

# Energy dependence of Delayed-Neutron Data

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*DEN/CAD/DER/SPRC/LEPh*



**WONDER 2018**

October 9<sup>th</sup>, 2018

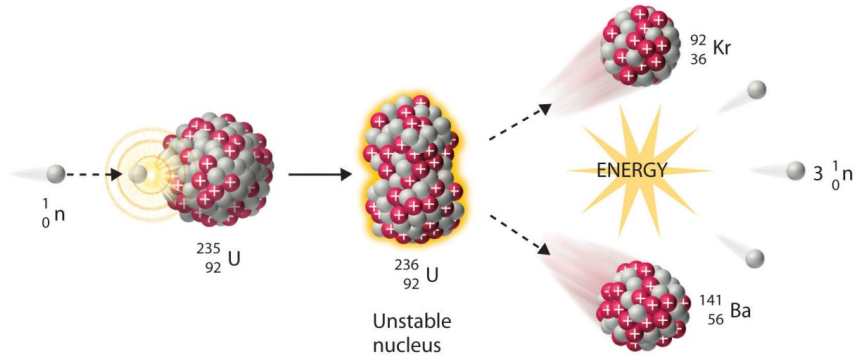
# OUTLINE:

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- Context
- State of the art
- Alternative method
- Results:  $v_d$
- Results:  $a_i$ ,  $\langle T_{1/2} \rangle$
- Conclusions
- Open questions

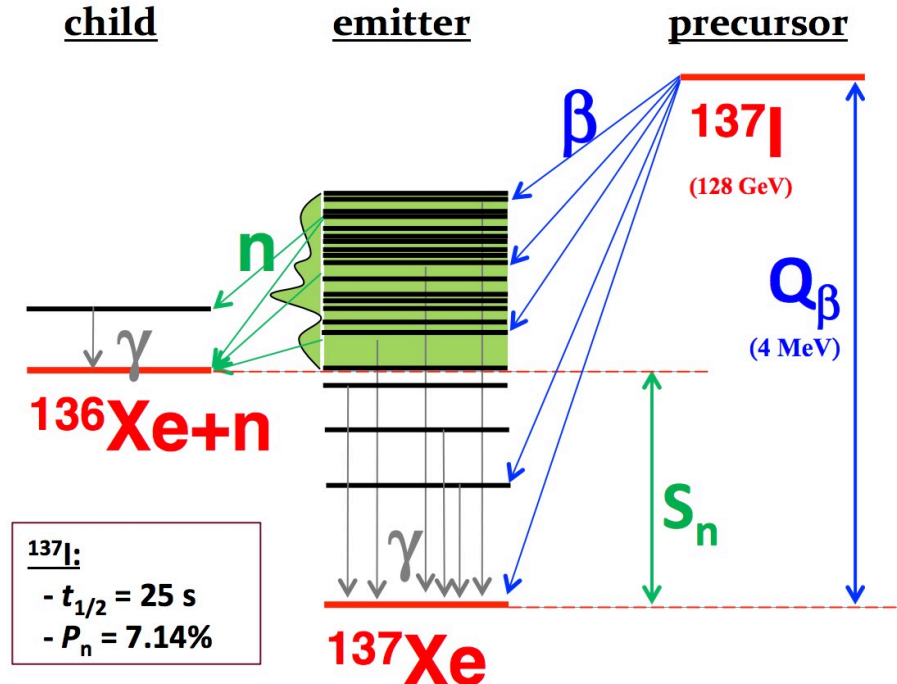
# Context

## Physics of Delayed Neutrons



Delayed neutrons are emitted by  $\beta^-$  - decay of fission fragments

The neutron emission (from the **emitter**) is almost instantaneous. The delay in the appearance of the **delayed neutron** is therefore linked to the half-life of the **precursor**!



$$\rho = \frac{\Lambda}{T} + \beta_{eff} \sum_i \frac{a_i}{1 + \lambda_i T} \approx \beta_{eff} \frac{\overline{T_{1/2}}}{T}$$

$$\beta_{eff} = \frac{\sum_k \int_0^\infty \Phi^+(E') \chi_{d,k}(E') dE' \int_0^\infty \nu_{d,k}(E) \Sigma_{f,k}(E) dE}{\sum_k \int_0^\infty \Phi^+(E') \chi_{t,k}(E') dE' \int_0^\infty \nu_{t,k}(E) \Sigma_{f,k}(E) dE}$$

$$\overline{T_{1/2}} = \sum_i \frac{a_i}{\lambda_i}$$

### Quantities of interest:

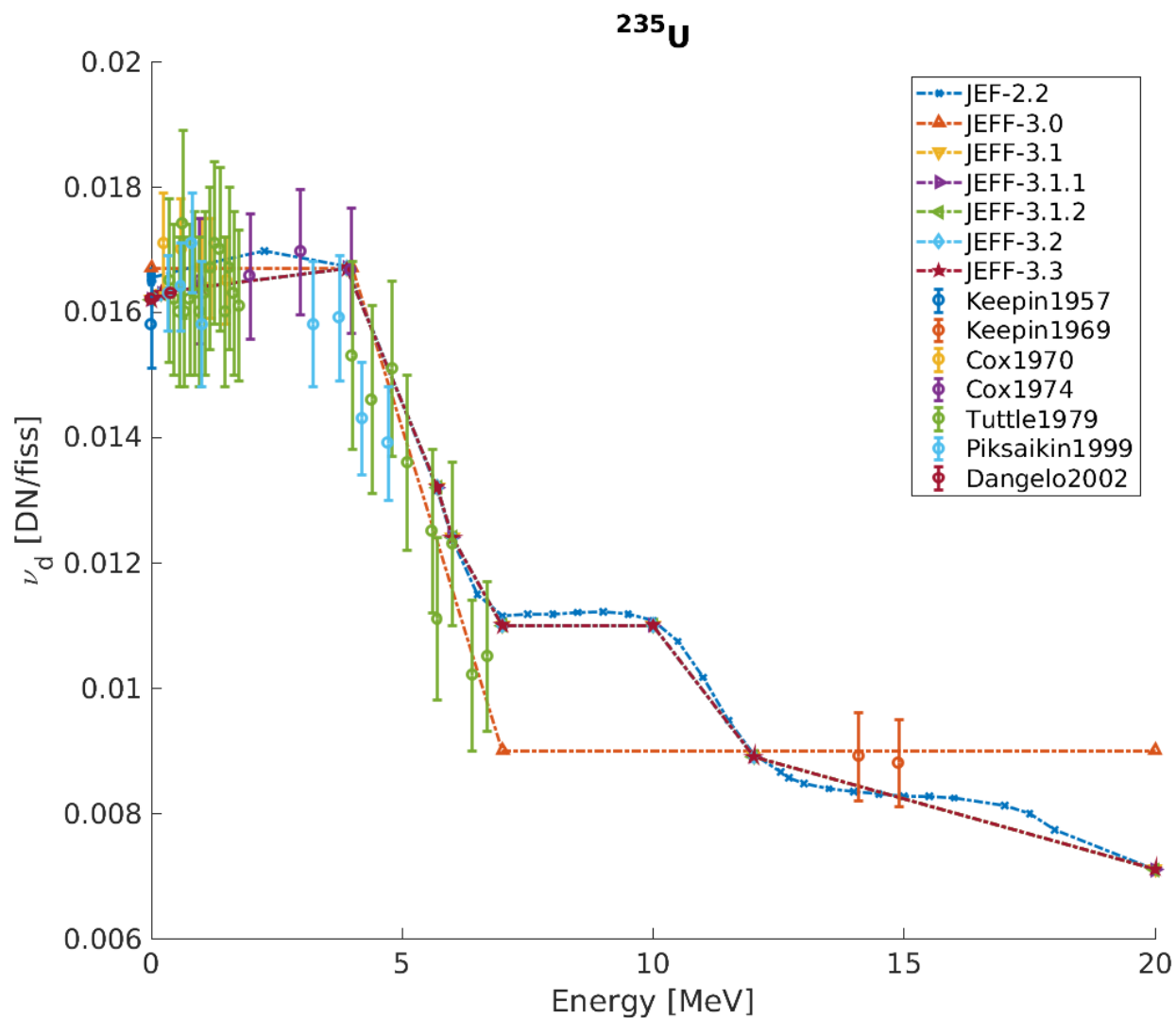
- Average DN yield  $\nu_{d,k}(E)$
- DN group kinetic parameters  $a_i(E)$   $\lambda_i$
- DN spectra  $\chi_{d,k}(E')$

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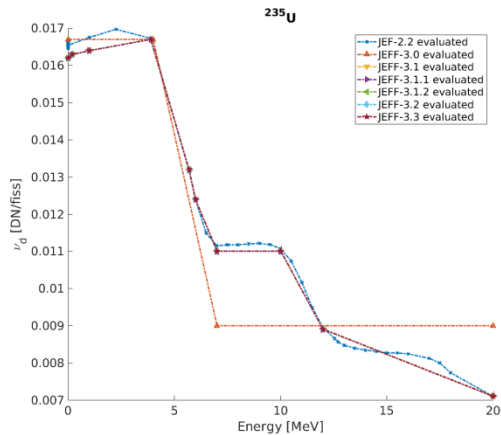
# State of the art

## JEFF evaluated library



JEF2

LIBRARY, DUMMY TAPE HEADER



MF=1

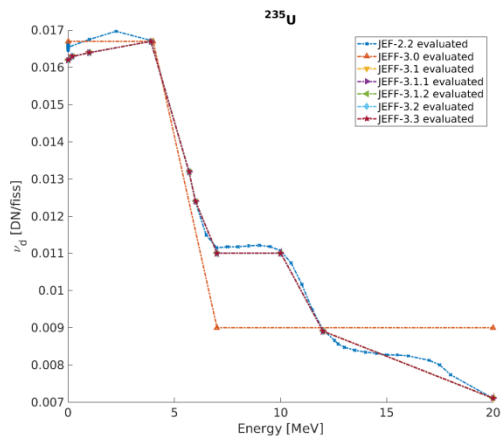
MT=452 NU. SYSTEMATICS. (REF. 5)

MT=458 ENERGY OF FISSION. SYSTEMATICS OF SHER (REF. 6)

→ No info about MT=455

In producing the JEF-2.2 evaluations for  $^{235}\text{U}$  and  $^{239}\text{Pu}$  Fort and Long (1989) calculated the energy dependence using the theoretical model of Lendel *et al* (1986).

