Neutrino physics and CP violation in three generation seesaw model with four-zero textures

Akihiro Yuu (Hiroshima University)

This work is in collaboration with

T. Morozumi (Hiroshima U.)

Y. Shimizu (Hiroshima U.)

H. Umeeda (Shimane U.)

Introduction

- seesaw mechanism (type I)
- model and parameters
- CP asymmetry

Contents

- Four-zero texture model
- numerical analysis
- summary

Seesaw mechanism (Type I)

Minkowski, '77; Gell-Mann, Ramond, Slansky, Yanagida; Glashow; Mohapatra, Senjanovic '79

- heavy right-handed neutrinos ν_R
- Majorana particle

Lagrangian

$$\mathcal{L}_m = -\frac{1}{2}m_R(\overline{\nu}_R)^c\nu_R - \frac{1}{2}m_L(\overline{\nu}_L)^c\nu_L - m_D\overline{\nu}_R\nu_L + h.c.$$
$$= -\frac{1}{2}((\overline{\nu}_L)^c, \overline{\nu}_R)\begin{pmatrix} m_L & m_D \\ m_D & m_R^* \end{pmatrix}\begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c.$$

$$= -\frac{1}{2}m_s(\overline{\nu}_s)^c\nu_s - \frac{1}{2}m_a(\overline{\nu}_a)^c\nu_a$$

Only the neutrino with tiny masse remains in effective low energy theory

m_R,m_L : Majorana masses

m_D: Dirac mass

$$m_s \gg m_a$$

If one of the mass eigenvalues gets heavier, the other one becomes lighter, and vice versa.

three generation model (e, μ, τ)

6×6 mass matrix

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & m_{De1} & m_{De2} & m_{De3} \\ 0 & 0 & 0 & m_{D\mu1} & m_{D\mu2} & m_{D\mu3} \\ 0 & 0 & 0 & m_{D\tau1} & m_{D\tau2} & m_{D\tau3} \\ m_{De1} & m_{D\mu1} & m_{D\tau1} & M_1 & 0 & 0 \\ m_{De2} & m_{D\mu2} & m_{D\tau2} & 0 & M_2 & 0 \\ m_{De3} & m_{D\mu3} & m_{D\tau3} & 0 & 0 & M_3 \end{pmatrix}$$

$$\begin{aligned}
\mathcal{L}_m &= -\frac{1}{2}((\overline{\nu_L})^c, \overline{\nu_R}) \begin{pmatrix} 0 & m_D^t \\ m_D & m_R^* \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c. \\
&= -\frac{1}{2}((\overline{\nu_L})^c, \overline{\nu_R}) U^* U^t \begin{pmatrix} 0 & m_D^t \\ m_D & m_R^* \end{pmatrix} U U^\dagger \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c. \\
&\simeq -\frac{1}{2}(-i(\overline{\nu_L})^c, \overline{\nu_R}) \begin{pmatrix} -m_{eff} & 0 \\ 0 & m_R^* \end{pmatrix} \begin{pmatrix} -i\nu_L \\ (\nu_R)^c \end{pmatrix} + h.c. \\
&= -\frac{1}{2}\{(\overline{\nu_L})^c m_{eff} \nu_L + \overline{\nu_R} m_R^* (\nu_R)^c\} + h.c.
\end{aligned}$$

We define the effective mass

$$\begin{aligned} m_{eff} &\equiv -m_D (m_R^*)^{-1} m_D^t \\ &= -m_D \frac{1}{M_R} m_D^t \end{aligned}$$

$$V^{PMNS} m_{eff} (V^{PMNS})^t = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

m_i: mass eigenvalues

Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix) diagonalizes m_{eff}.

PMNS matrix describes the flavor mixing between weak-basis and mass-basis.

$$V^{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

A conventional parametrization for MNS matrix V^{PMNS} is

$$V^{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{13} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

- θ_{12}, θ_{23}, θ_{13} are mixing angles.
- δ is a Dirac CP phase, α_{21} and α_{31} are Majorana phases.

Parameters and notations in this model

We define m_{Di} $(i = 1,2,3)$ to be the magnitude of each column vector of the 3 by 3 Dirac mass matrix m_D.

