Predicting δ^{PMNS} and θ_{23}^{PMNS} from GUT flavour models with CSD2

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Ongoing work

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Outline

- Neutrino mass-mixing in Flavour GUTs
- Implementing CSD2 in GUTs
- Predictive set-up (SU(5) based models)
- Outlook and Conclusions

(Neutrino) Flavor puzzle

Lepton Sector

$\theta_{23}^{L} \approx 47.2^{\circ}, \theta_{13}^{L} \approx 8.5^{\circ}, \theta_{12}^{L} \approx 33.6^{\circ}$

$m_e \approx 0.51\,\text{MeV}, m_\mu \approx 105.6\,\text{MeV}, m_\tau \approx 1.77\,\text{GeV}$

$\Delta m_{21}^2 \approx 7.40\times 10^{-5}\; eV^2, \Delta m_{31}^2 \approx 2.49\times 10^{-3}\; eV^2$

Quark Sector

$\theta_{23}^{Q} \approx 2.5^{\circ}, \theta_{13}^{Q} \approx 0.2^{\circ}, \theta_{12}^{Q} \approx 13^{\circ}$

$m_u \approx 2.2\,\text{MeV}, m_c \approx 1.2\,\text{GeV}, m_t \approx 173\,\text{GeV}$

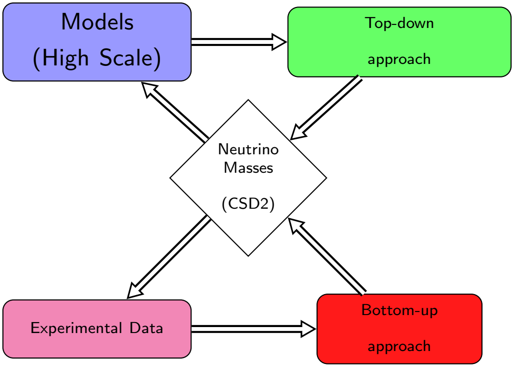
$m_d \approx 4.7\,\text{MeV}, m_s \approx 95\,\text{MeV}, m_b \approx 4.18\,\text{GeV}$

Additional questions for neutrinos

- Mass generation : New Physics Scale ?
- Dirac or Majorana
- Neutrino Hierarchy : Normal versus Inverted

Neutrino mixing in Flavor GUTs[1]

- **Top-down approach** : **Flavor GUTs** and many more
- **Bottom up approach** : Data guidance



[1] Review by S. F. King, Prog. Part. Nucl. Phys. **94**, 217 (2017) (and references therein)

Bottom up approach

Tri-bimaximal mixing :

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

After $\theta_{13}^{\text{PMNS}}$ measurement

1. If not TB mixing, alternative structures : Trimaximal TM_1 or TM_2, the reactor angle is a free parameter
2. Consider TB as a leading order pattern, include charged lepton corrections (more natural possibility in GUTs)

Constrained Sequential Dominance[2]

$$Y_\nu = \begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_A & 0 & 0 \\ 0 & M_B & 0 \\ 0 & 0 & M_C \end{pmatrix}$$

Type I see-saw :

$$M_\nu = v^2 Y_\nu M_R^{-1} Y_\nu^T = v^2 \left[\frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C} \right]$$

- Sequential dominance ($M_A \ll M_B \ll M_C$)
- The ordering $\frac{AA^T}{M_A} \gg \frac{BB^T}{M_B} \gg \frac{CC^T}{M_C}$ corresponds to strong normal hierarchy i.e. $m_3^\nu \gg m_2^\nu \gg m_1^\nu$

$$\tan\theta_{23}^\nu \approx \frac{|A_2|}{|A_3|} \quad \tan\theta_{12}^\nu \approx \frac{|B_1|}{c_{23}^\nu |B_2| \cos(\phi'_{B_2}) - s_{23}^\nu |B_3| \cos(\phi'_{B_3})} \quad \theta_{13}^\nu \approx \frac{|B_1|(A_2^* B_2 + A_3^* B_3) M_A}{[|A_2|^2 + |A_3|^2]^{3/2} M_B}$$

$$|B_1| = |B_2| = |B_3|, |A_1| = 0, |A_2| = |A_3|, \phi'_{B_2} = 0, \phi'_{B_3} = \pi(\phi_{B_3} - \phi_{B_2} + \phi_{A_2} - \phi_{A_3} = \pi)$$

