Fully-Constrained Majorana Neutrino Mass Matrices using $\Sigma(72 \times 3)$

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[1] with R. Krishnan and W. G. Scott, Eur. Phys. J. C (2018) 78:74

Outline

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Beyond Tribimaximal Mixing

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

PFH, DH Perkins, WG Scott
hep-ph/0202074

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Altering TBM with a 2×2 unitary matrix $\rightarrow$ "S3 Group Mixing":

$$\text{eg.} \quad U_{S3GM} = \begin{pmatrix} ? & \frac{1}{\sqrt{3}} & ? \\ ? & \frac{1}{\sqrt{3}} & ? \\ ? & \frac{1}{\sqrt{3}} & ? \end{pmatrix}$$

PFH, WG Scott
hep-ph/0302025

Also named TM_i, TM^i etc.

C. H. Albright et al.
1004.2798

Tribimaximal Mixing

$$U_{TM_2} = \begin{pmatrix} ? & \frac{1}{\sqrt{3}} & ? \\ ? & \frac{1}{\sqrt{3}} & ? \\ ? & \frac{1}{\sqrt{3}} & ? \end{pmatrix}$$

- Maximally CP violating - Triχmaximal mixing:

$$U_{\mathrm{T}\chi\mathrm{M}} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \end{pmatrix}$$

PFH, WGS
hep-ph/0203209

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PFH, WGS
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- CP conserving - Triϕmaximal mixing:

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General Formalism

- Effective see-saw mass matrix:

$$M_{ss} = -M_D M_{Maj}^{-1} M_D^T$$

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- The leptonic mixing matrix:

$$U_{PMNS} = U_L U_\nu$$

A Working Ansatz

- Neutrino sector:

$M_D \propto I$

$$M_{\mathrm{Maj}} \propto \begin{pmatrix} (2-\sqrt{2}) & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \qquad \propto \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & (2-\sqrt{2}) \end{pmatrix},$$

$$M_{\mathrm{Maj}} \propto \begin{pmatrix} i+\frac{1-i}{\sqrt{2}} & 0 & 1-\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1-\frac{1}{\sqrt{2}} & 0 & -i+\frac{1+i}{\sqrt{2}} \end{pmatrix}, \quad \propto \begin{pmatrix} -i+\frac{1+i}{\sqrt{2}} & 0 & 1-\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1-\frac{1}{\sqrt{2}} & 0 & i+\frac{1-i}{\sqrt{2}} \end{pmatrix}$$

RK
1211.3364

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- Charged-lepton sector (mixing contribution):

RK
1211.3364

$$3 \times 3 \text{ Trimaximal, } U_L = -\frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \bar{\omega} \\ 1 & \bar{\omega} & \omega \end{pmatrix}$$

A Working Ansatz

- The mixing Observables:

	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ
nu-fit	$0.0198 - 0.0244$	$0.272 - 0.346$	$0.418 - 0.613$	
TχM	$\frac{2}{3}\sin^2\chi$	$\frac{1}{\left(3-2\sin^2\chi\right)}$	$\frac{1}{2}$	$\pm\frac{\pi}{2}$
$\chi = \pm\frac{\pi}{16}$	0.0254	0.342	0.5	$\pm\frac{\pi}{2}$
TϕM	$\frac{2}{3}\sin^2\phi$	$\frac{1}{\left(3-2\sin^2\phi\right)}$	$\frac{2\sin^2\left(\frac{2\pi}{3}+\phi\right)}{\left(3-2\sin^2\phi\right)}$	$0,\ \pi$
$\phi = \pm\frac{\pi}{16}$	0.0254	0.342	$0.387, 0.613$	$0,\ \pi$

nufit: I. Esteban et al.
1611.01514

A Working Ansatz

- The neutrino masses:

  $m_1 : m_2 : m_3 =$

  $2\sqrt{B}-B \ : \ 1 \ : \ 2\sqrt{B}+B$

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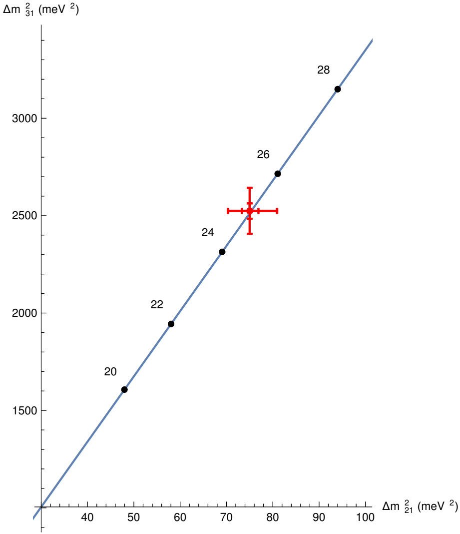

