Influence of vertical orbit distortions on energy calibration accuracy

A. Bogomyagkov

Budker Institute of Nuclear Physics
Novosibirsk

January 11th, 2018

References

- A.M. Kondratenko. Doctoral Thesis. Novosibirsk, 1982.
- R. Assmann, J.P. Koutchouk, CERN SL/94-13 (AP).
- A.V. Bogomyagkov, S.A. Nikitin, A.G. Shamov, MOAP02,RuPAC 2006, Novosibirsk, Russia, http://accelconf.web.cern.ch/AccelConf/r06/PAPERS/MOAP02.PDF
- https://arxiv.org/abs/1801.01227

Introduction

For flat orbits only

$$E[MeV] = 440.64843(3) \times \nu \, .$$

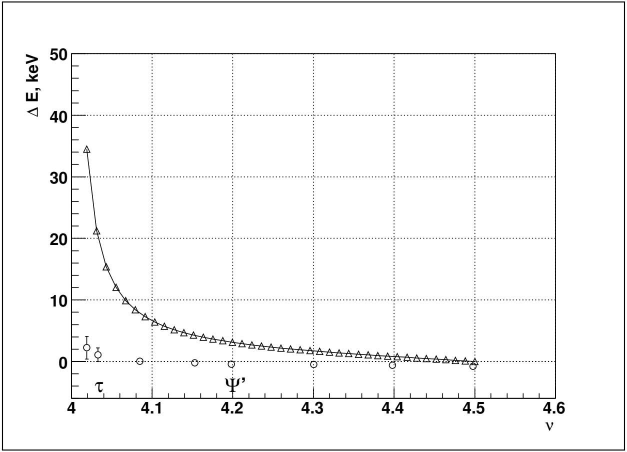
Approximation (R. Assmann, J.P. Koutchouk)

$$\Delta\nu = \frac{\nu^2 \cot \pi\nu}{8\pi} \sum \alpha_i^2 \, ,$$

α_i are the orbit rotation angles.
Using observed vertical orbit RMS $\left\langle z^2 \right\rangle$ (assuming that $\langle z \rangle = 0$), number of quadrupole lenses N with average focal length F

$$\Delta\nu = \frac{\nu^2 \cot(\pi\nu)}{8\pi} \frac{N \left\langle z^2 \right\rangle}{F^2} \, .$$

Validity of approximation



Energy shift versus spin tune at 1 mm vertical orbit RMS for VEPP-4M. Triangles are calculations by approximate expression, circles with error bars are results of the simulation.

General approach

Spin tune shift (Kodratenko)

$$\Delta\nu = \frac{1}{2}\sum_k \frac{|\omega_k|^2}{\nu - k}$$

Spin harmonics

$$\omega_k = \frac{1}{2\pi}\int\limits_0^{2\pi} \nu z'' \exp\left[-i(\Phi(\theta) - \nu\theta) - ik\theta\right] d\theta\,,$$

$$z'' = \frac{1}{R}\frac{d^2 z}{d\theta^2}\,,$$

$$\Phi(\theta) = \int_0^\theta \nu R K_0(\theta') d\theta'$$

Approximation of general approach

Assumptions and definitions

- No straight sections: $\Phi(\theta) = \nu\theta$
- Constant vertical beta function: $\beta_z = const = \langle\beta_z\rangle$
- Average over circumference $\langle\rangle$, average over orbits $\bar{\ }$

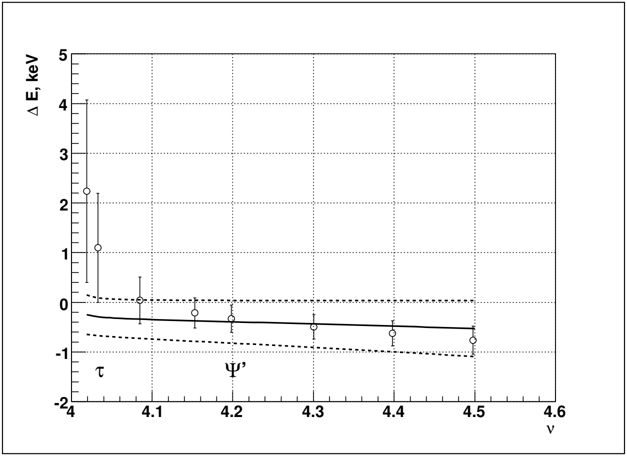
Results

$$\overline{\Delta\nu} = \frac{\nu^2}{2}\frac{\overline{\langle z^2\rangle}}{Q}\sum_{k=-\infty}^{\infty}\frac{k^4}{(\nu_z^2 - k^2)^2(\nu - k)}$$