- Combining SD and vacuum alignment → **CSD**
- Provides TB mixing ($\theta_{13}^\nu = 0$)

[2] S. F. King, JHEP **0508**, 105 (2005),[hep-ph/0506297]

CSD2[3]

Consider the same dominant flavon VEV but different sub dominant flavon choice ϕ_{120} or ϕ_{102}, the neutrino Yukawa matrix will be

$$Y_\nu^{(120)} = \begin{pmatrix} 0 & b \\ a & 2b \\ -a & 0 \end{pmatrix} \quad \text{or} \quad Y_\nu^{(102)} = \begin{pmatrix} 0 & b \\ a & 0 \\ -a & 2b \end{pmatrix}$$

then, mass matrix for the left handed neutrinos

$$M_\nu = v^2 \left[\frac{AA^T}{M_A} + \frac{BB^T}{M_B} \right] = m_a \begin{pmatrix} \epsilon e^{i\alpha} & 2\epsilon e^{i\alpha} & 0 \\ 2\epsilon e^{i\alpha} & 1 + 4\epsilon e^{i\alpha} & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

where $m_a = \frac{v^2 a^2}{M_A}$ and $m_b = \frac{v^2 b^2}{M_B}$, $\frac{m_b}{m_a} = \epsilon e^{i\alpha}$ ($M_A \ll M_B$ we get $|m_b| \ll |m_a|$)

$$\theta_{23}^\nu \approx \frac{\pi}{4} + \epsilon \cos\alpha + \epsilon^2 \left(\frac{3}{2} - \cos 2\alpha \right)$$

$$\theta_{12}^\nu \approx \arcsin \frac{1}{\sqrt{3}} - \frac{\epsilon^2}{2\sqrt{2}}$$

$$\theta_{13}^\nu \approx \frac{\epsilon}{\sqrt{2}} + \frac{\epsilon^2}{2\sqrt{2}} \cos\alpha \sim 5^\circ - 6^\circ$$ **(include charged lepton corrections)**

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[3] S. Antusch, S. F. King, C. Luhn and M. Spinrath, Nucl. Phys. B **856** (2012) 328, [arXiv:1108.4278 [hep-ph]].

Charged lepton corrections in CSD2[4]

$$\theta_{23}^{\text{PMNS}} \approx 45^0 - \epsilon \cos \alpha$$
$$\theta_{13}^{\text{PMNS}} \approx \frac{1}{\sqrt{2}} \left(\epsilon^2 + {\theta_{12}^{e}}^2 + 2\epsilon\theta_{12}^{e} \cos\left(\alpha - \beta\right)\right)^{1/2}$$
$$\theta_{12}^{\text{PMNS}} \approx 35.3^0 - \cos \beta \frac{\theta_{12}^{e}}{\sqrt{2}}$$

[4] S. Antusch, S. F. King and M. Spinrath, Phys. Rev. D **87**, no. 9, 096018 (2013), [arXiv:1301.6764 [hep-ph]].

CSD2 embedding in GUTs

GUTs

- Natural home for neutrino masses (For example SO(10))
- Joint representation for quark and leptons : **quark and lepton Yukawas are related**

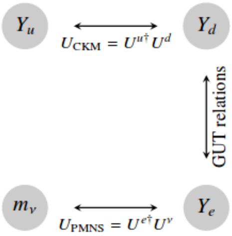


Figure : Fermion Yukawas in GUTs [5]

- **Flavor symmetries + GUTs** : Higher dimensional operators needed in some cases to generate fermion masses

[5] Figure taken from S. Antusch, Nucl. Phys. Proc. Suppl. **235-236**, 303 (2013), [arXiv:1301.5511 [hep-ph]]

Example Clebsch factor in SU(5)[6]

$$5_F = (d^1 \ d^2 \ d^3 \ e^c \ -\nu^c)$$

$$24_h = v_{24} diag(2, 2, 2, -3, -3)$$

$$A = 5_F \quad B = 24_H$$

$$C = \overline{10}_F \quad D = 45_H$$

$\rightarrow \frac{y_e}{y_d} = \frac{9}{2}$,
Left vertex : vev of 24_H, relative factor of $\frac{-3}{2}$
Right vertex : vev of 45_H provide factor of -3

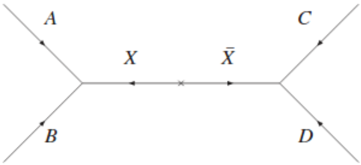


Figure : Higher dimensional operator which could lead to GUT relations between quark and lepton masses

[6] S. Antusch and M. Spinrath, Phys. Rev. D **79**, 095004 (2009), [arXiv:0902.4644 [hep-ph]].

