Assessing the Viability of A_4, S_4 and A_5 Flavour Symmetries for Description of Neutrino Mixing

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Outline

- 3-neutrino mixing
- Discrete symmetry approach to flavour
- Neutrino mixing sum rules
- Groups A_4, S_4 and A_5
- Viability of A_4, S_4 and A_5 flavour symmetries
- Conclusions

3-neutrino mixing

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \overline{\ell_L}(x)\,\gamma_\alpha\,\nu_{\ell L}(x)\,W^{\alpha\dagger}(x) + \mathrm{h.c.}$$

charged current weak interactions

$$\nu_{\ell L}(x) = \sum_{j=1}^{3} U_{\ell j}\,\nu_{jL}(x)$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix (3 × 3, unitary)

The standard parametrisation:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}\,, \quad s_{ij} \equiv \sin\theta_{ij}$$

3-neutrino mixing

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θ_{23}
atmospheric
mixing angle

θ_{13}
reactor
mixing angle

δ
Dirac phase

θ_{12}
solar
mixing angle

α_{21}, α_{31}
Majorana
phases

(only if neutrinos are Majorana)

3-neutrino mixing

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Leptons:

$\theta_{23} \approx 47^\circ$

$\theta_{13} \approx 8.5^\circ$

$\delta \approx 234^\circ \; (278^\circ)$ **?**

$\theta_{12} \approx 33.6^\circ$

$\alpha_{21}, \; \alpha_{31}$

?

NuFIT 3.2 (January 2018), www.nu-fit.org

3-neutrino mixing

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$\alpha_{21},\ \alpha_{31}$

?

NuFIT 3.2 (January 2018), www.nu-fit.org

Quarks:

$\theta^q_{23} \approx 2.4^\circ$

$\theta^q_{13} \approx 0.21^\circ$

$\delta^q \approx 66^\circ$

$\theta^q_{12} \approx 13^\circ$

No Majorana phases (Dirac particles)

Utfit (Summer 2016), www.utfit.org

3-neutrino mixing

Parameter	Best fit	3σ range
$\sin^2\theta_{12}$	0.307	$0.272 - 0.346$
$\sin^2\theta_{23}$ (NO)	0.538	$0.418 - 0.613$
$\sin^2\theta_{23}$ (IO)	0.554	$0.435 - 0.616$
$\sin^2\theta_{13}$ (NO)	0.02206	$0.01981 - 0.02436$
$\sin^2\theta_{13}$ (IO)	0.02227	$0.02006 - 0.02452$
δ [°] (NO)	234	$144 - 374$
δ [°] (IO)	278	$192 - 354$

NuFIT 3.2 (January 2018), www.nu-fit.org

Best fit	3σ range
0.304	$0.265 - 0.346$
0.551	$0.430 - 0.602$
0.557	$0.444 - 0.603$
0.0214	$0.0190 - 0.0239$
0.0218	$0.0195 - 0.0243$
238	$149 - 358$
274	$193 - 346$

Capozzi, Lisi, Marrone, Palazzo
arXiv:1804.09678 (April 2018)

NO = *normal ordering* of the neutrino mass spectrum: $m_1 < m_2 < m_3$

IO = *inverted ordering* of the neutrino mass spectrum: $m_3 < m_1 < m_2$

3-neutrino mixing

Parameter	Best fit	3σ range
$\sin^2\theta_{12}$	0.307	$0.272 - 0.346$
$\sin^2\theta_{23}$ (NO)	0.538	$0.418 - 0.613$
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- Preference for the *second octant*
- *Maximal mixing* ($\sin^2\theta_{23} = 0.5$) is compatible with the global data at 1σ (2σ) for NO (IO)

3-neutrino mixing

Parameter	Best fit	3σ range	Best fit	3σ range
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NuFIT 3.2 (January 2018), www.nu-fit.org

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- Nearly *maximal CP violation*: $\delta \sim 270°$
- CP-conserving value $\delta = 180°$ is disfavoured at $\sim 2\sigma$ (3σ) for NO (IO) and $\delta = 0°$ is disfavoured at $\sim 3\sigma$
- Significant part of the interval $0° - 180°$ is disfavoured at $> 3\sigma$

3-neutrino mixing

Parameter	Best fit	3σ range		Best fit	3σ range
$\sin^2\theta_{12}$	0.307	0.272 – 0.346	$\sim 1/3$	0.304	0.265 – 0.346
$\sin^2\theta_{23}$ (NO)	0.538	0.418 – 0.613	$\sim 1/2$	0.551	0.430 – 0.602
$\sin^2\theta_{23}$ (IO)	0.554	0.435 – 0.616		0.557	0.444 – 0.603
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NuFIT 3.2 (January 2018), www.nu-fit.org

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Is there any symmetry behind the observed pattern of neutrino mixing?

Lepton masses and mixing

Charged lepton mass term:

$$\overline{\ell_L}\, M_e\, \ell_R + \text{h.c.}, \quad \ell = (e, \mu, \tau)^T$$

Neutrino Majorana mass term (if neutrinos are Majorana particles):

$$\overline{(\nu_L)^c}\, M_\nu\, \nu_L + \text{h.c.}, \quad \nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T, \quad (\nu_{\ell L})^c = C\, \overline{\nu_{\ell L}}^T$$

Neutrino Dirac mass term (if right-handed neutrinos exist):

$$\overline{\nu_R}\, M_\nu^{\text{D}}\, \nu_L + \text{h.c.}, \quad \nu_R = (\nu_{1R}, \nu_{2R}, \nu_{3R})^T$$

Lepton masses and mixing originate from the mass matrices:

$$U_e^\dagger\, M_e\, V_e = \text{diag}(m_e, m_\mu, m_\tau)$$

$$U_\nu^T\, M_\nu\, U_\nu = \text{diag}(m_1, m_2, m_3)$$

The diagonalising matrices are 3×3 unitary matrices
The PMNS matrix:

$$\boxed{U = U_e^\dagger\, U_\nu}$$

Discrete symmetry approach to flavour

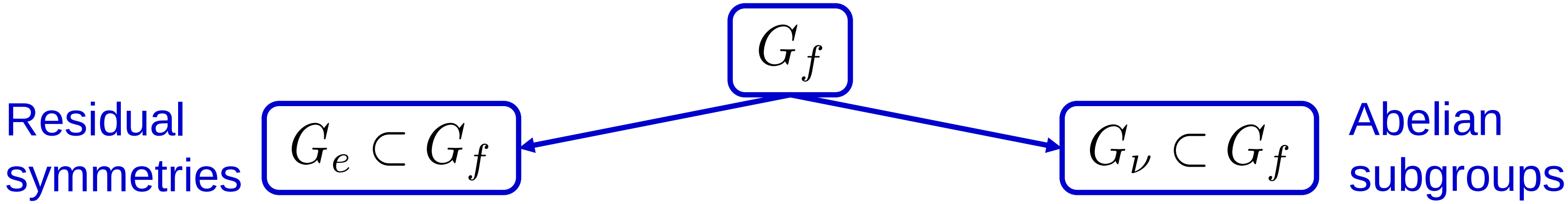
(Lepton) flavour symmetry $\leftrightarrow$ non-Abelian discrete (finite) group G_f

