Quark and lepton mixing from flavor and CP symmetries

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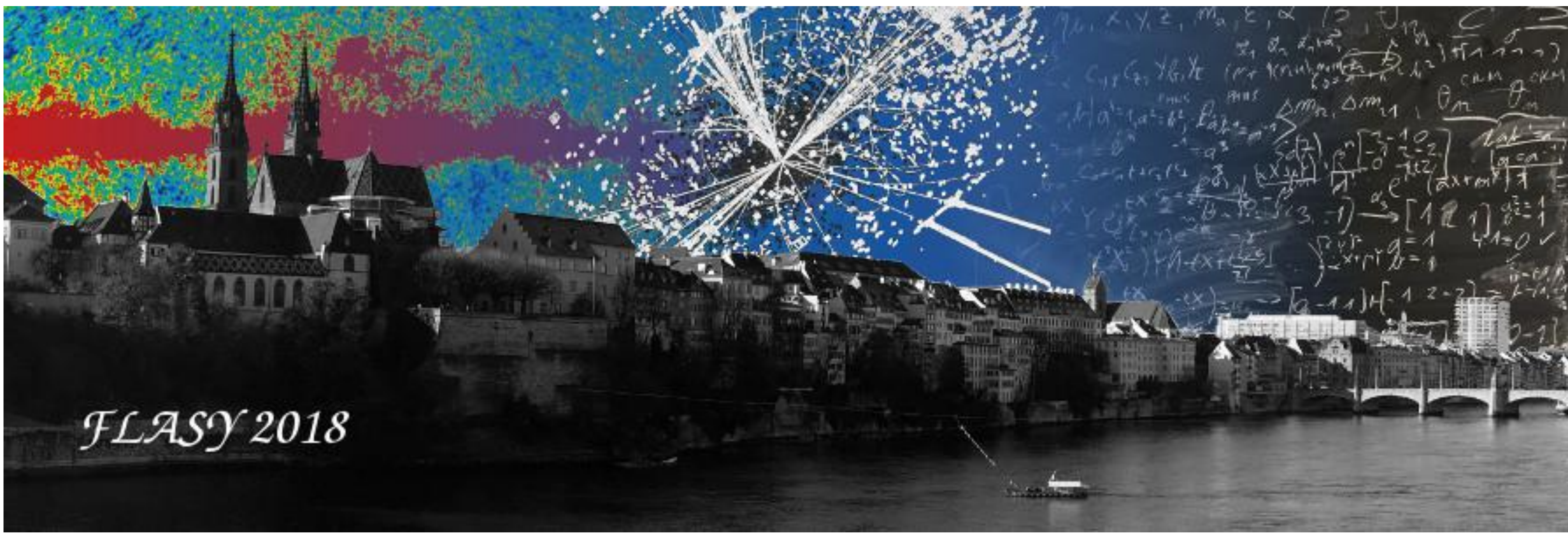
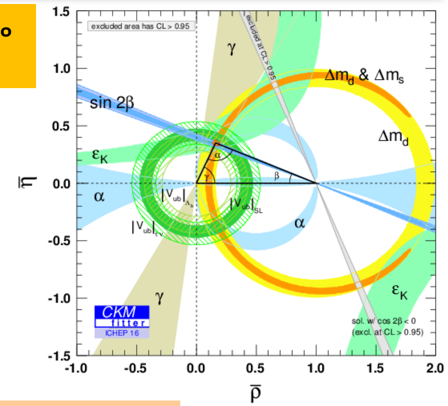


FLASY18, July 2nd-July 5th, 2018, University of Basel,Switzerland

"Old" flavor mixing puzzle in SM

[Particle Data Group 2018]

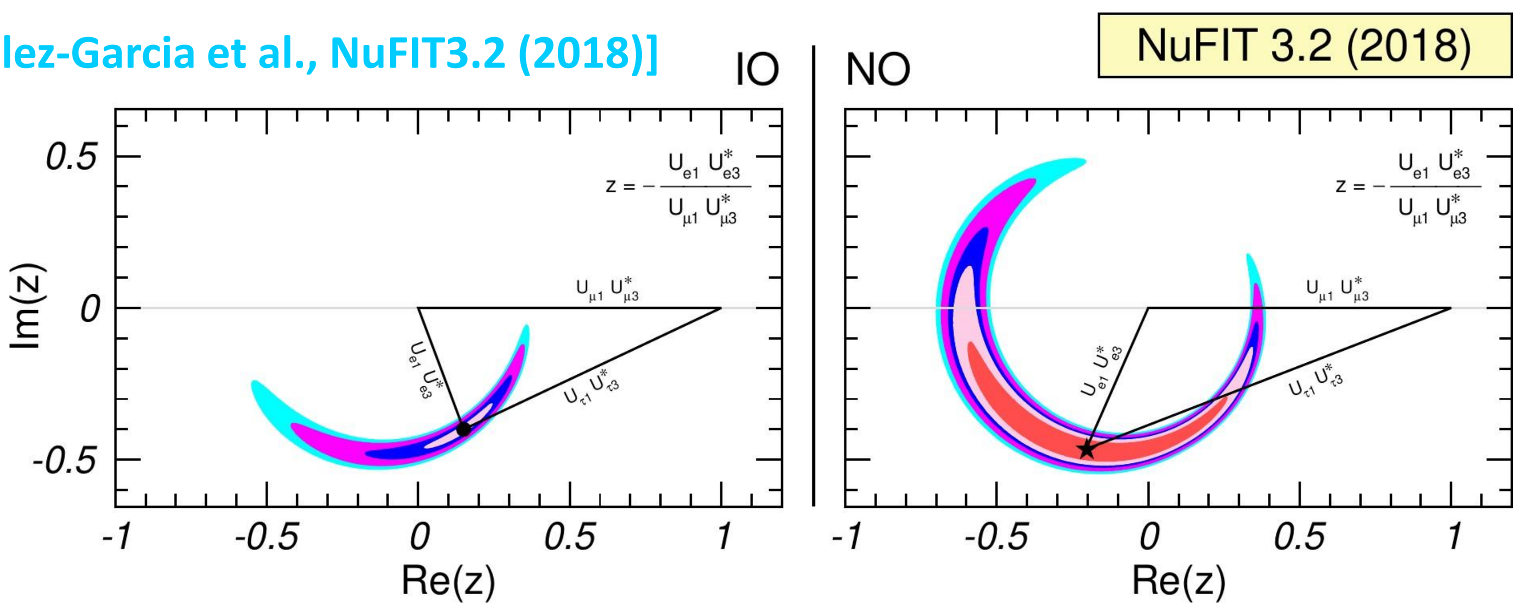
$\alpha=(88.8\pm2.3)^\circ$



Quarks: $$\|V_{CKM}\| \approx \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix}$$

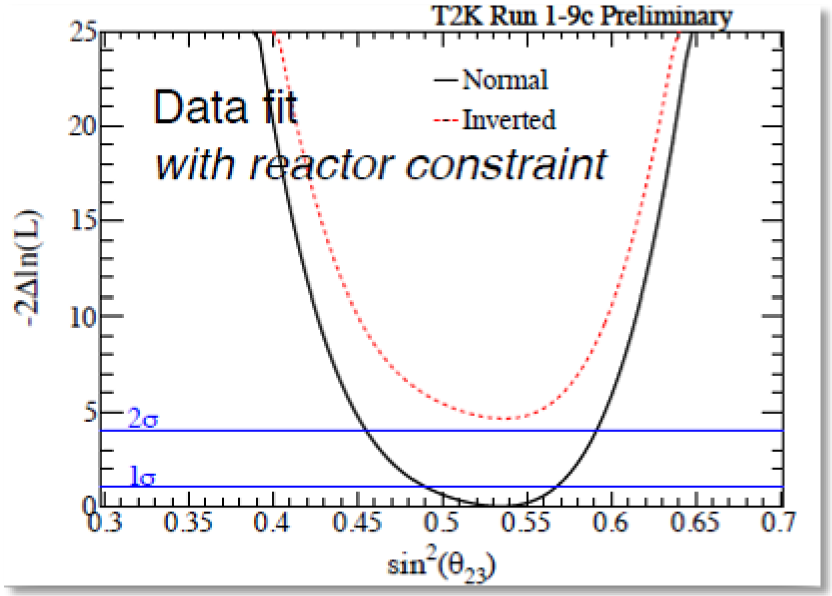
Leptons: $$\|U_{PMNS}\| = \begin{pmatrix} 0.799 \sim 0.844 & 0.516 \sim 0.582 & 0.141 \sim 0.156 \\ 0.242 \sim 0.494 & 0.467 \sim 0.678 & 0.639 \sim 0.774 \\ 0.284 \sim 0.521 & 0.490 \sim 0.695 & 0.615 \sim 0.754 \end{pmatrix}$$

[Gonzalez-Garcia et al., NuFIT3.2 (2018)]