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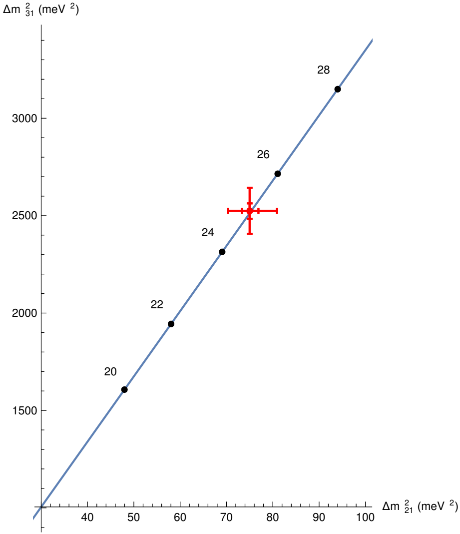
  $\Delta m^2_{21}=70.3-80.9\ \mathrm{meV}^2$,
  $|\Delta m^2_{31}|=2407-2643\ \mathrm{meV}^2$

- Observables:
  - $m_1 = 25.04^{+0.17}_{-0.15}$ meV
    $m_2 = 26.50^{+0.18}_{-0.16}$ meV
    $m_3 = 56.09^{+0.37}_{-0.34}$ meV
  - $\sum_i m_i = 107.6^{+0.71}_{-0.65}$ meV
  - $m_{\beta\beta} = 23.47^{+0.16}_{-0.14}$ meV



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- $\nu_R \to$ fermion triplet $(\mathbf{3})$ under a discrete group.
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- eg. A_4

$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \cancel{\mathbf{3}}$$

Invariants:

$$(\nu_R.\nu_R)_{\mathbf{1}}\phi_{\mathbf{1}}, \quad (\nu_R.\nu_R)_{\mathbf{1'}}\phi_{\mathbf{1'}}, \quad (\nu_R.\nu_R)_{\mathbf{1''}}\phi_{\mathbf{1''}}, \quad (\nu_R.\nu_R)_{\mathbf{3}}\phi_{\mathbf{3}}$$

Trace, Traceless diagonal, Off-diagonal

- Coupling constants for the invariants are unconstrained

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Invariant: $(\nu_R . \nu_R)_{\mathbf{6}} \, \xi$

Full Complex-symmetric mass matrix.

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Full Complex-symmetric mass matrix.

- $(\nu_R.\nu_R)_{\mathbf{6}} = \Big(\nu_{R1}.\nu_{R1}, \nu_{R2}.\nu_{R2}, \nu_{R3}.\nu_{R3}, \frac{1}{\sqrt{2}}\left(\nu_{R2}.\nu_{R3} + \nu_{R3}.\nu_{R2}\right), \frac{1}{\sqrt{2}}\left(\nu_{R3}.\nu_{R1} + \nu_{R1}.\nu_{R3}\right), \frac{1}{\sqrt{2}}\left(\nu_{R1}.\nu_{R2} + \nu_{R2}.\nu_{R1}\right)\Big)$

  $\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$

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  $$\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$$

$$(\nu_R.\nu_R)_{\mathbf{6}}\,\xi = \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}^T \cdot \begin{pmatrix} \xi_1 & \frac{1}{\sqrt{2}}\xi_6 & \frac{1}{\sqrt{2}}\xi_5 \\ \frac{1}{\sqrt{2}}\xi_6 & \xi_2 & \frac{1}{\sqrt{2}}\xi_4 \\ \frac{1}{\sqrt{2}}\xi_5 & \frac{1}{\sqrt{2}}\xi_4 & \xi_3 \end{pmatrix} \cdot \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix}$$