Model Set-up

- SU(5) GUT based SUSY flavour models
- CSD2 form for neutrino masses
- Predictive Yukawa texture
- Single operator dominance for each Yukawa relation

Yukawa Texture

$$Y_d = \begin{pmatrix} 0 & z & 0 \\ ye^{i\tilde{\beta}} & x & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = c_{ij} Y_d^T = \begin{pmatrix} 0 & c_y ye^{i\tilde{\beta}} & 0 \\ c_z z & c_x x & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$Y_u = \begin{pmatrix} \star & i\star & 0 \\ \star & \star & \star \\ \star & \star & y_t \end{pmatrix}$$

$$y_d \approx \left|\frac{yz}{x}\right|, \quad y_s \approx |x|, \quad y_e \approx \left|\frac{c_y c_z}{c_x}\right|\left|\frac{yz}{x}\right|, \quad y_\mu \approx |c_x||x|$$

The CSD2 mechanism provides two choices of flavon VEVs which determine the neutrino Yukawa matrices, i.e. $Y_\nu^{(102)}$ and $Y_\nu^{(120)}$,

$$Y_\nu^{(120)} = \begin{pmatrix} 0 & b \\ a & 2b \\ -a & 0 \end{pmatrix} \quad \text{or} \quad Y_\nu^{(102)} = \begin{pmatrix} 0 & b \\ a & 0 \\ -a & 2b \end{pmatrix}$$

$$M_\nu^{(102)} = m_a \begin{pmatrix} \epsilon e^{i\alpha} & 0 & 2\epsilon e^{i\alpha} \\ 0 & 1 & -1 \\ 2\epsilon e^{i\alpha} & -1 & 1+4\epsilon e^{i\alpha} \end{pmatrix}, \quad M_\nu^{(120)} = m_a \begin{pmatrix} \epsilon e^{i\alpha} & 2\epsilon e^{i\alpha} & 0 \\ 2\epsilon e^{i\alpha} & 1+4\epsilon e^{i\alpha} & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Clebsches considered[7]

Double ratio

$$d = \frac{y_\mu y_d}{y_e y_s} = \left| \frac{c_x^2}{c_y c_z} \right| \approx 10.7$$

c_x, c_y, c_z	c_x, c_y, c_z	c_x, c_y, c_z
$3, \frac{1}{6}, \frac{9}{2}$	$\frac{9}{2}, \frac{1}{2}, \frac{9}{2}$	$6, \frac{1}{2}, 6$
$3, \frac{1}{6}, 6$	$\frac{9}{2}, \frac{2}{3}, 3$	$6, \frac{2}{3}, \frac{9}{2}$
$3, \frac{1}{2}, \frac{3}{2}$	$\frac{9}{2}, 1, 2$	$6, \frac{2}{3}, 6$
$3, \frac{1}{2}, 2$	$\frac{9}{2}, \frac{3}{2}, \frac{3}{2}$	$6, 1, 3$
$3, \frac{2}{3}, \frac{3}{2}$	$\frac{9}{2}, 2, 1$	$6, \frac{3}{2}, 2$
$3, 1, 1$	$\frac{9}{2}, 3, \frac{2}{3}$	$6, 2, \frac{3}{2}$
$3, \frac{3}{2}, \frac{1}{2}$	$\frac{9}{2}, \frac{9}{2}, \frac{1}{2}$	$6, 2, 2$
$3, \frac{3}{2}, \frac{2}{3}$		$6, 3, 1$
$3, 2, \frac{1}{2}$		$6, \frac{9}{2}, \frac{2}{3}$
$3, \frac{9}{2}, \frac{1}{6}$		$6, 6, \frac{1}{2}$
$3, 6, \frac{1}{6}$		$6, 6, \frac{2}{3}$