Latest results on θ_{23} octant

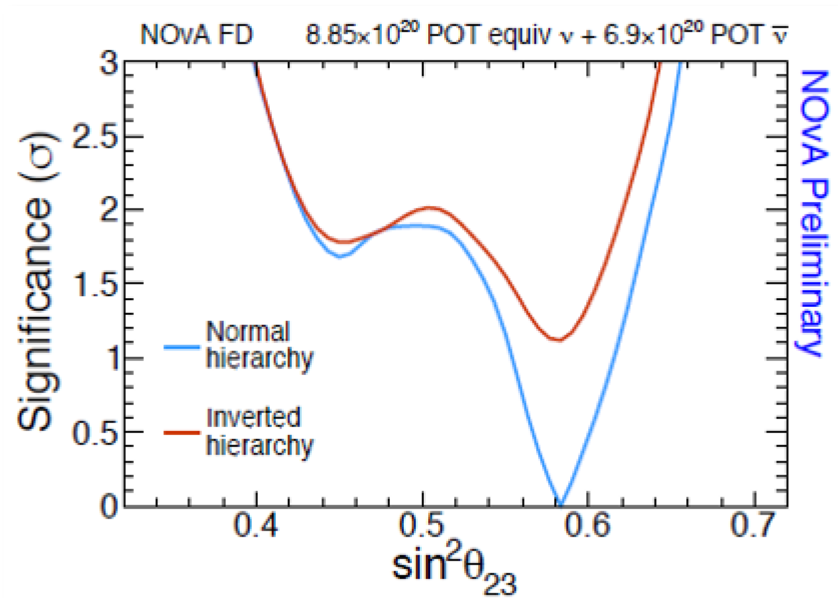
T2K Wascko@ Neutrino 18



- Best fit values:

	NO	IO
$\sin^2\theta_{23}$	$0.536^{+0.031}_{-0.046}$	$0.536^{+0.031}_{-0.041}$
$\lvert\Delta m^2\rvert$	2.434 ± 0.064	$2.410^{+0.062}_{-0.063}$

NOvA Sanchez @ Neutrino 18

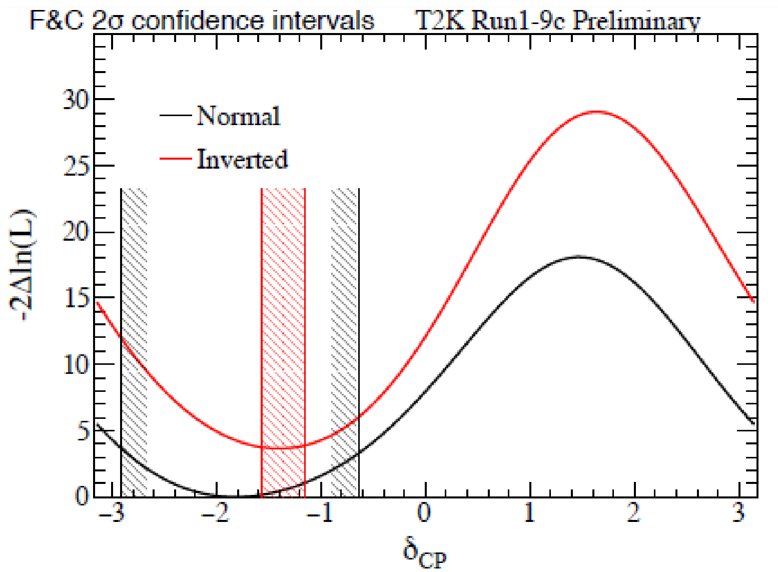


- **Best fit: $\sin^2\theta_{23}$=0.58±0.03 for NO**
- **Prefer non-maximal at 1.8σ, exclude lower octant at similar level**

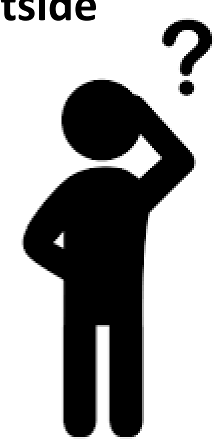
more disfavored lower octant?

Latest results on δ_{CP}

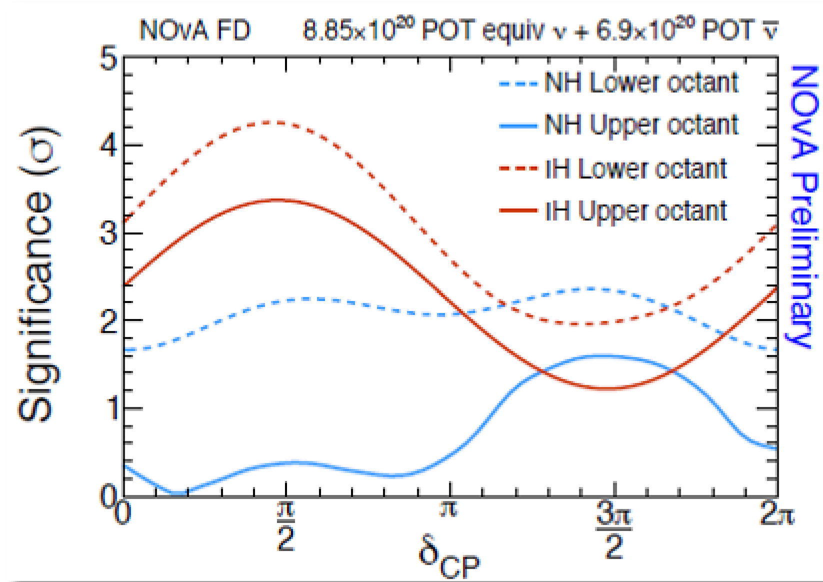
T2K Wascko@ Neutrino 18



- CP conserving values δ_{CP} =0, π outside 2σ region for NO & IO
- – π< δ_{CP} <0 is favored



NOvA Sanchez @ Neutrino 18



- Best fit: δ_{CP}≈0.17π for NO, δ_{CP}≈1.5π for IO
- Prefer NO by 1.8σ, exclude δ_{CP}=π/2 in the IO at > 3σ

a simple predictive CP symmetry: μτ reflection

μτ reflection =**μτ exchange+canonical CP**

See talk by Celso Nishi

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to \begin{pmatrix} \nu_e^c \\ \nu_\tau^c \\ \nu_\mu^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}$$

This CP transformation is **not** a unit matrix.

If the neutrino mass matrix is **invariant** under the μτ reflection

$$m_\nu = \begin{pmatrix} a & b & b^* \\ b & c & d \\ b^* & d & c^* \end{pmatrix}$$

$$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$$

$$\theta_{23} = \frac{\pi}{4}, \quad \delta_{CP} = \pm\frac{\pi}{2}$$

[Harrison, Scott, 2002;Grimus, Lavoura,2004]

A general residual CP transformation

Definition of generalized CP transformation [Chen, Li, Ding, 2014]

$$\psi \stackrel{CP}{\longmapsto} iX\gamma^0\mathcal{C}\bar{\psi}^T\,,$$

X is unitary and symmetric

Invariance of the ψ mass matrix under X

$$\begin{cases} X^T m_\psi X = m_\psi^*, & \text{for Majorana } \psi \\ X^\dagger m_\psi^\dagger m_\psi X = (m_\psi^\dagger m_\psi)^*, & \text{for Dirac } \psi \end{cases}$$

Performing Takagi factorization of X

$$X = \Sigma\Sigma^T \;\Rightarrow\; \begin{cases} \Sigma^T m_\psi \Sigma = (\Sigma^T m_\psi \Sigma)^*, & \text{for Majorana } \psi \\ \Sigma^\dagger m_\psi^\dagger m_\psi \Sigma = (\Sigma^\dagger m_\psi^\dagger m_\psi \Sigma)^*, & \text{for Dirac } \psi \end{cases}$$

The mass matrix m_ψ is diagonalized by

$$U_\psi = \Sigma O_{3\times3} P, \qquad P \equiv \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$$

with $\alpha_{1,2,3}=0, \pm\pi/2, \pi$ for Majorana ψ

Determined up to a real orthogonal matrix O_{3x3}

Generalized μτ reflection on neutrinos

the generalized μτ reflection of neutrino

[Chen, Ding, Gonzalez-Canales, Valle,2015]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & i\sin\Theta \\ 0 & i\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix},$$