A theory at high energies is invariant under

$$\varphi(x) \xrightarrow{G_f} \rho_{\mathbf{r}}(g)\,\varphi(x)\,, \qquad g \in G_f$$

$\rho_{\mathbf{r}}(g)$ is the unitary representation matrix for g in the irrep $\mathbf{r}$

Usually $\mathbf{r} = \mathbf{3}$ for the left-handed charged lepton and neutrino fields



$$\ell_L(x) \xrightarrow{G_e} \rho_{\mathbf{3}}(g_e)\,\ell_L(x)\,, \quad g_e \in G_e$$

$$\rho_{\mathbf{3}}(g_e)^\dagger\, M_e M_e^\dagger\, \rho_{\mathbf{3}}(g_e) = M_e M_e^\dagger$$

$$U_e^\dagger\, M_e M_e^\dagger\, U_e = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

$$U_e^\dagger\, \rho_{\mathbf{3}}(g_e)\, U_e = \rho_{\mathbf{3}}(g_e)^{\mathrm{diag}}$$

$$\nu_L(x) \xrightarrow{G_\nu} \rho_{\mathbf{3}}(g_\nu)\,\nu_L(x)\,, \quad g_\nu \in G_\nu$$

$$\rho_{\mathbf{3}}(g_\nu)^T\, M_\nu\, \rho_{\mathbf{3}}(g_\nu) = M_\nu$$

$$U_\nu^T\, M_\nu\, U_\nu = \mathrm{diag}(m_1, m_2, m_3)$$

$$U_\nu^\dagger\, \rho_{\mathbf{3}}(g_\nu)\, U_\nu = \rho_{\mathbf{3}}(g_\nu)^{\mathrm{diag}}$$

Discrete symmetry approach to flavour

- G_e and G_ν are both $> Z_2 \Rightarrow U$ is fixed
  (up to Majorana phases and permutations of rows and columns)

  Example: tri-bimaximal (TBM) mixing from the S_4 group

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = 1/3 \qquad \theta_{12} \approx 35^\circ$$
$$\sin^2\theta_{23} = 1/2 \qquad \theta_{23} = 45^\circ$$
$$\sin^2\theta_{13} = 0 \qquad \theta_{13} = 0^\circ$$

- G_e, G_ν or both $= Z_2 \Rightarrow U$ contains free parameters (angles and phases)

$$\rho_{\mathbf{3}}\left(g_{e(\nu)}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad g_{e(\nu)}^2 = E$$

E is the identity of G_f

  This freedom leads to correlations between the mixing angles
  and/or the mixing angles and the Dirac phase,
  which are called neutrino mixing sum rules

Neutrino mixing sum rules

$$(\mathrm{A})\ \ G_e = Z_2 \text{ and } G_\nu = Z_k,\ k > 2 \text{ or } Z_m \times Z_n,\ m,n \geq 2$$

Girardi, Petcov, Stuart, AVT
NPB **902** (2016) 1

$$U = U_{ij}(\theta^e_{ij}, \delta^e_{ij})\, U^\circ(\theta^\circ_{12}, \theta^\circ_{13}, \theta^\circ_{23}, \delta^\circ_{kl})\, Q_0$$

Free complex rotation in the *i-j* plane

$U^\circ = (U^\circ_e)^\dagger U^\circ_\nu$ is *fixed* by symmetries

Contains *2 free phases* contributing to the *Majorana phases*

- Case **A1**: $(ij) = (12)$

$$\sin^2\theta_{23} = 1 - \frac{\cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}}{1 - \sin^2\theta_{13}}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{23} - \cos^2\theta_{12}) + \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\circ_{13}\cos\theta^\circ_{23}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23})^{\frac{1}{2}}}$$

- Case **A2**: $(ij) = (13)$

Analogous sum rules for $\sin^2\theta_{23}$ and $\cos\delta$

- Case **A3**: $(ij) = (23)$

$$\sin^2\theta_{13} = \sin^2\theta^\circ_{13} \qquad \sin^2\theta_{12} = \sin^2\theta^\circ_{12}$$

Neutrino mixing sum rules

$$\text{(B)}\ \ G_e = Z_k,\ k > 2 \text{ or } Z_m \times Z_n,\ m, n \geq 2 \text{ and } G_\nu = Z_2$$

Girardi, Petcov, Stuart, AVT
NPB **902** (2016) 1

$$U = U^\circ(\theta^\circ_{12}, \theta^\circ_{13}, \theta^\circ_{23}, \delta^\circ_{kl})\, U_{ij}(\theta^\nu_{ij}, \delta^\nu_{ij})\, Q_0$$

$U^\circ = (U^\circ_e)^\dagger U^\circ_\nu$ is *fixed* by symmetries

Free complex rotation in the *i-j* plane

Contains *2 free phases* contributing to the *Majorana phases*

- Case **B1**: $(ij) = (13)$

$$\sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1 - \sin^2\theta_{13}}$$

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{23}) + \sin^2\theta^\circ_{12}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\sin\theta^\circ_{12}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}$$

- Case **B2**: $(ij) = (23)$

Analogous sum rules for $\sin^2\theta_{12}$ and $\cos\delta$

- Case **B3**: $(ij) = (12)$

$$\sin^2\theta_{13} = \sin^2\theta^\circ_{13} \qquad \sin^2\theta_{23} = \sin^2\theta^\circ_{23}$$

Neutrino mixing sum rules

(C) $G_e = Z_2$ and $G_\nu = Z_2$ — Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1

$$U = U_{ij}(\theta^e_{ij}, \delta^e_{ij})\, U^\circ(\theta^\circ_{12}, \theta^\circ_{13}, \theta^\circ_{23}, \delta^\circ_{kl})\, U_{rs}(\theta^\nu_{rs}, \delta^\nu_{rs})\, Q_0$$

- $U_{ij}(\theta^e_{ij}, \delta^e_{ij})$: *Free* complex rotation in the *i-j* plane
- U°: $U^\circ = (U^\circ_e)^\dagger U^\circ_\nu$ is *fixed* by symmetries
- $U_{rs}(\theta^\nu_{rs}, \delta^\nu_{rs})$: *Free* complex rotation in the *r-s* plane
- Q_0: *2 free phases* contributing to the *Majorana phases*

Sum rules for $\cos\delta$:
- **C1**: $(ij, rs) = (12,13)$
- **C3**: $(ij, rs) = (12,23)$
- **C4**: $(ij, rs) = (13,23)$
- **C8**: $(ij, rs) = (13,13)$

Sum rules for $\sin^2\theta_{12}$:
- **C5**: $(ij, rs) = (23,13)$
- **C9**: $(ij, rs) = (23,23)$

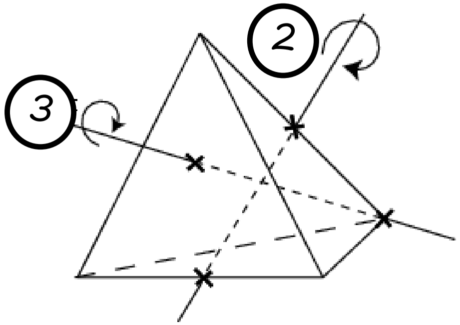
Sum rules for $\sin^2\theta_{23}$:
- **C2**: $(ij, rs) = (13,12)$
- **C7**: $(ij, rs) = (12,12)$