Table : Possible $\mathrm{SU}(5)$ predictions for the GUT scale ratios $\frac{y_\mu y_d}{y_e y_s}$

[7] S. Antusch and M. Spinrath, Phys. Rev. D **79**, 095004 (2009), [arXiv:0902.4644 [hep-ph]], S. Antusch and V. Maurer, JHEP **1311**, 115 (2013),[arXiv:1306.6879 [hep-ph]]

Numerical Analysis

1. **Parameters** : For fixed Clebsches c_x, c_y, c_z : Model parameters x, y, z, $\tilde{\beta}$, θ_u^{12}, m_a, ϵ, α, $\tan\beta$, η_b and η_q
2. **Observables** : y_e, y_μ, y_d, y_s, the CKM angles θ_{12}^{CKM}, θ_{13}^{CKM}, the PMNS angles $\theta_{12}^{\text{PMNS}}$, $\theta_{13}^{\text{PMNS}}$, $\theta_{23}^{\text{PMNS}}$ and Δm^2_{sol}, Δm^2_{atm}
3. Fitting data including SUSY threshold corrections :

$$Y_u^{\text{MSSM}} \approx \frac{Y_u^{\text{SM}}}{\sin\beta}$$

$$Y_d^{\text{MSSM}} \approx \text{diag}\left(\frac{1}{1+\eta_q}, \frac{1}{1+\eta_q}, \frac{1}{1+\eta_b}\right)\frac{Y_d^{\text{SM}}}{\cos\beta}$$

$$Y_e^{\text{MSSM}} \approx \frac{Y_e^{\text{SM}}}{\cos\beta}$$

- The two threshold parameters $\tan\beta$ and η_b have a minor impact on the observables
- The parameters x, y, z, θ_u^{12} and η_q are used to fit the four Yukawa couplings in the down-type quark and charged lepton sector and the two CKM angles
- $\tilde{\beta}$, m_a, ϵ and α determine the PMNS angles and the neutrino mass square differences

Label	$\{c_x, c_y, c_z\}$	χ^2_{Tot}	χ^2_{Yuk}	χ^2_{PMNS}	$\theta_{23}^{\text{PMNS}}[^\circ]$	$\delta^{\text{PMNS}}[^\circ]$	$\tilde{\beta}[^\circ]$	$\alpha[^\circ]$
(1)	$\left\{\mathbf{3}, \frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}}\right\}$							
a_1	(102)	0.165	0.0484	0.116	47.9	92.7	68.7	126.9
a_2		0.165	0.0484	0.116	47.9	267.3	291.3	233.1
b_1	(120)	4.06	0.0471	4.01	41.6	120.1	71.6	211.7
b_2		4.06	0.0471	4.01	41.6	239.9	288.4	148.3
(2)	$\{\mathbf{6}, \mathbf{3}, \mathbf{1}\}$							
a_1	(102)	0.185	0.0476	0.137	47.9	93.7	67.7	126.1
a_2		0.185	0.0476	0.137	47.9	266.3	292.3	233.9
b_1	(120)	4.20	0.0475	4.15	41.5	118.9	72.6	212.9
b_2		4.20	0.0475	4.15	41.5	241.1	287.4	147.1
(3)	$\left\{\frac{\mathbf{9}}{\mathbf{2}}, \mathbf{2}, \mathbf{1}\right\}$							
a_1	(102)	1.61	1.05	0.566	43.9	103.0	72.9	96.8
a_2		1.61	1.05	0.566	43.9	257.0	287.1	263.2
b_1	(120)	1.05	1.05	0.000147	47.2	90.2	71.0	269.5
b_2		1.05	1.05	0.000147	47.2	269.8	289.0	90.5
(4)	$\left\{\frac{\mathbf{9}}{\mathbf{2}}, \mathbf{3}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$							
a_1	(102)	1.63	0.905	0.724	48.8	83.9	72.5	144.7
a_2		1.63	0.905	0.724	48.8	276.1	287.5	215.3
b_1	(120)	9.69	0.906	8.79	40.4	117.0	78.5	205.0
b_2		9.69	0.906	8.79	40.4	243.0	281.5	155.0