$\boldsymbol{u}_i$ are the normalized column vectors of m_D.

$$m_D = (m_{D1}\boldsymbol{u}_1, m_{D2}\boldsymbol{u}_2, m_{D3}\boldsymbol{u}_3) \Rightarrow \boxed{U \equiv (\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3) = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu 1} & u_{\mu 2} & u_{\mu 3} \\ u_{\tau 1} & u_{\tau 2} & u_{\tau 3} \end{pmatrix}}$$

such that $|\boldsymbol{u}_i| = 1$

U is <u>not</u> unitary

Defining mass scales X_i written in terms of m_{Di} and the Majorana mass M_i.

$$\boxed{X_i \equiv \frac{m_{Di}^2}{M_i}}$$

The effective mass matrix m_{eff} is expressed as

$$\boxed{m_{eff} \equiv -m_D \frac{1}{M_R} m_D^t = -UXU^t}$$

In order to find the mass eigenvalues and the PMNS mixing matrix, we have to solve the characteristic equation (solutions λ are mass squares).

the characteristic equation

$$det(m_{eff}m_{eff}^{\dagger} - \lambda) = 0$$

↕

This equation is identical to three independent algebraic equations of mass squares and seesaw parameters (elements of U and X).

$$= 16J \sin(\frac{\Delta m_{12}^2}{2E}t) \sin(\frac{\Delta m_{23}^2}{2E}t) \sin(\frac{\Delta m_{31}^2}{2E}t)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

$$\Delta = Im\{(m_{eff}m_{eff}^{\dagger})_{e\mu}(m_{eff}m_{eff}^{\dagger})_{\mu\tau}(m_{eff}m_{eff}^{\dagger})_{\tau e}\}$$

defining a new symbol

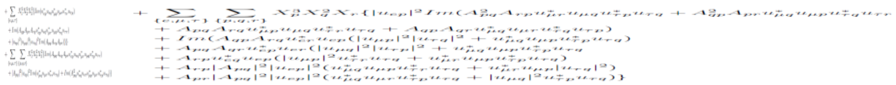
$$A_{ij} \equiv u_i^{\dagger}u_j$$

CP asymmetry in neutrino sector

Transition probabilities via neutrino oscillation

Transition probabilities via anti-neutrino oscillation

CP asymmetry



$$\Delta = \sum_{\{e,\mu,\tau\}} \sum_{\{p,q,r\}} X_p^4 X_q^2 (1 - |A_{pq}|^2) |u_{ep}|^2 Im(u_{\mu p}^* u_{\mu q} u_{\tau q}^* u_{\tau p})$$

Mass squared differences

$$\Delta m_{ij} = m_i^2 - m_i^2$$

CP asymmetry is proportional to J written with mixing angles and CP phase.

Jarlskog parameter

(in terms of mixing angles and δ CP phase)

$$J \equiv s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 sin\delta$$

C.Jarlskog ‘85

J can be expressed in terms of m_{eff} and mass eigenvalues.

$$J = \frac{Im\{(m_{eff}m_{eff}^{\dagger})_{e\mu}(m_{eff}m_{eff}^{\dagger})_{\mu\tau}(m_{eff}m_{eff}^{\dagger})_{\tau e}\}}{\Delta m_{12}^2 \Delta m_{23}^2 \Delta m_{31}^2}$$

We define Δ as the product of J and mass squared differences.

$$\begin{aligned}\Delta &\equiv J\Delta m_{12}^2 \Delta m_{23}^2 \Delta m_{31}^2 \\ &= Im\{(m_{eff}m_{eff}^{\dagger})_{e\mu}(m_{eff}m_{eff}^{\dagger})_{\mu\tau}(m_{eff}m_{eff}^{\dagger})_{\tau e}\}\end{aligned}$$

For $A^{CP} \neq 0, \quad \Delta \neq 0.$

Expressing it in terms of U and X, it will be complicated.