- **C6**: $(ij, rs) = (23,12)$ — $\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$

Groups A_4, S_4 and A_5

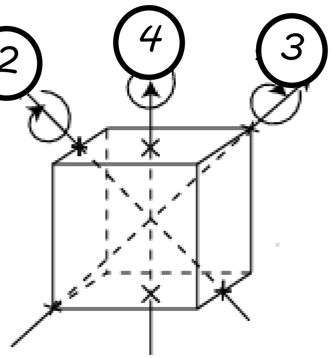
A_4 is the group of even permutations on 4 objects
$\cong$ the group of rotational symmetries
of a regular tetrahedron (12 elements)

$$S^2 = T^3 = (ST)^3 = E$$



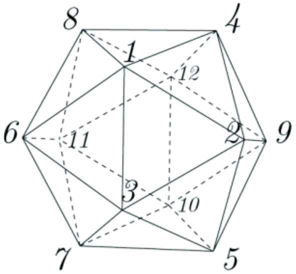
S_4 is the group of permutations on 4 objects
$\cong$ the group of rotational symmetries
of a cube (24 elements)

$$S^2 = T^3 = U^2 = (ST)^3$$
$$= (SU)^2 = (TU)^2 = (STU)^4 = E$$



A_5 is the group of even permutations on 5 objects
$\cong$ the group of rotational symmetries
of a regular icosahedron (60 elements)

$$S^2 = T^5 = (ST)^3 = E$$



Figures are adapted from Ishimori et al., *PTPS* **183** (2010) 1

Groups A_4, S_4 and A_5

Abelian subgroups

- A_4: 3 Z_2, 4 Z_3, 1 $K_4 \cong Z_2 \times Z_2$ (Klein)
- S_4: 9 Z_2, 4 Z_3, 3 Z_4, 4 $Z_2 \times Z_2$
- A_5: 15 Z_2, 10 Z_3, 5 $Z_2 \times Z_2$, 6 Z_5

For each pair of the residual symmetries (G_e, G_ν)

$$(U_e^\circ)^\dagger \, \rho_{\mathbf{3}}(g_e) \, U_e^\circ = \rho_{\mathbf{3}}(g_e)^{\mathrm{diag}} \qquad (U_\nu^\circ)^\dagger \, \rho_{\mathbf{3}}(g_\nu) \, U_\nu^\circ = \rho_{\mathbf{3}}(g_\nu)^{\mathrm{diag}}$$

$$U^\circ = (U_e^\circ)^\dagger U_\nu^\circ$$

Suitable parametrisation of U^{o} ⇒ values of the fixed parameters $\sin^2 \theta_{ij}^{\mathrm{o}}$

Results for A_4, S_4 and A_5

- A_4: *only 1* phenomenologically viable case using NuFIT 3.2 (January 2018) data for NO

Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1
Petcov, AVT, *PRD* **97** (2018) 115045

(G_e, G_ν)	Case	$\sin^2\theta^\circ_{ij}$	$\cos\delta$	$\sin^2\theta_{ij}$
(Z_3, Z_2)	B1	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	-0.353	$\sin^2\theta_{12} = 0.341$

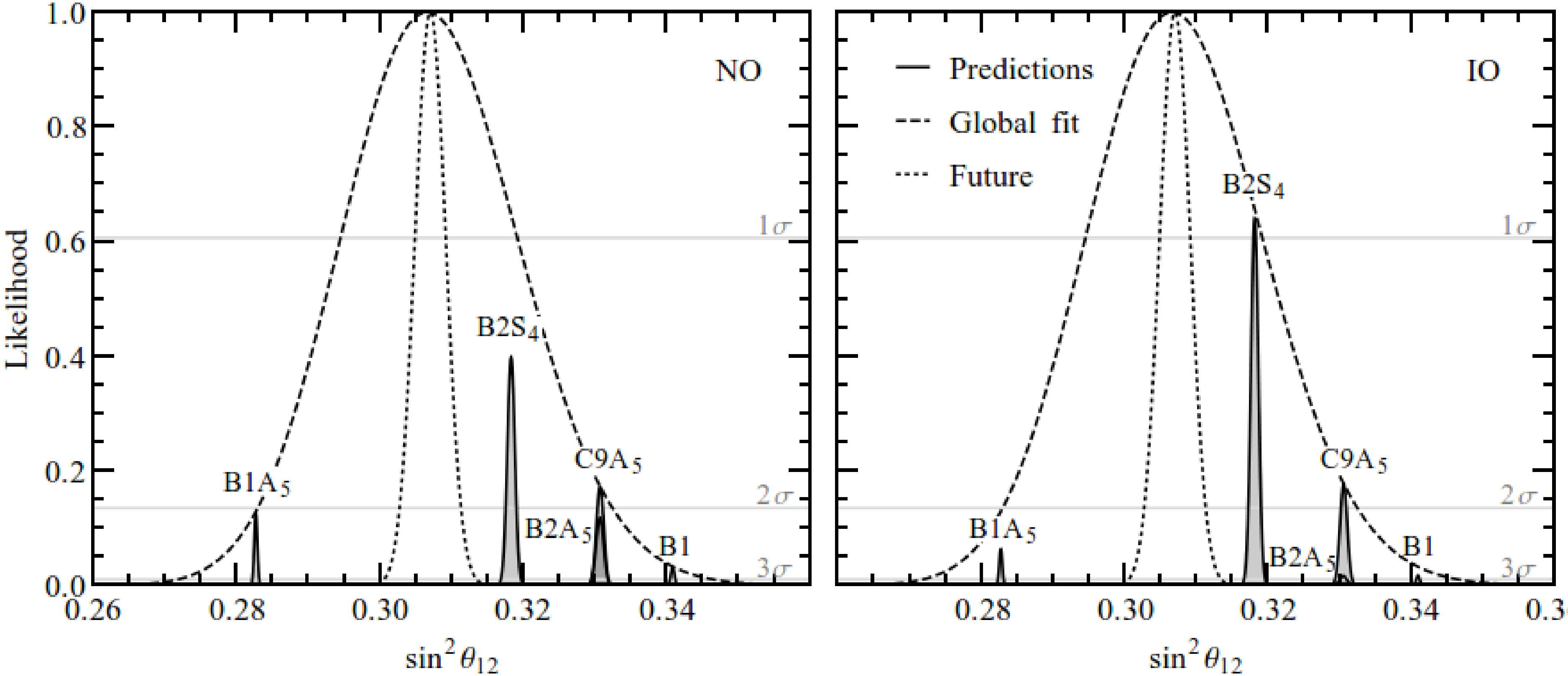
- S_4: *6 more* phenomenologically viable cases