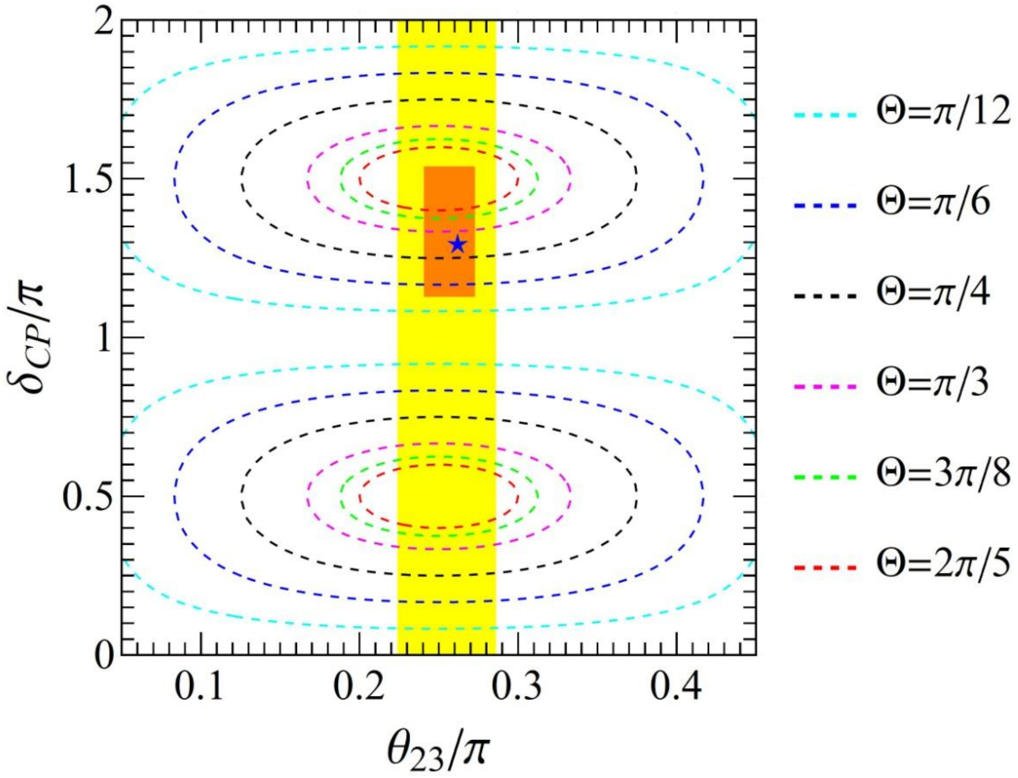
μτ reflection: $\Theta = \pi/2$

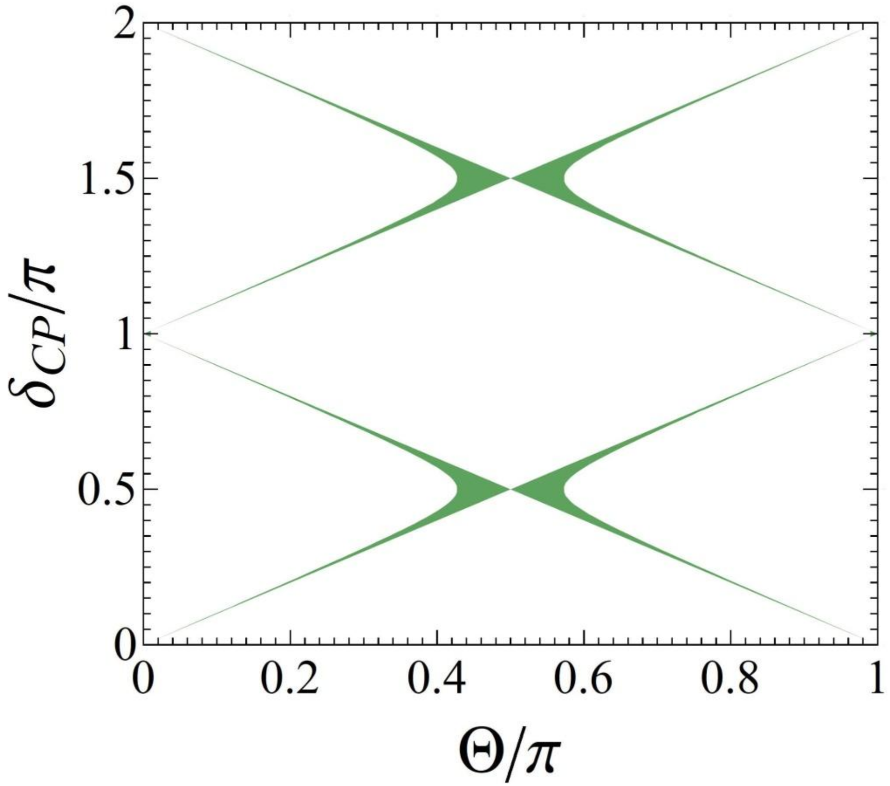
In the charged lepton diagonal basis, the lepton mixing matrix is

$$U = \Sigma_{\mu\tau} O_{3\times3} diag(1, i^{k_1}, i^{k_2}), \qquad \Sigma_{\mu\tau} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\Theta}{2} & i\sin\frac{\Theta}{2} \\ 0 & i\sin\frac{\Theta}{2} & \cos\frac{\Theta}{2} \end{pmatrix}$$

Predictions for lepton mixing parameters:

$$\sin^2\delta_{CP}\sin^2 2\theta_{23} = \sin^2\Theta, \qquad \sin\alpha_{21} = \sin\alpha'_{31} = 0$$



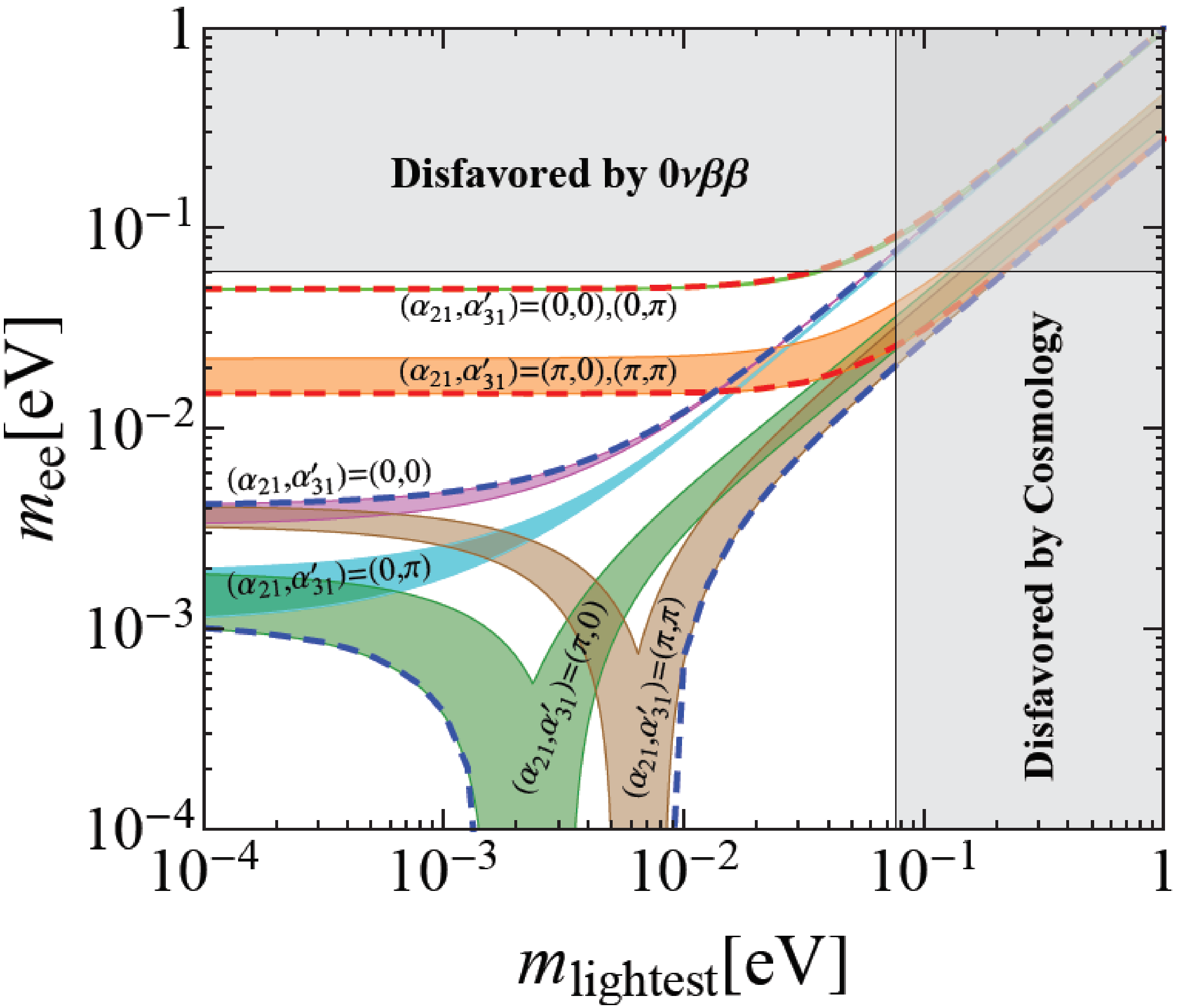


Numerical example:

$$\Theta = \frac{\pi}{3}, \quad \theta_1 = 0.274\pi, \quad \theta_2 = 0.0475\pi, \quad \theta_3 = 0.813\pi,$$

$$\sin^2\theta_{12} = 0.307, \quad \sin^2\theta_{13} = 0.0221, \quad \sin^2\theta_{23} = 0.538, \quad \delta_{CP} = 1.335\pi$$

Predictions for the effective mass of 0ν2β decay



Generalized eμ reflection on charged leptons

the generalized eμ reflection of charged lepton

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to \begin{pmatrix} \cos\Theta & i\sin\Theta & 0 \\ i\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix}$$

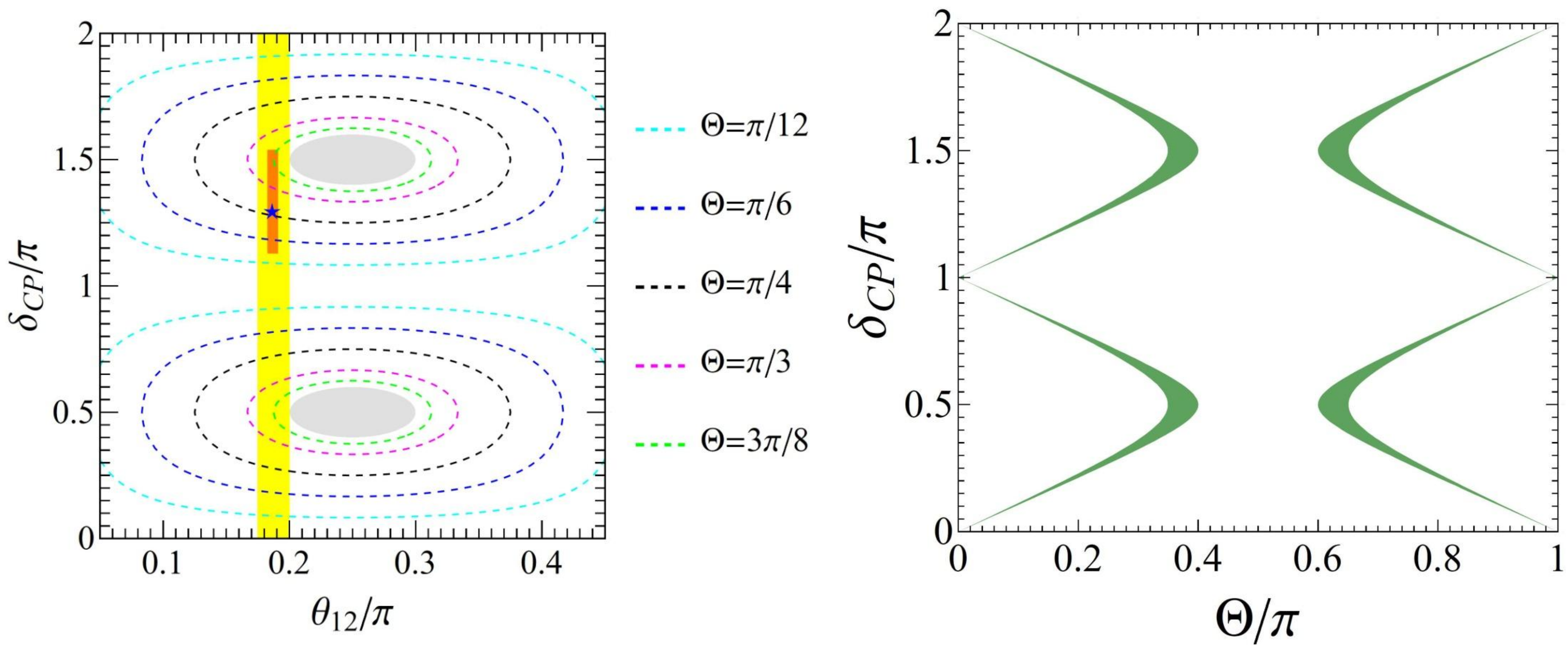
[Chen,Chulia,Ding,Srivastava, Valle,2018]