The most general form of Δ in three generations model in terms of seesaw parameters U and X.

$$m_{eff} = -\begin{pmatrix} X_1 \sin^2\theta_1 + X_2 \sin^2\theta_2 e^{2i\phi_1} + X_3 e^{2i\phi_2} & X_2 \sin\theta_2 \cos\theta_2 & X_1 \sin\theta_1 \cos\theta_1 \\ X_2 \sin\theta_2 \cos\theta_2 & X_2 \cos^2\theta_2 & 0 \\ X_1 \sin\theta_1 \cos\theta_1 & 0 & X_1 \cos^2\theta_1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & \cos\theta_2 & 0 \\ \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ \cos\theta_1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix}$$

$$PUQ$$

$$QXQ^T$$

$$m_{eff} = -P\{U(QXQ^T)U^T\}P^T$$

$$P^T V_{PMNS}$$

$$U$$

$$X = \begin{pmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{pmatrix}$$

$$m_{eff} = -UXU^T$$

$$V_{PMNS}$$

- notation

p, q, r denote 1, 2, or 3.

$\sum_{\{p,q,r\}}$ means the sum of all 6 patterns of substitution among p, q, r .

$p \neq q \neq r$.

e, μ, τ are each flavor.

$\sum_{\{e,\mu,\tau\}}$ means the sum of 3 patterns of periodic substitution for (e, μ, τ) .

Four zero texture model of Dirac mass matrix

To reduce the number of parameters, we replace some elements of the Dirac mass matrix with 0 .

$$U=\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ 0 & * & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix} \quad \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix} \quad \cdots\cdots$$

The configuration of the four zero texture has $_9C_4=126$ patterns.

Classification of four zero textures

${}_9C_4$=126 patterns.

U

Type I $\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ 0 & * & 0 \end{pmatrix}$ There is only one row component all in which the elements are not zero, and $rank\ U = 3$. (18 patterns)

Type II $\begin{pmatrix} * & * & 0 \\ * & 0 & * \\ * & 0 & 0 \end{pmatrix}$ There is only one column component all in which the elements are not zero, and $rank\ U = 3$. (18 patterns)

Type III $\begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ 0 & * & * \end{pmatrix}$ The numbers of non-zero elements on any row and column components are 1 or 2, and $rank\ U = 3$. (36 patterns)

Type IV $\begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & 0 \end{pmatrix}$ There is only one column component all in which the elements are zero. (18 patterns)

Type V $\begin{pmatrix} * & * & * \\ * & * & 0 \\ 0 & 0 & 0 \end{pmatrix}$ There is only one row component all in which the elements are zero. (18 patterns)

Type VI $\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}$ There is one row component and one column component all in which the elements are not zero. (9 patterns)

Type VII $\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$ There are one row and column components in which two elements are zero, and the common element on the both of such row and column components is not zero. (9 patterns)

Assumption

- The lightest neutrino mass is also non-zero.

$$m_1 \neq 0 \quad (m_3 \neq 0)$$

- Three masses are not degenerated.

$$m_1 < m_2 \ll m_3 \quad \text{or} \quad m_3 \ll m_1 < m_2$$

- CP symmetry is violated in the leptonic sector.

$$\Delta \neq 0$$

We adopt such textures satisfying the assumptions.

We suppose the CP asymmetry in neutrino sector and three massive neutrinos.

	U		
Type Ⅰ	$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ 0 & * & 0 \end{pmatrix}$	There is only one row component all in which the elements are not zero, and $rank\ U = 3$.	(18 patterns)
Type Ⅱ	$\begin{pmatrix} * & * & 0 \\ * & 0 & * \\ * & 0 & 0 \end{pmatrix}$	There is only one column component all in which the elements are not zero, and $rank\ U = 3$.	(18 patterns)
Type Ⅲ	$\begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ 0 & * & * \end{pmatrix}$	The numbers of non-zero elements on any row and column components are 1 or 2, and $rank\ U = 3$.	(36 patterns)
Type Ⅳ	$\begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & 0 \end{pmatrix}$	There is only one column component all in which the elements are zero.	(18 patterns)
Type Ⅴ	$\begin{pmatrix} * & * & * \\ * & * & 0 \\ 0 & 0 & 0 \end{pmatrix}$	There is only one row component all in which the elements are zero.	(18 patterns)
Type Ⅵ	$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}$	There is one row component and one column component all in which the elements are not zero.	(9 patterns)
Type Ⅶ	$\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$	There are one row and column components in which two elements are zero, and the common element on the both of such row and column components is not zero.	(9 patterns)

Types Ⅰ–Ⅳ: $\Delta \neq 0$

Types Ⅰ–Ⅲ: We adopt first three types.