(G_e, G_ν)	Case	$\sin^2\theta^\circ_{ij}$	$\cos\delta$	$\sin^2\theta_{ij}$
(Z_3, Z_2)	B1	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	-0.353	$\sin^2\theta_{12} = 0.341$
	B2S$_4$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (1/6, 1/5)$	0.167	$\sin^2\theta_{12} = 0.318$
(Z_2, Z_2)	C1	$\sin^2\theta^\circ_{23} = 1/4$	-1^*	not fixed
	C2S$_4$	$\sin^2\theta^\circ_{23} = 1/2$	not fixed	$\sin^2\theta_{23} = 0.511$
	C3	$\sin^2\theta^\circ_{13} = 1/4$	-1^*	not fixed
	C7S$_4$	$\sin^2\theta^\circ_{23} = 1/2$	not fixed	$\sin^2\theta_{23} = 0.489$
	C8	$\sin^2\theta^\circ_{23} = 3/4$	1^*	not fixed

- A_5: *7 more* phenomenologically viable cases

Cases predicting $\sin^2\theta_{12}$: present

Petcov, AVT, *PRD* **97** (2018) 115045

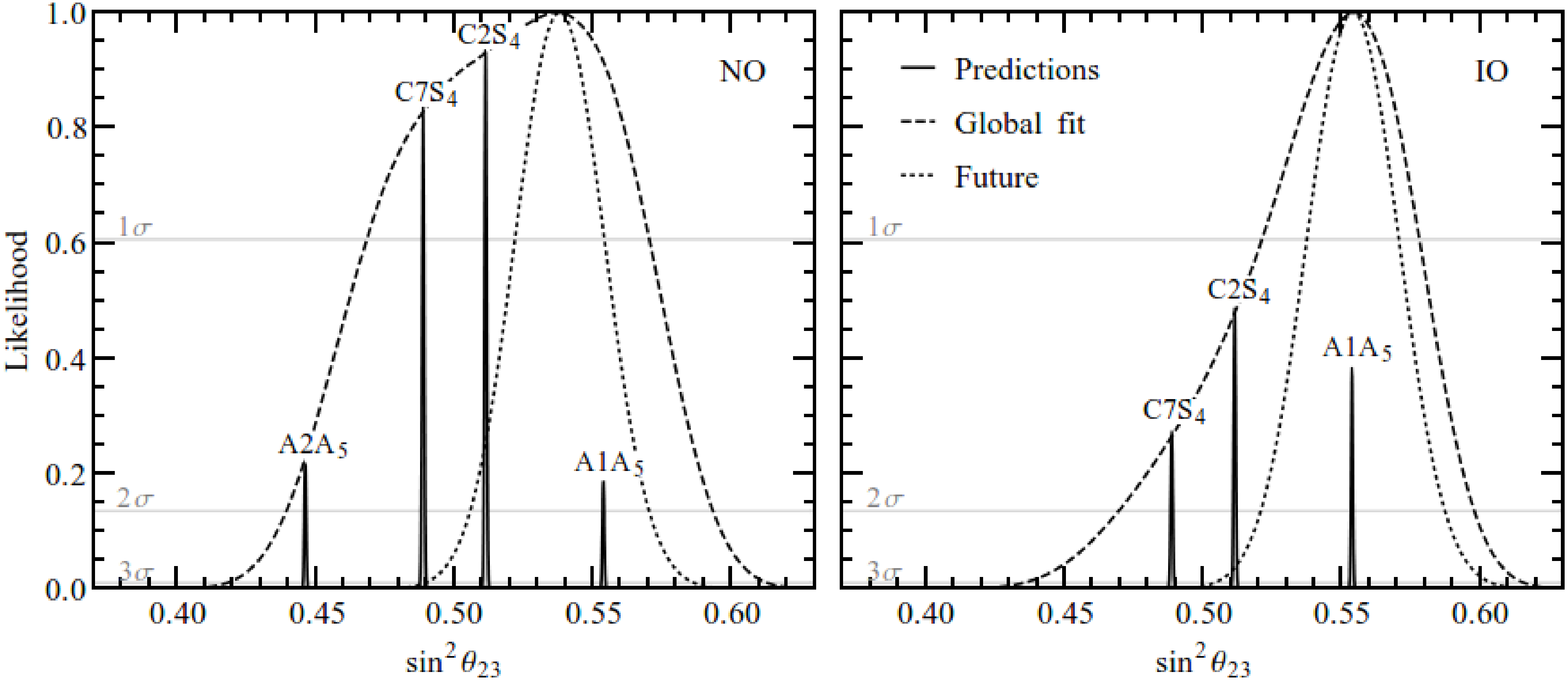


Future: $\sin^2\theta_{12}^{\text{true}} = 0.307$ (current best fit value)

$\sigma(\sin^2\theta_{12}) = 0.007 \times \sin^2\theta_{12}^{\text{true}}$ (medium-baseline JUNO experiment)

Cases predicting $\sin^2\theta_{23}$: present

Petcov, AVT, *PRD* **97** (2018) 115045

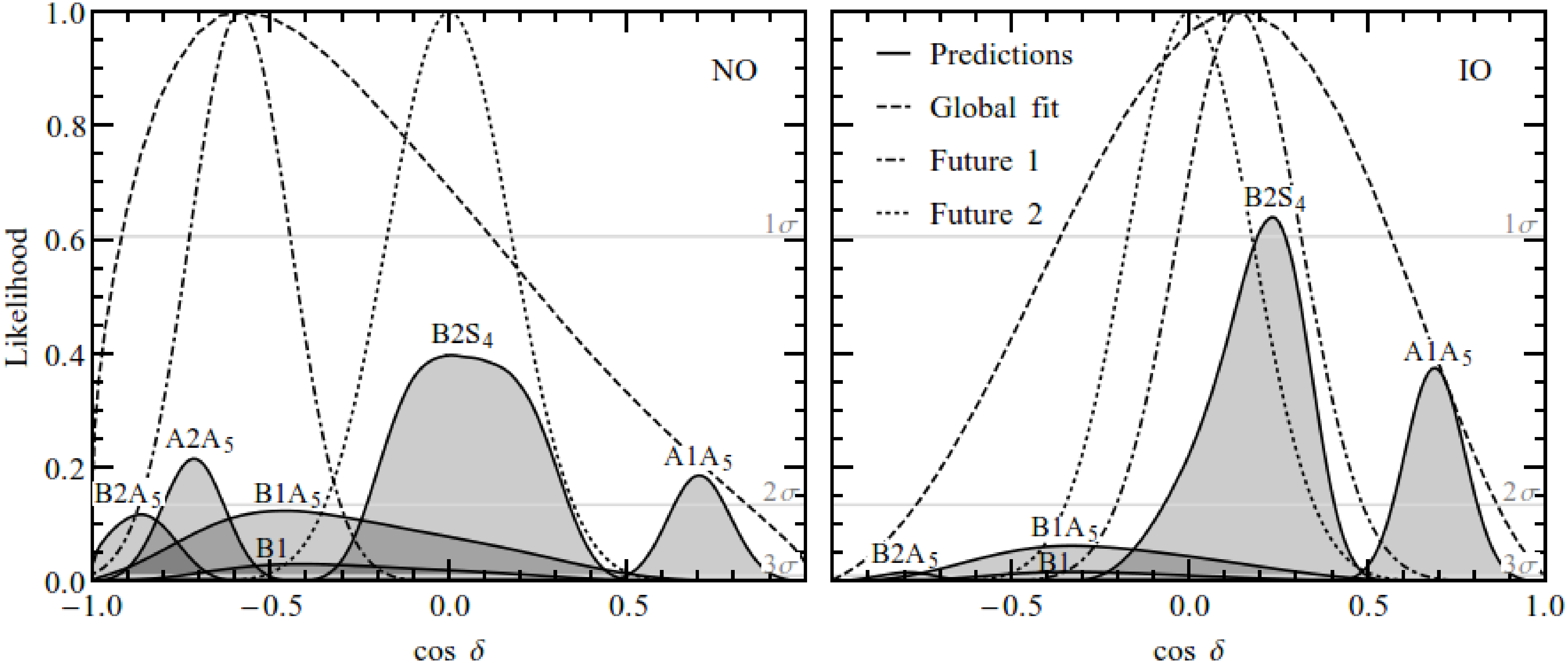


Future: $\sin^2\theta_{23}^{\text{true}} = 0.538\ (0.554)$ for NO (IO) (current best fit value)