In the diagonal neutrino basis, the lepton mixing matrix reads

$$U = diag(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}) O_{3\times3} \Sigma_{e\mu}^{\dagger}, \qquad \Sigma_{e\mu} = \begin{pmatrix} \cos\frac{\Theta}{2} & i\sin\frac{\Theta}{2} & 0 \\ i\sin\frac{\Theta}{2} & \cos\frac{\Theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Sum rule for lepton mixing parameters:

$$\sin^2\delta_{CP}\sin^2 2\theta_{12} = \sin^2\Theta$$



Numerical example:

$$\Theta = \frac{\pi}{3}, \quad \theta_1 = 0.262\pi, \quad \theta_2 = 0.0475\pi, \quad \theta_3 = 0.890\pi,$$

$$\sin^2\theta_{12} = 0.307, \quad \sin^2\theta_{13} = 0.0221, \quad \sin^2\theta_{23} = 0.538, \quad \delta_{CP} = 1.388\pi$$

Universal flavor symmetry for quark and lepton mixing

$$G_f \rtimes H_{CP}$$

[Lu, Ding, 2016,2018;Li, Lu, Ding, 2017]

charged lepton

neutrino

$$Z_2^{g_l} \times X_l$$

$$Z_2^{g_\nu} \times X_\nu$$

$$X_l = \Sigma_l \Sigma_l^T,$$
$$\Sigma_l^\dagger \rho_3(g_l)\Sigma_l = \pm\mathrm{diag}(1,-1,-1)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$
$$\Sigma_\nu^\dagger \rho_3(g_\nu)\Sigma_\nu = \pm\mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu$$

$$U_{PMNS}(\theta_l, \theta_\nu) = R_{23}^T(\theta_l)\Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu$$

- All mixing angles and CP phases are expressed in terms of two free parameters $\theta_{l,\nu} \in [0,\pi)$
- This scheme can be extended to quark sector, and the CKM mixing matrix is of similar form

CKM matrix from $\Delta(6n^2)$ and CP

- $\Delta(6n^2)$ is isomorphic to $(Z_n \times Z_n) \rtimes S_3$ with the multiplication rules

$$a^3 = b^2 = (ab)^2 = 1,$$

$S_3 =< a, b >$

$$c^n = d^n = 1, \qquad cd = dc,$$

$Z_n \times Z_n =< c, d >$

$$aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}$$

$\rtimes$

for triplet representation $\eta \equiv e^{2\pi i/n}$ [Escobar,Luhn,2008]

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

Only one viable quark mixing pattern is obtained up to row and column permutations

Residual symmetry:

$$Z_2^{g_u} = Z_2^{bc^x d^x}, \; X_u = \left\{ c^\gamma d^{-2x-\gamma}, bc^{x+\gamma} d^{-x-\gamma} \right\},$$

$$Z_2^{g_d} = Z_2^{bc^y d^y}, X_d = \left\{ c^\delta d^{-2y-\delta}, bc^{y+\delta} d^{-y-\delta} \right\}$$

[Lu, Ding, 2016,2018;Li, Lu, Ding, 2017]

- The CKM matrix is determined to be

$$V_{CKM} = \begin{pmatrix} s_d \sin\varphi_1 & \cos\varphi_1 & -c_d \sin\varphi_1 \\ c_u c_d e^{i\varphi_2} + s_u s_d \cos\varphi_1 & -s_u \sin\varphi_1 & c_u s_d e^{i\varphi_2} - c_d s_u \cos\varphi_1 \\ c_d s_u e^{i\varphi_2} - c_u s_d \cos\varphi_1 & c_u \sin\varphi_1 & s_u s_d e^{i\varphi_2} + c_u c_d \cos\varphi_1 \end{pmatrix}$$

with

$$\varphi_1 = \frac{x-y}{n}\pi, \quad \varphi_2 = \frac{3(x-y+\gamma-\delta)}{n}\pi$$ ⟵ **fixed by residual symmetry**

- Sum rules

$$\begin{cases} \cos^2\theta_{13}^q \sin^2\theta_{12}^q = \cos^2\varphi_1, \\ J_{CP}^q \approx \frac{1}{2}\sin 2\varphi_1 \sin\varphi_2 \sin\theta_{13}^q \sin\theta_{23}^q \end{cases}$$

- Example: $\varphi_1 = \varphi_2 = 3\pi/7$ which can be achieved in **$\Delta(6\cdot7^2) = \Delta(294)$** group

	θ_u^{bf}/π	θ_d^{bf}/π	$\sin\theta_{12}^q$	$\sin\theta_{23}^q$	$\sin\theta_{13}^q$	J_{CP}^q
Our	0.4867	0.4988	0.22252	0.04204	0.00359	3.202×10^{-5}
Data	—	—	0.22497 ± 0.00069	0.04229 ± 0.00057	0.00368 ± 0.00010	$(3.115\pm0.093)\times10^{-5}$

The measured quark mixing angles and CP violation phases can be accommodated.

- Lepton sector : $g_l = b$, $X_l = cd^{-1}$, $g_\nu = abc^{-2}$, $X_\nu = cd^{-2}$ in **$\Delta(6\cdot 7^2) = \Delta(294)$**

$$U_{PMNS} = R_{13}^T(\theta_l)\frac{1}{2}\begin{pmatrix} -\sqrt{2}e^{i\pi/7} & -1 & 1 \\ -\sqrt{2}e^{i\pi/7} & 1 & -1 \\ 0 & \sqrt{2}e^{-i\pi/7} & \sqrt{2}e^{-i\pi/7} \end{pmatrix} R_{13}(\theta_\nu)Q_\nu$$

Sum rule between $\cos\delta_{CP}$ and mixing angles :

$$\cos\delta_{CP} = \frac{4\cos^2\theta_{12}\cos^2\theta_{23} + 4\sin^2\theta_{12}\sin^2\theta_{23}\sin^2\theta_{13} - 1}{2\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}}$$

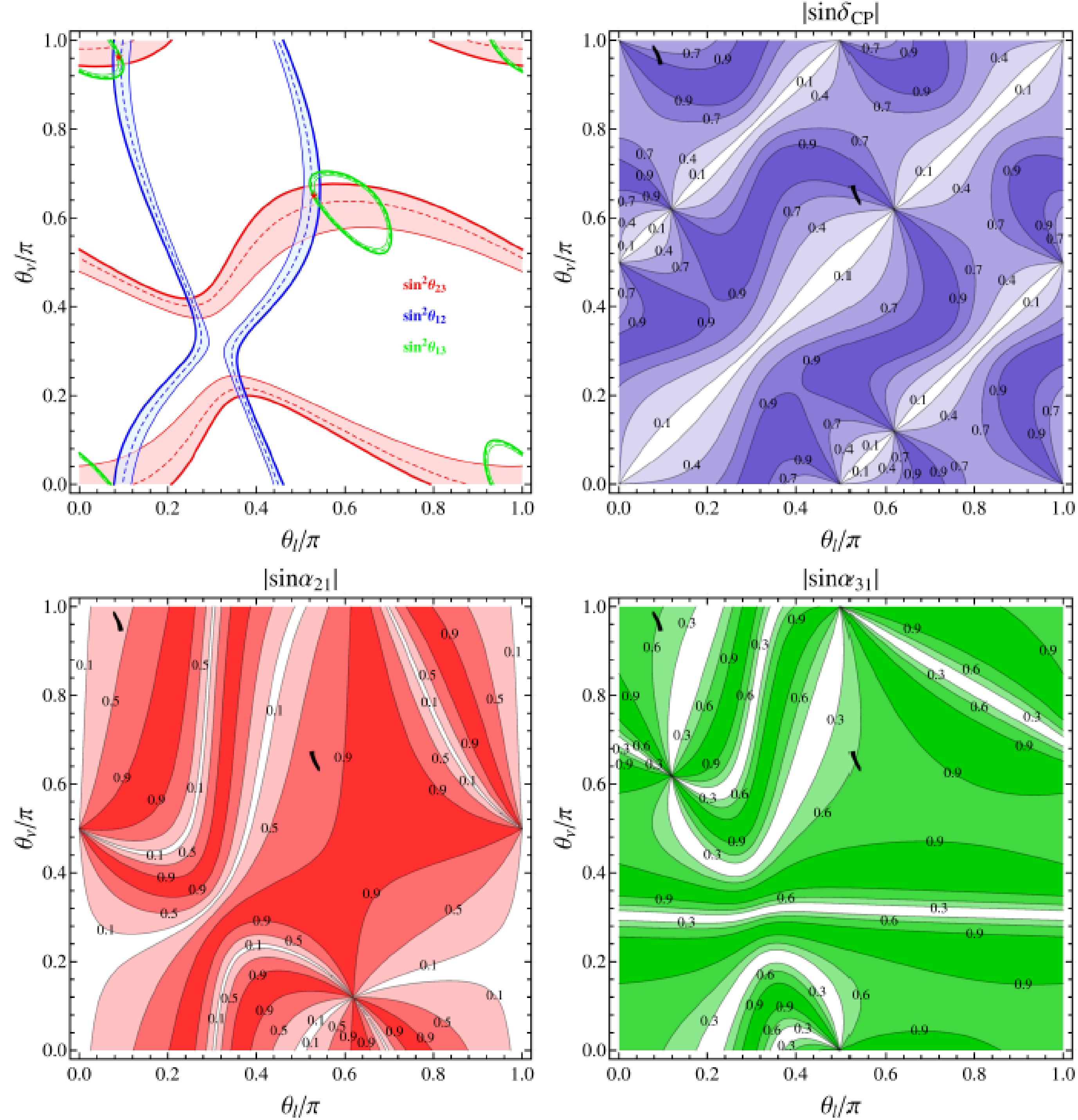
Best fit values:

$$\theta_l = 0.089\pi, \quad \theta_\nu = 0.965\pi, \quad \chi^2_{\min} = 2.416,$$

$$\sin^2\theta_{13} = 0.0222, \quad \sin^2\theta_{12} = 0.319, \quad \sin^2\theta_{23} = 0.579,$$

$$\delta_{CP} = 1.204\pi, \quad |\sin\alpha_{21}| = 0.391, \quad |\sin\alpha_{31}| = 0.596$$

[Li, Lu, Ding, 2017]


$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
$\sin^2\theta_{13}$
$|\sin\delta_{CP}|$
$|\sin\alpha_{21}|$
$|\sin\alpha_{31}|$
θ_ν/π
θ_l/π

A model building paradigm: Littlest Seesaw

- Three lepton doublets transform as triplet of flavor symmetry $L \sim 3$

$$\mathcal{L} = -y_{\rm atm}(L.\Phi_{\rm atm})N^c_{\rm atm} - y_{\rm sol}(L.\Phi_{\rm sol})N^c_{\rm sol} - \frac{1}{2}M_{\rm atm}N^c_{\rm atm}N^c_{\rm atm} - \frac{1}{2}M_{\rm sol}N^c_{\rm sol}N^c_{\rm sol}$$

- Vacuum alignment **CSD(n)** [King,2015;Luhn, King, 2016]

$$\langle\Phi_{\bf atm}\rangle = v_{\bf atm}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \langle\Phi_{\bf sol}\rangle = v_{\bf sol}\begin{pmatrix}1\\n\\n-2\end{pmatrix}$$

See talk by Steve King

- Neutrino mass matrix

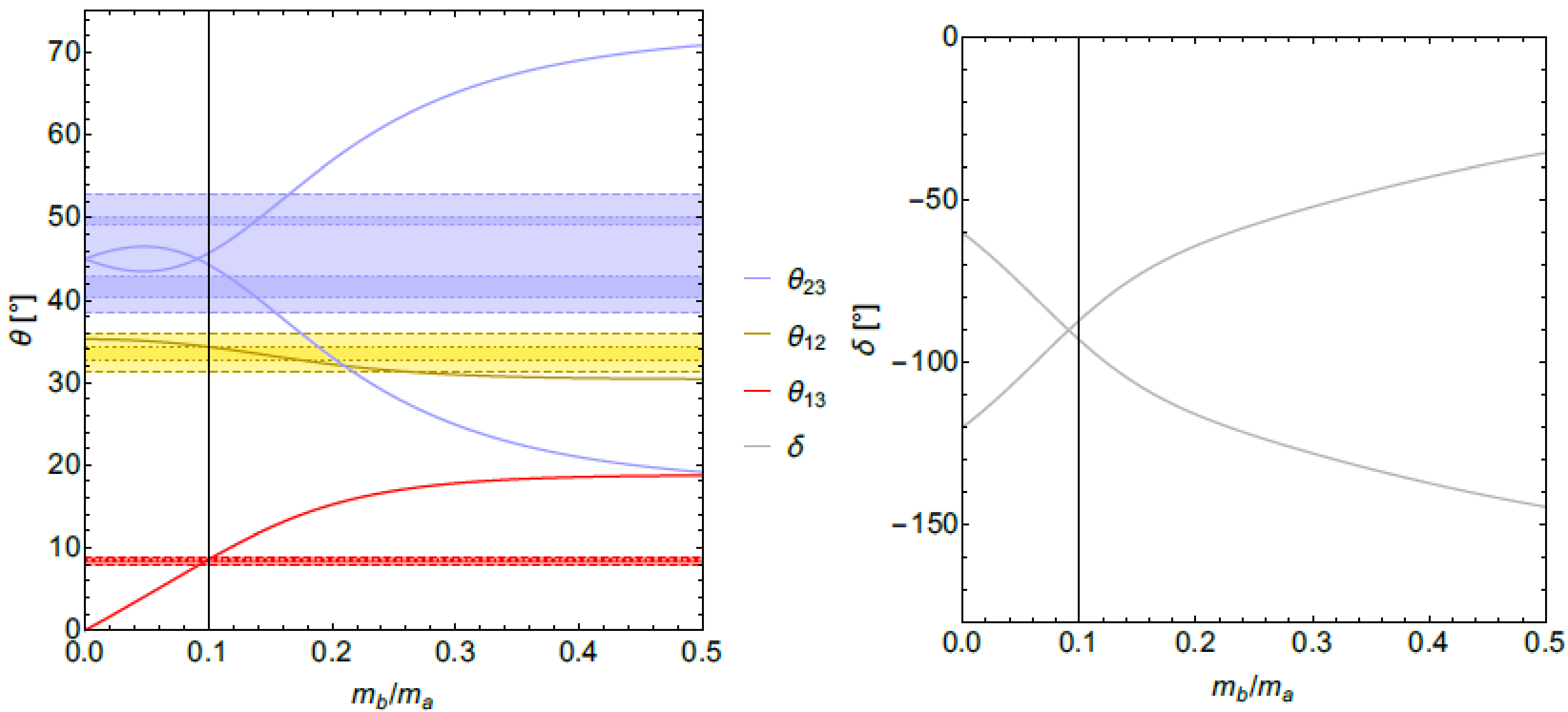
Only one phase

$$m_\nu = m_a\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + m_b e^{i\eta}\begin{pmatrix}1&n&n-2\\n&n^2&n(n-2)\\n-2&n(n-2)&(n-2)^2\end{pmatrix}$$

m_a and m_b are fixed by the neutrino mass squared differences

➢ Numerical benchmark: **n=3, η=2π/3** [Ballett,King,Pascoli et al,2016]

m_a (meV)	m_b (meV)	η (rad)	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	δ_{CP} (°)	m_1 (meV)	m_2 (meV)	m_3 (meV)
26.57	2.684	$\frac{2\pi}{3}$	34.3	8.67	45.8	-86.7	0	8.59	49.8
Value	from	[25]	$33.48^{+0.78}_{-0.75}$	$8.50^{+0.20}_{-0.21}$	$42.3^{+3.0}_{-1.6}$	-54^{+39}_{-70}	0	8.66±0.10	49.57±0.47

