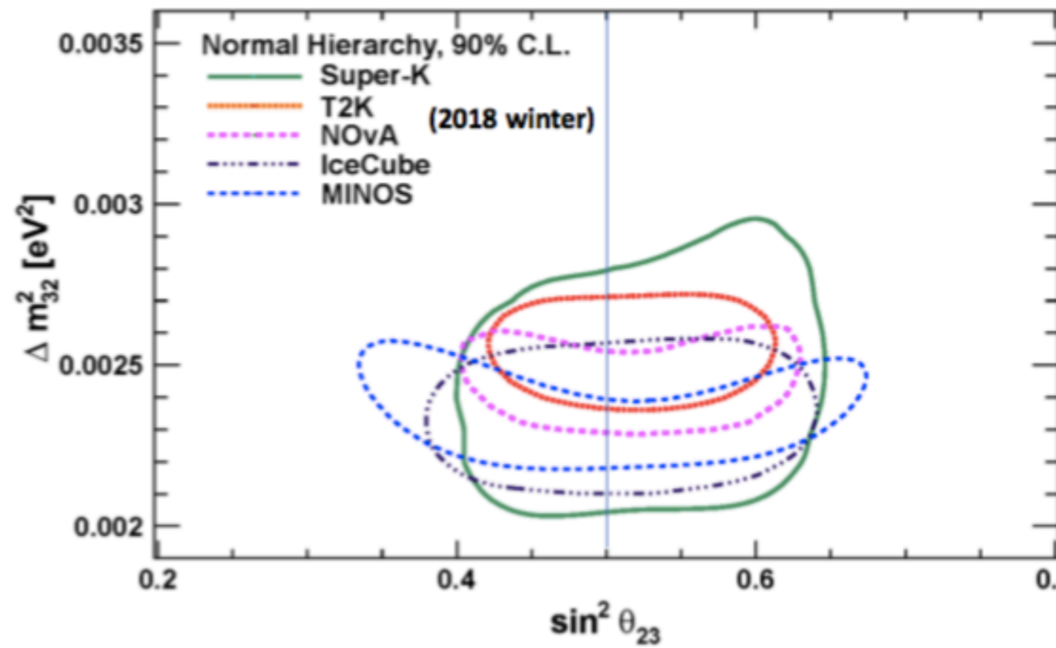
Phys. Rev. D 97, 072001 (2018)

$$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\text{ev}^2,$$
$$\sin^2\theta_{12} = 0.304 \pm 0.014,$$
$$\sin^2\theta_{13} = 0.0219 \pm 0.012$$





90%



$$|\Delta m^2_{32}| = 2.50^{+0.13}_{-0.20} \times 10^{-3}\text{eV}^2$$
$$\sin^2\theta_{23} = 0.588 \pm^{0.031}_{0.067}$$

$(\chi^2_{NH,min} - \chi^2_{IH,min} = -4.34)$

Summary

- Daya Bay is releasing three new results this summer:

new oscillation results with 1958 days →

$$\sin^2 2\theta_{13} = 0.0856 \pm 0.0029$$
$$|\Delta m^2_{ee}| = (2.52 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$$
$$\Delta m^2_{32} = (2.47 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2\ \mathrm{(NH)}$$

Articles in preparation

absolute reactor antineutrino flux (wrt Huber+Mueller) with 1230 days →

$$R_{\mathrm{data/pred}} = 0.952 \pm 0.014(\mathrm{exp.}) \pm 0.023(\mathrm{model})$$

also a search for a time-varying electron antineutrino signal.

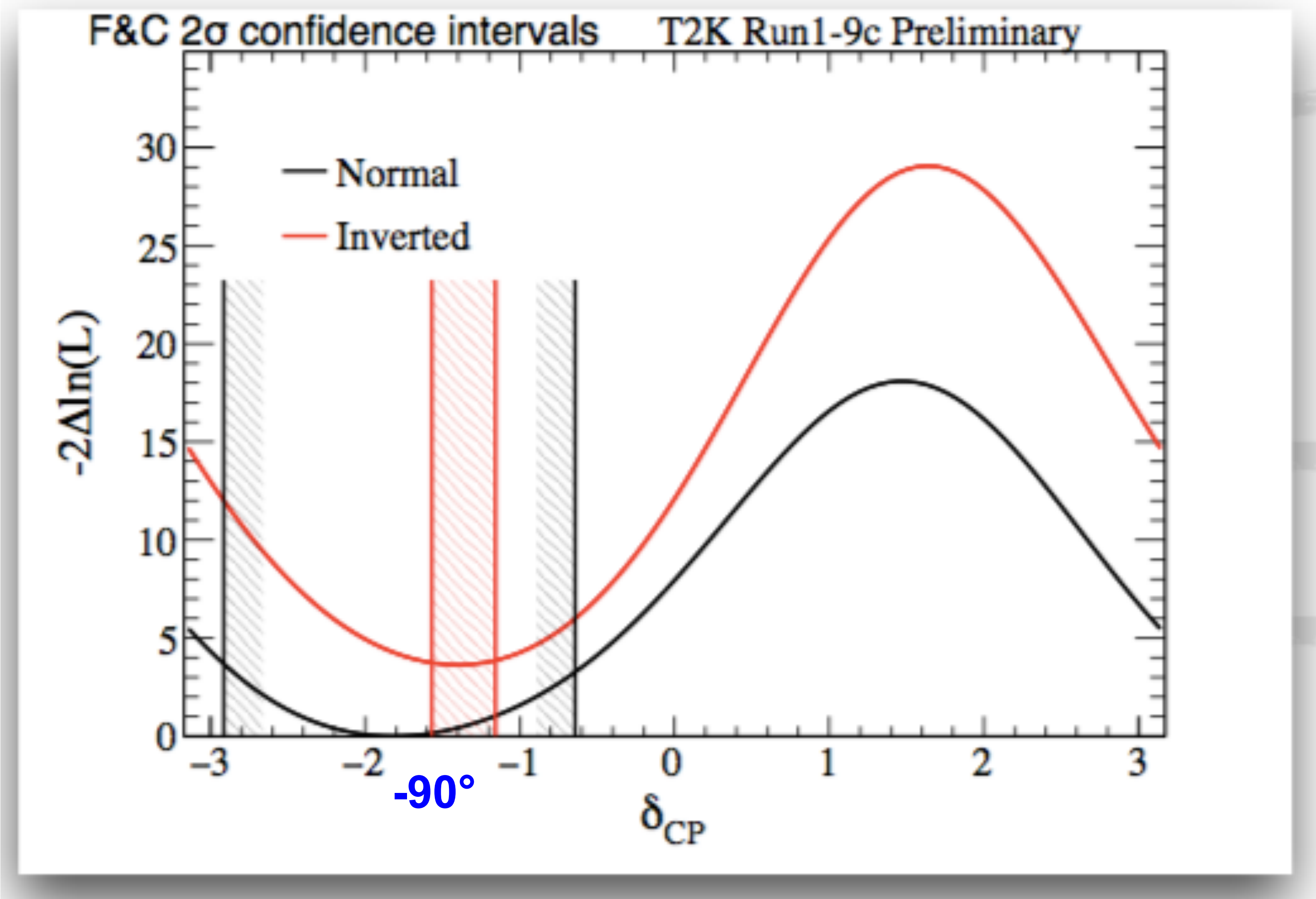
- We also have many other recent results in other areas