$\sigma(\sin^2\theta_{23}) = 0.03 \times \sin^2\theta_{23}^{\text{true}}$ (long-baseline T2HK and DUNE)

Cases predicting $\cos\delta$: present

Petcov, AVT, *PRD* **97** (2018) 115045

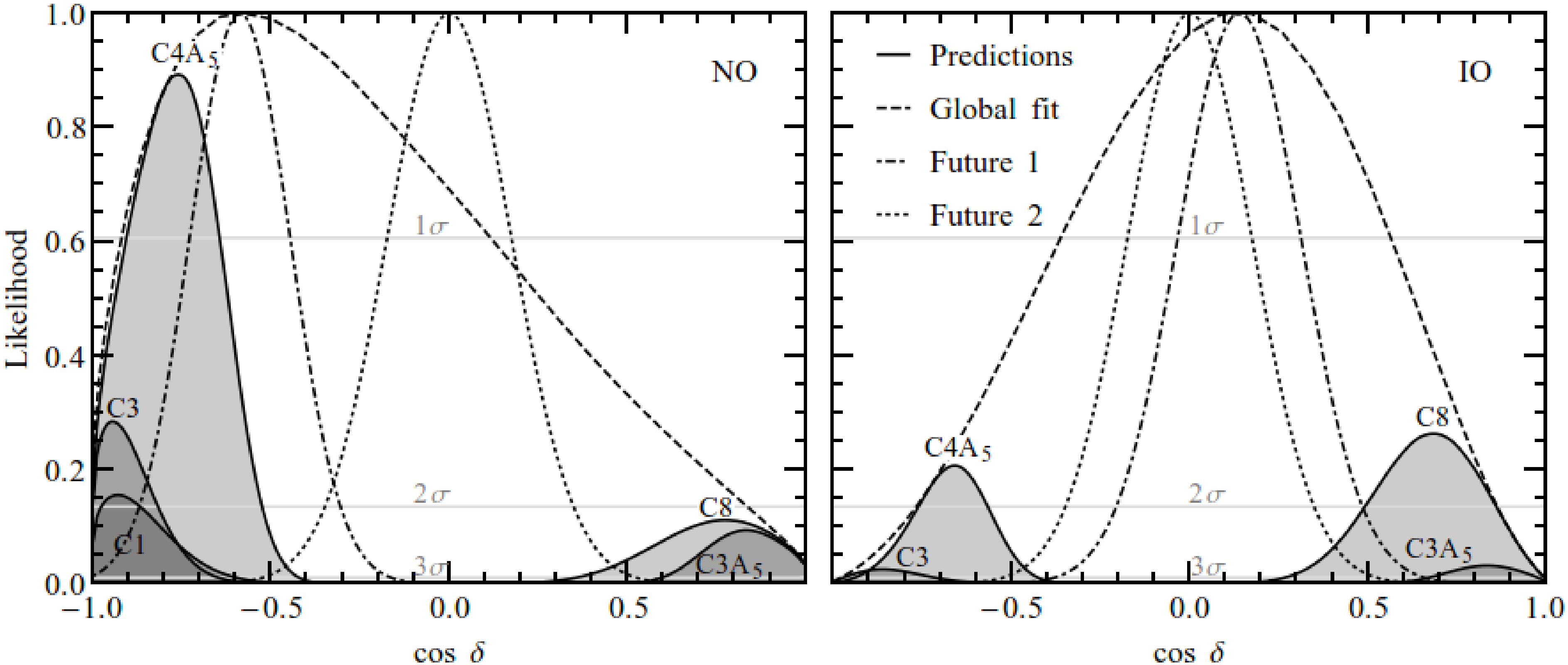


Future 1: $\delta^{\text{true}} = 234°$ $(278°)$ for NO (IO) (current b.f.v.), $\sigma(\delta) = 10°$

Future 2: $\delta^{\text{true}} = 270°$, $\sigma(\delta) = 10°$

Cases predicting $\cos\delta$: present

Petcov, AVT, *PRD* **97** (2018) 115045



Future 1: $\delta^{\text{true}} = 234°$ ($278°$) for NO (IO) (current b.f.v.), $\sigma(\delta) = 10°$

Future 2: $\delta^{\text{true}} = 270°$, $\sigma(\delta) = 10°$

Cases predicting $\sin^2\theta_{23}$: future

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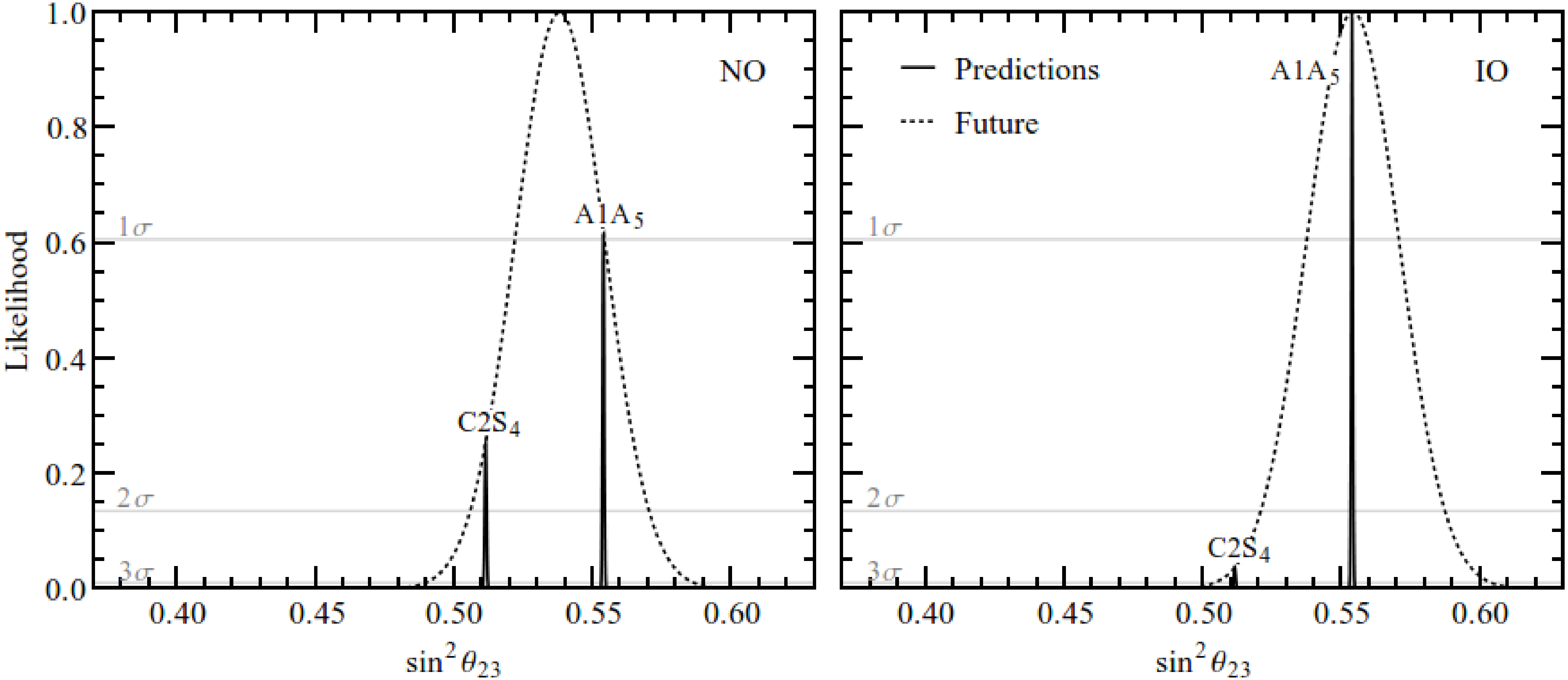

