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What physics does the charged lepton mass relation tell us?

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The story begins from the following formula:

Charged Lepton Mass Relation

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

The formula was proposed 1982 by myself:

Y.K, Lett.Nuvo Cim. 34, 201 (1982); Phys. Lett. B 120, 161, (1983)

It is well known that the formula is excellently satisfied by the observed charged lepton masses.

$$K^{obs} = (2/3) \times (0.999989 \pm 0.000014)$$

1982, the formula predicted a tau lepton mass

$$m_\tau = 1776.97 \ \mathrm{MeV}$$

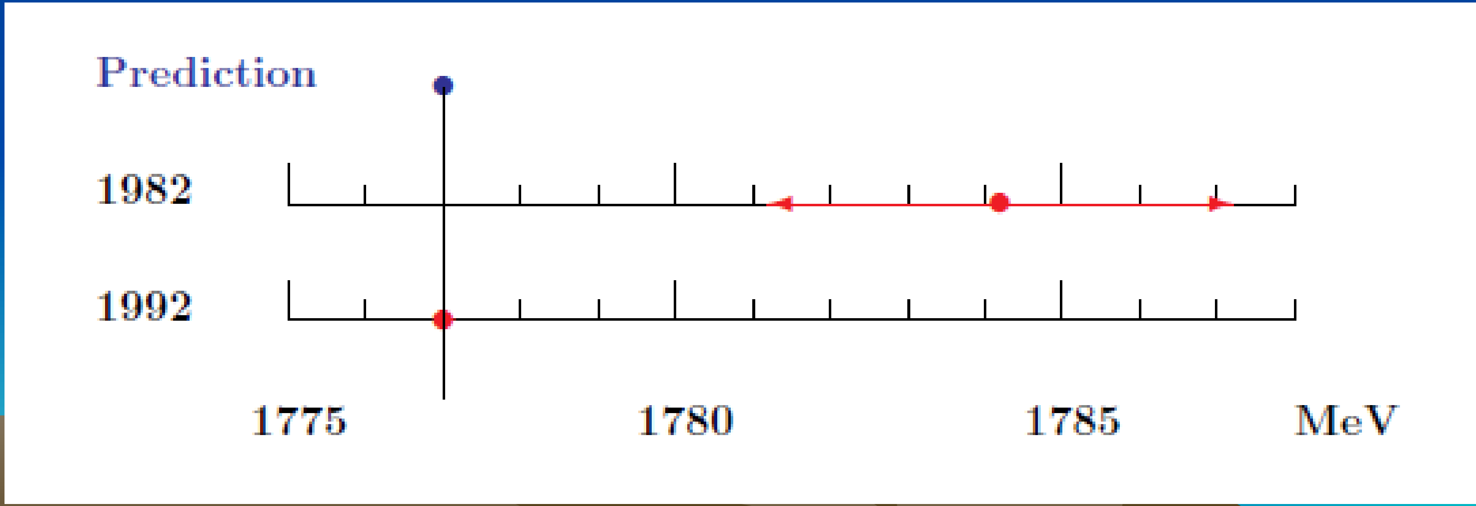
by inputting m_e & m_μ

On the other hand, the observed mass at 1982:

$$(m_\tau^{exp})_{old} = 1784.2 \pm 3.2 \quad \mathrm{MeV}$$

Ten years after, an accurate value was reported by ARGUS, BES, CLEO (1992)

$$(m_\tau^{exp})_{new} = 1776.99^{+0.29}_{-0.26} \quad \mathrm{MeV}$$



In general, the ``mass'' in the relation derived in a field theoretical model means the ``running'' mass, instead of the ``pole'' mass. Therefore, the charged lepton mass relation should be never satisfied by pole masses. Nevertheless, the relation is excellently satisfied by the pole masses.

This accuracy is excellent enough to believe that the coincidence is not accidental, but suggests a nontrivial physics behind it.

Thus, if we take the coincidence seriously, we should treat the renormalization group (RG) effects carefully. This was first pointed by Sumino

The present topic is not phenomenological one.

The purpose of my talk is to review of a field theoretical study by Sumino and recent development.