Type Ⅳ causes one massless neutrino automatically.

Types Ⅴ–Ⅶ: $\Delta = 0$

Type Ⅴ, Ⅵ and Ⅶ do not explain the CP violation.

They are ruled out.

Parametrization of four zero texture model

$$U = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ 0 & u_{\mu 2} & 0 \\ u_{\tau 1} & 0 & 0 \end{pmatrix}$$

Parameterization ➡

$$U = \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\Phi_1} & e^{i\Phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix}$$

4 parameters

$\theta_1, \theta_2, \Phi_1, \Phi_2$

in total 7 parameters $X_1, X_2, X_3, \theta_1, \theta_2, \Phi_1, \Phi_2$

m_{eff} in this case

$$m_{eff} = -\begin{pmatrix} X_1 u_{e1}^2 + X_2 u_{e2}^2 + X_3 u_{e3}^2 & X_2 u_{e2} u_{\mu 2} & X_1 u_{e1} u_{\tau 1} \\ X_2 u_{e2} u_{\mu 2} & X_2 u_{\mu 2}^2 & 0 \\ X_1 u_{e1} u_{\tau 1} & 0 & X_1 u_{\tau 1}^2 \end{pmatrix}$$

Δ in this case

$$\Delta \quad m_{eff} = -\begin{pmatrix} X_2 \sin^2\theta_1 + X_1 \sin^2\theta_2 e^{2i\phi_1} + X_3 e^{2i\phi_2} & X_1 \sin\theta_2 \cos\theta_2 & X_2 \sin\theta_1 \cos\theta_1 \\ X_1 \sin\theta_2 \cos\theta_2 & X_1 \cos^2\theta_2 & 0 \\ X_2 \sin\theta_1 \cos\theta_1 & 0 & X_2 \cos^2\theta_1 \end{pmatrix}$$

Numerical analysis

A texture in Type I, for example

$$U=\begin{pmatrix} sin\theta_1 & sin\theta_2 e^{i\Phi_1} & e^{i\Phi_2} \\ 0 & cos\theta_2 & 0 \\ cos\theta_1 & 0 & 0 \end{pmatrix}$$

Allocating parameters randomly

model angles in U ; θ_1, θ_2 from $-\pi$ to π

model phases in U ; Φ_1, Φ_1 from $-\pi$ to π

the lightest neutrino mass ; m_1 (m_3) from 0 to 0.046 [eV]

	Best-fit	3 σ
Δm^2_{21} $[10^{-5} eV^2]$	7.30	6.39 - 7.97
Δm^2_{31} $[10^{-3} eV^2]$	2.50	2.37 – 2.63

Reconstructing the m_{eff} and solving the algebraic equations

$$\det(m_{eff} m^{\dagger}_{eff} - \lambda) = 0 \ ,$$

we obtain X_1, X_2 and X_3 .

Giving the values to parameters$(\theta_1, \theta_2, \Phi_1, \Phi_2, m_1, X_1, X_2, X_3)$,

We determine the m_{eff}.

The m_{eff} produces flavor mixing angles $(\theta_{12}, \theta_{23}$, and $\theta_{13})$, the Dirac CP phase δ, and Majorana phases (α_{21}, α_{31}).

If all three mixing angles $(\theta_{12}, \theta_{23}$, and $\theta_{13})$ are within 3σ of the experimentally measured values, we collect the set of parameters as a possible model.

	Best-fit	3σ
$sin^2\theta_{12}$	0.297	0.250 - 0.357
$sin^2\theta_{23}$	0.437	0.379 - 0.616
$sin^2\theta_{13}$	0.0214	0.0185 - 0.0246

Results

- Each solved X_i is smaller than 0.5 [eV].

  After assuming the heavy right handed neutrino masses M_i, we can determine the Dirac mass matrix m_D from the solutions X_i and the other parameters θ_1, θ_2, Φ_1, Φ_2.