- current best fit values of $s_{12}^2, s_{13}^2, s_{23}^2$
- 0.7% on s_{12}^2 (JUNO), 3% on s_{13}^2 (Daya Bay), 3% on s_{23}^2 (T2HK/DUNE)
- no experimental information on δ

Cases predicting $\cos\delta$: future

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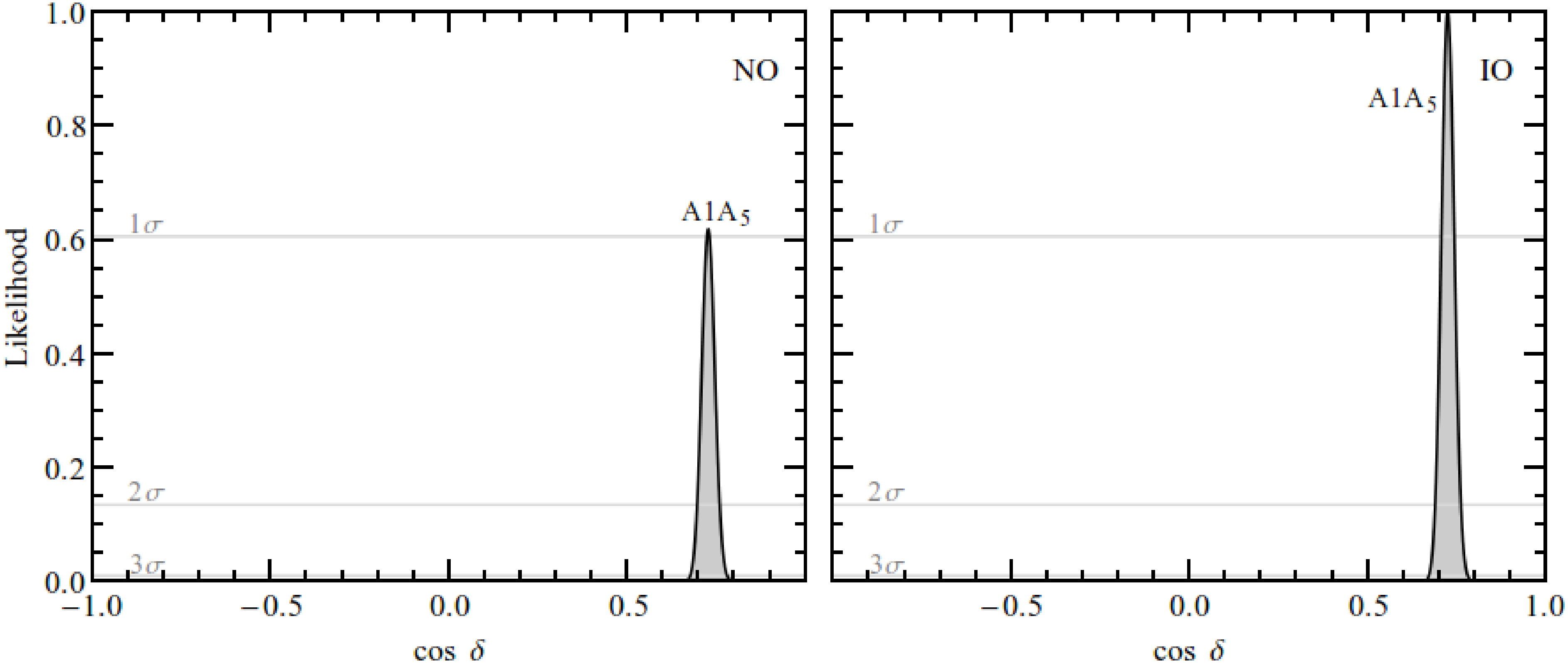
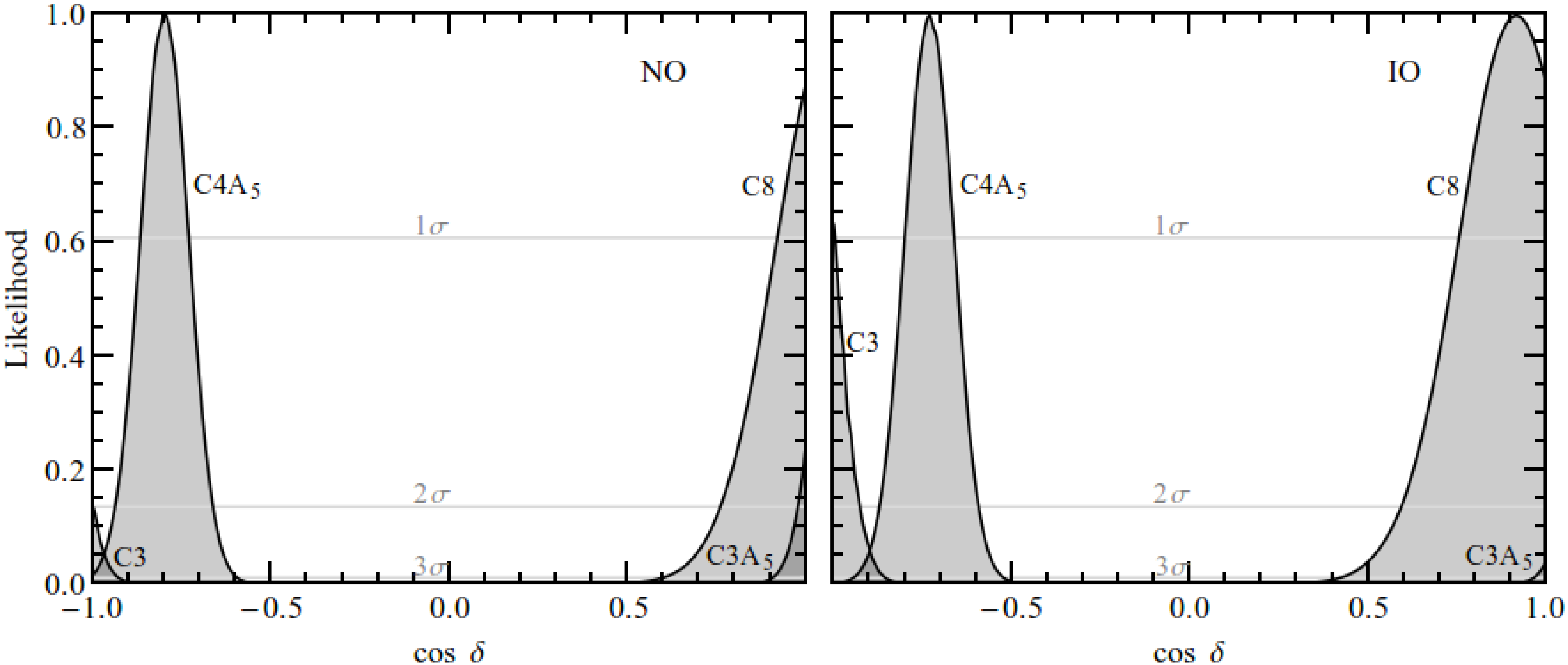