We encourage you to look at the 9 posters from Daya Bay in this conference

- Much work is going into better understanding and improving our systematics, given the statistical precision we have achieved with a > 5 year data set

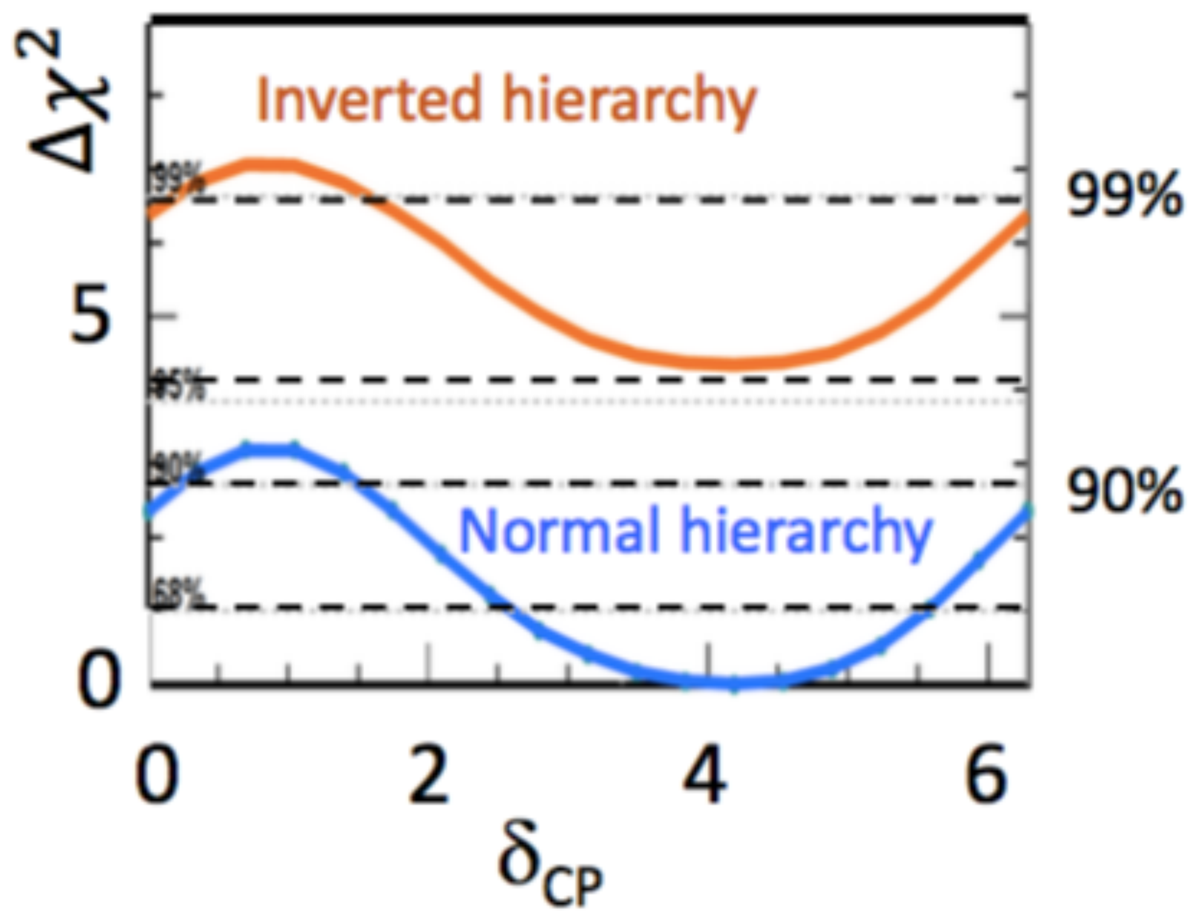
- Future looks bright ahead with ~2.5 more years of data taking, as well as many new and improved results in the works