**A. D'angelo and J. Rowlands** Conclusions concerning the delayed neutron data for the major actinides (2002)



### JEF2 LIBRARY, DUMMY TAPE HEADER

JEFF-3.0 file header. Release April 2002

JEFF-3.1 General Purpose Neutron File, May 2005.

JEFF-3.1.1 general purpose neutron file update release Jan. 2009

JEFF-3.1.2 general purpose neutron file update released Oct.2011

MF=1

MT=452 NU. SYSTEMATICS. (REF. 5)

MT=458 ENERGY OF FISSION. SYSTEMATICS OF SHER (REF. 6)

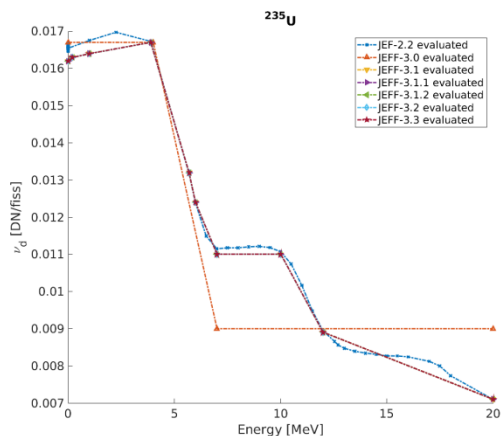
File 1 Descriptive and Nubar Information

MT=455 Delayed neutron yields from England [EN89] ★

★ [En89] T.R. England et al, Los Alamos National Laboratory  
reports: LA-11151-MS(88)  
LA-11534T(89)  
LAUR-88-4118 to be published in Nucl.Sci.Eng. (1989)

# State of the art

## Recursive reference



JEF2 LIBRARY, DUMMY TAPE HEADER

JEFF-3.0 file header. Release April 2002

JEFF-3.1 General Purpose Neutron File, May 2005.

JEFF-3.1.1 general purpose neutron file update release Jan. 2009

JEFF-3.1.2 general purpose neutron file update released Oct.2011

JEFF-3.2 Release - Neutron File March 2014

JEFF-3.3 Incident Neutron File

MF=1

MT=452 NU. SYSTEMATICS. (REF. 5)

MT=458 ENERGY OF FISSION. SYSTEMATICS OF SHER (REF. 6)

File 1 Descriptive and Nubar Information

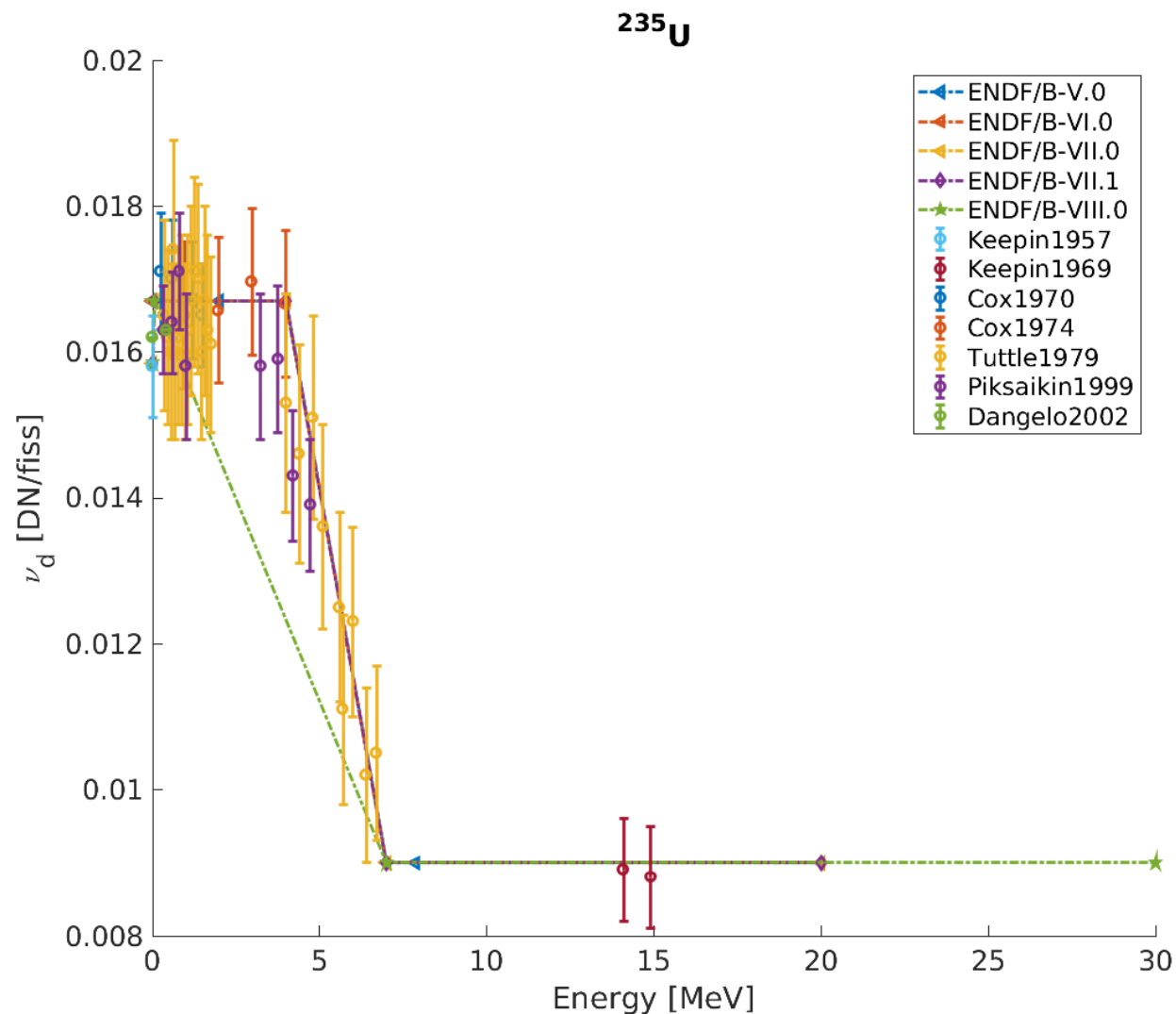
MT=455 Delayed neutron yields from England [EN89]

MF=1 General Information

MT=455 Delayed Neutron Yields : JEFF-3.1

MF 1 MT 452,455: restoring delayed neutron data from JEFF-311  
total is modified accordingly (CEA/DEN)

# State of the art ENDF/B evaluated library



*Can't we do better than recycling over  
and over again the same data?*



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There is another way to compute macroscopic quantities:

### Summation Calculations

Average DN yield

$$v_d = \sum_i^N CY_i \cdot P_{n,i} \cdot x_i$$

Kinetic Parameters

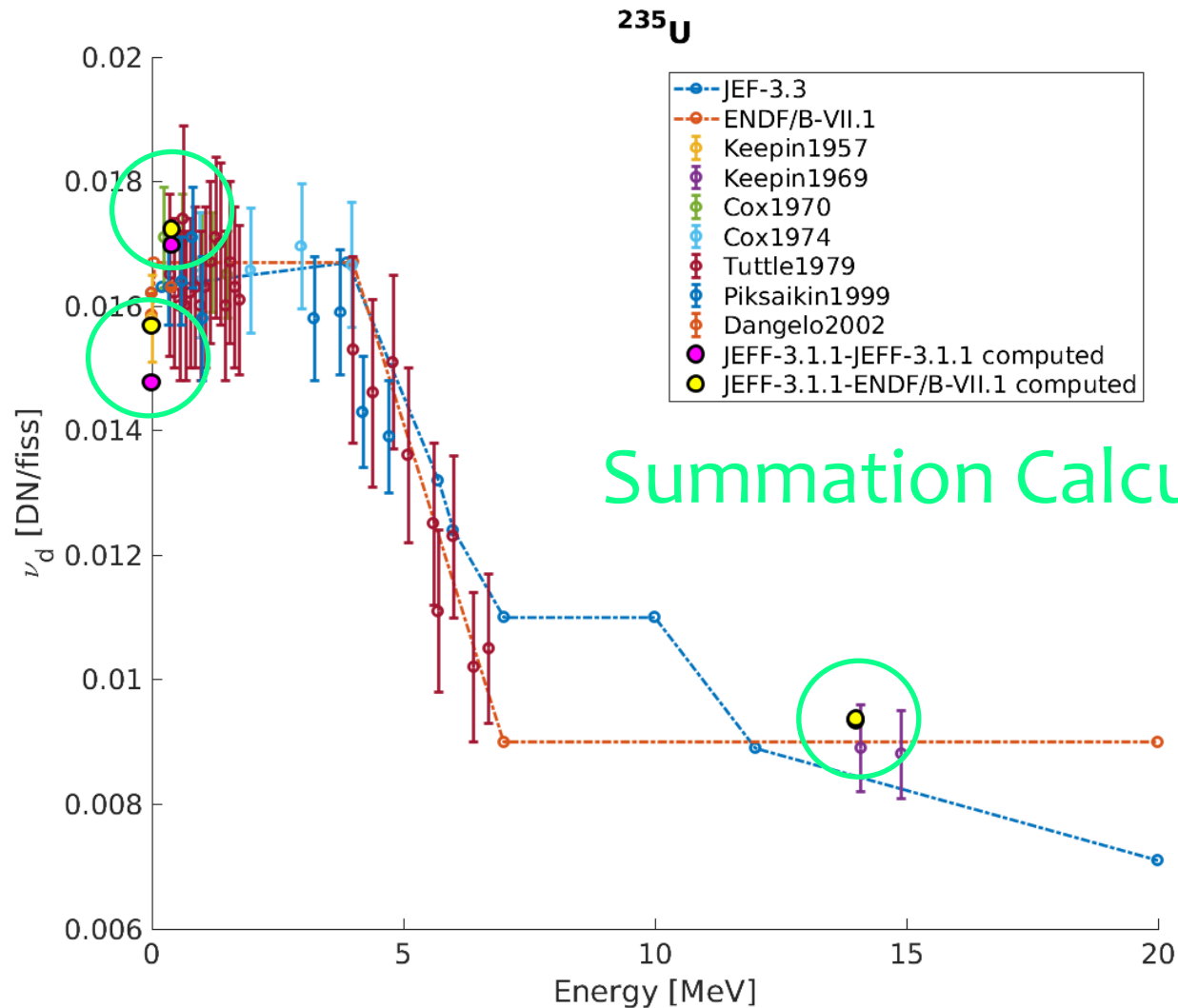
$$\frac{n_d(t)}{n_d(t_\infty)} - \sum_{i=1}^8 \left( 1 - \sum_{\substack{j=1 \\ j \neq i}}^8 a_j \right) (1 - e^{-\lambda_i t_{irr}}) e^{-\lambda_i (t - t_{irr})} = 0$$