Table : Complete table of Clebsch-Gordan coefficients (c_x, c_y, c_z) with $\chi^2 < 15$, ordered according to their best χ^2 value. χ^2_{Tot} indicates the χ^2 of the model, which includes all observables. χ^2_{Yuk} contains the contributions of the χ^2 coming from the quark and charged lepton Yukawa couplings and the CKM angles, whereas in χ^2_{PMNS} the remaining contributions to the χ^2 from the neutrino masses and the PMNS angles are incorporated. In the last four columns the values in the different minima of the two observables $\theta_{23}^{\text{PMNS}}$, δ^{PMNS} and the two parameters $\tilde{\beta}$, α are listed.

Label	$\{c_x, c_y, c_z\}$	χ^2_{Tot}	χ^2_{Yuk}	χ^2_{PMNS}	$\theta^{\text{PMNS}}_{23}[^\circ]$	$\delta^{\text{PMNS}}[^\circ]$	$\tilde{\beta}[^\circ]$	$\alpha[^\circ]$
(5)	$\left\{\mathbf{6}, \mathbf{2}, \frac{\mathbf{3}}{\mathbf{2}}\right\}$							
a_1	(102)	2.99	0.0277	2.96	42.3	117.8	64.3	76.2
a_2		2.99	0.0277	2.96	42.3	242.2	295.7	283.8
b_1	(120)	1.77	0.0283	1.74	48.3	84.7	77.9	272.4
b_2		1.77	0.0283	1.74	48.3	275.3	282.1	87.6
(6)	$\left\{\mathbf{6}, \mathbf{6}, \frac{\mathbf{1}}{\mathbf{2}}\right\}$							
a_1	(102)	2.37	0.359	2.01	49.6	84.4	71.5	212.5
a_2		2.37	0.359	2.01	49.6	275.6	288.5	147.5
a_3		8.67	0.359	8.32	40.9	134.2	64.4	296.6
a_4		8.67	0.359	8.32	40.9	225.8	295.6	63.4
b_1	(120)	3.11	0.358	2.75	42.1	121.6	68.9	141.6
b_2		3.11	0.358	2.75	42.1	238.4	291.1	218.4
(7)	$\left\{\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}\right\}$							
a_1	(102)	3.23	3.11	0.117	47.9	87.4	72.7	133.1
a_2		3.23	3.11	0.117	47.9	272.6	287.3	226.9
b_1	(120)	8.61	3.12	5.50	41.2	114.2	77.5	215.8
b_2		8.61	3.12	5.50	41.2	245.8	282.5	144.2
b_3		11.7	3.12	8.60	49.2	97.0	49.1	285.0
b_4		11.7	3.12	8.60	49.2	263.0	310.9	75.0
(8)	$\left\{\mathbf{3}, \frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$							
a_1	(102)	3.75	3.26	0.491	44.0	102.7	72.6	97.7
a_2		3.75	3.26	0.491	44.0	257.3	287.4	262.3
b_1	(120)	3.27	3.26	0.00622	47.1	91.4	69.8	268.5
b_2		3.27	3.26	0.00622	47.1	268.6	290.2	91.5