- Some models do not reproduce the correct mixing angles (normal hierarchical case).

- There are some correlations among the parameters, and physical quantities.

- In normal hierarchical case, any texture which produces 0 on μ- τ element (and also τ -μ element) of m_{eff}, does not explain the experimental results.

Textures, which makes $m_{eff} = \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$, are consistent with experiments.

$$m_{eff} = -UXU^t$$

For example, $U = \begin{pmatrix} * & 0 & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}, \begin{pmatrix} * & 0 & 0 \\ * & * & * \\ 0 & 0 & * \end{pmatrix}, \begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ 0 & * & * \end{pmatrix}, \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix} \cdots\cdots$

These 18 textures are ruled out.

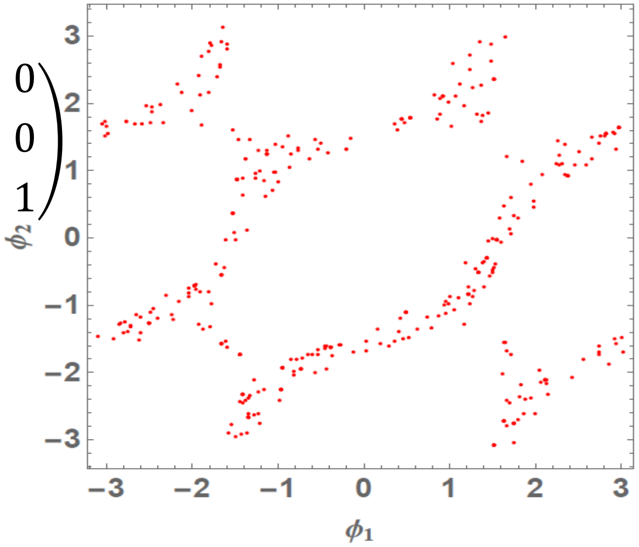
While, In inverted hierarchical case, all textures are possible to reproduce the experimental results.

・A correlation between parameters (Normal)

Holding up two specific textures in typeⅢ as examples,

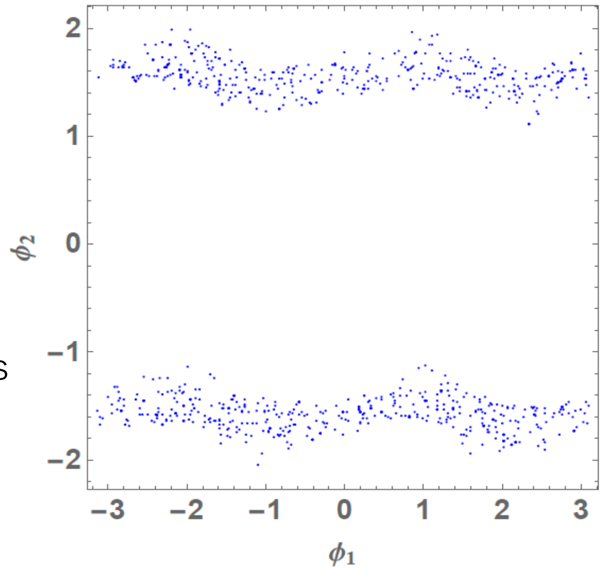
$$U = \begin{pmatrix} sin\theta_1 & 0 & 0 \\ cos\theta_1 & sin\theta_2 e^{i\Phi_1} & 0 \\ 0 & cos\theta_2 e^{i\Phi_2} & 1 \end{pmatrix}$$

There are some kind of correlations between parameters Φ_1 and Φ_2.