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1. Why is the excellent coincidence so problematic?

There are two kind of "masses":

"pole mass" and "running mass",

The charged lepton mass relation was derived based on a field theoretical model. (see the next slide.)

Y.K. Mod. Phys. Lett. A, 2319 (1990)

Therefore, we have to use the running masses for the formula, not the pole masses.

However, then, we obtain

$$K(\mu) = (2/3) \times (1.00189 \pm 0.000002) \quad \text{at } \mu = m_Z$$

The agreement is not so excellent.

2. Derivation of the mass formula

Y.K. Mod. Phys. Lett. A, 2319 (1990),

We introduce U(3)-family nonet scalar $(\Phi)_i^{\ j}$ $(i, j = 1, 2, 3)$

In the model, the charged lepton mass matrix is given by

$$M_e \propto \langle\Phi\rangle\langle\Phi\rangle$$

We consider the following scalar potential:

$$V = \mu^2 \mathrm{Tr}[\Phi\Phi] + \lambda \mathrm{Tr}[\Phi\Phi\Phi\Phi] + \lambda' \mathrm{Tr}[\Phi_8\Phi_8]\mathrm{Tr}[\Phi]^2,$$

$$\Phi_8 \equiv \Phi - \frac{1}{3}[\Phi]1,$$

Octet part of nonet scalar Φ

Here and hereafter, for convenience,
we denote Tr[A] as [A] simply.

Then, the condition $\partial V/\partial \Phi = 0$ leads to

$$\frac{\partial V}{\partial \Phi} = 2\left(\mu^2 + \lambda[\Phi\Phi] + \lambda'[\Phi]^2\right)\Phi + 2\lambda'\left([\Phi\Phi] - \frac{2}{3}[\Phi]^2\right)\mathbf{1}.$$

We want a solution $\Phi \neq \mathbf{1}$, so that the coefficients of Φ and $\mathbf{1}$ must be zero.
Then, we obtain

$$\mu^2 + \lambda[\Phi\Phi] + \lambda'[\Phi]^2 = 0,$$

$$[\Phi\Phi] - (2/3)[\Phi]^2 = 0$$

Note that the second equation is independent of the potential parameters μ and λ.
Thus, we obtain the relation

$$K = \frac{[\Phi\Phi]}{[\Phi]^2} = \frac{2}{3}.$$

Also, recently, we have obtained another mass formula

$$\kappa \equiv \frac{\det\Phi}{[\Phi]^3} = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^3} = \frac{1}{2\cdot 3^5} = \frac{1}{486}$$

Y. Koide, Phys.Lett. B 777, 131 (2018)

in addition to the formula

$$K \equiv \frac{[\Phi\Phi]}{[\Phi]^2} = \frac{m_e+m_\mu+m_\tau}{(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^2} = \frac{2}{3}$$

Note that those relations are invariant under a transformation

$$(m_e, m_\mu, m_\tau) \to (\lambda m_e, \lambda m_\mu, \lambda m_\tau)$$

3. Sumino mechanism

(Y. Sumino, Phys. Lett. B 677, 477 (2009))

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

The deviation of $K(m_{ei}^{running})$ **from** $K(m_{ei}^{pole})$
is caused by the logarithmic term of the QED correction

$$m_i(\mu) = m_i \left\{ 1 - \frac{\alpha(\mu)}{\pi} \left(1 + \frac{3}{4} \boxed{\log \frac{\mu^2}{m_i^2}} \right) \right\}$$

In 2009, Sumino proposed an attractive mechanism:

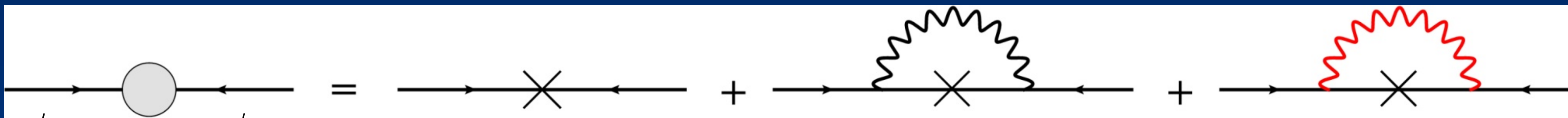
(a) Assume U(3) family gauge bosons

(b) Their masses M_{ii} are given by $M_{ii}^2 \propto m_{ei}$

Then, the unwelcome term $\log(m_{ei}/\mu)$ **is canceled by the factor** $\log(M_{ii}/\mu)$ **in the FGB contribution.**