DATA FIT with reactor constraint



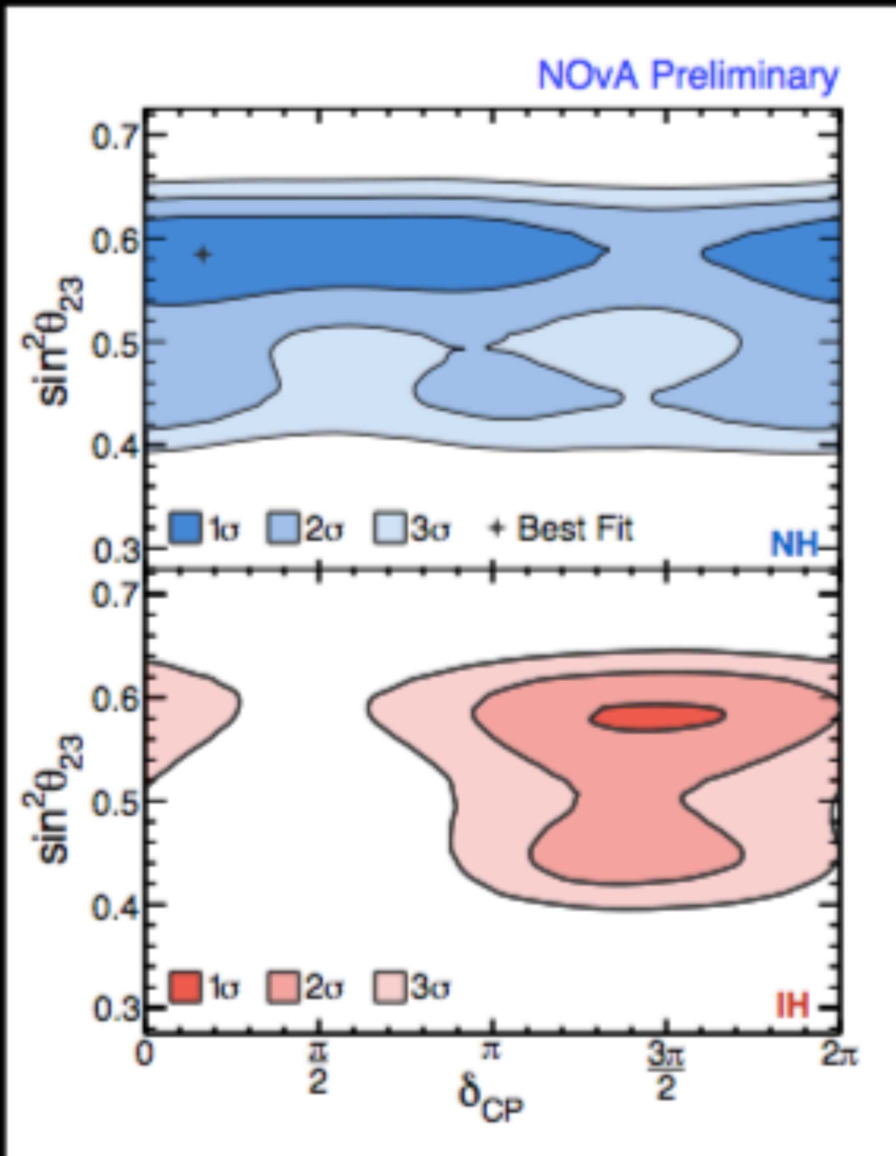
- **CP conserving values of δ_{CP} lie outside 2σ region.**

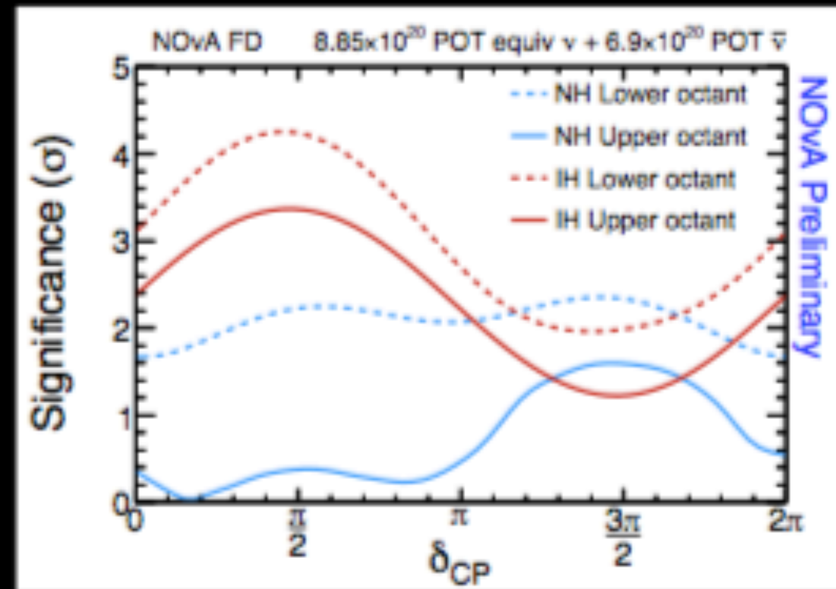
CP violation parameter δ_{CP}



SK I - IV

ALLOWED OSCILLATION PARAMETERS





- Best fit: Normal Hierarchy
  δ_{CP}= 0.17π
  $\sin^2\theta_{23}$ = 0.58±0.03 (UO)
  $\Delta m^2_{32} = (2.51^{+0.12}_{-0.08})\cdot 10^{-3}$ eV^2

Prefer NH by 1.8σ
Exclude δ=π/2 in the IH at > 3σ

If θ_{23} is rather less than 45°
it could be related neutrino masses.

For example,

$$\sin^2\theta_{23} \simeq \sqrt[4]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.40 \sim 0.43$$

FTY(2003), FSTY(2012)

Just like GST relation

$$M_{\rm d} = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \Rightarrow \theta_{12} \simeq \sqrt{\frac{m_d}{m_s}}$$

GST 1968 Weinberg 1977

However, the closer θ_{23} =45° or > 45°
the more likely that some symmetry/structure behind it.

Also the closer δ_{CP} =-90° the more likely
that some symmetry/structure behind it.

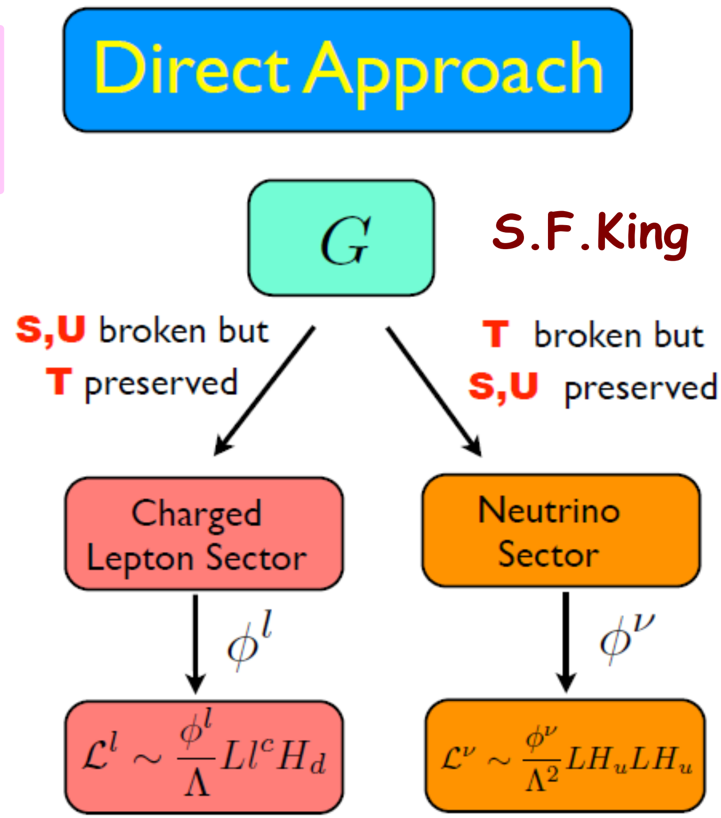
2 Neutrino mixing and Flavor Symmetry

Footprint of the non-Abelian discrete symmetry is expected to be seen in the neutrino mixing matrix, which is the imprint of generators of finite groups.