Mean precursors' half-life

$$\overline{T_{1/2}} = \frac{\sum_i^8 a_i \cdot T_{1/2,i}}{\sum_i^8 a_i}$$

# Alternative method

## Summation calculations – 3 points only



### Use GEF to compute JEFF

The idea is to apply the GEF energy dependence to JEFF

$$FY_{JEFF}(E) = \frac{FY_{GEF}(E)}{FY_{GEF}(E_{ref})} FY_{JEFF}(E_{ref})$$

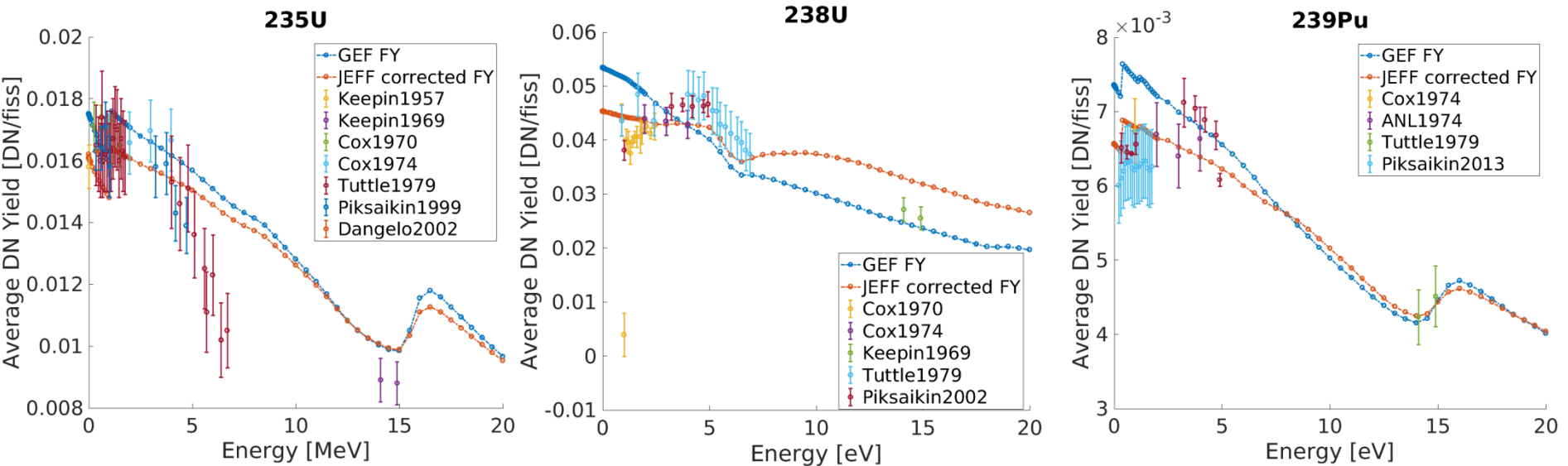
Fissioning System	E <sub>ref</sub>
<sup>235</sup> U, <sup>239</sup> Pu	0,0253 eV
<sup>238</sup> U	400 keV

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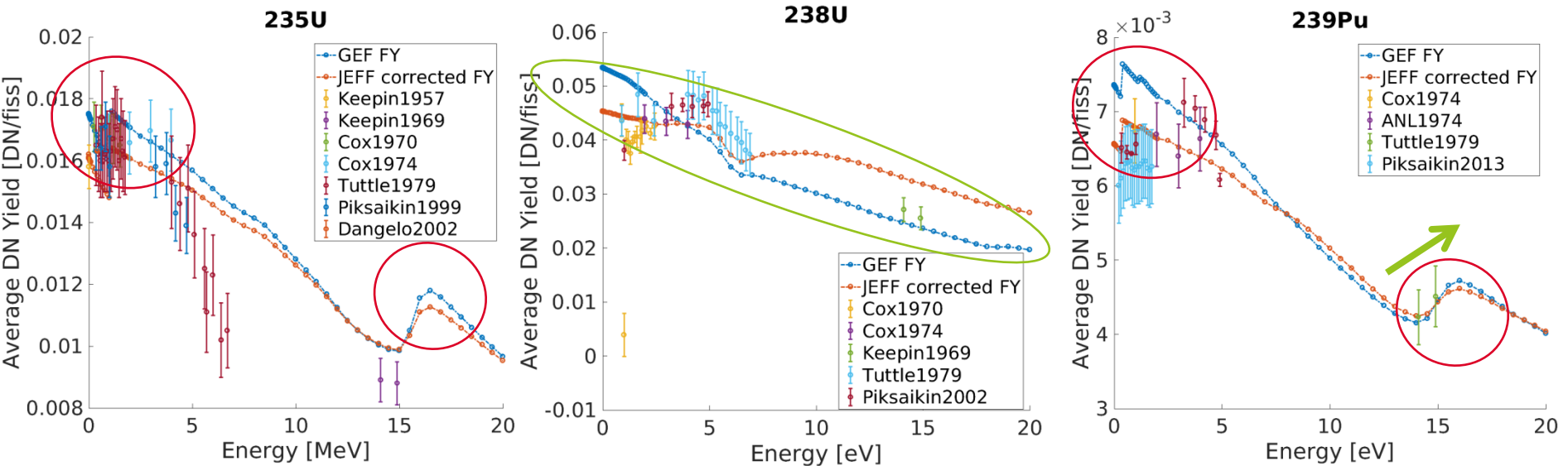
# Results: $v_d$

GEF-6.1 versus the « corrected » JEFF-3.1.1



# Results: $\nu_d$

## GEF-6.1 versus the « corrected » JEFF-3.1.1



Seems good: - 238U: step-like behavior  
- 238U: visible second-chance fission effect  
- 238U, 239Pu: agreement with experiments

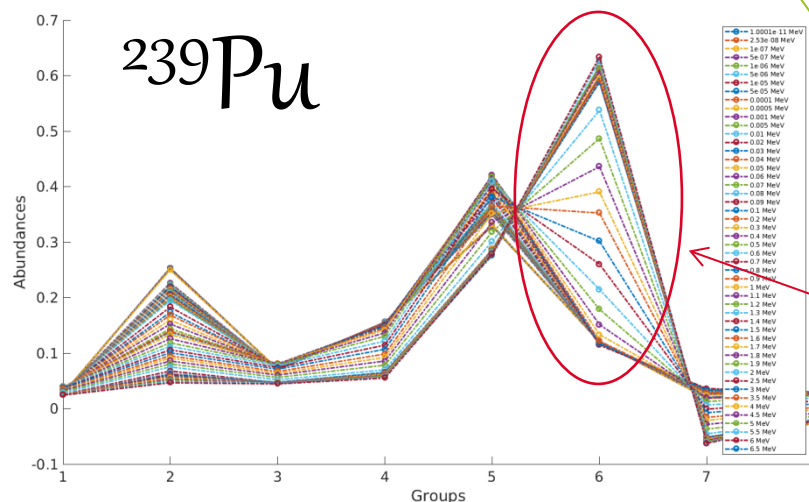
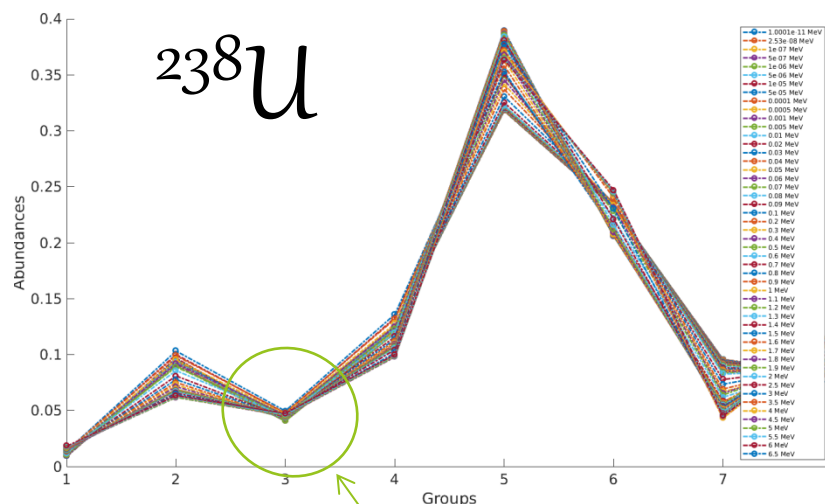
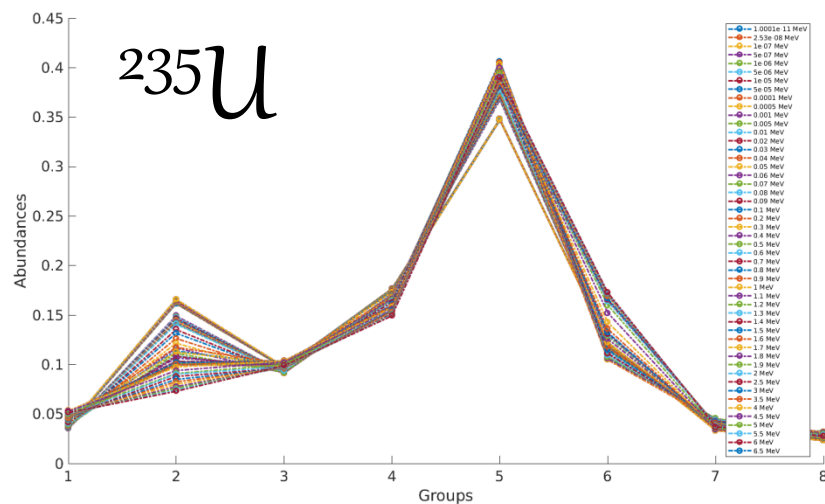
Seems weird: - 235U, 239Pu: bump after 15 MeV  
- 235U, 239Pu: uneven behavior below 4 MeV  
- 235U: different slope