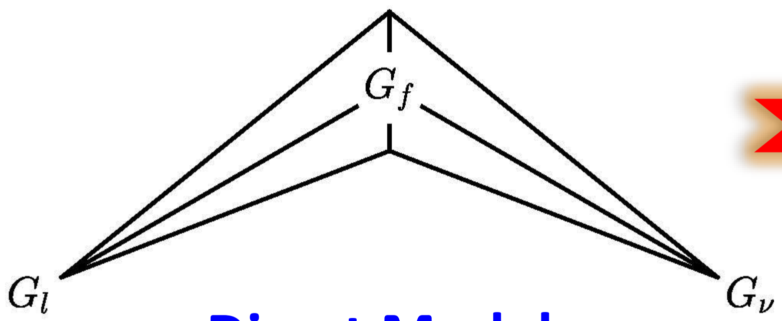


Littlest Seesaw from flavor and CP symmetries

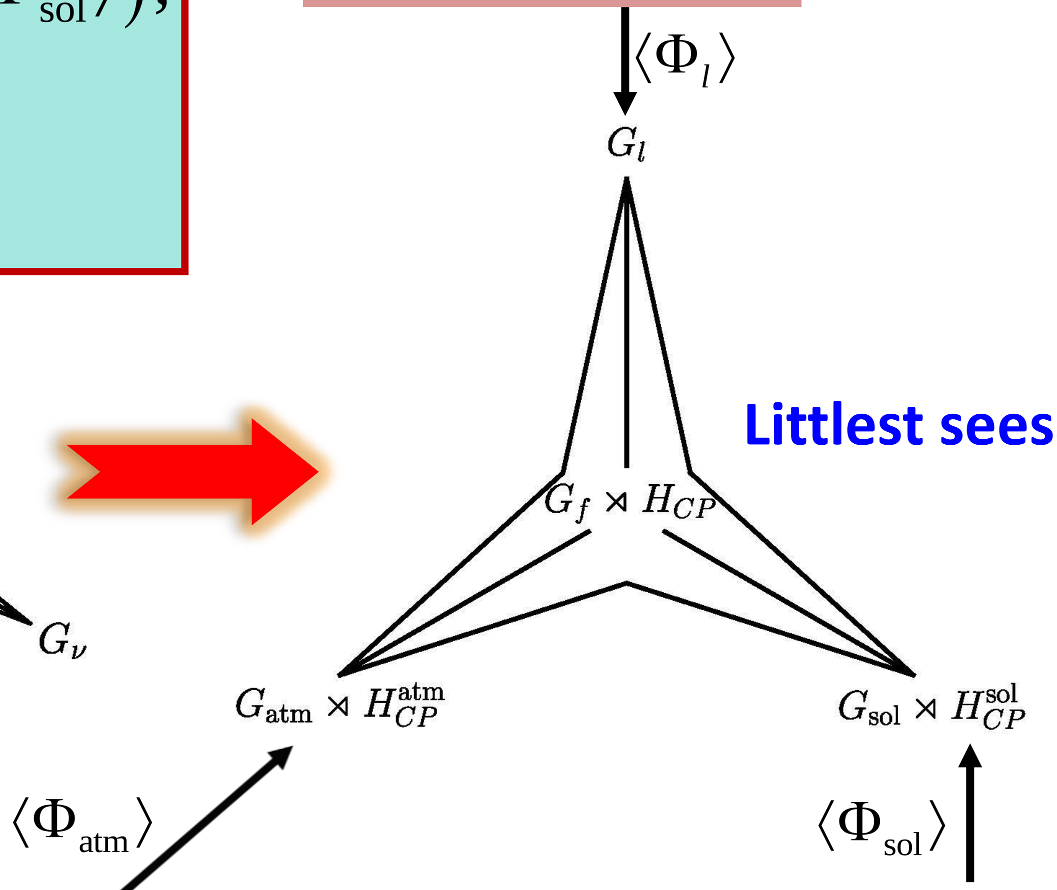
$$m_D = \left(y_{\rm atm} \langle \Phi_{\rm atm} \rangle, \quad y_{\rm sol} \langle \Phi_{\rm sol} \rangle \right),$$

$$m_N = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$

$$\mathcal{L}_c = -y_l (L.\Phi_l) l^c$$



Direct Model



Littlest seesaw

$$\mathcal{L}_{\rm atm} = -\frac{1}{2} M_{\rm atm} N^c_{\rm atm} N^c_{\rm atm} - y_{\rm atm} (L.\Phi_{\rm atm}) N^c_{\rm atm}$$

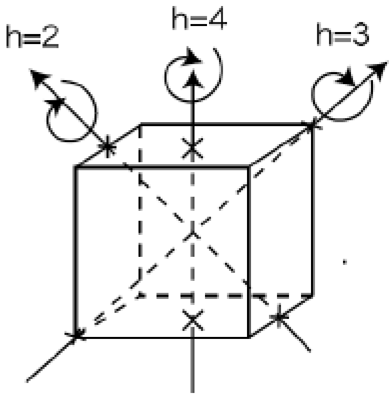
$$\mathcal{L}_{\rm sol} = -\frac{1}{2} M_{\rm sol} N^c_{\rm sol} N^c_{\rm sol} - y_{\rm sol} (L.\Phi_{\rm sol}) N^c_{\rm sol}$$

Example of new Littlest seesaw in S_4

- Flavor symmetry is S_4, its generators in the triplet representation 3 and $3'$ are:

$$\omega \equiv e^{2\pi i/3}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad U = \mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$



Symmetry of a cube

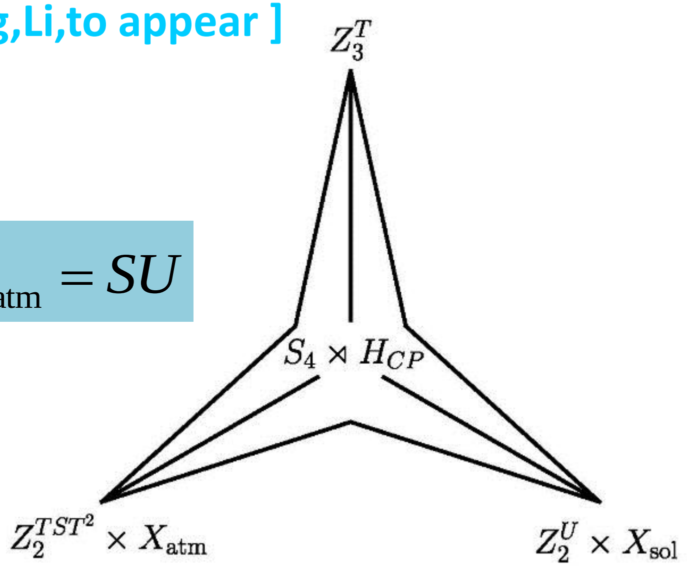
- assignment $L \sim 3,\ N^c_{\rm atm} \sim 1,\ N^c_{\rm sol} \sim 1',\ \Phi_{\rm atm} \sim 3,\ \Phi_{\rm sol} \sim 3'$

- S_4 is broken to different subgroups [Ding,King,Li,to appear]

Charged lepton sector: $G_l = Z_3^T$

Atmospheric neutrino sector: $G_{\rm atm} = Z_2^{TST^2},\ X_{\rm atm} = SU$

Solar neutrino sector: $G_{\rm sol} = Z_2^U,\ X_{\rm sol} = U$



- Vacuum alignment

$$\begin{cases} G_{\text{atm}}\langle\Phi_{\text{atm}}\rangle = \langle\Phi_{\text{atm}}\rangle \\ X_{\text{atm}}\langle\Phi_{\text{atm}}\rangle = \langle\Phi_{\text{atm}}\rangle^* \end{cases} \Rightarrow \langle\Phi_{\text{atm}}\rangle \propto \left(1, \quad \omega^2, \quad \omega\right)^T$$

$$\begin{cases} G_{\text{sol}}\langle\Phi_{\text{sol}}\rangle = \langle\Phi_{\text{sol}}\rangle \\ X_{\text{sol}}\langle\Phi_{\text{sol}}\rangle = \langle\Phi_{\text{sol}}\rangle^* \end{cases} \Rightarrow \langle\Phi_{\text{sol}}\rangle \propto \left(1, \quad n, \quad n\right)^T$$

- Mass matrices

$$m_D = \begin{pmatrix} a & b \\ \omega a & nb \\ \omega^2 a & nb \end{pmatrix}, \qquad m_N = \begin{pmatrix} M_{\text{atm}} & 0 \\ 0 & M_{\text{sol}} \end{pmatrix}$$

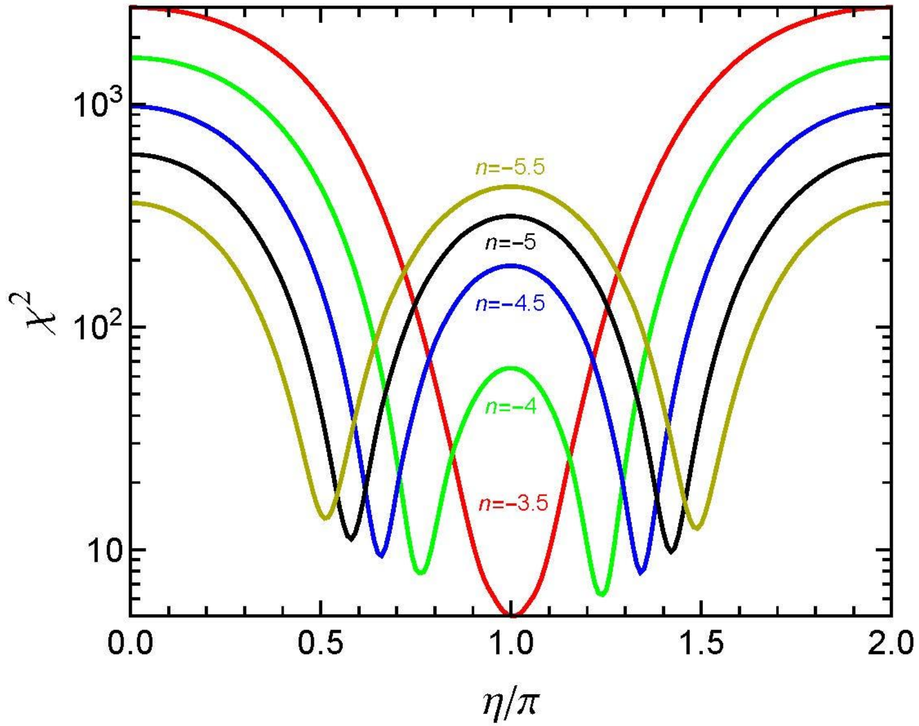
After seesaw

$$m_\nu = m_a \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & n & n \\ n & n^2 & n^2 \\ n & n^2 & n^2 \end{pmatrix}$$

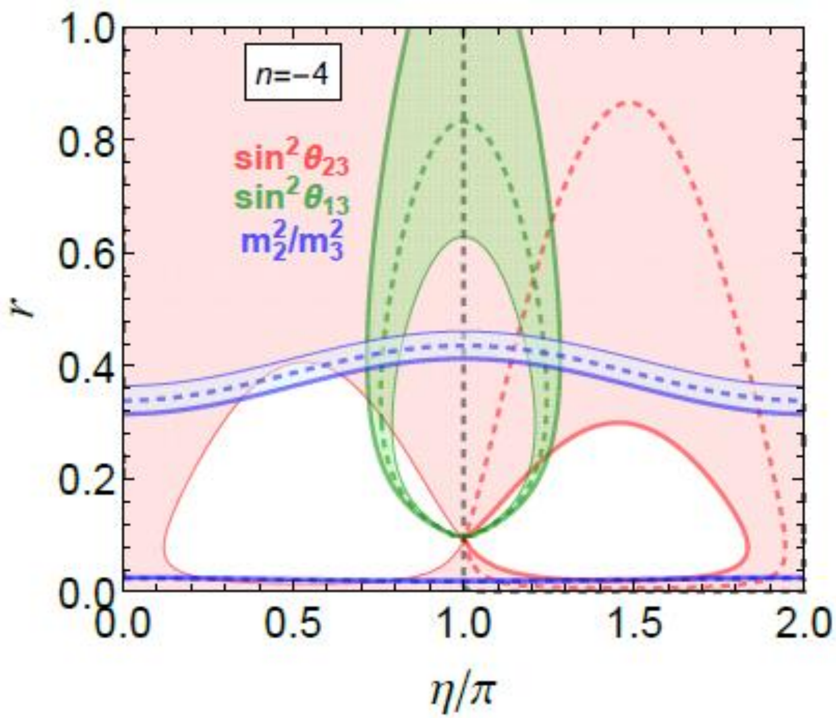
Four parameters m_a , m_s , η, n explain nine observables (3 neutrino masses+ 3 mixing angles+3 CP phases).