Note: $M_{ii}^2 = \lambda m_{ei}$ **then** $2 \log M_{ii} = \log m_{ei} + \log \lambda$

$$\frac{1}{2} g_F = e = g_w \sin^2 \theta_W$$



ψ_L ψ_R

$m_i(\mu)$ m_i^{pole} $e^2 \log m_i^2$ $-g^2 \log M_{ii}^2$

Of course, this does not mean $m_i(\mu) = m_i^{pole}$

Note that, in order to guarantee the cancellation,
we must take the coupling constant *g* as
+*g* for e_L but -*g* for e_R, $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$
in other word, we must assign the U(3) family as
3 for e_L, but 3* for e_R :

Shortcomings of the Sumino model

(i) The Sumino model is not anomaly free model because of the assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$
c.f. QED charge is assigned as -e for e_L and -e for e_R
An anomaly non-free model can not be renormalizable.

(ii) In his model, unwelcome decay modes $\Delta N_{fam} = 2$ inevitably appear.

(iii) The K-relation cannot be derived simply in his model. The relation is derived from a family symmetry U(9), not U(3). The symmetry breaking is very complicated.

(iv) Against his hope, his FGB masses are still heavy because of the severe constraint from the observed K^0-$\bar{K}^0$ mixing data.

4. Modified Sumino Model

Such defects in the original Sumino model are due to the family number assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$

In order to avoid this defect, Yamashita and YK proposed a modified Sumino model: $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3})$

YK and T.Yamashita, PLB 711, 384 (2012)

In this model, the minus sign comes from the following idea:

The family gauge bosons have an inverted mass hierarchy.

i.e.

$$M_{ij}^2 = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right)$$

Then, we can obtain the minus sign from

$$2\log M_{ii} = \log(k/m_{ei}) = \log k - \log m_{ei}$$

Merit of the modified Sumino model

The family gauge boson with lightest mass is A11, which couples electron, but with b quark.

Lepton-Quark correspondence:

$$(e_1, e_2, e_3) = (e, \mu, \tau) \leftrightarrow (d_1, d_2, d_3) = (b, s, d)$$

(i) The inverted family number assignment for quarks weakens the severe constraint from K^0-$\bar{K}^0$ mixing data, so that we can obtain considerably low FGB masses. *YK, PLB (2014)*