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# Results: $a_i, \langle T_{1/2} \rangle$

## Abundances' sensitivity to an energy change



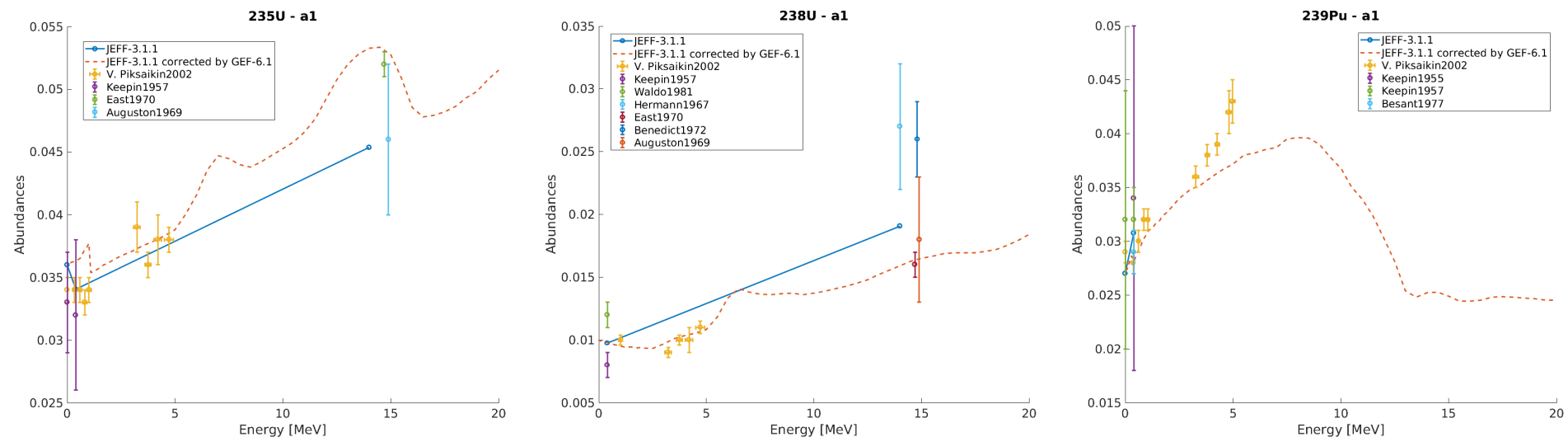
### HOW TO READ IT:

X-axis: Group number

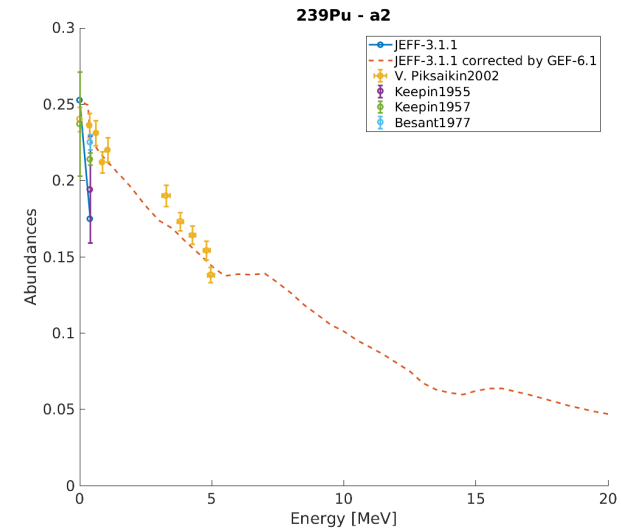
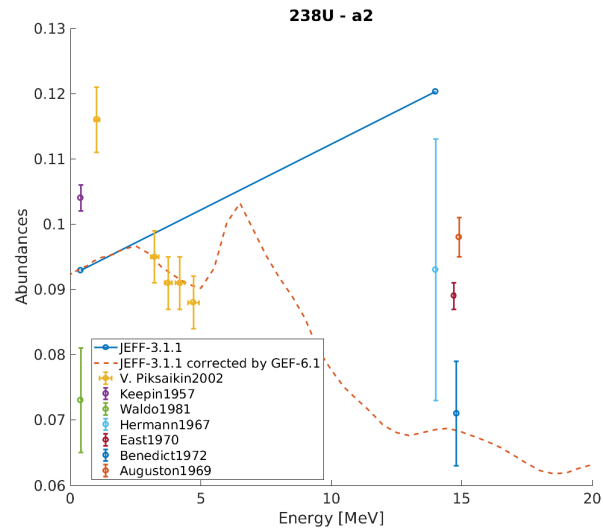
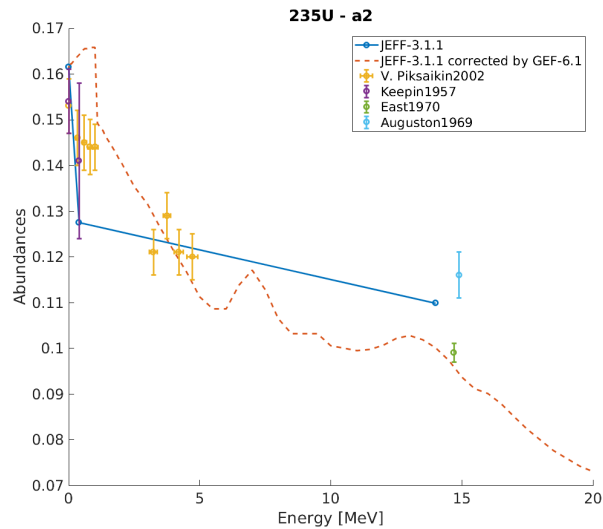
Y-axis: Relative abundance

Lines: Energy

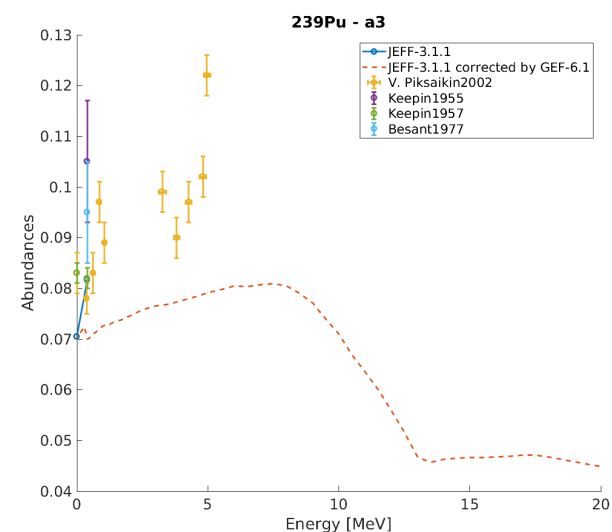
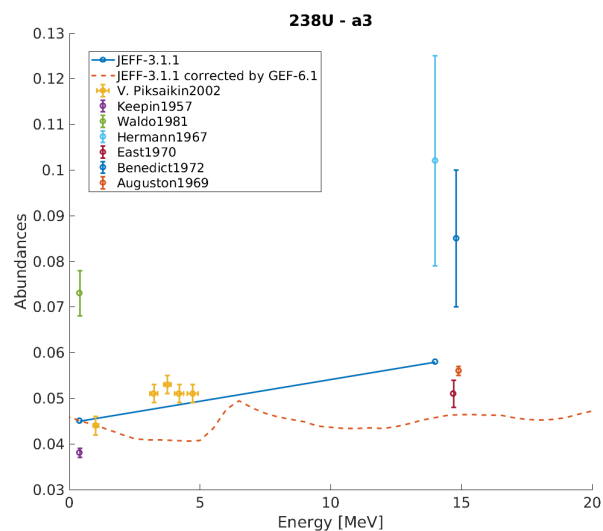
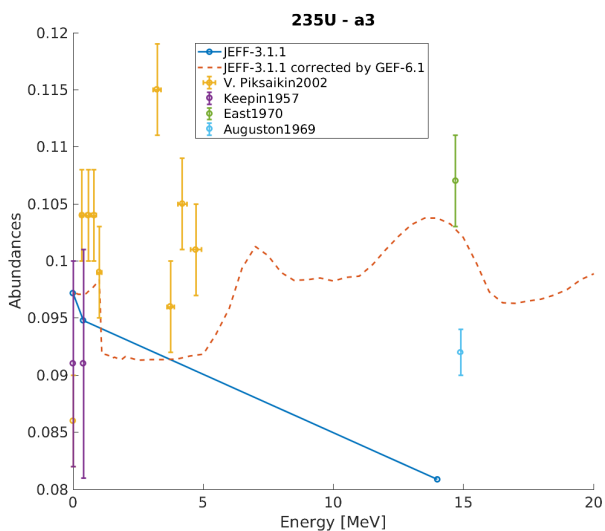
# Results: $a_i$ , $\langle T_{1/2} \rangle$ First abundance



