W mass studies - Kinematic Fit

WG1 & WG2 working meeting

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February 6, 2018

Kinematic Fit - Principle (1)

The aim is ...

...to improve the resolution of an object slightly changing the measured quantities (e.g four-vector) within their uncertainty to fulfil kinematic constraints.

To solve a problem with **n measured parameters y**, **p unmeasured parameters a** and **m constraints f**, a kinematic fit will determine the corrections Δy on parameters y, such that :

1. $S(y) = \Delta y^T V^{-1} \Delta y = \chi^2$ is minimal V = cov. matrix y params.
2. All constraints are fulfilled :

$$
\begin{aligned}
f_1(a_1, ..., a_p, y_1, ...y_n) &= 0 \\
f_2(a_1, ..., a_p, y_1, ...y_n) &= 0 \\
&\vdots \\
f_m(a_1, ..., a_p, y_1, ...y_n) &= 0
\end{aligned}
$$

Kinematic Fit - Principle (2)

To determine the optimal Δy, the Lagrange Multipliers definition is used :

$$L(y, a, \lambda) = S(y) + 2\sum_{i=1}^{m} \lambda_i f_i(y, a)$$

one multiplier by constraints

L(y,a,λ) minimal when S(y) minimal and $f_i(y, a) = 0$

If the constraints are not linear, the minimization is done iteratively using the Taylor development at the first order :

$$f(y') \sim f_i(y^*) + \sum_{i=1}^{4} \frac{\partial f_i}{\partial y_i}(\Delta y_i - \Delta y_i^*) \sim 0$$

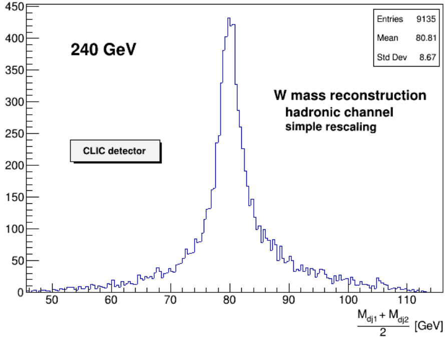
Fit output :

- $\chi^2 =$ probability that the event kinematic hypothesis are true
- Resolution improvement on reconstructed object

Kinematic Fit - W mass reconstruction (1)

W mass reconstruction improvement in the *hadronic decay*

- Samples : Simulation with Pythia and reconstruction with Heppy (CLIC detector, ee-kt algorithm, 4 jets per event) at 162.3, 240 and 350 GeV.
- Kinematic fit :
  - **Measured parameters** : rescaling coefficient α, jets angles θ and ϕ and jets velocity v
  - **Constraints** :
    - energy-momentum conservation (4C) : $\Sigma p_x = \Sigma p_y = \Sigma p_z = 0$ and $\Sigma E = \sqrt{s}$
    - mass equality (1C) : $M_{dj1} = M_{dj2}$



Kinematic Fit - W mass reconstruction (2) - Likelihood

The likelihood is defined as follow :

$$L = \frac{\sum_{i=1}^{4}(\alpha_i - \alpha_{i0})^2}{\sigma_\alpha^2} + \frac{\sum_{i=1}^{4}(\theta_i - \theta_{i0})^2}{\sigma_\theta^2} + \frac{\sum_{i=1}^{4}(\phi_i - \phi_{i0})^2}{\sigma_\phi^2} + \frac{\sum_{i=1}^{4}(v_i - v_{i0})^2}{\sigma_v^2}$$

$$+ 2\lambda_X \Sigma p_x$$

$$+ 2\lambda_Y \Sigma p_y$$

$$+ 2\lambda_Z \Sigma p_z$$

$$+ 2\lambda_E (\Sigma E - \sqrt{s})$$

$$+ 2\lambda_M (M_{dj1} - M_{dj2})$$

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Kinematic Fit - W mass reconstruction (2) - Likelihood

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$\Delta y = y_{meas} - y_0$

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$$+ 2\lambda_M (M_{dj1} - M_{dj2})$$

$$V^{-1} = \begin{pmatrix} \sigma_\alpha^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{pmatrix}$$

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Lagrange Multipliers

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constraints $f_i(y)$

Jets parametrization

Four-vector parametrization as function of the measured parameters :

- $p_x = \alpha E_0 v \sin\theta \cos\phi$
- $p_y = \alpha E_0 v \sin\theta \sin\phi$
- $p_z = \alpha E_0 v \cos\theta$
- $E = \alpha E_0$

Advantage = measurements independent (covariance matrix diagonal)
Disadvantage = non-linear constraints, derivatives more complicated