$\Sigma(72 \times 3)$

- $SU(3) : \mathbf{3} \times \mathbf{3} = \mathbf{6} + \cancel{\bar{\mathbf{3}}}$
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- Why $\Sigma(72 \times 3)$?
  - Smallest group with a complex sextet from two triplets
  - No Goldstone Bosons from SSB, unlike SU(3)
  - Residual Symmetries of the VEVs (arising from breaking the flavour group into its subgroups) $\rightarrow$ Symmetries of the Mass matrix

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- Principal series: $\{e\} \triangleleft Z_3 \triangleleft \Delta(27) \triangleleft \Delta(54) \triangleleft \Sigma(72 \times 3)$

$\Sigma(72\times 3)$

- The generators:

$$C \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \bar{\omega} \end{pmatrix}, \qquad E \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$V \equiv -\frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \bar{\omega} \\ 1 & \bar{\omega} & \omega \end{pmatrix}, \quad X \equiv -\frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \bar{\omega} \\ 1 & \omega & \omega \\ \omega & 1 & \omega \end{pmatrix}$$

W. Grimus, P. O. Ludl
1006.0098

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- C and E generate $\Delta(27)$

  V and $E^2VXVE^2 = -\frac{i}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega \\ \bar{\omega} & \omega & \bar{\omega} \\ \bar{\omega} & \bar{\omega} & \omega \end{pmatrix}$ generate Q_8

- $\Sigma(72 \times 3) = \Delta(27) \rtimes Q_8$

VEVs -The Neutrino Sector

Majorana mass matrices and the corresponding VEVs

TχM:

$$M_{\mathrm{Maj}} \propto \begin{pmatrix} (2-\sqrt{2}) & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \qquad \propto \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & (2-\sqrt{2}) \end{pmatrix}$$

$$\langle \xi \rangle \propto \left((2-\sqrt{2}), 1, 0, 0, 1, 0\right), \qquad \propto \left(0, 1, (2-\sqrt{2}), 0, 1, 0\right)$$

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$$M_{\mathrm{Maj}} \propto \begin{pmatrix} i+\frac{1-i}{\sqrt{2}} & 0 & 1-\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1-\frac{1}{\sqrt{2}} & 0 & -i+\frac{1+i}{\sqrt{2}} \end{pmatrix}, \quad \propto \begin{pmatrix} -i+\frac{1+i}{\sqrt{2}} & 0 & 1-\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1-\frac{1}{\sqrt{2}} & 0 & i+\frac{1-i}{\sqrt{2}} \end{pmatrix}$$

$$\langle\xi\rangle \propto \left(i+\frac{1-i}{\sqrt{2}}, 1, -i+\frac{1+i}{\sqrt{2}}, 0, (\sqrt{2}-1), 0\right),$$

$$\propto \left(-i+\frac{1+i}{\sqrt{2}}, 1, i+\frac{1-i}{\sqrt{2}}, 0, (\sqrt{2}-1), 0\right)$$

Diagonalisation

- All cases:

$$U_\nu^\dagger M_{\text{Maj}}^{-1} U_\nu^* \;\propto\; \text{Diag}\left(2\sqrt{B} - B, 1, 2\sqrt{B} + B\right)$$

with $B = (2 - \sqrt{2})$.

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with $B=(2-\sqrt{2})$.

- 2×2 contribution to mixing:

$$U_\nu \propto \begin{pmatrix} \cos\left(\frac{3\pi}{16}\right) & 0 & -i\sin\left(\frac{3\pi}{16}\right) \\ 0 & 1 & 0 \\ \sin\left(\frac{3\pi}{16}\right) & 0 & i\cos\left(\frac{3\pi}{16}\right) \end{pmatrix}, \quad \propto \begin{pmatrix} \cos\left(\frac{5\pi}{16}\right) & 0 & i\sin\left(\frac{5\pi}{16}\right) \\ 0 & 1 & 0 \\ \sin\left(\frac{5\pi}{16}\right) & 0 & -i\cos\left(\frac{5\pi}{16}\right) \end{pmatrix},$$

$$U_\nu \propto \begin{pmatrix} \frac{1}{\sqrt{2}}e^{-i\frac{\pi}{16}} & 0 & -\frac{1}{\sqrt{2}}e^{-i\frac{\pi}{16}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}}e^{i\frac{\pi}{16}} & 0 & \frac{1}{\sqrt{2}}e^{i\frac{\pi}{16}} \end{pmatrix}, \quad \propto \begin{pmatrix} \frac{1}{\sqrt{2}}e^{i\frac{\pi}{16}} & 0 & \frac{1}{\sqrt{2}}e^{i\frac{\pi}{16}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}}e^{-i\frac{\pi}{16}} & 0 & -\frac{1}{\sqrt{2}}e^{-i\frac{\pi}{16}} \end{pmatrix}.$$