Label	$\{c_x, c_y, c_z\}$	χ^2_{Tot}	χ^2_{Yuk}	χ^2_{PMNS}	$\theta_{23}^{\text{PMNS}}[^\circ]$	$\delta^{\text{PMNS}}[^\circ]$	$\tilde{\beta}[^\circ]$	$\alpha[^\circ]$
(9)	$\left\{\mathbf{6}, \frac{\mathbf{9}}{\mathbf{2}}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$							
a_1	(102)	4.87	0.135	4.73	50.6	82.0	70.5	172.9
a_2		4.87	0.135	4.73	50.6	278.0	289.5	187.1
b_1	(120)	8.15	0.134	8.01	40.5	128.9	69.9	171.2
b_2		8.15	0.134	8.01	40.5	231.1	290.1	188.8
(10)	$\left\{\mathbf{6}, \mathbf{6}, \frac{\mathbf{2}}{\mathbf{3}}\right\}$							
a_1	(102)	5.97	2.64	3.33	50.2	78.0	73.6	177.9
a_2		5.97	2.64	3.33	50.2	282.0	286.4	182.1
b_1	(120)	14.6	2.64	11.9	39.8	125.1	72.8	172.7
b_2		14.6	2.64	11.9	39.8	234.9	287.2	187.3
(11)	$\left\{\frac{\mathbf{9}}{\mathbf{2}}, \frac{\mathbf{9}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}}\right\}$							
a_1	(102)	6.12	2.65	3.48	50.2	78.4	73.2	177.7
a_2		6.12	2.65	3.48	50.2	281.6	286.8	182.3
b_1	(120)	14.6	2.64	12.0	39.8	125.0	72.8	172.8
b_2		14.6	2.64	12.0	39.8	235.0	287.2	187.2
(12)	$\left\{\mathbf{6}, \frac{\mathbf{3}}{\mathbf{2}}, \mathbf{2}\right\}$							
b_1	(120)	11.6	0.0259	11.6	50.3	67.0	99.8	285.5
b_2		11.6	0.0259	11.6	50.3	293.0	260.2	74.5
(13)	$\left\{\frac{\mathbf{9}}{\mathbf{2}}, \frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{3}}{\mathbf{2}}\right\}$							
b_1	(120)	13.3	3.31	10.0	50.2	61.9	103.9	291.7
b_2		13.3	3.31	10.0	50.2	298.1	256.1	68.3
(14)	$\{\mathbf{6}, \mathbf{2}, \mathbf{2}\}$							
b_1	(120)	13.4	3.32	10.1	50.2	62.5	103.3	291.0
b_2		13.4	3.32	10.1	50.2	297.5	256.7	69.0

Analytical consideration

$$\theta_{13}^{\text{PMNS}} e^{i\delta^{\text{PMNS}}} \approx \frac{\epsilon}{\sqrt{2}} e^{i(\pi+\alpha)} + \frac{\theta_{12}^{e}}{\sqrt{2}} e^{i(\pi-\tilde{\beta})}$$

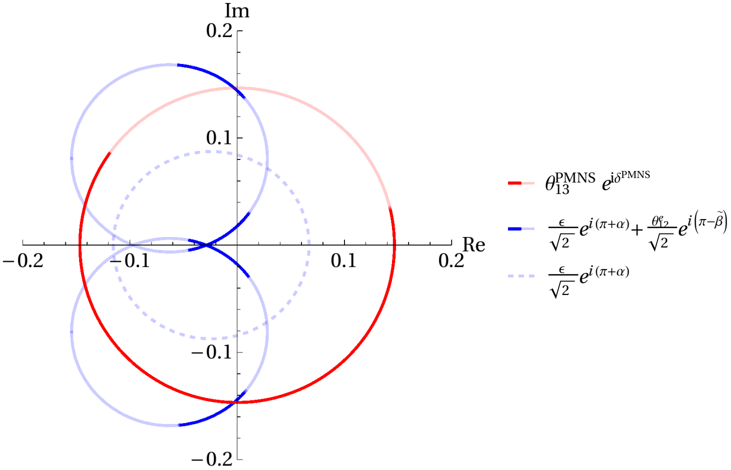


Figure : $(c_x, c_y, c_z) = (3, \frac{3}{2}, \frac{1}{2})$ and the CSD2 scenario $Y_\nu^{(102)}$, $\theta_{12}^{e} = \left|\frac{c_y}{c_x} \frac{y}{x}\right|$. The dark blue lines represent the experimental 1σ range of $\theta_{23}^{\text{PMNS}}$.