Generators of G (S,T,U) deterimine the flavor mixing directly.

Suppose group G for flavors at high energy.

At low energy, different subgroups of G are preserved in Yukawa sectors of Neutrinos and Charged leptons, respectively.



Consider the case of A_4 flavor symmetry:

A_4 has subgroups:

three Z_2, four Z_3, one $Z_2 \times Z_2$ (klein four-group)

$S^2=T^3=(ST)^3=1$

Z_2: {1,S}, {1,T^2ST}, {1,TST^2}
Z_3: {1,T,T^2}, {1,ST,T^2S}, {1,TS, ST^2}, {1,STS,ST^2S}
K_4: {1,S,T^2ST,TST^2}

Suppose A_4 is spontaneously broken to one of subgroups:
Neutrino sector preserves Z_2: {1,S}
Charged lepton sector preserves Z_3: {1,T,T^2}

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

⇩

$$[S, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

Mixing matrices diagonalise m^ν_{LL}, $Y_e Y_e^\dagger$ also diagonalize S and T, respectively !

For the triplet, the representations are given as

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$V_\nu^T S V_\nu = \mathrm{diag}(-1, 1, -1)$$

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Independent of mass eigenvalues !

Freedom of the rotation between 1st and 3rd column because a column corresponds to a mass eigenvalue.

Then, we obtain PMNS matrix.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}-s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

$c = \cos\theta \quad s = \sin\theta e^{-i\sigma}$

CP violating phase appears accidentally.

Tri-maximal mixing : so called TM_2

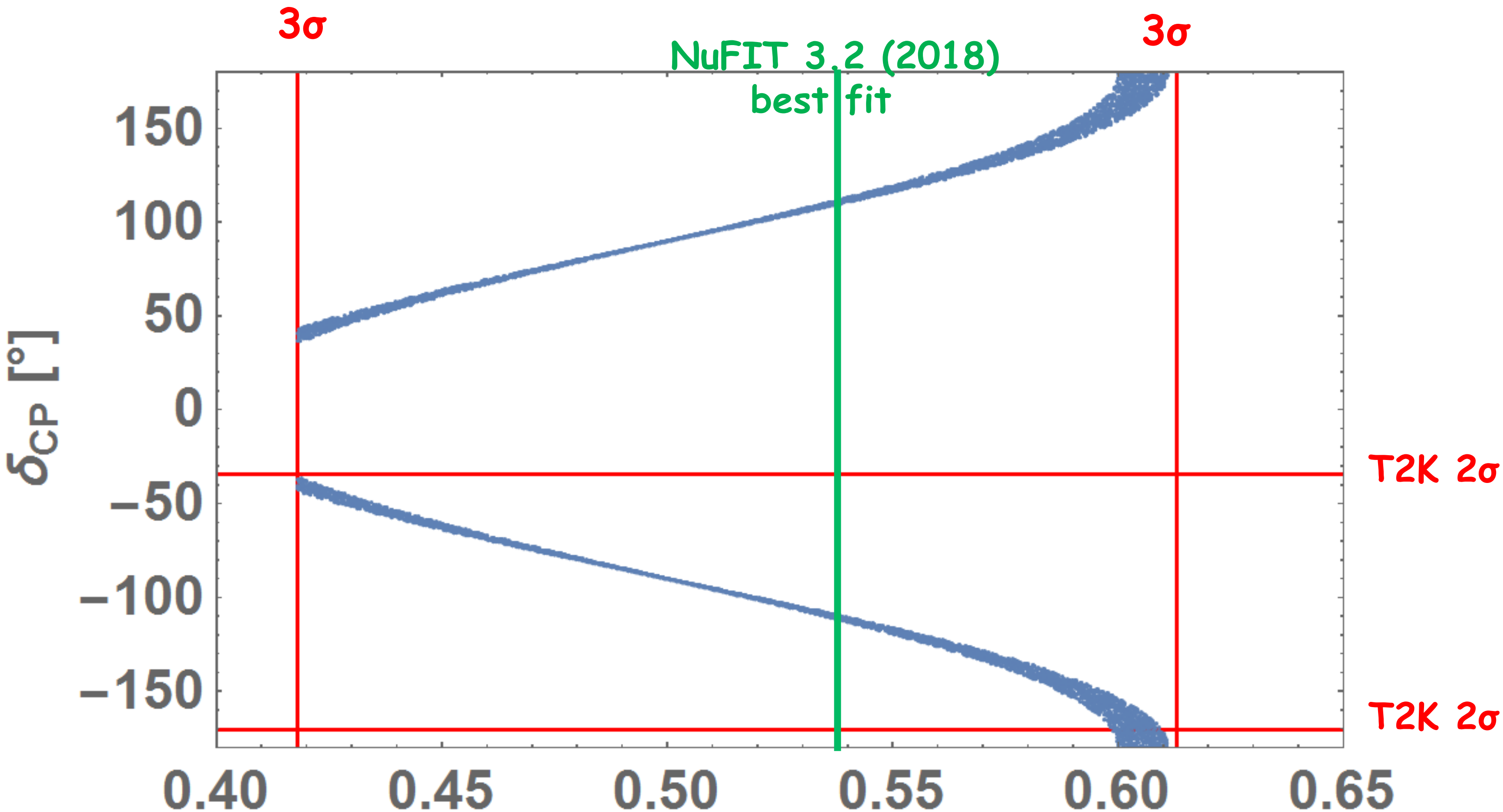
θ and σ are not fixed.