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- Two triplet flavons: ϕ_α, ϕ_β

  VEVs $\langle\phi_\alpha\rangle \propto (1,1,1)$, $\langle\phi_\beta\rangle \propto (1,\omega,\bar{\omega})$

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- ϕ_α, ϕ_β and higher order products $\rightarrow$ hierarchical charged-lepton mass matrix:

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^\dagger V^\dagger \begin{pmatrix} \mathcal{O}(\epsilon^4) & \mathcal{O}(\epsilon^4) & 0 \\ 0 & y_\mu h_o \epsilon^2 + \mathcal{O}(\epsilon^4) & 0 \\ \mathcal{O}(\epsilon^4) & \mathcal{O}(\epsilon^4) & y_\tau h_o \epsilon + \mathcal{O}(\epsilon^3) \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$\left(\frac{m_e}{m_\mu}\right) \approx \left(\frac{m_\mu}{m_\tau}\right)^2 = \mathcal{O}(\epsilon^2)$$

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$$\left(\frac{m_e}{m_\mu}\right) \approx \left(\frac{m_\mu}{m_\tau}\right)^2 = \mathcal{O}(\epsilon^2)$$

- $U_L \approx V \rightarrow$ 3×3 trimaximal mixing contribution to mixing. (The VEVs $\langle \phi_\alpha \rangle$, $\langle \phi_\beta \rangle$ have their origin in the group generator V)
- $\mathcal{O}(\epsilon^2)$ correction to $\sin\theta_{13}$

Symmetries of VEVs (Mass Matrices)

- $U_{PMNS} = U_L U_\nu \;\rightarrow\; \mathrm{T}\chi\mathrm{M}_{(\chi=\pm\frac{\pi}{16})}, \, \mathrm{T}\phi\mathrm{M}_{(\phi=\pm\frac{\pi}{16})}$

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- TχM case:
  - Conjugation symmetry of M_{Maj} and 3×3 - trimaximal U_L
    $\rightarrow$ Generalised CP ($\mu-\tau$ reflection symmetry) of U_{PMNS}
    eg. F. Feruglio et al. 1211.5560 PFH, WGS hep-ph/0210197

Symmetries of VEVs (Mass Matrices)

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- $\mathrm{Diag}(-1, 1, -1)$ is not an element of $\Sigma(72 \times 3)$, so this symmetry is 'additional' in the model
- Some models implement the generalised CP and trimaximal column as the residual Klein symmetries of the flavour group

eg. C. S. Lam
0708.3665, 1104.0055 etc.

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- These are not elements of $\Sigma(72 \times 3)$, so this symmetry is also additional in the model
- Some models implement this symmetry as the residual symmetry of the flavour group $\Delta(6.16^2)$

M. Holthausen et al. 1212.2411 S. F. King et al 1305.3200

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- Flavon $(\xi, \phi_\alpha, \phi_\beta)$ potentials constructed for obtaining the required VEVs
- Additional symmetries arise from the features of the potential
- $\Sigma(72 \times 3)$: a frame work on which general fully-constrained complex-symmetric mass matrices can be constructed: only the first step in constructing such mass matrices
- Combining $\Sigma(72 \times 3)$ with additional groups to "explain" the full symmetries of the mass matrix: manuscript in preparation.