- current best fit values of $s^2_{12}, s^2_{13}, s^2_{23}$
- 0.7% on s^2_{12} (JUNO), 3% on s^2_{13} (Daya Bay), 3% on s^2_{23} (T2HK/DUNE)
- no experimental information on δ

Cases predicting $\cos\delta$: future

Petcov, AVT, *PRD* **97** (2018) 115045



- current best fit values of $s_{12}^2, s_{13}^2, s_{23}^2$
- 0.7% on s_{12}^2 (JUNO), 3% on s_{13}^2 (Daya Bay), 3% on s_{23}^2 (T2HK/DUNE)
- no experimental information on δ

Conclusions

- A_4, S_4 and A_5 discrete flavour symmetries broken down to non-trivial residual symmetries in such a way that at least one of them is a Z_2 represent a viable possibility

- 14 cases in total are compatible at 3σ with the present global neutrino oscillation data

- 6 cases survive the prospective constraints on the neutrino mixing angles

- The number of viable cases is likely to be further reduced by a high precision measurement of δ

Backup slides

Summary of sum rules for $\sin^2\theta_{ij}$

Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1

Case	Parametrisation of the PMNS matrix U	Sum rule for $\sin^2\theta_i$
A1	$U_{12}(\theta^e_{12},\delta^e_{12})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,R_{23}(\theta^\circ_{23})\,R_{13}(\theta^\circ_{13})\,Q_0$	$\sin^2\theta_{23} = \dfrac{\sin^2\theta^\circ_{13} - \sin^2\theta_{13} + \cos^2\theta^\circ_{13}\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}$
A2	$U_{13}(\theta^e_{13},\delta^e_{13})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,R_{23}(\theta^\circ_{23})\,R_{12}(\theta^\circ_{12})\,Q_0$	$\sin^2\theta_{23} = \dfrac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}$
A3	$U_{23}(\theta^e_{23},\delta^e_{23})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,R_{13}(\theta^\circ_{13})\,R_{12}(\theta^\circ_{12})\,Q_0$	$\sin^2\theta_{13} = \sin^2\theta^\circ_{13}\,,\quad \sin^2\theta_{12} = \sin^2\theta^\circ_{12}$
B1	$R_{23}(\theta^\circ_{23})\,R_{12}(\theta^\circ_{12})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,U_{13}(\theta^\nu_{13},\delta^\nu_{13})\,Q_0$	$\sin^2\theta_{12} = \dfrac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}$
B2	$R_{13}(\theta^\circ_{13})\,R_{12}(\theta^\circ_{12})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,U_{23}(\theta^\nu_{23},\delta^\nu_{23})\,Q_0$	$\sin^2\theta_{12} = \dfrac{\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}}{1-\sin^2\theta_{13}}$
B3	$R_{23}(\theta^\circ_{23})\,R_{13}(\theta^\circ_{13})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,U_{12}(\theta^\nu_{12},\delta^\nu_{12})\,Q_0$	$\sin^2\theta_{13} = \sin^2\theta^\circ_{13}\,,\quad \sin^2\theta_{23} = \sin^2\theta^\circ_{23}$

$$\text{(A)}\;\; G_e = Z_2 \text{ and } G_\nu = Z_n,\; n>2 \text{ or } Z_n\times Z_m,\; n,m\geq 2$$

$$\text{(B)}\;\; G_e = Z_n,\; n>2 \text{ or } Z_n\times Z_m,\; n,m\geq 2 \text{ and } G_\nu = Z_2$$

Summary of sum rules for $\cos\delta$

Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1

Case	Sum rule for $\cos\delta$
A1	$\frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{23} - \cos^2\theta_{12}) + \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}\lvert\cos\theta^\circ_{13}\cos\theta^\circ_{23}\rvert(\cos^2\theta_{13} - \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23})^{\frac{1}{2}}}$
A2	$-\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{12}) + \sin^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}\lvert\sin\theta^\circ_{23}\rvert(\cos^2\theta_{13} - \sin^2\theta^\circ_{23})^{\frac{1}{2}}}$
A3	$\pm\cos\hat{\delta}_{23}$
B1	$-\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{23}) + \sin^2\theta^\circ_{12}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}\lvert\sin\theta^\circ_{12}\rvert(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}$
B2	$\frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{12} - \cos^2\theta_{23}) + \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}\lvert\cos\theta^\circ_{12}\cos\theta^\circ_{13}\rvert(\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13})^{\frac{1}{2}}}$
B3	$\pm\cos\hat{\delta}_{12}$