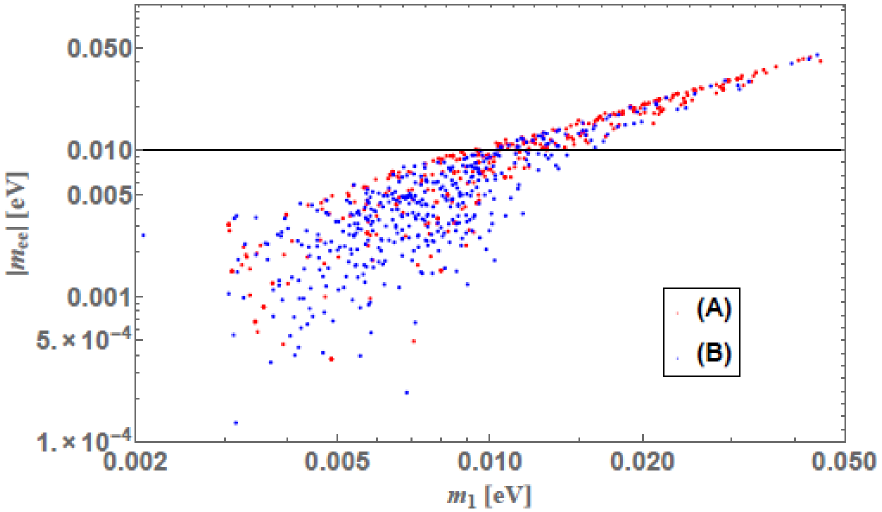


$$U = \begin{pmatrix} 0 & cos\theta_2 e^{i\Phi_1} & 1 \\ sin\theta_1 & 0 & 0 \\ cos\theta_1 & sin\theta_2 e^{i\Phi_2} & 0 \end{pmatrix}$$

There is no particularly features for Φ_1 , while Φ_2 is restricted in the specific range.



・The correlation between m_1 and $|m_{ee}|$ (for two textures as above)



x axis : the lightest neutrino mass

y axis : the absolute value of the e-e element of m_{eff}

The correlation between m_1 and $|m_{ee}|$ is analytically comprehensible for both two examples.

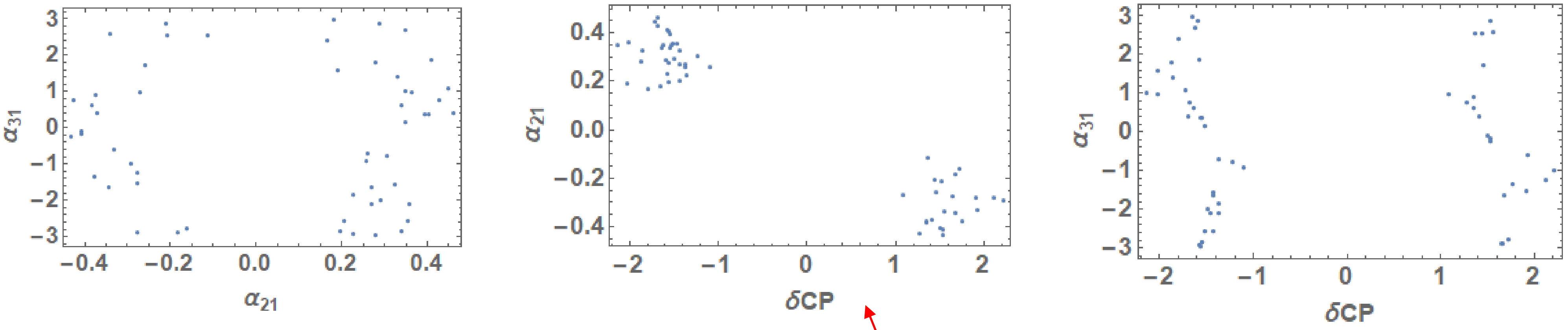
From the experimental results of the neutrinoless double beta decay, for example, it gives a prediction for the mass of the lightest neutrino mass.

- Correlations among CP phase and Majorana phases (inverted)

$$U = \begin{pmatrix} sin\theta_1 & 0 & 0 \\ cos\theta_1 & sin\theta_2 e^{i\Phi_1} & 0 \\ 0 & cos\theta_2 e^{i\Phi_2} & 1 \end{pmatrix}$$

α_{31} is not determined and it does not have correlation with α_{21} nor δ.

While α_{21} and δ appear in the limited range.



The sign of δ and the sign of α_{21} depend on each other.

In some cases of the model, the CP phase δ is outputted around $-\frac{\pi}{2}$ or $\frac{\pi}{2}$.