Outline

Summary

- A set of fully-constrained Majorana neutrino masses consistent with the neutrino masses and the mixing data
- $\Sigma(72 \times 3)$: the smallest discrete group which produces a complex sextet from two triplets
- The fully-constrained complex-symmetric mass matrices mapped to the VEVs of the sextets
- Flavon potential constructed, their spontaneous symmetry breaking leads to the VEVs
- Some residual symmetries of $\Sigma(72 \times 3)$ manifests as the symmetries of the VEVs (and mass matrices)

Character Table

$\Sigma(72\times 3)$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
$\#C_k$	1	1	1	24	9	9	9	18	18	18	18	18	18	18	18	18
$ord(C_k)$	1	3	3	3	2	6	6	4	12	12	4	12	12	4	12	12
$\mathbf{1}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}^{(0,1)}$	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
$\mathbf{1}^{(1,0)}$	1	1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
$\mathbf{1}^{(1,1)}$	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1
$\mathbf{2}$	2	2	2	2	-2	-2	-2	0	0	0	0	0	0	0	0	0
$\mathbf{3}$	3	3ω	$3\bar{\omega}$	0	-1	$-\omega$	$-\bar{\omega}$	1	ω	$\bar{\omega}$	1	ω	$\bar{\omega}$	1	ω	$\bar{\omega}$
$\mathbf{3}^{(0,1)}$	3	3ω	$3\bar{\omega}$	0	-1	$-\omega$	$-\bar{\omega}$	1	ω	$\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$
$\mathbf{3}^{(1,0)}$	3	3ω	$3\bar{\omega}$	0	-1	$-\omega$	$-\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$	1	ω	$\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$
$\mathbf{3}^{(1,1)}$	3	3ω	$3\bar{\omega}$	0	-1	$-\omega$	$-\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$	-1	$-\omega$	$-\bar{\omega}$	1	ω	$\bar{\omega}$
$\bar{\mathbf{3}}$	3	$3\bar{\omega}$	3ω	0	-1	$-\bar{\omega}$	$-\omega$	1	$\bar{\omega}$	ω	1	$\bar{\omega}$	ω	1	$\bar{\omega}$	ω
$\bar{\mathbf{3}}^{(0,1)}$	3	$3\bar{\omega}$	3ω	0	-1	$-\bar{\omega}$	$-\omega$	1	$\bar{\omega}$	ω	-1	$-\bar{\omega}$	$-\omega$	-1	$-\bar{\omega}$	$-\omega$
$\bar{\mathbf{3}}^{(1,0)}$	3	$3\bar{\omega}$	3ω	0	-1	$-\bar{\omega}$	$-\omega$	-1	$-\bar{\omega}$	$-\omega$	1	$\bar{\omega}$	ω	-1	$-\bar{\omega}$	$-\omega$
$\bar{\mathbf{3}}^{(1,1)}$	3	$3\bar{\omega}$	3ω	0	-1	$-\bar{\omega}$	$-\omega$	-1	$-\bar{\omega}$	$-\omega$	-1	$-\bar{\omega}$	$-\omega$	1	$\bar{\omega}$	ω
$\mathbf{6}$	6	$6\bar{\omega}$	6ω	0	2	$2\bar{\omega}$	2ω	0	0	0	0	0	0	0	0	0
$\bar{\mathbf{6}}$	6	6ω	$6\bar{\omega}$	0	2	2ω	$2\bar{\omega}$	0	0	0	0	0	0	0	0	0
$\mathbf{8}$	8	8	8	-1	0	0	0	0	0	0	0	0	0	0	0	0

Tensor Product Expansions

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\mathbf{6} \otimes \mathbf{3} = \mathbf{2} \oplus \mathbf{8} \oplus \mathbf{8}$$

$$\mathbf{6} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{3}^{(\mathbf{0,1})} \oplus \mathbf{3}^{(\mathbf{1,0})} \oplus \mathbf{3}^{(\mathbf{1,1})}$$

$$\mathbf{2} \otimes \bar{\mathbf{3}} = \mathbf{6}$$

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}^{(\mathbf{0,1})} \oplus \mathbf{1}^{(\mathbf{1,0})} \oplus \mathbf{1}^{(\mathbf{1,1})}$$

$$\mathbf{6} \otimes \mathbf{6} = \bar{\mathbf{6}} \oplus \bar{\mathbf{6}} \oplus \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{3}^{(\mathbf{0,1})} \oplus \mathbf{3}^{(\mathbf{1,0})} \oplus \mathbf{3}^{(\mathbf{1,1})}$$

$$\mathbf{6} \otimes \bar{\mathbf{6}} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{1}^{(\mathbf{0,1})} \oplus \mathbf{1}^{(\mathbf{1,0})} \oplus \mathbf{1}^{(\mathbf{1,1})} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8}$$