Since two parameters appear, there are two relations among mixing angles and CP violating phase.

Mixing sum rules

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1-\frac{5}{4}\sin^2\theta_{13}\right)$$

Prediction CP violating phase by using sum rules.



3σ: 0.272-0.346

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3} \ ,$$

$$\cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1 - \frac{5}{4}\sin^2\theta_{13}\right)$$

Direct Approach

☆ Flavor Structure of Yukawa Interactions is directly related with the Generators of Finite groups. Predictions are clear.

★ One cannot discuss the related phenomena without Lagrangian.
Leptogenesis, Quark CP violation, Lepton flavor violation

Model building is required.

☆ Introduce flavons (gauge singlet scalars) to discuss dynamics of flavors, so write down Lagrangian.
Flavor symmetry is broken spontaneously.
Also investigate the vacuum structure in the broken symmetry.

★ The number of parameters of Yukawa interactions increases.
Predictivity of models is less than the Direct approach.

3 Minimal flavor model with A_4

Flavor symmetry G is broken by flavon (SU_2 singlet scalors)VEV's.
Flavor symmetry controls Yukaw couplings
among leptons and flavons with special vacuum alignments.

A_4 model: E. Ma, G. Rajasekaran (2001)
K.S.Babu, E.Ma, J.W.F.Valle(2004) M.Hirsch et al(2004)

A_4 group is the minimal one including a triplet of ir.r.

	Leptons	flavons	
A_4 triplets	(L_e, L_μ, L_τ)	$\phi_\nu(\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$	couple to neutrino sector
		$\phi_E(\phi_{E1}, \phi_{E2}, \phi_{E3})$	couple to charged lepton sector
A_4 singlets	$e_R : \mathbf{1}$ $\mu_R : \mathbf{1}''$ $\tau_R : \mathbf{1}'$		

Mass matrices are given by A_4 invariant couplings with flavons

$$\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{3}_{flavon} \rightarrow \mathbf{1}\ , \qquad \mathbf{3}_L \times \mathbf{1}_R^{(')('')} \times \mathbf{3}_{flavon} \rightarrow \mathbf{1}$$

Majorana neutrino

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

Flavor symmetry G is broken by VEV of flavons

$3_L \times 3_L \times 3_{flavon} \to 1$ $\qquad$ $3_L \times 1_R (1_R', 1_R'') \times 3_{flavon} \to 1$

$$m_{\nu LL} \sim y \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} \qquad m_E \sim \begin{pmatrix} y_e\langle\phi_{E1}\rangle & y_e\langle\phi_{E3}\rangle & y_e\langle\phi_{E2}\rangle \\ y_\mu\langle\phi_{E2}\rangle & y_\mu\langle\phi_{E1}\rangle & y_\mu\langle\phi_{E3}\rangle \\ y_\tau\langle\phi_{E3}\rangle & y_\tau\langle\phi_{E2}\rangle & y_\tau\langle\phi_{E1}\rangle \end{pmatrix}$$

Residual symmetries lead to specific Vacuum Alingnments

Z_2 (1, S) in neutrinos $\quad \langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$

Z_3 (1, T, T^2) in charged leptons $\quad \langle\phi_{E2}\rangle = \langle\phi_{E3}\rangle = 0$

$$\Rightarrow \quad \langle\phi_\nu\rangle \sim (1,1,1)^T, \qquad \langle\phi_E\rangle \sim (1,0,0)^T$$

$$S\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

m_E is a diagonal matrix, on the other hand, $m_{\nu LL}$ is

$$m_{\nu LL} \sim 3y \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} - y \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$$

Rank 2

two generated masses and one massless neutrinos !
(0, 3y, 3y)
Flavor mixing is not fixed !

Z_2 (1,S) is preserved

Adding A_4 singlet flavon $\xi : \mathbf{1}$ ➡ flavor mixing matrix is fixed.

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}} \to \mathbf{1}$

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} + y_2\langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$, which preserves S symmetry.

$$m_{\nu LL} = 3a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Flavor mixing is determined: Tri-bimaximal mixing. $\theta_{13}=0$

$$m_\nu = 3a + b,\ b,\ 3a - b \ \Rightarrow m_{\nu_1} - m_{\nu_3} = 2m_{\nu_2}$$

There appears a Neutrino Mass Sum Rule.

This is a minimal framework of A_4 symmetry predicting mixing angles and masses.

Prototype A_4 flavor model has been modified !

Three directions:

- Another flavon in A_4 flavor model: my talk
- Larger symmetry S_4 ··· : Y.Shimizu, many works
- Another aspect of A_4 **modulei** :
  F. Feruglio, T.Tatsuishi, J. Penedo (S_4)

S_3, A_4, S_4, A_5 are congruence subgroups of the modular symmetry.
Couplings Ys are not constant, but in the modular form.
Flavor structure is determined essentially
by the modular parameter τ without flavons.

A_4 model easily realizes non-vanishing θ_{13} .

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

Add 1' or 1" flavon

LL $\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$

LL $\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}' \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$

LL $\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}'' \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$

ξ $\qquad \xi'$

$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \quad \mathbf{1}'' \times \mathbf{1}' \Rightarrow \mathbf{1}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \quad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \quad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \quad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda} \qquad a = -3b$$

Both normal and inverted mass hierarchies are possible.