$$Q = \frac{\pi}{2\nu_z^3}\cot\pi\nu_z + \frac{\pi^2}{2\nu_z^2}\csc^2\pi\nu_z$$

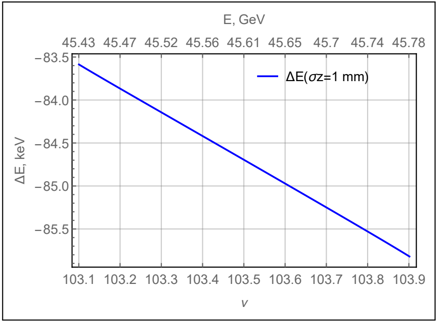
$$\sigma_{\overline{\Delta\nu}} = \frac{\nu^2\sqrt{3}}{2}\frac{\overline{\langle z^2\rangle}}{Q}\sqrt{2\nu\sum_{k=-\infty}^{\infty}\frac{k^8}{(\nu_z^2 - k^2)^4(\nu - k)^2(\nu + k)}}$$

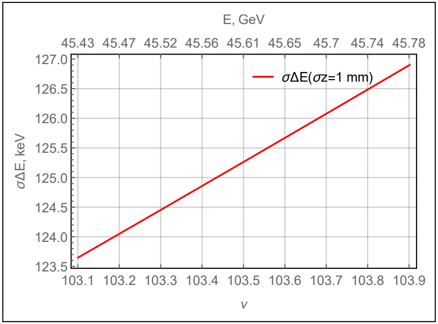
Validity of approximation of general approach



Energy shift versus spin tune at 1 mm vertical orbit RMS for VEPP-4M. Solid and dashed lines are the spin tune shift and its uncertainty, circles with error bars are results of the simulation.

FCCee at $E = 45.6$ GeV, $\sigma_z = 1$ mm





Tables for Z and W

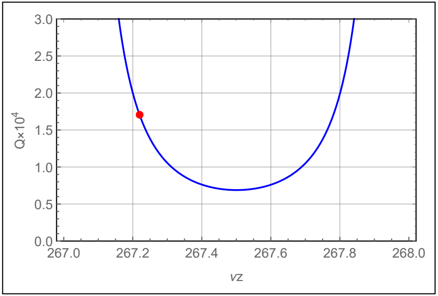
E, GeV	45.6	78.65	81.3
σ_z, mm	1		
ν_z	267.22		
ν	103.484	178.487	184.5
$\Delta\nu$	$-1.9 \cdot 10^{-4}$	$-1.5 \cdot 10^{-3}$	$-1.8 \cdot 10^{-3}$
$\sigma\Delta\nu$	$2.8 \cdot 10^{-4}$	$2.2 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$
ΔE, keV	-84.65	-667.116	-779.992
$\sigma\Delta E$, keV	125.197	986.9	1153.9
$\frac{\Delta E}{E}$	$-1.9 \cdot 10^{-6}$	$-8.5 \cdot 10^{-6}$	$-9.6 \cdot 10^{-6}$
$\frac{\sigma\Delta E}{E}$	$2.7 \cdot 10^{-6}$	$1.3 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$

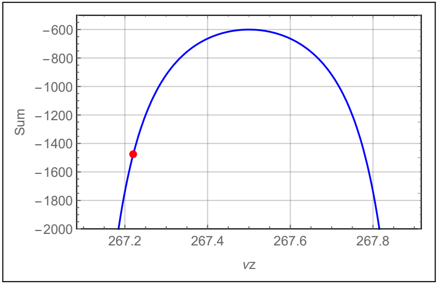
Beam energy shift needs to be added to the actual value of the beam energy, uncertainty is unavoidable and sets the minimum error.

Choice of ν_z

$$\overline{\Delta\nu} = \frac{\nu^2}{2}\frac{\overline{\left\langle z^2\right\rangle}}{Q}\sum_{k=-\infty}^{\infty}\frac{k^4}{(\nu_z^2 - k^2)^2(\nu - k)}$$

$$Q = \frac{\pi}{2\nu_z^3}\cot\pi\nu_z + \frac{\pi^2}{2\nu_z^2}\csc^2\pi\nu_z$$





Conclusion

- Vertical orbit distortions produce beam energy shift.
- Vertical orbit distortions produce uncertainty of the beam energy.
- Beam energy shift dependence on vertical betatron frequency is small.