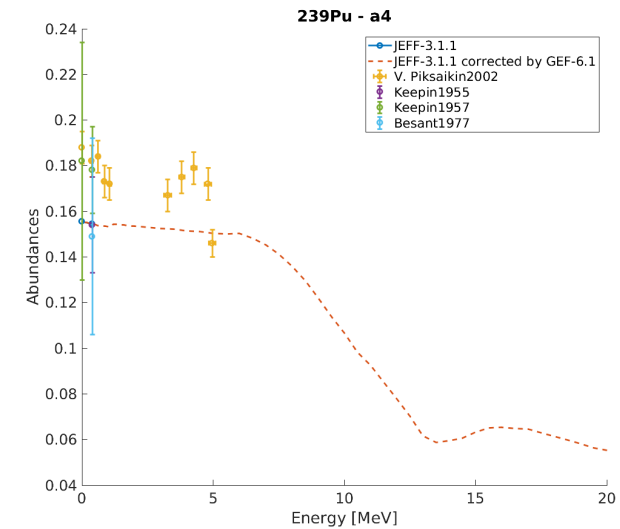
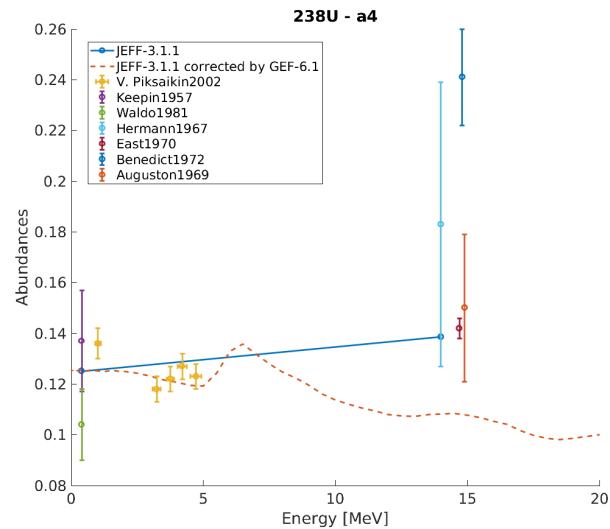
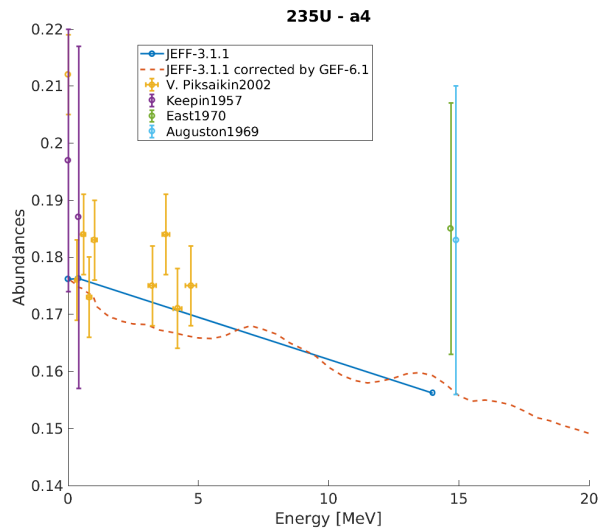
# Results: $a_i$ , $\langle T_{1/2} \rangle$ First abundance



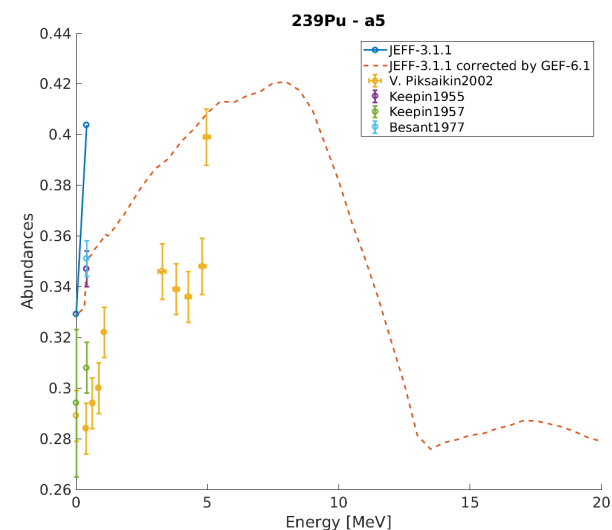
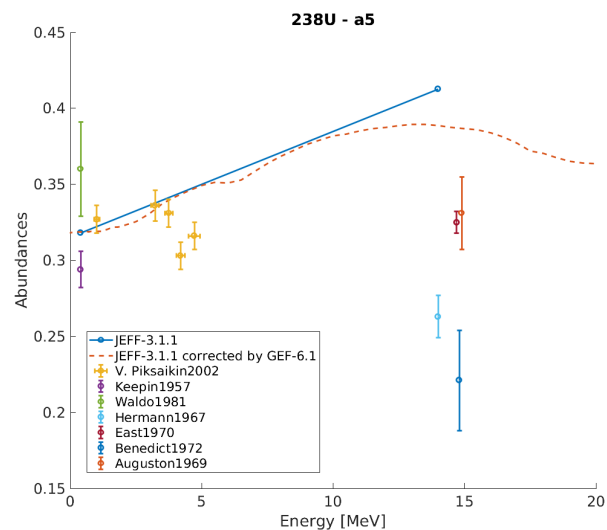
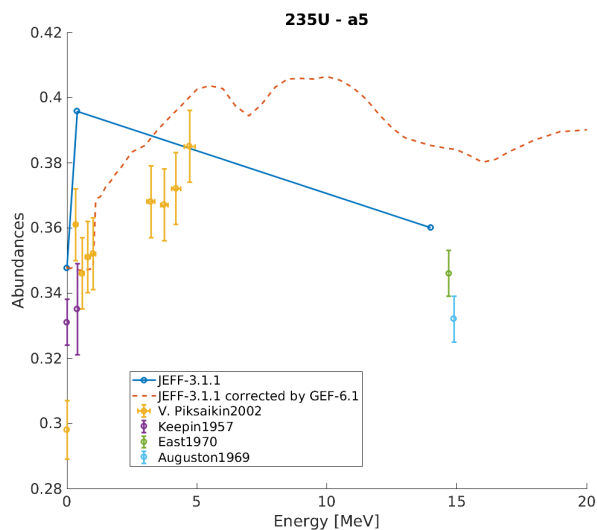
# Results: $a_i$ , $\langle T_{1/2} \rangle$ First abundance



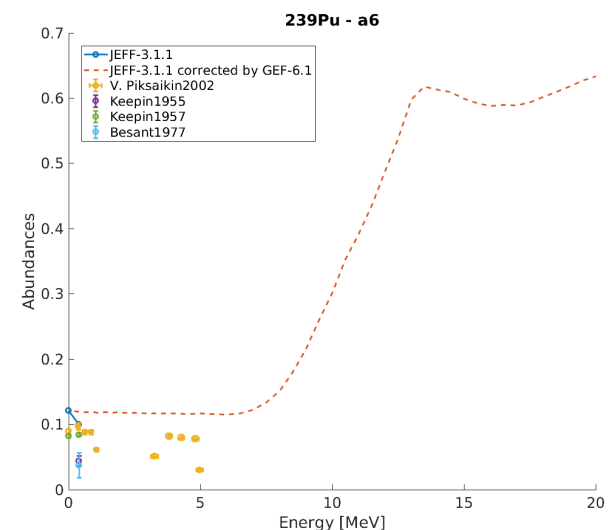
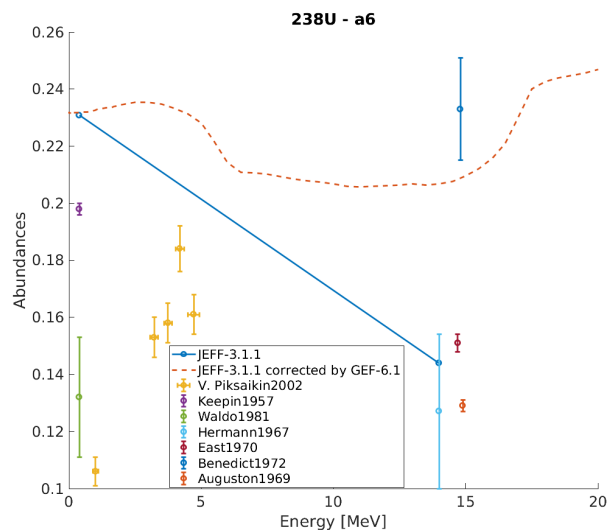
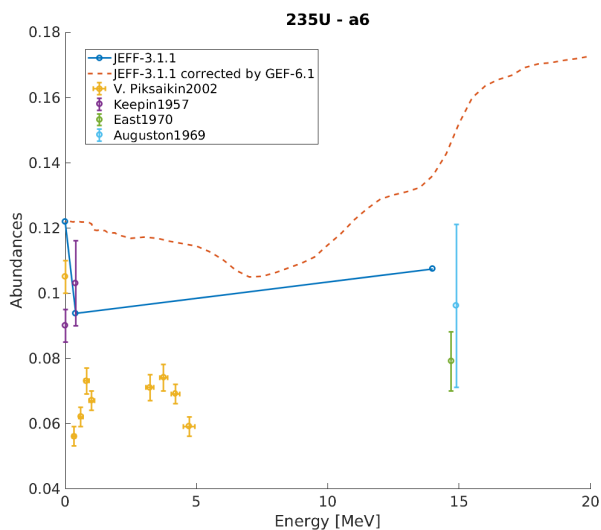
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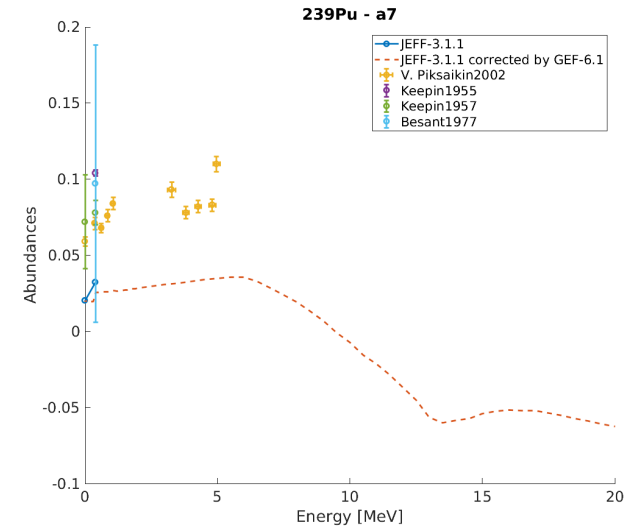
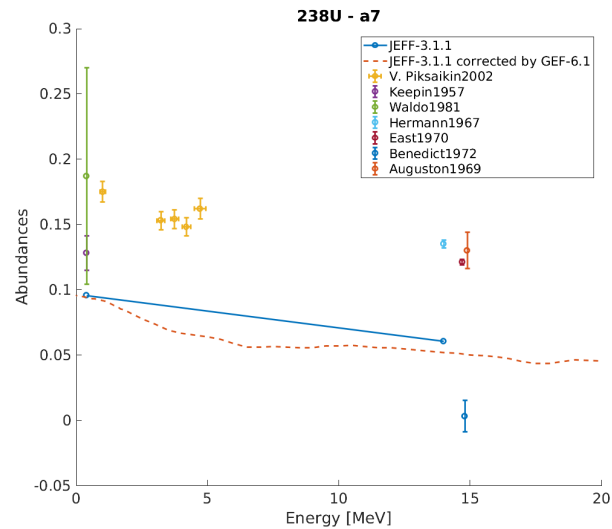
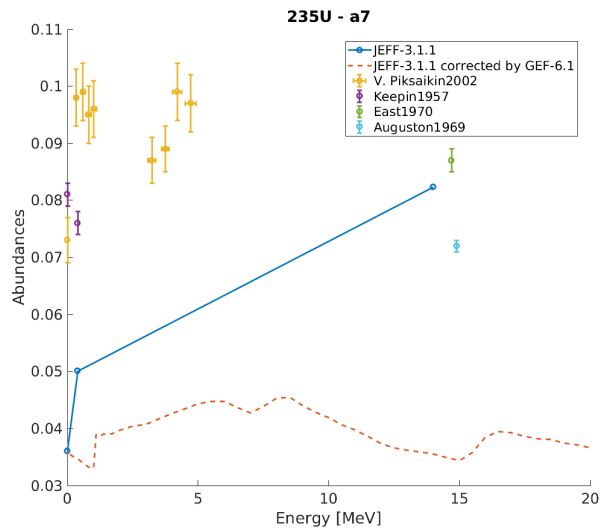
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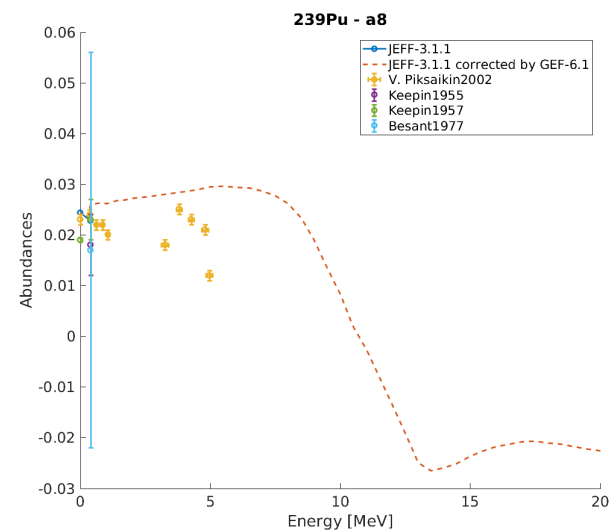
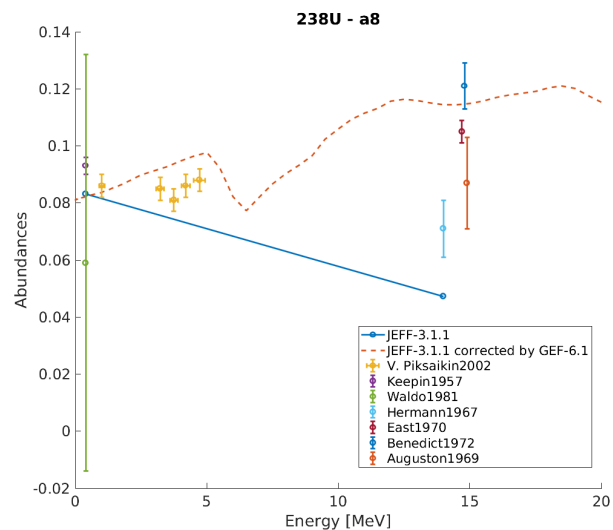
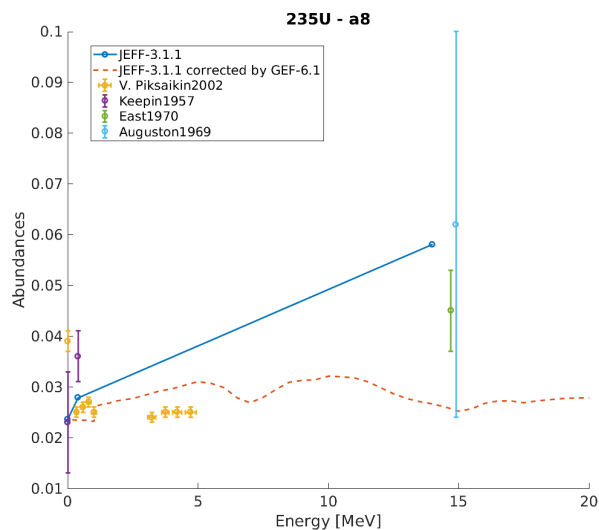
# Results: $a_i$ , $\langle T_{1/2} \rangle$ First abundance