Best Fit points

Label	$\tan\beta$	η_b	η_q	x	y	z	$\tilde{\beta}[^\circ]$	θ_{12}^u	$m_a[\mathrm{eV}]$	ϵ	$\alpha[^\circ]$
$1a_2$	46.9	0.449	−0.344	0.0072	0.00183	0.00164	291.3	0.0872	0.0284	0.103	233.1
$3b_2$	33.4	−0.170	0.017	0.0035	0.00075	0.00078	289.0	0.0872	0.0262	0.119	90.5
$3a_2$	31.1	−0.147	0.016	0.0032	0.00069	0.00071	287.1	0.0872	0.0264	0.116	263.2
$4a_2$	31.0	−0.141	0.021	0.0031	0.00069	0.00070	287.5	0.0872	0.0286	0.099	215.3
$5b_2$	48.0	0.395	0.310	0.0037	0.00094	0.00085	282.1	0.0871	0.0264	0.121	87.6
$6a_2$	48.7	0.568	0.328	0.0036	0.00097	0.00083	288.5	0.0871	0.0291	0.098	147.5
$5a_2$	49.1	0.494	0.309	0.0038	0.00096	0.00087	295.7	0.0871	0.0259	0.125	283.8
$6b_2$	49.6	0.590	0.328	0.0037	0.00099	0.00085	291.1	0.0871	0.0293	0.097	218.4
$7a_2$	32.5	−0.167	−0.308	0.0050	0.00098	0.00112	287.3	0.0872	0.0283	0.102	226.9
$8b_2$	35.0	−0.078	−0.310	0.0054	0.00105	0.00120	290.2	0.0872	0.0263	0.119	91.5
$8a_2$	32.2	−0.121	−0.309	0.0049	0.00096	0.00109	287.4	0.0872	0.0265	0.115	262.3
$1b_2$	49.7	0.596	−0.344	0.0077	0.00195	0.00174	288.4	0.0872	0.0295	0.096	148.3

Table : The model parameters for the best fit points

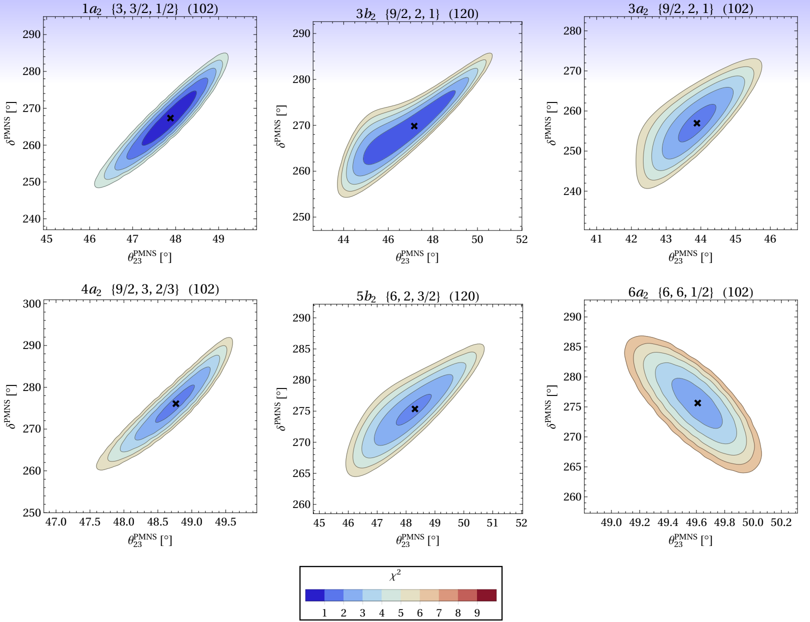
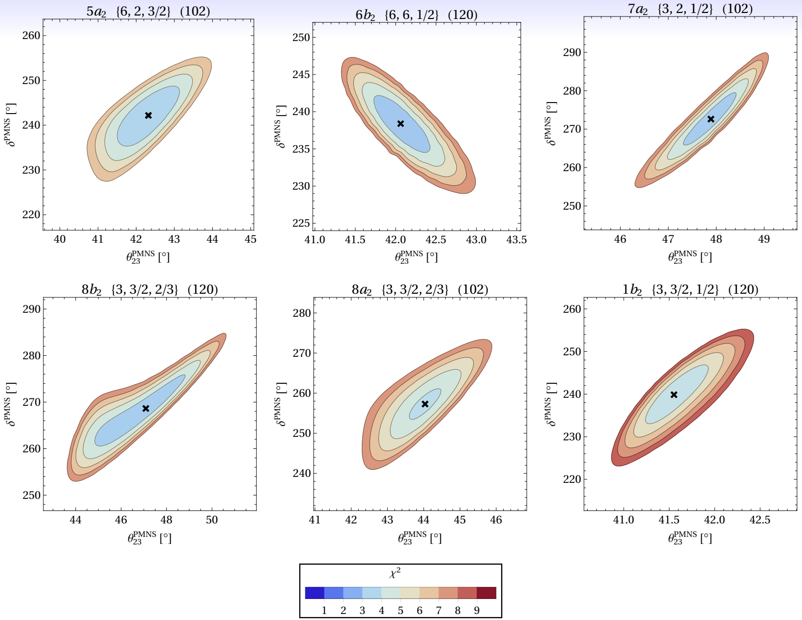