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix}V^T_{\text{tri-bi}} \qquad V_{\text{tri-bi}} = \begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

Realization of Tri-maximal mixing: TM_2

3σ: 0.272-0.346

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1-\frac{5}{4}\sin^2\theta_{13}\right)$$

TM_2 may be soon excluded …. $Z_2(1,S)$ is broken by 1' and 1"

Further modification of A_4 Model

Kang, Simizu, Takagi, Takahashi, TM : arXiv: 1804.10468

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

$\xi' \rightarrow \eta$: η, 1”, 1

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 8(2011)

Next-leading couplings

For Charged leptons

$$(\phi_T l)_{\mathbf{1}'(\mathbf{1}'')}\, e^c h_d \eta$$

$$(\phi_T l)_{\mathbf{1}''(\mathbf{1})}\, \mu^c h_d \eta$$

$$(\phi_T l)_{\mathbf{1}(\mathbf{1}')}\, \tau^c h_d \eta$$

Neutrinos for

$$(ll)_{\mathbf{1}'(\mathbf{1}'')} h_u h_u \xi \eta / \Lambda^3$$

Scalar potential

V=V_T+V_S

$$V_T = \sum_i \left|\frac{\partial w_d^T}{\partial \phi_{0i}^T}\right|^2 + h.c.$$
$$= 2\left|-M\phi_{T1} + \lambda\phi_{T2}\tilde{\eta} + \frac{2g}{3}\left(\phi_{T1}^2 - \phi_{T2}\phi_{T3}\right)\right|^2$$
$$+ 2\left|-M\phi_{T3} + \lambda\phi_{T1}\tilde{\eta} + \frac{2g}{3}\left(\phi_{T2}^2 - \phi_{T1}\phi_{T3}\right)\right|^2$$
$$+ 2\left|-M\phi_{T2} + \lambda\phi_{T3}\tilde{\eta} + \frac{2g}{3}\left(\phi_{T3}^2 - \phi_{T1}\phi_{T2}\right)\right|^2$$
$$+ 2\left|-\lambda_1\left(\phi_{T2}\phi_{S2} + \phi_{T1}\phi_{S3} + \phi_{T3}\phi_{S1}\right) + \lambda_2\eta\xi + \lambda_3\eta\tilde{\xi} + \lambda_4\tilde{\eta}\xi + \lambda_5\tilde{\eta}\tilde{\xi}\right|^2$$

$$V_S = \sum \left|\frac{\partial w_d^S}{\partial X}\right|^2 + h.c.$$
$$= 2\left|\frac{2g_1}{3}\left(\phi_{S1}^2 - \phi_{S2}\phi_{S3}\right) + g_2\phi_{S1}\tilde{\xi}\right|^2 + 2\left|\frac{2g_1}{3}\left(\phi_{S2}^2 - \phi_{S1}\phi_{S3}\right) + g_2\phi_{S3}\tilde{\xi}\right|^2$$
$$+ 2\left|\frac{2g_1}{3}\left(\phi_{S3}^2 - \phi_{S1}\phi_{S2}\right) + g_2\phi_{S2}\tilde{\xi}\right|^2$$
$$+ 2\left|-g_3\left(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3}\right) + g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2\right|^2 .$$

Vacuum alignments

$$\langle\phi_T\rangle = v_T(1,0,0), \quad \langle\phi_S\rangle = v_S(1,1,1), \quad \langle\eta\rangle = q, \quad \langle\tilde{\eta}\rangle = 0, \quad \langle\xi\rangle = u, \quad \langle\tilde{\xi}\rangle = 0,$$

$$v_T = \frac{3M}{2g}, \qquad v_S^2 = \frac{g_4}{3g_3}u^2, \qquad q = \frac{\lambda_1 v_T v_S}{\lambda_2 u} = \frac{\lambda_1}{\lambda_2}\sqrt{\frac{g_4}{3g_3}}v_T$$

$\langle\eta\rangle \sim |\langle\Phi_T\rangle|$

VEVs of flavons give Mass matrices

$$M_\ell = v_d \alpha_\ell \begin{pmatrix} y_e \lambda^4 & 0 & y'_\tau \alpha_\eta \\ y'_e \alpha_\eta \lambda^4 & y_\mu \lambda^2 & 0 \\ 0 & y'_\mu \alpha_\eta \lambda^2 & y_\tau \end{pmatrix} \qquad m_\tau = |y_\tau| \alpha_\ell v_d$$

Next-leading couplings of η

$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

a+3b=0

$$a = \frac{y_S \alpha_\nu}{\Lambda} v_u^2, \quad b = -\frac{y_S \alpha_\nu}{3\Lambda} v_u^2, \quad c = \frac{y_\xi \alpha_\xi}{\Lambda} v_u^2, \quad d = \frac{y'_7 \alpha_\xi \alpha_\eta}{\Lambda} v_u^2$$

$$\alpha_\ell \equiv \frac{\langle \phi_T \rangle}{\Lambda} = \frac{v_T}{\Lambda}, \quad \boxed{\alpha_\eta \equiv \frac{\langle \eta \rangle}{\Lambda} = \frac{q}{\Lambda},} \quad \alpha_\nu \equiv \frac{\langle \phi_S \rangle}{\Lambda} = \frac{v_S}{\Lambda}, \quad \alpha_\xi \equiv \frac{\langle \xi \rangle}{\Lambda} = \frac{u}{\Lambda}$$

α_η is expected to be O(0.1) .

λ is FN suppression coefficient ~0.2.

$$U_{\rm PMNS} = U_\ell \, V_{\rm TBM} \, U_\nu^\dagger \, P$$

$$U_\ell^\dagger \simeq \frac{1}{\sqrt{1+\alpha_\eta^{\tau 2}}} \begin{pmatrix} 1 & -\mathcal{O}(\alpha_\eta^2) & \alpha_\eta^\tau e^{i\varphi} \\ \mathcal{O}(\alpha_\eta^2) & \sqrt{1+\alpha_\eta^{\tau 2}} & \mathcal{O}(\alpha_\eta \lambda^4) \\ -\alpha_\eta^\tau e^{-i\varphi} & \mathcal{O}(\alpha_\eta^3) & 1 \end{pmatrix}$$

$$\alpha_\eta^\tau e^{i\varphi} \equiv \frac{y'_\tau}{y_\tau} \alpha_\eta$$

$$V_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_\nu^\dagger = \begin{pmatrix} \cos\theta & 0 & \sin\theta e^{-i\sigma} \\ 0 & 1 & 0 \\ -\sin\theta e^{i\sigma} & 0 & \cos\theta \end{pmatrix}$$

$$\tan 2\theta = \sqrt{3}\, \frac{d\sqrt{a^2\cos^2\phi_d + c^2\sin^2(\phi_c-\phi_d)}}{a(d\cos\phi_d - 2c\cos\phi_c)} \,, \qquad \sigma = -\frac{c\sin(\phi_c-\phi_d)}{a\cos\phi_d} \,.$$

$$\frac{d}{c} = \left|\frac{y'_7}{y_\xi}\right| \alpha_\eta \equiv \alpha_\eta^\nu$$

Both neutrinos and charged leptons have extra (1-3) family rotations from Tri-bimaximal mixing.