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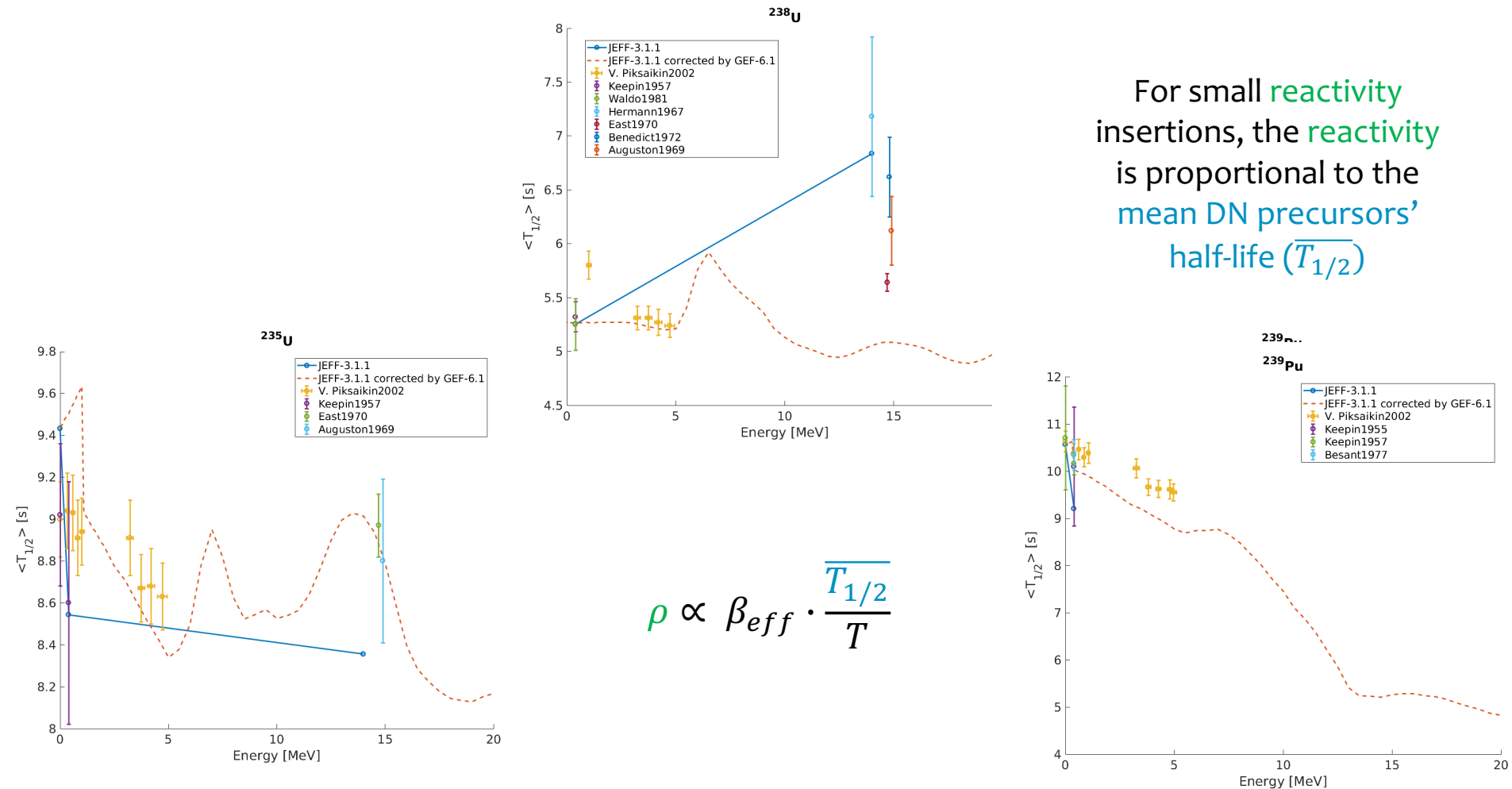


# Results: $\alpha_i, \langle T_{1/2} \rangle$

## $^{235}\text{U}$ – Mean precursors' half-life

For small reactivity insertions, the reactivity is proportional to the mean DN precursors' half-life ( $\overline{T_{1/2}}$ )

$$\rho \propto \beta_{eff} \cdot \frac{\overline{T_{1/2}}}{T}$$

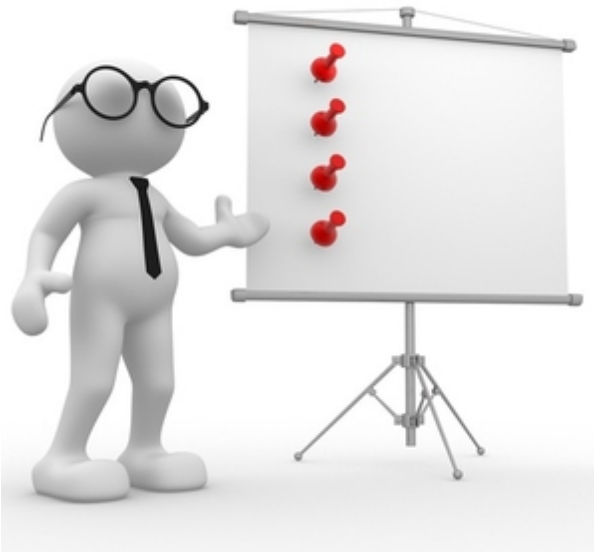


**V.M. Pikaikin et al. Energy Dependence of Relative Abundances and Periods of Delayed Neutrons from neutron-induced fission of  $^{235}\text{U}$ ,  $^{238}\text{U}$ ,  $^{239}\text{Pu}$  in 6- and 8-group model representation (2002)**