(A) $G_e = Z_2$ and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$

(B) $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and $G_\nu = Z_2$

Summary of sum rules for $\sin^2\theta_{ij}$

Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1

Case	Parametrisation of the PMNS matrix U	Sum rule for $\sin^2\theta_{ij}$
C1	$U_{12}(\theta^e_{12},\delta^e_{12})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,R_{23}(\theta^\circ_{23})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,U_{13}(\theta^\nu_{13},\delta^\nu_{13})\,Q_0$	not fixed
C2	$U_{13}(\theta^e_{13},\delta^e_{13})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,R_{23}(\theta^\circ_{23})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,U_{12}(\theta^\nu_{12},\delta^\nu_{12})\,Q_0$	$\sin^2\theta_{23} = \dfrac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}$
C3	$U_{12}(\theta^e_{12},\delta^e_{12})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,R_{13}(\theta^\circ_{13})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,U_{23}(\theta^\nu_{23},\delta^\nu_{23})\,Q_0$	not fixed
C4	$U_{13}(\theta^e_{13},\delta^e_{13})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,R_{12}(\theta^\circ_{12})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,U_{23}(\theta^\nu_{23},\delta^\nu_{23})\,Q_0$	not fixed
C5	$U_{23}(\theta^e_{23},\delta^e_{23})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,R_{12}(\theta^\circ_{12})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,U_{13}(\theta^\nu_{13},\delta^\nu_{13})\,Q_0$	$\sin^2\theta_{12} = \dfrac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}$
C6	$U_{23}(\theta^e_{23},\delta^e_{23})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,R_{13}(\theta^\circ_{13})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,U_{12}(\theta^\nu_{12},\delta^\nu_{12})\,Q_0$	$\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$
C7	$U_{12}(\theta^e_{12},\delta^e_{12})\,U_{12}(\theta^\circ_{12},\delta^\circ_{12})\,R_{23}(\theta^\circ_{23})\,U_{12}(\tilde\theta^\circ_{12},\tilde\delta^\circ_{12})\,U_{12}(\theta^\nu_{12},\delta^\nu_{12})\,Q_0$	$\sin^2\theta_{23} = \dfrac{\sin^2\theta^\circ_{23}-\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$
C8	$U_{13}(\theta^e_{13},\delta^e_{13})\,U_{13}(\theta^\circ_{13},\delta^\circ_{13})\,R_{23}(\theta^\circ_{23})\,U_{13}(\tilde\theta^\circ_{13},\tilde\delta^\circ_{13})\,U_{13}(\theta^\nu_{13},\delta^\nu_{13})\,Q_0$	not fixed
C9	$U_{23}(\theta^e_{23},\delta^e_{23})\,U_{23}(\theta^\circ_{23},\delta^\circ_{23})\,R_{12}(\theta^\circ_{12})\,U_{23}(\tilde\theta^\circ_{23},\tilde\delta^\circ_{23})\,U_{23}(\theta^\nu_{23},\delta^\nu_{23})\,Q_0$	$\sin^2\theta_{12} = \dfrac{\sin^2\theta^\circ_{12}-\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$

(C) $G_e = Z_2$ and $G_\nu = Z_2$

Summary of sum rules for $\cos\delta$

Girardi, Petcov, Stuart, AVT, *NPB* **902** (2016) 1

Case	Sum rule for $\cos\delta$
C1	$\dfrac{\sin^2\theta^\circ_{23} - \cos^2\theta_{12}\sin^2\theta_{23} - \cos^2\theta_{23}\sin^2\theta_{12}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}$
C2	$\dfrac{\cos^2\theta_{13}(\cos^2\theta^\circ_{23}\sin^2\hat{\theta}^\nu_{12} - \sin^2\theta_{12}) + \sin^2\theta^\circ_{23}(\sin^2\theta_{12} - \cos^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}\lvert\sin\theta^\circ_{23}\rvert(\cos^2\theta_{13} - \sin^2\theta^\circ_{23})^{\frac{1}{2}}}$
C3	$\dfrac{\sin^2\theta_{12}\sin^2\theta_{23} - \sin^2\theta^\circ_{13} + \cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}$
C4	$\dfrac{\sin^2\theta^\circ_{12} - \cos^2\theta_{23}\sin^2\theta_{12} - \cos^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}$
C5	$\dfrac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\sin^2\hat{\theta}^e_{23} - \sin^2\theta_{23}) + \sin^2\theta^\circ_{12}(\sin^2\theta_{23} - \cos^2\theta_{23}\sin^2\theta_{13})}{\sin 2\theta_{23}\sin\theta_{13}\lvert\sin\theta^\circ_{12}\rvert(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}$
C6	$\pm\cos\hat{\delta}$
C7	$\dfrac{\sin^2\theta_{13}(\cos^2\theta_{12}\cos^2\theta^\circ_{23} - \sin^2\theta_{12}) + \sin^2\theta^\circ_{23}(\sin^2\theta_{12} - \cos^2\theta_{13}\sin^2\hat{\theta}^\nu_{12})}{\sin 2\theta_{12}\sin\theta_{13}\lvert\cos\theta^\circ_{23}\rvert(\sin^2\theta^\circ_{23} - \sin^2\theta_{13})^{\frac{1}{2}}}$
C8	$\dfrac{\cos^2\theta_{12}\cos^2\theta_{23} - \cos^2\theta^\circ_{23} + \sin^2\theta_{12}\sin^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}$
C9	$\dfrac{\sin^2\theta_{13}(\cos^2\theta_{23}\cos^2\theta^\circ_{12} - \sin^2\theta_{23}) + \sin^2\theta^\circ_{12}(\sin^2\theta_{23} - \cos^2\theta_{13}\sin^2\hat{\theta}^e_{23})}{\sin 2\theta_{23}\sin\theta_{13}\lvert\cos\theta^\circ_{12}\rvert(\sin^2\theta^\circ_{12} - \sin^2\theta_{13})^{\frac{1}{2}}}$

Results for A_5

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Using NuFIT 3.2 (January 2018) data for NO

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(G_e, G_ν)	Case	$\sin^2\theta^\circ_{ij}$	$\cos\delta$	$\sin^2\theta_{ij}$
(Z_2, Z_3)	$\mathrm{A1A_5}$	$(\sin^2\theta^\circ_{13}, \sin^2\theta^\circ_{23}) = (0.226, 0.436)$	0.727	$\sin^2\theta_{23} = 0.554$
	$\mathrm{A2A_5}$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.226, 0.436)$	-0.727	$\sin^2\theta_{23} = 0.446$
(Z_3, Z_2)	B1	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	-0.353	$\sin^2\theta_{12} = 0.341$
(Z_5, Z_2)	$\mathrm{B1A_5}$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.276, 1/2)$	-0.405	$\sin^2\theta_{12} = 0.283$
$(Z_2 \times Z_2, Z_2)$	$\mathrm{B2A_5}$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (0.095, 0.276)$	-0.936	$\sin^2\theta_{12} = 0.331$
(Z_2, Z_2)	C1	$\sin^2\theta^\circ_{23} = 1/4$	-1^*	not fixed
	$\mathrm{C3A_5}$	$\sin^2\theta^\circ_{13} = 0.095$	1^*	not fixed
	C3	$\sin^2\theta^\circ_{13} = 1/4$	-1^*	not fixed
	$\mathrm{C4A_5}$	$\sin^2\theta^\circ_{12} = 0.095$	-0.799	not fixed
	C8	$\sin^2\theta^\circ_{23} = 3/4$	1^*	not fixed
	$\mathrm{C9A_5}$	$\sin^2\theta^\circ_{12} = 0.345$	not fixed	$\sin^2\theta_{12} = 0.331$

Details of statistical analysis

Total χ^2 function (present): $\chi^2(\vec{x}) = \sum_{i=1}^{4} \chi_i^2(x_i)$

$\vec{x} = (\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, \delta)$

χ_i^2 are the 1-dimensional projections from a global analysis

Total χ^2 function (future): $\chi^2_{\text{future}}(\vec{y}) = \sum_{i=1}^{3} \frac{(y_i - \overline{y}_i)^2}{\sigma_{y_i}^2}$

$\vec{y} = (\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23})$, $\overline{y}_i$ are the potential best fit values

σ_{y_i} are the prospective 1σ uncertainties

Minimisation of total χ^2 for a fixed value of α ($\alpha = \sin^2\theta_{12}, \sin^2\theta_{23}$ or $\cos\delta$):

$$\chi^2(\alpha) = \min \left[\chi^2(\vec{x}) \Big|_{\substack{\text{sum rules} \\ \alpha = \text{const}}} \right]$$

Likelihood:

$$L(\alpha) = \exp\left(-\frac{\chi^2(\alpha)}{2} \right)$$