Mixing angles

charged lepton

$$\sin\theta_{12} \simeq \frac{1}{\sqrt{1+\alpha_\eta^{\tau 2}}}\frac{1}{\sqrt{3}}\left|1-\alpha_\eta^\tau e^{i\varphi}\right| \ ,$$

$$\sin\theta_{13} \simeq \frac{1}{\sqrt{1+\alpha_\eta^{\tau 2}}}\left|\frac{2}{\sqrt{6}}\sin\theta e^{-i\sigma}-\frac{1}{\sqrt{2}}\alpha_\eta^\tau\cos\theta e^{i\varphi}\right| \ ,$$

$$\sin\theta_{23} \simeq \left|-\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{6}}\sin\theta e^{-i\sigma}\right| \ ,$$

neutrino

$$\alpha_\eta^\tau e^{i\varphi} \equiv \frac{y'_\tau}{y_\tau}\alpha_\eta$$

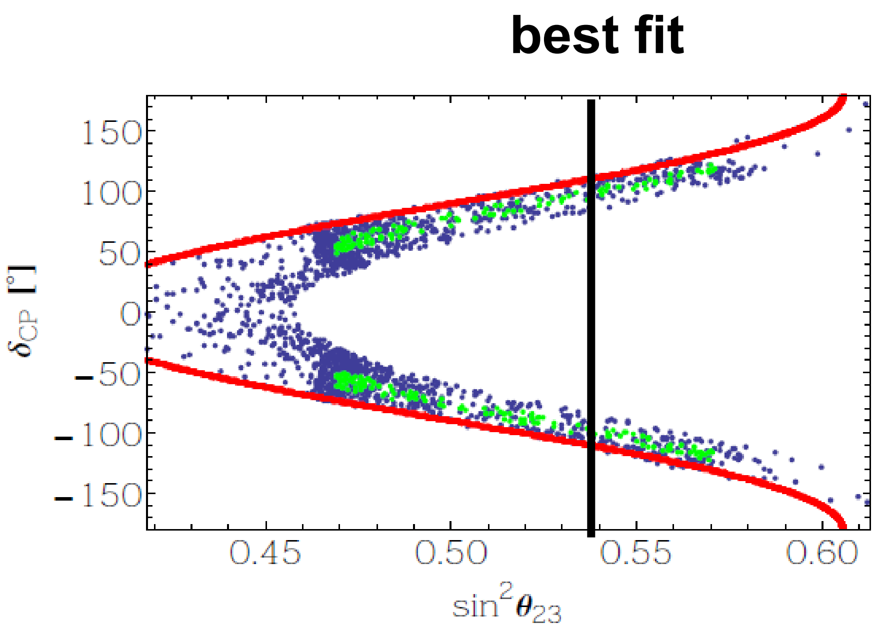
$$\frac{d}{c} = \left|\frac{y'_7}{y_\xi}\right|\alpha_\eta \equiv \alpha_\eta^\nu$$

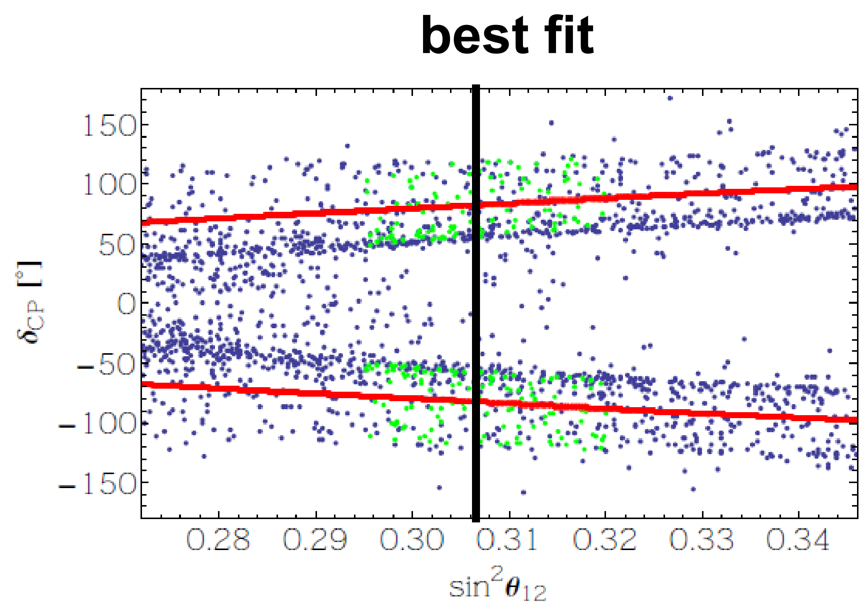
Put $\alpha_\eta = \alpha_\eta^\tau = \alpha_\eta^\nu$

5 parameters of model: $\alpha_\eta \equiv \langle\eta\rangle/\Lambda$, m_l , φ , ϕ_c , ϕ_d

Since $\langle\eta\rangle \sim |\langle\Phi_T\rangle|$ is required in vacuum, $\alpha_\eta \sim \alpha_l = O(0.1)$ is put.

Blue dots: input of 3 σ data
green dots: input of 1 σ data



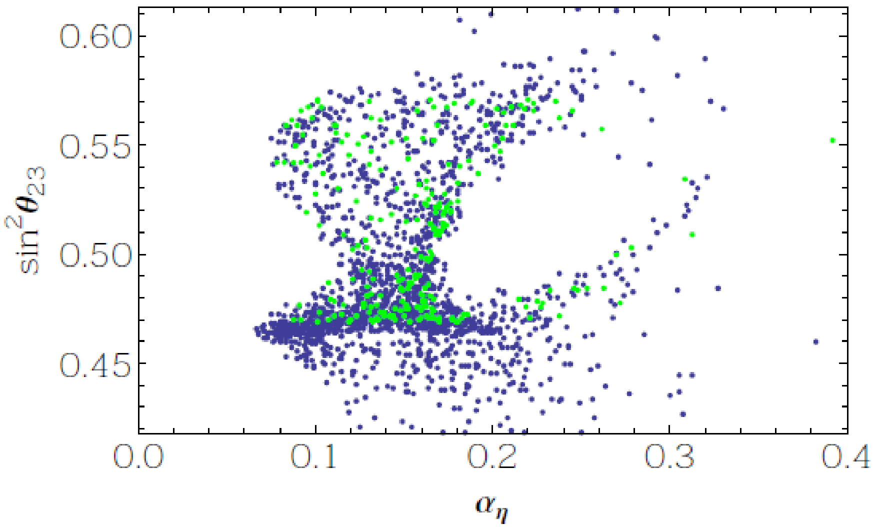


Red curve denotes the case of diagonal charged leptons (TM_2), where $\sin^2\theta_{12}>1/3$