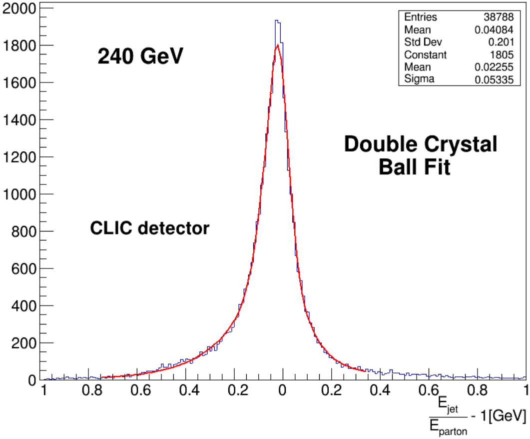
Kinematic fit - W mass reconstruction (3) - Covariance matrix

No correlation between parameters → diagonal matrix.

$$V^{-1} = \begin{pmatrix} \sigma_\alpha^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{pmatrix}$$

σ determined by fit:

- $\sigma_\alpha : \frac{E_{jets}}{E_{partons}} - 1$
- $\sigma_\theta : \theta_{genjets} - \theta_{recojets}$
- $\sigma_\phi : \phi_{genjets} - \phi_{recojets}$
- $\sigma_v : \frac{p}{E}_{genjets} - \frac{p}{E}_{recojets}$

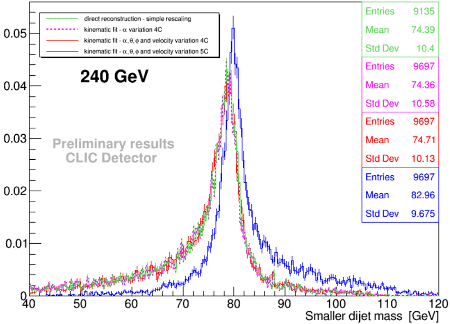


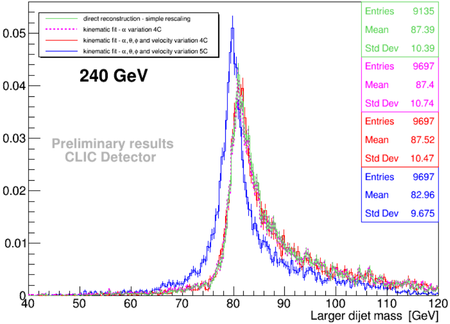
Kinematic fit - W mass reconstruction (4)

Resume

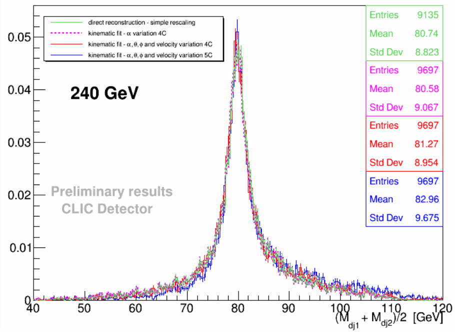
- Before fit : jets four-vector
- Kinematic fit : computation of Δy, the correction to apply on parameters in order to fulfil constraints taking into account their uncertainties.
- Fit output :
  - corrected jets four-vector
  - W mass given by the mass equality constraint

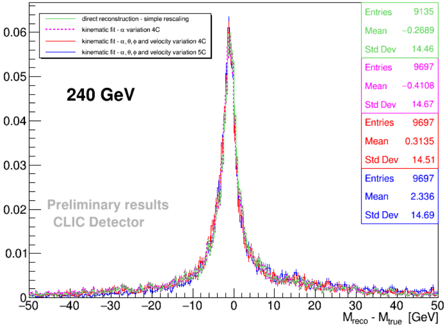
Kinematic fit - Results 240 GeV (1)



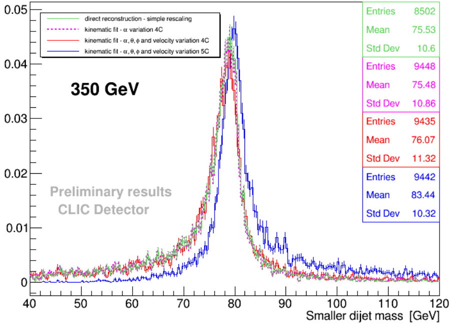


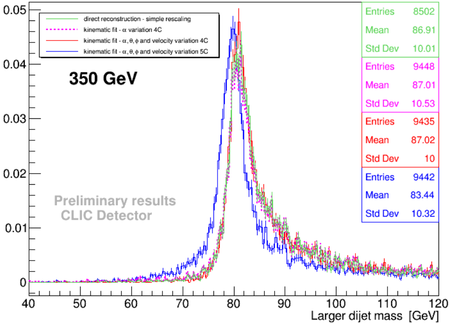
Kinematic fit - Results 240 GeV (2)



