Unified approach for multiband optical model in soft deformed even-even and odd-A nuclides

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Outline

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  • Effective deformations: fitting params, calc using structure model
  • SRM
• Problem: odd nuclides
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  • Collectivity – even-even core
  • Spins assignment
• Results
  • XS, total ratios
  • Multiband coupling: CN ratios, direct XS
  • Stretching
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Optical model for soft deformed nuclei

But actinides are both considerably deformed in GS and soft for vibrations

Explicit deformations $\rightarrow$ vibrations
bad convergence for big deformations

Good convergence for big static deformations
no explicit deformations $\rightarrow$ no vibrations!
Solution: Taylor expansion near axial static form

\[ R_i(\theta', \varphi') \]

\[ = R_{0i} \left\{ 1 + \sum_{\lambda=2,3; \text{even } \mu} \beta_{\lambda \mu} Y_{\lambda \mu}(\theta', \varphi') + \sum_{\lambda=4,6} \beta_{\lambda 0} Y_{\lambda 0}(\theta') \right\} \]

\[ = R_i^{\text{zero}}(\theta') + \delta R_i(\theta', \varphi'; \delta \beta_2, \gamma, \beta_3) \]

\[ = R_{0i} \left\{ 1 + \sum_{\lambda=2,4,6} \beta_{\lambda 0} Y_{\lambda 0}(\theta') \right\} \]

\[ \begin{align*}
\beta_2 &= \beta_{20} + \delta \beta_2 \\
\langle \delta \beta_2^2 \rangle, \langle \gamma^2 \rangle, \langle \delta \beta_3^2 \rangle &\ll \beta_{20}^2
\end{align*} \]

\[ + R_{0i} \left\{ \beta_{20} \left[ \frac{\delta \beta_2}{\beta_{20}} \cos \gamma + \cos \gamma - 1 \right] Y_{20}(\theta') \right. \\
+ \left( \beta_{20} + \delta \beta_2 \right) \sin \gamma \left[ Y_{22}(\theta', \varphi') + Y_{2-2}(\theta', \varphi') \right] \right. \\
+ \beta_3 Y_{30}(\theta') \right\} \]
Potential expansion near axially deformed shape

\[ V(r, R(\theta', \varphi')) \approx V(r, R^{\text{zero}}(\theta')) + \frac{\partial}{\partial R} V(r, R(\theta', \varphi')) \bigg|_{R^{\text{zero}}(\theta')} \delta R(\theta', \varphi'; \delta \beta_2, \gamma, \beta_3) \]

\[ \approx V(r, R^{\text{zero}}(\theta')) + \frac{v_2(r)}{R_0 \beta_2} \delta R(\theta', \varphi'; \delta \beta_2, \gamma, \beta_3) \]

\[ R^{\text{zero}}(\theta') = R_{0i} \left\{ 1 + \sum_{\lambda=2,4,6} \beta_{\lambda 0} Y_{\lambda 0}(\theta') \right\} \]

\[ v_2(r) = 2\pi \int_0^\pi V(r, R^{\text{zero}}(\theta')) Y_{20}(\theta') \sin \theta' \, d\theta' \]

E.S. Soukhovitskiï et al, PRC 94 (2016) 64605
Coupled channels matrix elements

\[
\langle i | V(r, \theta, \varphi) | f \rangle = \sum_{K} \sum_{K'} A_K^l A_{K'}^{l'} \left\{ \sum_{\alpha=0,2,4,...} \nu_\alpha(r) \langle IK || D_\alpha^2 || I'K' \rangle A \left( ljI; l'j'I'; \alpha J \frac{1}{2} \right) \delta_{KK'} 
+ \nu_2(r) \left[ [\beta_2]_{\text{eff}} + [\gamma_{20}]_{\text{eff}} \right] \langle IK || D_0^2 || I'K' \rangle A \left( ljI; l'j'I'; 2J \frac{1}{2} \right) \delta_{KK'} 
+ \left[ \gamma_{22} \right]_{\text{eff}} \langle IK || D_2^2 + D_{-2}^2 || I'K' \rangle A \left( ljI; l'j'I'; 2J \frac{1}{2} \right) 
+ [\beta_3]_{\text{eff}} \langle IK || D_0^3 || I'K' \rangle A \left( ljI; l'j'I'; 3J \frac{1}{2} \right) \delta_{KK'} 
+ [\beta_0]_{\text{eff}} \delta_{KK'} \delta_{l'l'} \delta_{jj'} \delta_{ll'} \right\}
\]

- **Rigid rotor**
- **\(\beta\)- and \(\gamma\)-vibrations and stretching**
- **\(K = 2\) band coupling**
- **Octupole coupling (negative parity band)**
- **Volume conservation correction**
Effective deformations

\[
\begin{align*}
[\beta_2]_{\text{eff}} &= \langle n_i(\beta_2) | \frac{\delta \beta_2}{\beta_{20}} | n_f(\beta_2) \rangle \\
[\beta_3]_{\text{eff}} &= \langle n_i(\beta_3) | \frac{\beta_3}{\beta_{20}} | n_f(\beta_3) \rangle \\
[\gamma_{20}]_{\text{eff}} &= \langle n_i(\gamma) | \cos \gamma - 1 | n_f(\gamma) \rangle \\
[\gamma_{22}]_{\text{eff}} &= \langle n_i(\gamma) | \frac{\sin \gamma}{\sqrt{2}} | n_f(\gamma) \rangle \\
[\beta_2^2]_{\text{eff}} &= \langle n_i(\beta_2) | \frac{\delta \beta_2^2}{\beta_{20}^2} | n_f(\beta_2) \rangle \\
[\beta_3^2]_{\text{eff}} &= \langle n_i(\beta_3) | \frac{\beta_3^2}{\beta_{20}^2} | n_f(\beta_3) \rangle \\
[\beta_0]_{\text{eff}} &= -\frac{\beta_{20}}{\sqrt{4\pi}} \left[ 2[\beta_2]_{\text{eff}} + [\beta_2^2]_{\text{eff}} + [\beta_3^2]_{\text{eff}} \right]
\end{align*}
\]
Softness effects