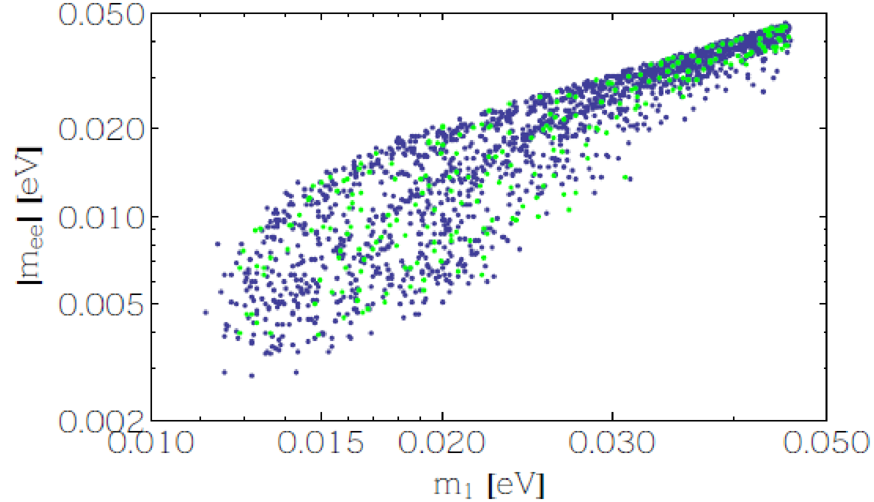
Red curve denotes the case of d=0, where $\sin^2\theta_{23} \fallingdotseq 0.51$

$$90^\circ \lesssim |\delta_{CP}| \lesssim 110^\circ \text{ at the best fit of } \sin^2\theta_{23} = 0.538$$

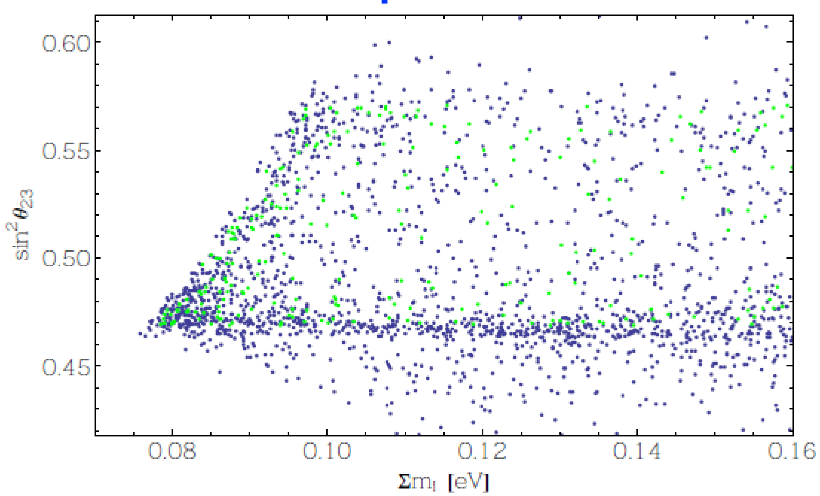
$\alpha_\eta > 0.07$



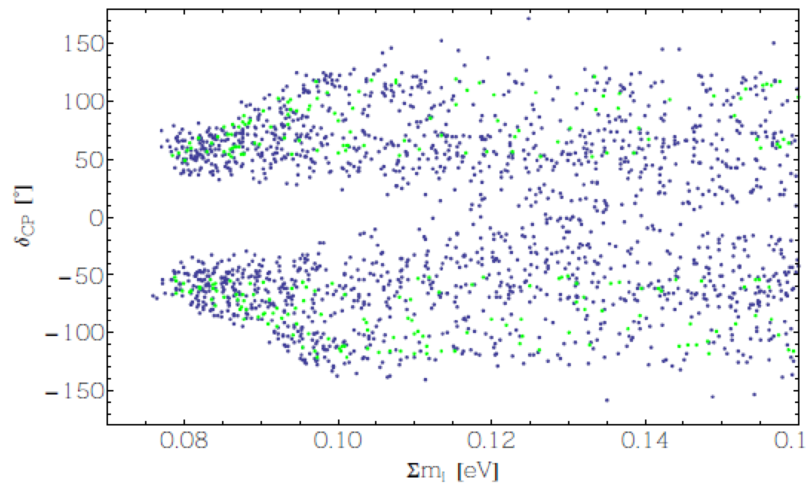
$\langle m_{ee} \rangle$ = (3 - 45) meV



$\Sigma\, m_i$ > 75meV



$\Sigma\, m_i$ > 75meV



m_1 > 12 meV

4 Summary

Imprint of the non-Abelian discrete symmetry is found in the neutrino flavor structure by using the direct approach of flavor symmetry.

Generators S, T, U of the residual symmetry

That will be precisely examined in the future experiments of flavor mixing angles and CP violation.

In order to discuss the flavor dynamics, models are build by introducing flavons (gauge singlet scalars).

Alignment of VEVs of flavons

Simplest flavor model is built based on the A_4 group, which is the smallest group including a triplet (order 12) in the irreducible representations.

Alignment of triplet flavons are (1,1,1) and (1,0,0) with singlet flavons 1 and 1" (1')

$$90^\circ \lesssim |\delta_{CP}| \lesssim 110^\circ \text{ at the best fit of } \sin^2\theta_{23} = 0.538$$

Alternative approach is the modular symmetry.

A_4 is a congruence subgroup of the modular symmetry.

Couplings Ys are not constant, but in the modular form.

Flavor structure is determined essentially by the modular parameter τ without flavons.

The phenomenology is going on (Tatsuishi's talk).

Thank you !