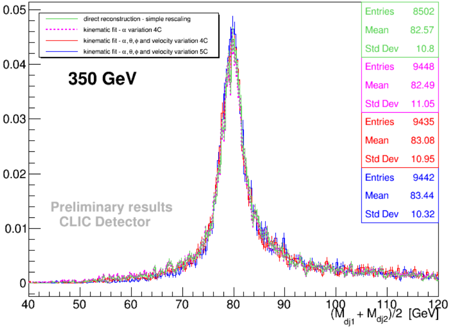


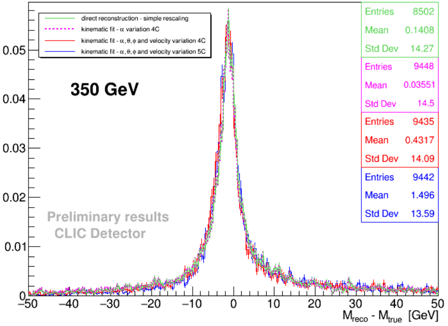
Kinematic fit - Results 350 GeV (1)



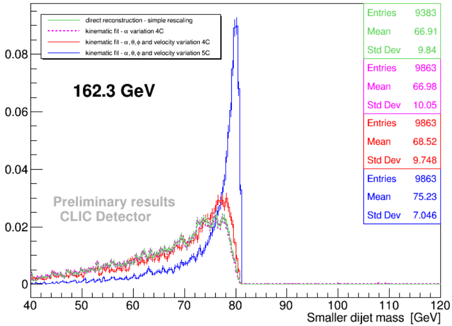


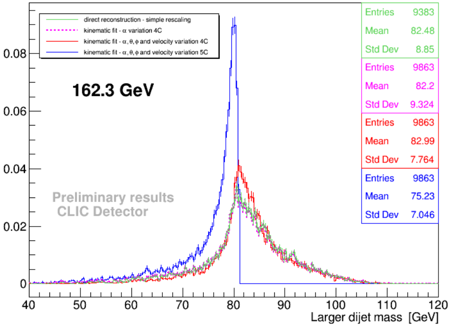
Kinematic fit - Results 350 GeV (2)



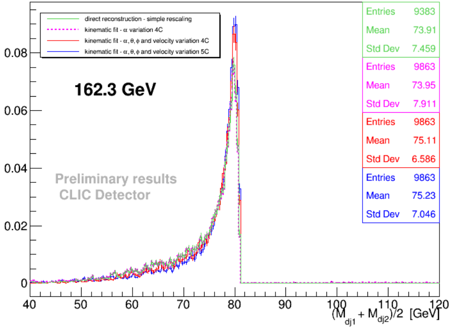


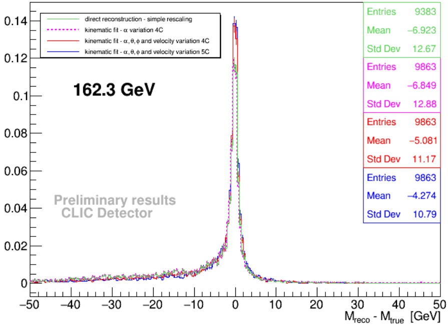
Kinematic fit - Results 162.3 GeV (1)





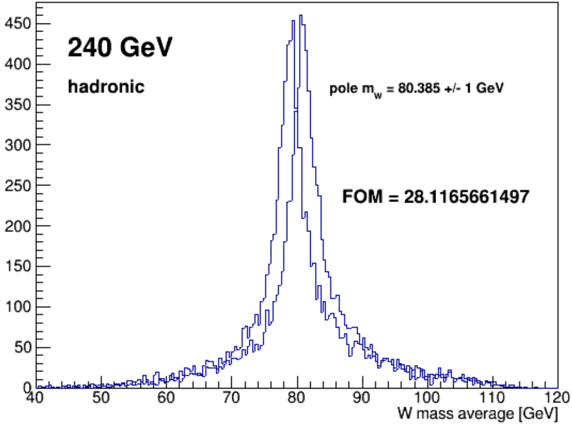
Kinematic fit - Results 162.3 GeV (2)





The W mass...

... is determined by comparison of the distribution with templates generated for $M_W \pm 1 GeV$. The figure of merit (FOM) is used to estimate the statistical uncertainty of M_W.