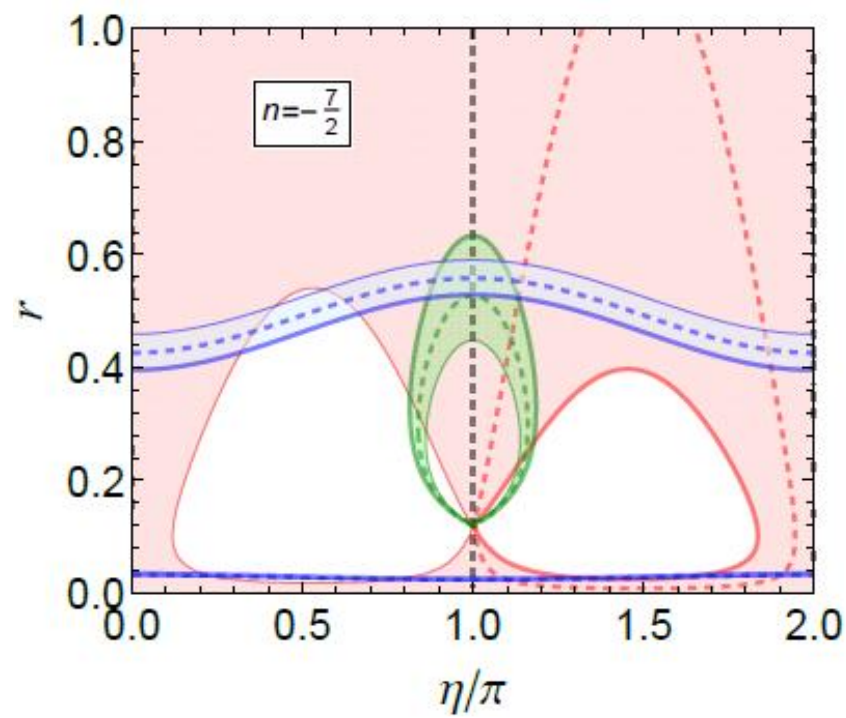
$$m_\nu \begin{pmatrix} -i\sqrt{3}\,n \\ n-\omega^2 \\ -n+\omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow U_{PMNS} = \begin{pmatrix} \frac{\sqrt{3}n}{\sqrt{5n^2+2n+2}} & \times & \times \\ \sqrt{\frac{n^2+n+1}{5n^2+2n+2}} & \times & \times \\ \sqrt{\frac{n^2+n+1}{5n^2+2n+2}} & \times & \times \end{pmatrix}$$

The neutrino mass spectrum is normal ordering and the lightest one is massless $m_1=0$



n	$\langle\Phi_{\rm sol}\rangle$	η	m_s/m_a	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}/π	β/π	m_2^2/m_3^2
-4	$(1,-4,-4)^T$	$\frac{5\pi}{4}$	0.422	6.140	0.0227	0.336	0.560	-0.412	0.264	0.0292
		$\frac{6\pi}{5}$	0.428	10.489	0.0204	0.338	0.549	-0.424	0.211	0.0290
-5	$(1,-5,-5)^T$	$\frac{7\pi}{5}$	0.266	11.880	0.0210	0.345	0.576	-0.379	0.422	0.0294
		$\frac{10\pi}{7}$	0.264	10.047	0.0225	0.344	0.578	-0.381	0.452	0.0291
$-\frac{7}{2}$	$(1,-\frac{7}{2},-\frac{7}{2})^T$	π	0.557	4.711	0.0227	0.331	0.5	-0.5	0	0.0294
		$\frac{10\pi}{9}$	0.550	11.708	0.0242	0.330	0.530	-0.459	0.117	0.0297
$-\frac{9}{2}$	$(1,-\frac{9}{2},-\frac{9}{2})^T$	$\frac{4\pi}{3}$	0.331	8.237	0.0216	0.341	0.571	-0.391	0.352	0.0293
		$\frac{11\pi}{8}$	0.329	13.713	0.0239	0.340	0.575	-0.390	0.396	0.0289





Model of new Littlest seesaw

➢ Field content

[Ding,King,Li,to appear]

matter	L	e^c	μ^c	τ^c	$\nu^c_{\rm atm}$	$\nu^c_{\rm sol}$	$H_{u,d}$
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1}$
Z_5	ω_5^4	ω_5	ω_5	ω_5	1	1	1
Z_8	ω_8^7	ω_8^6	ω_8^7	1	ω_8^5	ω_8	1
Z'_8	ω_8	ω_8^7	ω_8^7	ω_8^7	ω_8^5	1	1

driving	ξ_l^0	ϕ_l^0	ϕ_a^0	σ^0	ρ^0	η^0	χ^0	φ^0	Δ^0	κ^0	$\zeta^0_{1,2,3,4}$
S_4	$\mathbf{1}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{1}$	$\mathbf{1}$
Z_5	1	1	ω_5^3	ω_5^2	ω_5	ω_5^4	1	1	ω_5^2	ω_5^3	1
Z_8	ω_8^6	ω_8^6	1	1	1	1	1	1	1	1	1
Z'_8	1	1	ω_8^4	ω_8^6	ω_8^6	ω_8^6	ω_8^6	ω_8^7	ω_8^4	ω_8^2	1

flavon	η_l	ϕ_l	ξ_a	ϕ_a	ξ_s	η_s	χ_s	φ_s	Δ_s	ϕ_s	ψ_s
S_4	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{3'}$	$\mathbf{3'}$
Z_5	1	1	1	ω_5	1	ω_5^3	ω_5^2	1	ω_5^3	ω_5	ω_5^4
Z_8	ω_8	ω_8	ω_8^6	-1	ω_8^6	1	1	1	1	1	1
Z'_8	1	1	ω_8^6	ω_8^2	1	ω_8^5	ω_8^5	ω_8	ω_8^4	ω_8^7	ω_8^5

$$\langle\eta_l\rangle = (0, v_{\eta_l})^T, \quad \langle\phi_l\rangle = (0, v_{\phi_l}, 0)^T \Rightarrow G_l = Z_3^T$$

$$\langle\xi_a\rangle = v_{\xi_a}, \quad \langle\phi_a\rangle = v_{\phi_a}(1, \omega^2, \omega)^T \Rightarrow G_{\rm atm} = Z_2^{TST^2}$$

$$\begin{aligned} &\langle\chi_s\rangle = v_{\chi_s}(1,0,0)^T, \quad \langle\psi_s\rangle = v_{\psi_s}(1,-2,-2)^T \\ &\langle\varphi_s\rangle = v_{\varphi_s}(1,-1,-1)^T, \quad \langle\Delta_s\rangle = v_{\Delta_s}(1,2,2)^T, \\ &\langle\eta_s\rangle = v_{\eta_s}(1,1)^T, \quad \langle\xi_s\rangle = v_{\xi_s}, \quad \langle\phi_s\rangle = v_{\phi_s}(1,-4,-4)^T \end{aligned} \Rightarrow G_{\rm sol} = Z_2^U$$

- Charged lepton sector

$$w_l = \frac{y_\tau}{\Lambda}(L\phi_l)_{\mathbf{1}}\tau^c H_d + \frac{y_\mu}{\Lambda^2}(L(\eta_l\phi_l)_{\mathbf{3}})_{\mathbf{1}}\mu^c H_d + \frac{y_{e,1}}{\Lambda^3}(L\eta_l\eta_l\phi_l)e^c H_d + \frac{y_{e,2}}{\Lambda^3}(L\eta_l\phi_l\phi_l)e^c H_d + \frac{y_{e,3}}{\Lambda^3}(L\phi_l\phi_l\phi_l)e^c H_d$$

- ✓ Charged lepton mass matrix is diagonal
- ✓ Hierarchical charged lepton masses

$$m_\tau = \left|y_\tau \frac{v_{\phi_l}}{\Lambda}\right| v_d, \qquad m_\mu = \left|y_\mu \frac{v_{\eta_l} v_{\phi_l}}{\Lambda^2}\right| v_d\,,$$

$$m_e = \left|(y_{e,1} v_{\eta_l}^2 - 2y_{e,2} v_{\eta_l} v_{\phi_l} + y_{e,3} v_{\phi_l}^2)\frac{v_{\phi_l}}{\Lambda^3}\right| v_d$$

- ✓ Higher order corrections appear at relative order $1/\Lambda^3$

- Neutrino sector

$$w_\nu = \frac{y_a}{\Lambda}(L\phi_a)\,\nu^c_{\rm atm}H_u + \frac{y_s}{\Lambda}(L\phi_s)\,\nu^c_{\rm sol}H_u + x_a\xi_a\nu^c_{\rm atm}\nu^c_{\rm atm} + x_s\xi_s\nu^c_{\rm sol}\nu^c_{\rm sol}$$

After flavor and EW symmetry breaking, the neutrino Dirac and Majorana mass matrices read

$$m_D = \begin{pmatrix} a & b \\ \omega a & -4b \\ \omega^2 a & -4b \end{pmatrix}, \qquad m_N = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$

with

$$a = \frac{y_a v_{\phi_a}}{\Lambda}, \quad b = \frac{y_s v_{\phi_s}}{\Lambda}, \quad M_{\rm atm} = x_a v_{\xi_a}, \quad M_{\rm sol} = x_s v_{\xi_s}$$

new Littlest seesaw texture is reproduced exactly.