(ii) Therefore, we may expect various **observations of FGB effects**

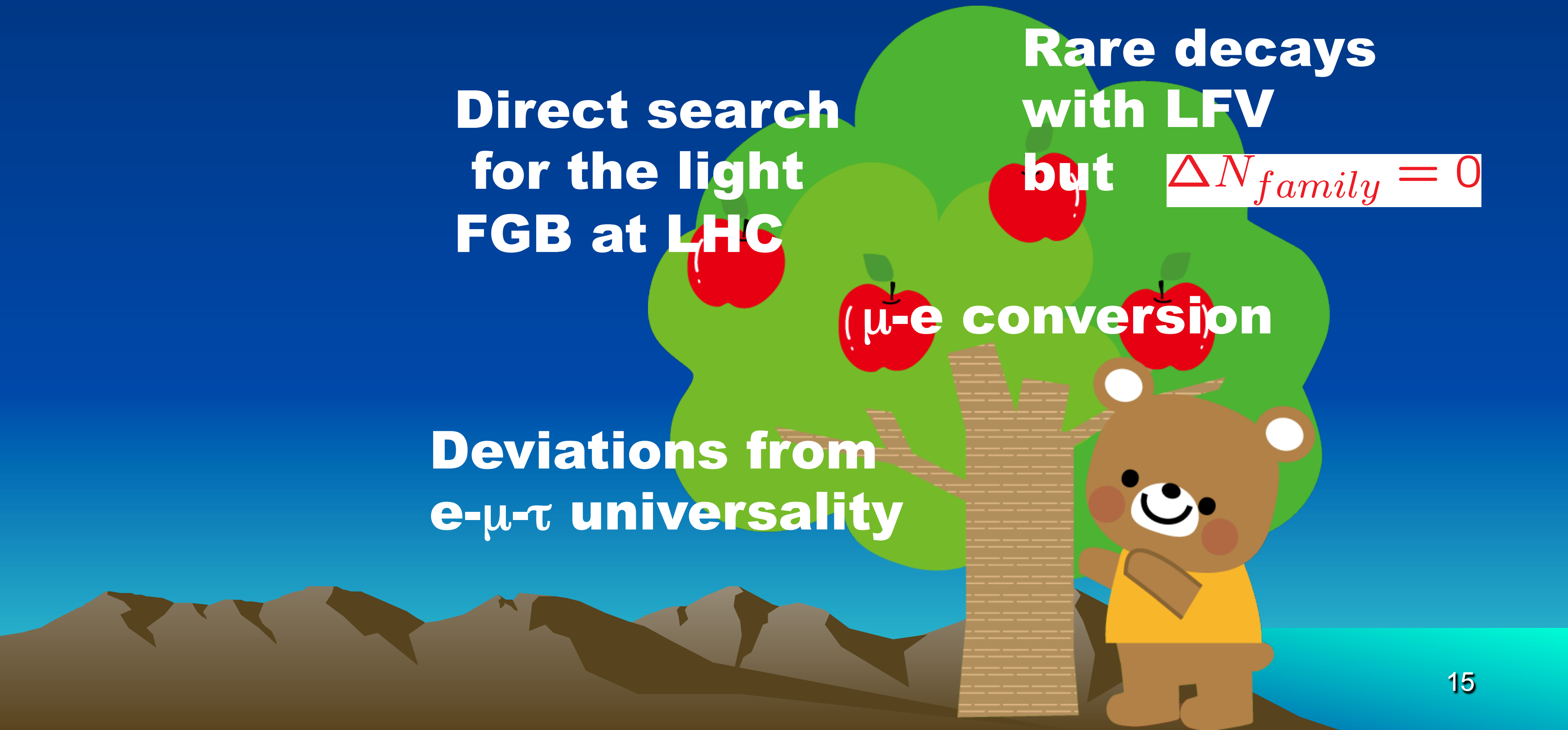
Now, we can expect fruitful and rich new events.

For example, see

μ - e conversion: *YK and M. Yamanaka, PLB (2016)*

A11 production at LHC: *YK, M. Yamanaka, H. Yokoya, PLB (2015)*

However , note that in our model the transition μ - > e + γ is exactly forbidden.



5. Recent development

There is another effect which changes of the potential form due to renormalization effect

(T. Yamashita, private communication)

The K and κ relations were derived from potential model.

Recall that there is no vertex correction in a SUSY model.

Therefore, if we derive the relations on the basis of SUSY scenario, then the problem will disappear.

Very recently, we succeeded to re-derive the K and κ relations on the basis of SUSY scenario.

(YK and T. Yamashita, arXiv:1805.09533 (hep-ph))

Thus, we can understand why the K- and κ-relations can keep the original forms.

6. Summary

We have discussed why the K relation is so beautifully satisfied by the pole masses, not the running masses.

Now we can understand the reason according to the Sumino's idea and the modified Sumino model.

Summary 2

My personal view of quarks and leptons

Charged leptons: only those are in the mass eigenstates

$\rightarrow$ We can observe simple mass relations.

Neutrinos: not in the mass eigenstates

$\rightarrow$ We can see the PMNS mixing

up-quarks: not in the mass eigenstates

$\rightarrow$ We can get U_u

down-qurarks: not in the mass eigenstates

$\rightarrow$ We can get U_d

So that we can observe the CKM mixing $V = U_u U_d^\dagger$


Thank you