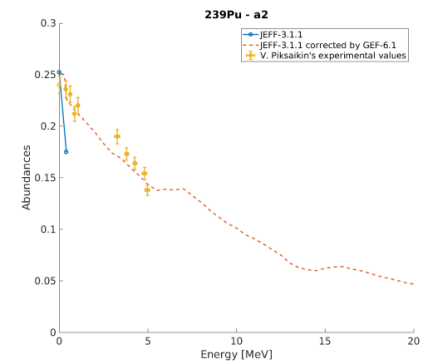
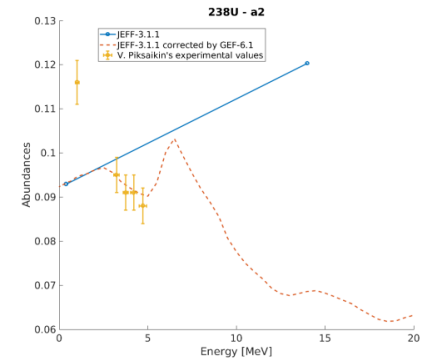
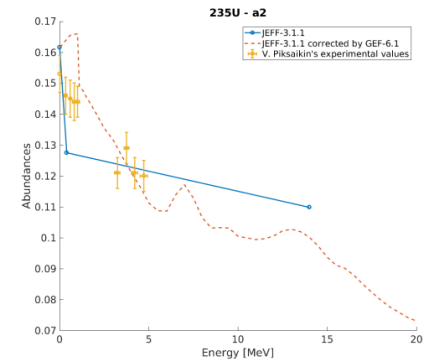
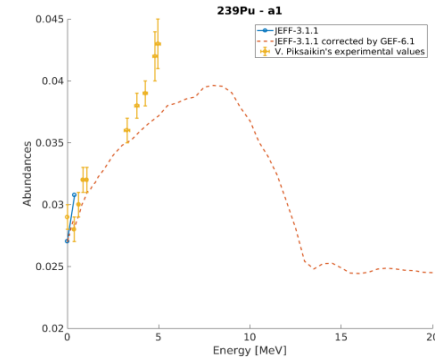
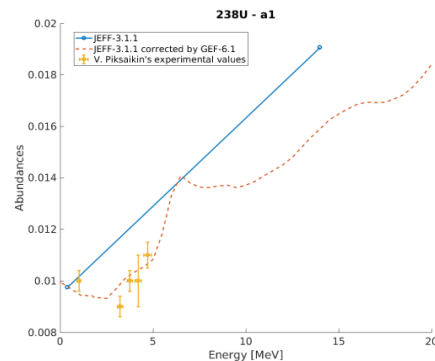
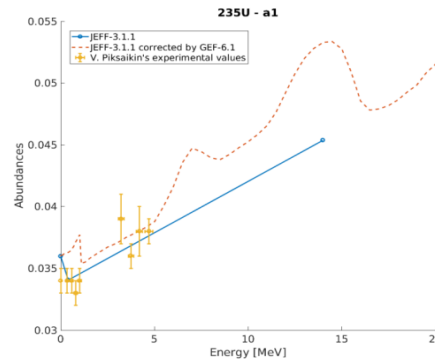
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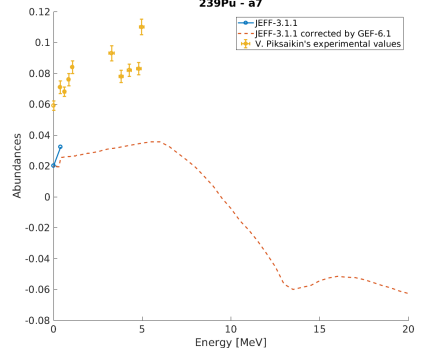
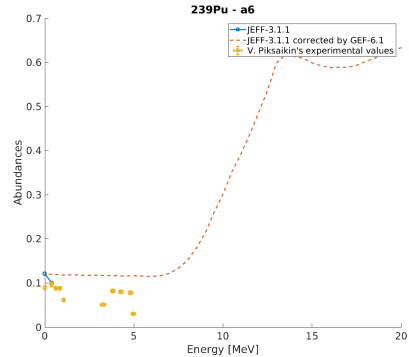
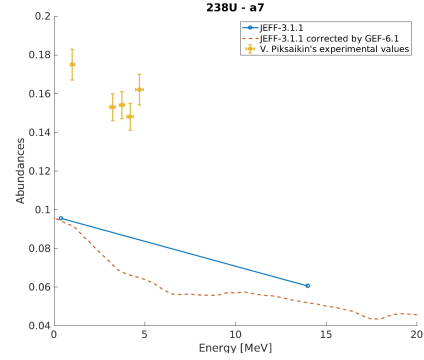
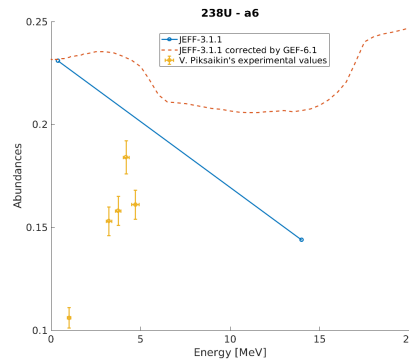
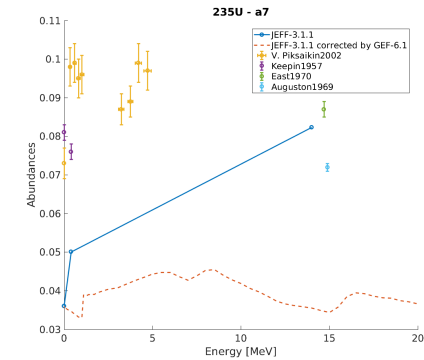
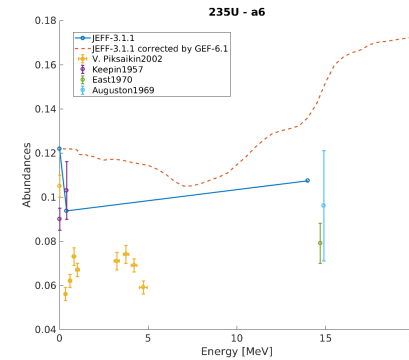
- DN data does not always has clear origins
- There is an evident need of a new evaluation
- The method seems promising for the estimation of  $\langle T_{1/2} \rangle$



- Up to 5 MeV, the first two abundances are better estimated with the *corrected* JEFF-3.1.1 than with the original JEFF-3.1.1



- Up to 5 MeV, the first two abundances are better estimated with the *corrected* JEFF-3.1.1 than with the original JEFF-3.1.1
- The short-lived groups are very badly estimated by both calculations, which means that a considerable effort should be done for improving short-lived precursors' FY



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- How to compute the uncertainty in the modified FY?
- What else could we do for investigating/improving the energy dependence of delayed-neutron data?



# Thank you!

## Questions?



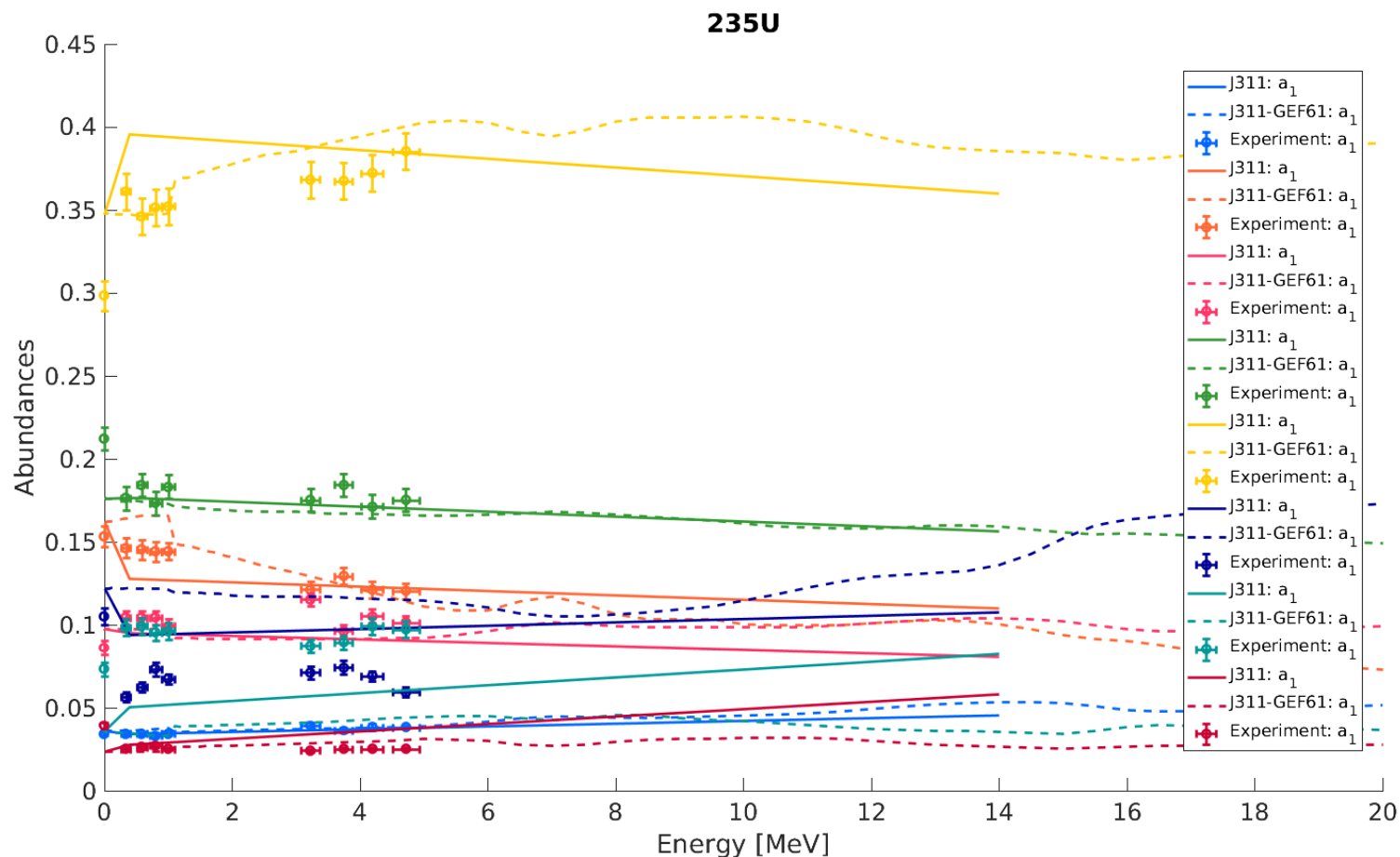
*Back-up*

$$(Eq. 1) \quad FY_{JEFF}(E) = \frac{FY_{GEF}(E)}{FY_{GEF}(E_{ref})} FY_{JEFF}(E_{ref})$$