Summary

・In three generation seesaw model, we reduced the number of model parameters as possible,

$$U = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \Rightarrow \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ * & 0 & 0 \end{pmatrix} = \begin{pmatrix} sin\theta_1 & sin\theta_2 e^{i\Phi_1} & e^{i\Phi_2} \\ 0 & cos\theta_2 & 0 \\ cos\theta_1 & 0 & 0 \end{pmatrix}$$

・We classify them and picked up the textures under the condition that the lightest neutrino mass is non-zero and CP is violated.

・There are some correlations among model parameters and physical quantities.

・This analysis makes some predictions for the phases δ, α_{21}, and α_{31} and also for relations among physical quantities.

Future works

・To give some explanation on the numerical results with the analytical method.

・To apply this model for other physical phenomena, such as neutrinoless double beta decay and leptogenesis.

Buck up

Textures in Type I

There are 18 different textures (patterns of configuration)

They are related on each other by replacing the column or row components.

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ 0 & * & 0 \end{pmatrix}, \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ * & 0 & 0 \end{pmatrix}, \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \begin{pmatrix} * & * & * \\ 0 & 0 & * \\ * & 0 & 0 \end{pmatrix}, \begin{pmatrix} * & * & * \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix},$$

$$\begin{pmatrix} * & 0 & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}, \begin{pmatrix} * & 0 & 0 \\ * & * & * \\ 0 & 0 & * \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ * & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & 0 & * \end{pmatrix}, \begin{pmatrix} 0 & 0 & * \\ * & * & * \\ * & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & * \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ * & * & * \end{pmatrix}, \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ * & 0 & 0 \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & 0 & * \\ * & 0 & 0 \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & * & * \end{pmatrix},$$

Textures in the same Type are related to each other by exchanging the column and row components in U.

$$U_b = PU_aQ$$

$$P, Q = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

Any criterial texture U_a

$$U$$

$$X = \begin{pmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{pmatrix}$$

$$m_{eff} = -UXU^T$$

$$V_{PMNS}$$

⟺

Another texture in the same type $U_b = PU_aQ$

$$PUQ$$

$$QXQ^T$$

$$m_{eff} = -P\{U(QXQ^T)U^T\}P^T$$

$$P^T V_{PMNS}$$

A texture in Type I

$$U = \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix} \qquad m_{eff} = -\begin{pmatrix} X_1\sin^2\theta_1 + X_2\sin^2\theta_2 e^{2i\phi_1} + X_3 e^{2i\phi_2} & X_2\sin\theta_2\cos\theta_2 & X_1\sin\theta_1\cos\theta_1 \\ X_2\sin\theta_2\cos\theta_2 & X_2\cos^2\theta_2 & 0 \\ X_1\sin\theta_1\cos\theta_1 & 0 & X_1\cos^2\theta_1 \end{pmatrix}$$

Another textures in Type I (exchanging the first and second row components)

$$\begin{pmatrix} 0 & \cos\theta_2 & 0 \\ \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ \cos\theta_1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix}$$

$$m_{eff} = -\begin{pmatrix} X_2\cos^2\theta_2 & X_2\sin\theta_2\cos\theta_2 & 0 \\ X_2\sin\theta_2\cos\theta_2 & X_1\sin^2\theta_1 + X_2\sin^2\theta_2 e^{2i\phi_1} + X_3 e^{2i\phi_2} & X_1\sin\theta_1\cos\theta_1 \\ 0 & X_1\sin\theta_1\cos\theta_1 & X_1\cos^2\theta_1 \end{pmatrix}$$

Another textures in Type I (exchanging the first and second column components)