• Multiband coupling (for bands, corresponding to collective excitations)

• Nucleus stretching due to rotation (centrifugal forces)

• Additional monopole coupling due to account of volume conservation in vibrating nucleus
Approach to effective deformations

**Effective deformations as fitting parameters**

- Rough model to keep minimal number of parameters, only multiband coupling accounted (rigid rotor coupling within each band)
- No additional knowledge needed

**Direct calculation**

- Nuclear structure model needed
- Gives all model effects

For even-even nuclides using SRM:

E.S. Soukhovitskii et al, PRC 94 (2016) 64605

Towards odd nuclides

No appropriate nuclear model (describing softness), but...

- Nuclear softness – collective effect, determined mainly by the even-even core, and varies smoothly from nucleus to nucleus
- \( \langle \psi_{\text{odd}} | \beta | \psi_{\text{odd}} \rangle \approx \langle \psi_{\text{core}} | \beta | \psi_{\text{core}} \rangle \)
- We may try to couple levels for bands built on single-particle state same as in GS
- We need to build appropriate core states
Core states assignment ($^{233}$U)

**Band (A):** $5/2^+[633]$, $\alpha=+1/2$ band

**Band (a):** $5/2^+[633]$, $\alpha=-1/2$ band

Core state: no vibrational excitation, only rotation

**Core $I^\pi$**

- GS band
- Octupole band

Core states assignment ($^{233}$U):

- **Core $I^\pi$**
  - First octupole excitation, rotation
  - In fact, we have also two subbands here!

- **Core $I^\pi$**
  - 2+
  - 6+
  - 4+
  - 2+
  - 0+

- **Core $I^\pi$**
  - 8+
  - 6+
  - 4+
  - 2+
  - 0+

**Core $I^\pi$**

- 2+
- 8+
- 6+
- 4+
- 2+
- 0+
Calculation algorithm

1. Build new regional OMP on $^{238}\text{U}$ and $^{232}\text{Th}$ experimental data

2. Use $^{233}\text{U}$ experimental data to fit its deformations with resulting OMP

3. Calculate all model predictions for $^{233}\text{U}$, $^{238}\text{U}$, and $^{232}\text{Th}$ for full and restricted (some options disabled or truncated coupling scheme) model
Coupling scheme

238U

- GS band
- NAX band
- γ-band
- β-band
- negative parity band

233U

- GS band
- Γ/2+
- β-band
- negative parity band
- γ-band
- 7/2+
OMP figure of merit: symmetrized total XS ratio for different nuclei $R(A, B) = \frac{1}{2} \frac{\sigma_A - \sigma_B}{\sigma_A + \sigma_B}$

$p_{233}U$ to $p_{238}U$

$p_{232}Th$ to $p_{238}U$
Multiband coupling: Direct level excitation $X_S$

![Graphs showing the excitation cross-sections for $^{233}$U and $^{238}$U](image)

- For $^{233}$U, the graphs display excitation cross-sections for various bands and states, including Octupole band, Beta band, GS $7/2^+$, GS $9/2^+$, GS $11/2^+$, GS $13/2^+$, GS $15/2^+$, GS $17/2^+$.
- For $^{238}$U, the graphs show excitation cross-sections for Octupole band, Beta band, Gamma band, Abnormal band, GS $2^+$, GS $4^+$, GS $6^+$, GS $8^+$.
Multiband coupling: CN XS change

![Graph showing the change in CN XS with different band states for 238U and 233U.](image)

- **R(CN)** for 238U without the β-band and with all bands.
- **R(CN)** for 238U without the octupole band and with all bands.
- **R(CN)** for 238U GS band and with all bands.

![Graph showing the change in CN XS with different band states for 233U.](image)

- **R(CN)** for 233U without the β-band and with all bands.
- **R(CN)** for 233U without the octupole band and with all bands.
- **R(CN)** for 233U GS band and with all bands.
Nucleus stretching: CN XS change

\[
\begin{array}{cccc}
10^{-3} & 10^{-2} & 10^{-1} & 10^0 \\
10^1 & 10^2 & \end{array}
\]

\[R_{\text{CN}}(\text{soft, rigid})\]

\[E_n (\text{MeV})\]

\[233\text{U, no VC}\]

\[238\text{U, axial, no VC}\]
Volume conservation: CN XS change

\[ R_{\text{CN}}(\text{no VC; full}) \]

\[ 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} \]

\[ E_n (\text{MeV}) \]

\[ ^{233}\text{U}, ^{238}\text{U} \]
Summary

• Softness and multiband coupling are important to reach accurate CCOM calculations results for odd nuclides

• We can build and use fully functional regional OMP for actinides, both even-even and odd!

• Have to look thoroughly to band assignment in odd actinides: very few rotational bands built on GS single-particle state + core vibration! (e.g. none for $^{235}\text{U}$)
Software

All calculations performed by two FORTRAN codes which have been being developed E. Soukhovitskii and coworkers for many years:

• optical model code **OPTMAN** (optical potential fitting, cross-section calculations)

• nuclear structure code **SHEMMAN** (soft-rotator model parameters fitting and levels prediction)

**OPTMAN**

• recommended to use with latest version of IAEA reference input parameter library (RIPL-3) for nuclear data evaluation

• used with the EMPIRE – nuclear reaction model code, one of the most used tools for basic research and evaluation of nuclear data


Thank you for the attention!
Comparison with the GS-band-only potential