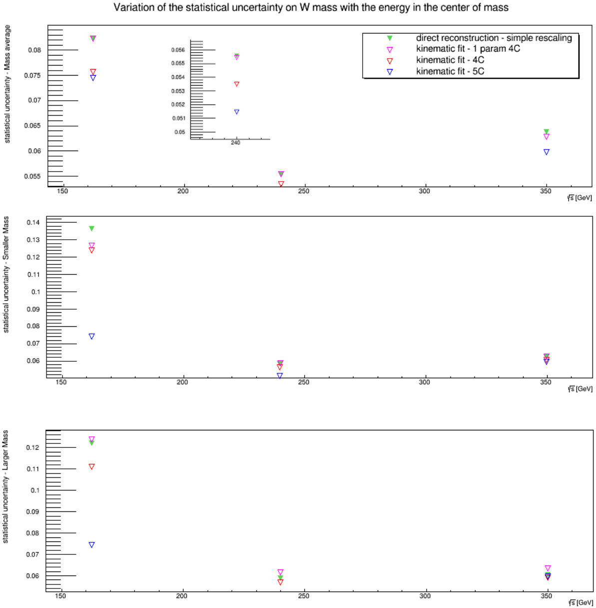


FOM computed like binned max. likelihood

$$FOM = \Delta M_w \frac{\Sigma(b_{79}+b_{81})}{\Sigma \frac{(b_{79}-b_{81})^2}{b_{79}+b_{81}}}$$

$$\sigma_{m_W,stat} = \frac{\sqrt{FOM}}{\sqrt{N_{event}}}$$

W mass measurement - Figure Of Merit (2)



10 000 events

Conclusion and Outlook

- Kinematic fit for the hadronic channel study is fully developed (4C and 5C fit)
- Improvement of W mass reconstruction for 240 and 350 GeV.
- The 5C fit at 162.3 GeV shows better W mass reconstruction but only the off-shell mass is well reconstructed. At this energy only the 4C fit will be used.

Next step : 2C fit for the semi-leptonic channel. The neutrino introduces unmeasured parameters to the fit.

Thanks for your attention

BACK-UP

Kinematic fit - Mathematics development (1)

Taylor development at first order :

$$f(y') \sim f_i(y^*) + \sum_{i=1}^{4} \frac{\partial f_i}{\partial y_i}(\Delta y_i - \Delta y_i^*) \sim 0 \quad (1)$$

With vector notation and $B = \frac{\partial f}{\partial y}$, this equation can be written

$$\vec{f^*} + B(\Delta\vec{y} - \Delta\vec{y^*}) \sim 0 \quad (2)$$

or

$$B\Delta\vec{y} - \vec{c} = 0 \quad with \quad \vec{c} = B\Delta\vec{y^*} - \vec{f^*} \quad (3)$$

Finally, the equation (1) becomes :

$$L = \Delta y^T \, V^{-1} \, \Delta y + 2\lambda^T (B\Delta\vec{y} - \vec{c}) \quad (4)$$

which is minimal if $\frac{\partial L}{\partial y} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$.

Kinematic fit - Mathematics development (2)

According to the equation (4), the equations to solve are :

$$\frac{\partial L}{\partial y} = V^{-1}\Delta y + B^T \lambda = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = B\Delta y - \vec{c} = 0 \tag{6}$$

With matrix notation this system becomes :

$$\begin{pmatrix} V^{-1} & B^T \\ B & 0 \end{pmatrix} \begin{pmatrix} \Delta y \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ \vec{c} \end{pmatrix} \tag{7}$$

then,

$$\begin{pmatrix} \Delta y \\ \lambda \end{pmatrix} = \begin{pmatrix} V^{-1} & B^T \\ B & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \vec{c} \end{pmatrix} \tag{8}$$

The tuned parameters α, θ, ϕ and v which are used to determined the fitted four-vector of each jets, can be computed with the Δy from equation (8)

Kinematic fit - Mathematics development (3)

$$B = \frac{\partial f}{\partial y} = \begin{pmatrix} \frac{\partial \Sigma p_x}{\partial \alpha} & \frac{\partial \Sigma p_x}{\partial v} & \frac{\partial \Sigma p_x}{\partial \phi} & \frac{\partial p_x}{\partial \theta} \\ \frac{\partial \Sigma p_y}{\partial \alpha} & \frac{\partial \Sigma p_y}{\partial v} & \frac{\partial \Sigma p_y}{\partial \phi} & \frac{\partial p_y}{\partial \theta} \\ \frac{\partial \Sigma p_z}{\partial \alpha} & \frac{\partial \Sigma p_z}{\partial v} & \frac{\partial \Sigma p_z}{\partial \phi} & \frac{\partial \Sigma p_z}{\partial \theta} \\ \frac{\partial (\Sigma E - \sqrt{s})}{\partial \alpha} & \frac{\partial (\Sigma E - \sqrt{s})}{\partial v} & \frac{\partial (\Sigma E - \sqrt{s})}{\partial \phi} & \frac{\partial (\Sigma E - \sqrt{s})}{\partial \theta} \end{pmatrix} \quad (9)$$

As the constraints depend directly on the four-vector components $P = (\vec{p}, E)$, each matrix term can be simplified with a derivative chain :

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial P} \frac{\partial P}{\partial y} \quad with \quad f = \Sigma p_i \quad (10)$$