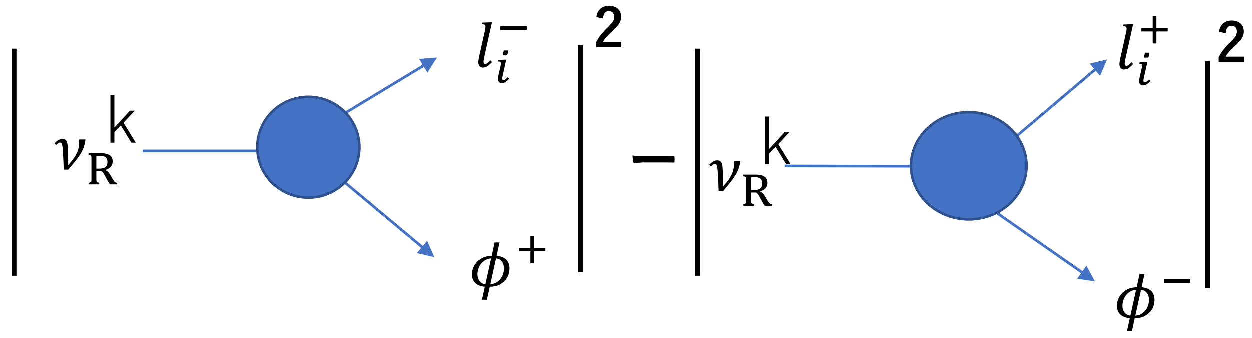
$$\begin{pmatrix} \sin\theta_2 e^{i\phi_1} & \sin\theta_1 & e^{i\phi_2} \\ \cos\theta_2 & 0 & 0 \\ 0 & \cos\theta_1 & 0 \end{pmatrix} = \begin{pmatrix} \sin\theta_1 & \sin\theta_2 e^{i\phi_1} & e^{i\phi_2} \\ 0 & \cos\theta_2 & 0 \\ \cos\theta_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_{eff} = -\begin{pmatrix} X_2\sin^2\theta_1 + X_1\sin^2\theta_2 e^{2i\phi_1} + X_3 e^{2i\phi_2} & X_1\sin\theta_2\cos\theta_2 & X_2\sin\theta_1\cos\theta_1 \\ X_1\sin\theta_2\cos\theta_2 & X_1\cos^2\theta_2 & 0 \\ X_2\sin\theta_1\cos\theta_1 & 0 & X_2\cos^2\theta_1 \end{pmatrix}$$

Application for leptogenesis

The lepton number asymmetry ε_i^k $(k = 1,2,3 \;\; i = e, \mu, \tau)$ is related to the difference of the partial decay width of heavy right handed neutrino.

$$\varepsilon_i^k = \frac{\Gamma(\nu_R^k \longrightarrow l_i^- \phi^+) - \Gamma(\nu_R^k \longrightarrow l_i^+ \phi^-)}{\Gamma(\nu_R^k \longrightarrow l_i^- \phi^+) + \Gamma(\nu_R^k \longrightarrow l_i^+ \phi^-)}$$



Electron number asymmetry $(i = e)$,

$$\varepsilon_e^k = \frac{v^2}{16\pi} \sum_{k' \neq k} \{ I(x_{k'k}) \frac{Im((m_D^\dagger m_D)_{kk'} (m_D^*)_{ek} (m_D)_{ek'})}{|(m_D)_{ek}|^2}$$

$$+ \frac{1}{1 - x_{k'k}} \frac{Im((m_D^\dagger m_D)_{k'k} (m_D^*)_{ek} (m_D)_{ek'})}{|(m_D)_{ek}|^2} \}$$

T.Endoh, T.Morozumi, Z.Xiong '2004

non-zero.

$$m_D = \begin{pmatrix} m_{De1} & m_{De2} & m_{De3} \\ m_{D\mu1} & m_{D\mu2} & m_{D\mu3} \\ m_{D\tau1} & m_{D\tau2} & m_{D\tau3} \end{pmatrix}$$

$$x_{kk'} = \frac{M_{k'}^2}{M_k^2}$$

In order to $\varepsilon_e^k \neq 0$, at least two of the matrix elements of $(m_{De1}, m_{De2}, m_{De3})$ must be non-zero.

For example, in the case of $m_{De1} = \begin{pmatrix} m_{De1} & m_{De2} & m_{De3} \\ m_{D\mu 1} & 0 & 0 \\ 0 & m_{D\tau 2} & 0 \end{pmatrix}$,

total number asymmetry is identical to the electron number asymmetry.

$$\sum_{i=e,\mu,\tau} \varepsilon_i^k = \varepsilon_e^k$$

ε_μ^k and ε_τ^k do not contribute to the total number asymmetry.

Textures in Type I have such feature.