Summary

- Generalized $\mu\tau$ reflection is a simple alternative to understand deviations of θ_{23} and δ_{CP} from maximal values.

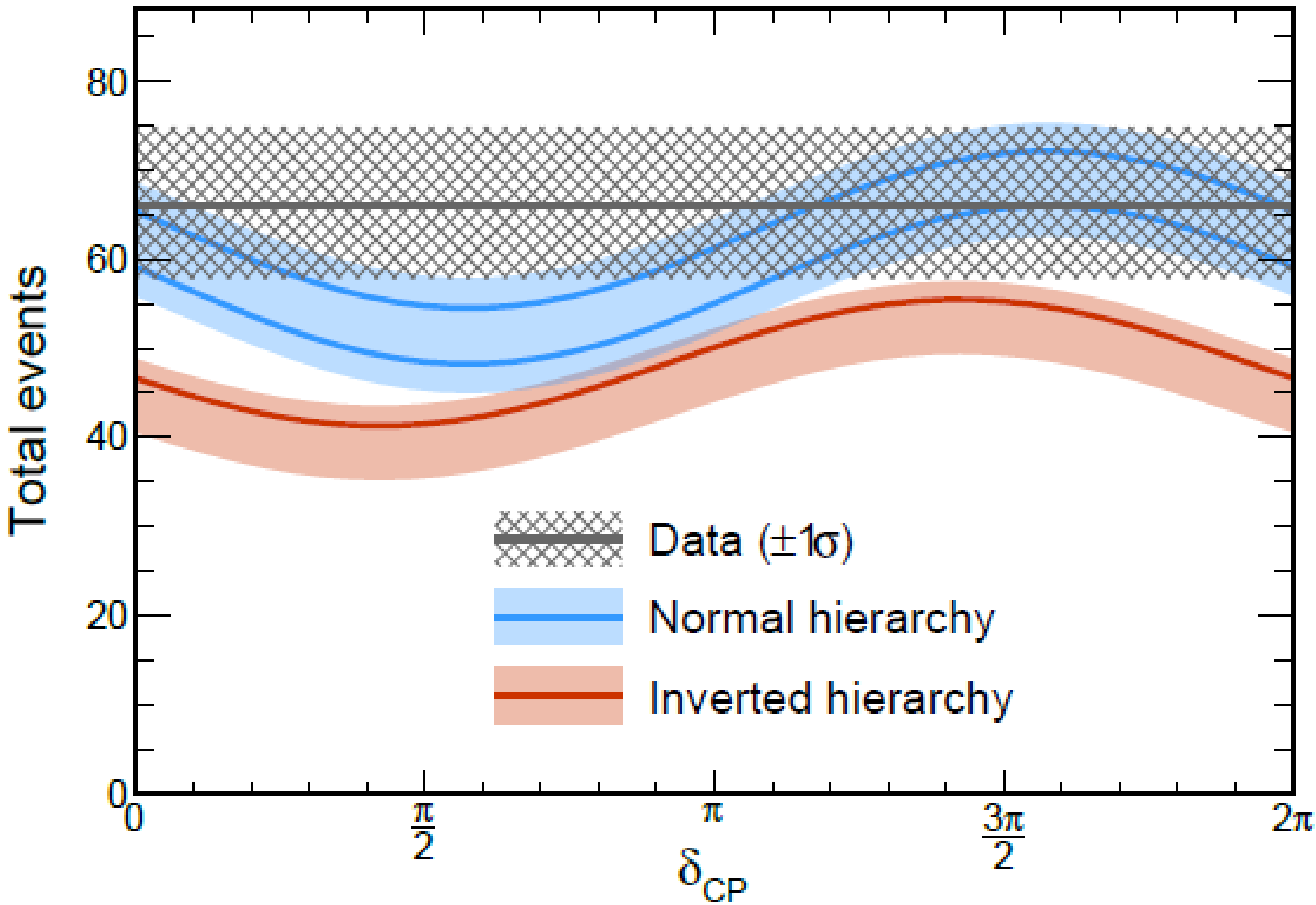
- The drastically different quark and lepton mixing patterns can be explained from the same flavor symmetry combined with CP, if the residual symmetry in quark sectors is Z_2xCP. $\Delta(6\cdot 7^2) = \Delta(294)$ is a good candidate for such flavor symmetry.

- Littlest seesaw inspired by flavor and CP symmetries (or indirect CP approach) provides new model building opportunity. New mixing patterns can be found, and an explicit model with S_4 and CP is constructed.

Thank you for your attention!

Backup

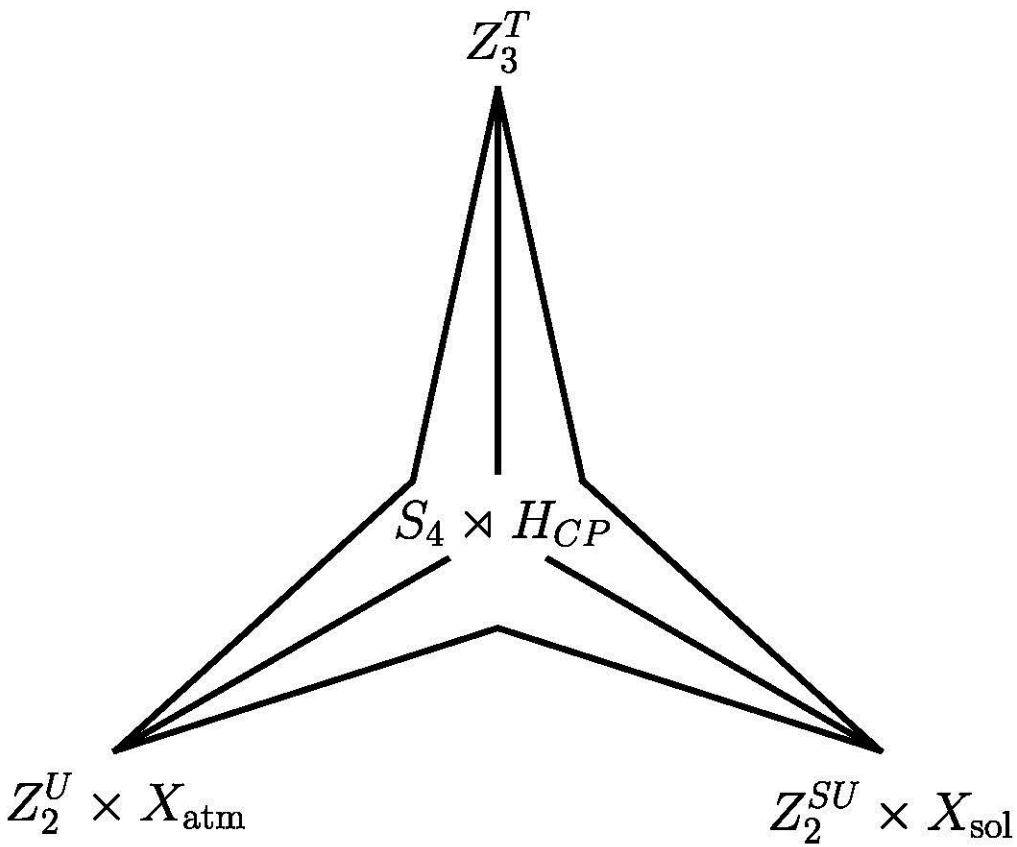
NOvA, Acero et al, 1806.00096



- Littlest seesaw

$$\langle\Phi_{\mathbf{atm}}\rangle = v_{\mathbf{atm}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \qquad \langle\Phi_{\mathbf{sol}}\rangle = v_{\mathbf{sol}}\begin{pmatrix}1\\n\\n-2\end{pmatrix}$$

$$X_{\text{atm}} = 1, \qquad X_{\text{sol}} = 1$$



The overall phase of the vacuum

$$
\begin{aligned}
w_d^{\mathrm{phase}} = {} & M_{\xi_s}^2 \zeta_1^0 + \frac{g_1}{\Lambda^2}\zeta_1^0 \xi_s^4 + M_{\xi_a}^2 \zeta_2^0 + \frac{g_2}{\Lambda^2}\zeta_2^0 \xi_a^4 + M_1^2 \zeta_3^0 + \frac{1}{\Lambda^2}\zeta_3^0 \left[g_3 \left(\eta_s \eta_s \varphi_s \psi_s\right)_{\mathbf{1}} \right. \\
& \left. + g_4 \left(\chi_s \varphi_s \psi_s \psi_s\right)_{\mathbf{1}} + g_5 \left(\phi_s \phi_s \psi_s \psi_s\right)_{\mathbf{1}} \right] + M_2^2 \zeta_4^0 + \frac{g_6}{\Lambda^2}\zeta_4^0 \left(\chi_s \phi_s \phi_a \phi_a\right)_{\mathbf{1}} \ ,
\end{aligned}
$$