Figure : From top left to bottom right the twelve best fit points with the lowest χ^2 from the previous table are presented in decreasing order. In each plot the minimal χ^2 for fixed $\theta_{23}^{\mathrm{PMNS}}$ (x-axis) and δ^{PMNS} (y-axis) is plotted as contours around the local minimum, indicated by a black cross.



5a2 {6, 2, 3/2} (102)
6b2 {6, 6, 1/2} (120)
7a2 {3, 2, 1/2} (102)
8b2 {3, 3/2, 2/3} (120)
8a2 {3, 3/2, 2/3} (102)
1b2 {3, 3/2, 1/2} (120)
δPMNS [°]
θ23PMNS [°]
χ2
1 2 3 4 5 6 7 8 9

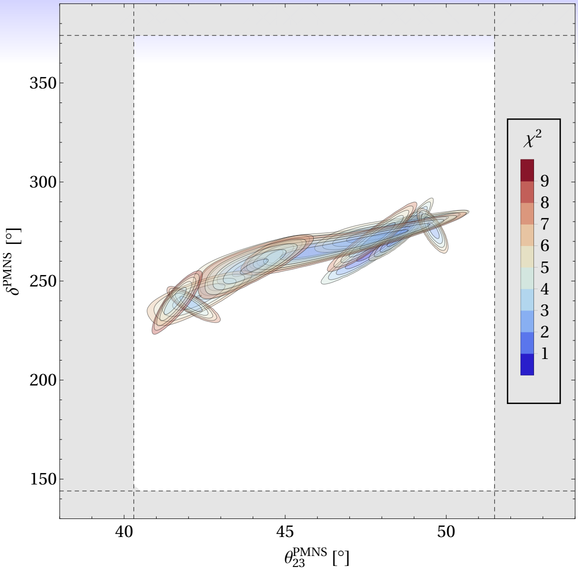


Figure : The plots corresponding to the twelve best fit point with lowest χ^2, are combined in this plot, with $\theta_{23}^{\rm PMNS}$ on the x-axis and $\delta^{\rm PMNS}$ on the y-axis. The grey areas represent the regions outside the experimental 3σ ranges of $\theta_{23}^{\rm PMNS}$ and $\delta^{\rm PMNS}$ which are given by $[40.3^\circ, 51.5^\circ]$ and $[144^\circ, 374^\circ]$, respectively [NuFit].

Summary

- We consider SUSY SU(5) models : predictive Yukawa texture and CSD2 for neutrino masses
- Sharp prediction for δ^{PMNS} : $240^\circ - 270^\circ$ (independent of the choice of clebsches)
- For a particular model, $\theta_{23}^{\mathrm{PMNS}}$ is predicted (better than experimental error) and more accurate future measurement can distinguish between these models

Including charge lepton corrections

$$s_{23} e^{-i\delta_{23}} \approx s^{\nu}_{23} e^{-i\delta^{\nu}_{23}} - \theta^{e}_{23} c^{\nu}_{23} e^{-i\delta^{e}_{23}}$$
$$s_{13} e^{-i\delta_{13}} \approx \theta^{\nu}_{13} e^{-i\delta^{\nu}_{13}} - \theta^{e}_{12} e^{-i(\delta^{\nu}_{23}+\delta^{e}_{12})}$$
$$s_{12} e^{-i\delta_{12}} \approx s^{\nu}_{12} e^{-i\delta^{\nu}_{12}} - \theta^{e}_{12} c^{\nu}_{23} c^{\nu}_{12} e^{-i\delta^{e}_{12}}$$

$$x, \theta^{e}_{12}, m_a \in [0, 0.1]\,, \quad y, z \in [0, 0.01]\,, \quad \tilde{\beta}, \alpha \in [0, 2\pi]\,, \quad \epsilon \in [0, 1]\,,$$
$$\tan\beta \in [20, 50]\,, \quad \eta_b, \eta_q \in [-0.6, 0.6]\,.$$