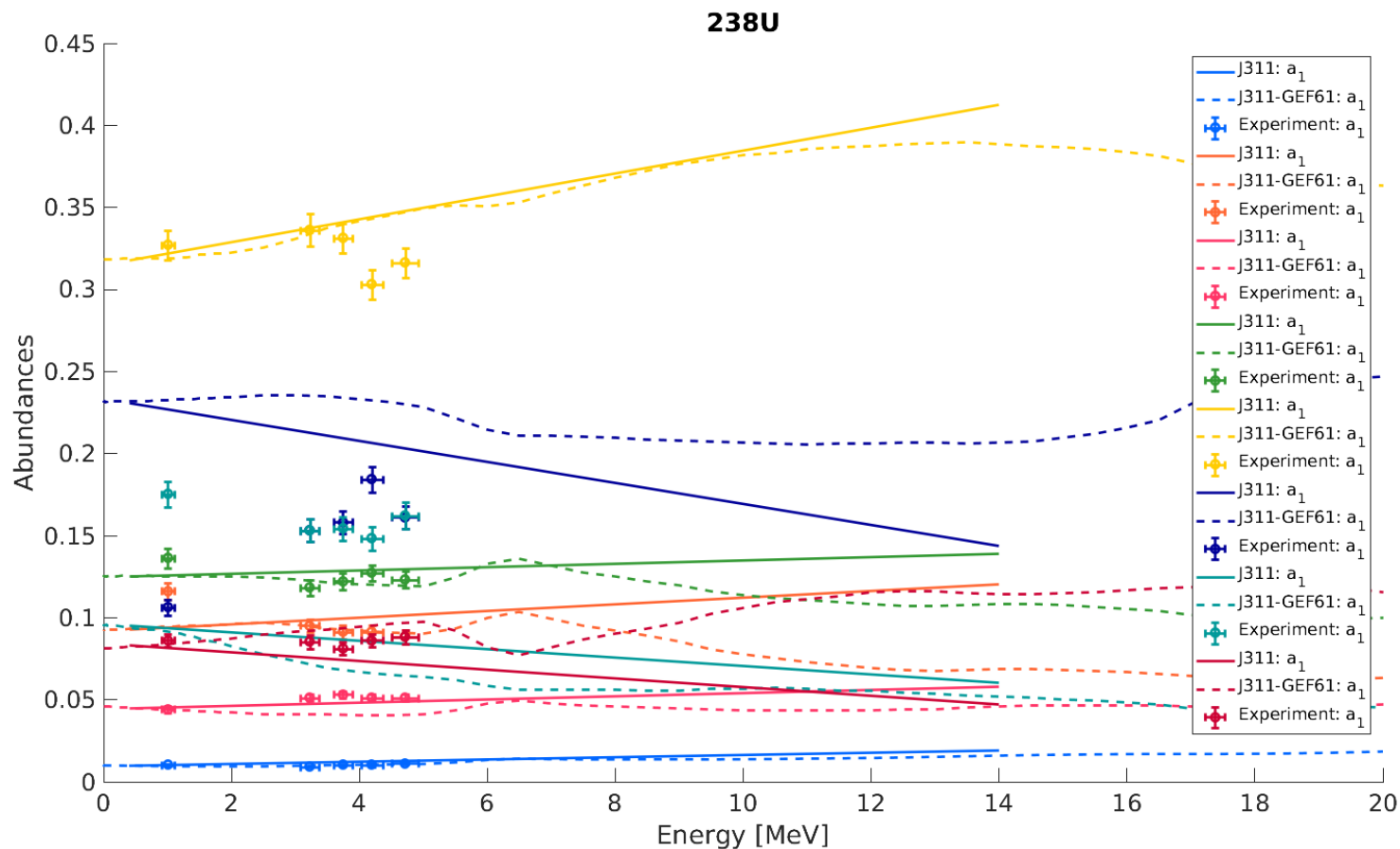
$$(Eq. 2) \quad FY_{JEFF}(E) = FY_{JEFF}(E_{ref})$$

GEF(E <sub>ref</sub> )	GEF(E)	JEFF(E)
≠ 0	≠ 0	Eq. 1
≠ 0	= 0	Eq. 1
= 0	≠ 0	Eq. 2
missing		Eq. 2

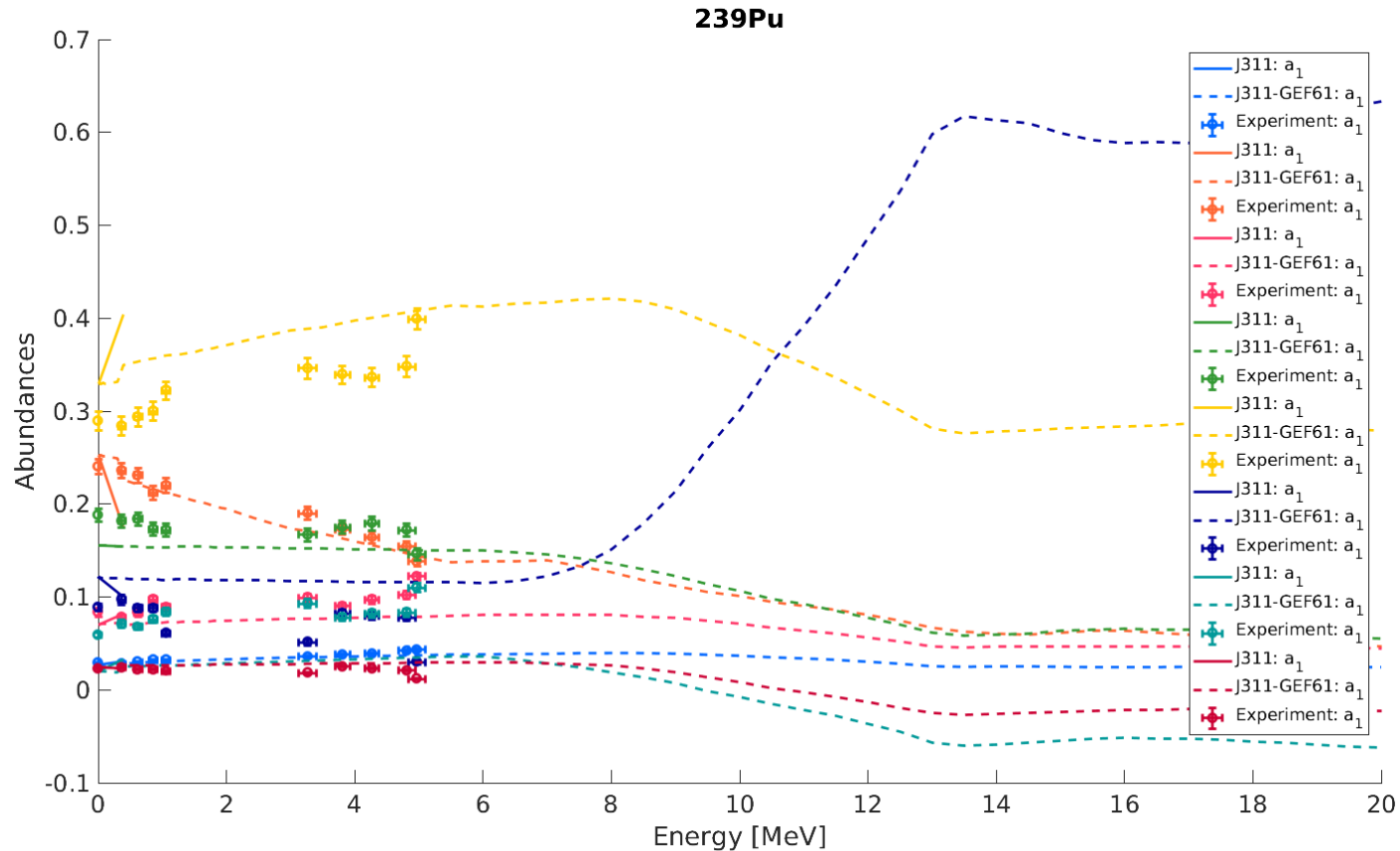




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## Semi-empirical Lendel model

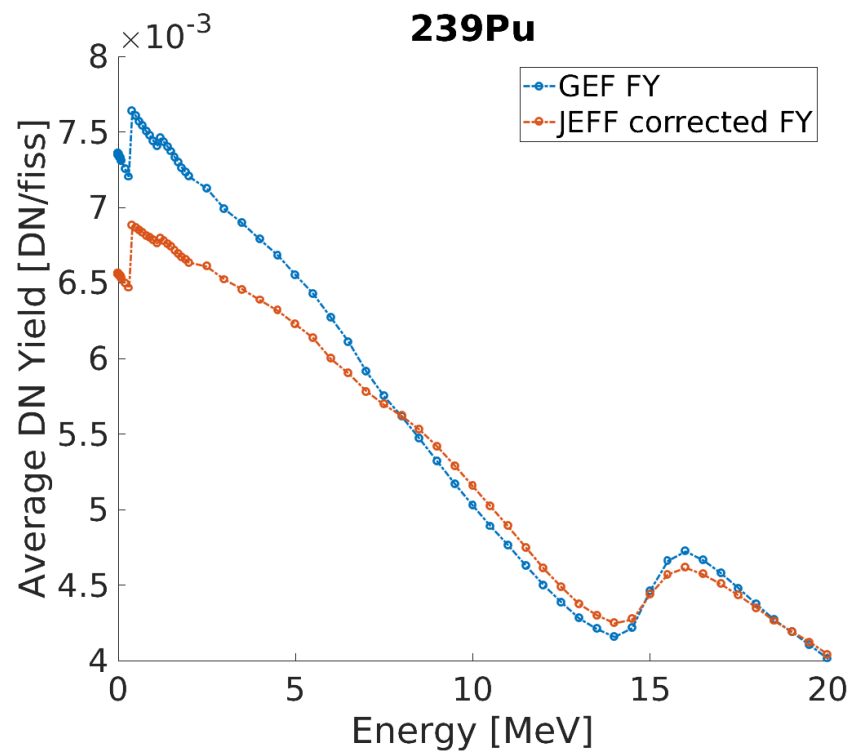
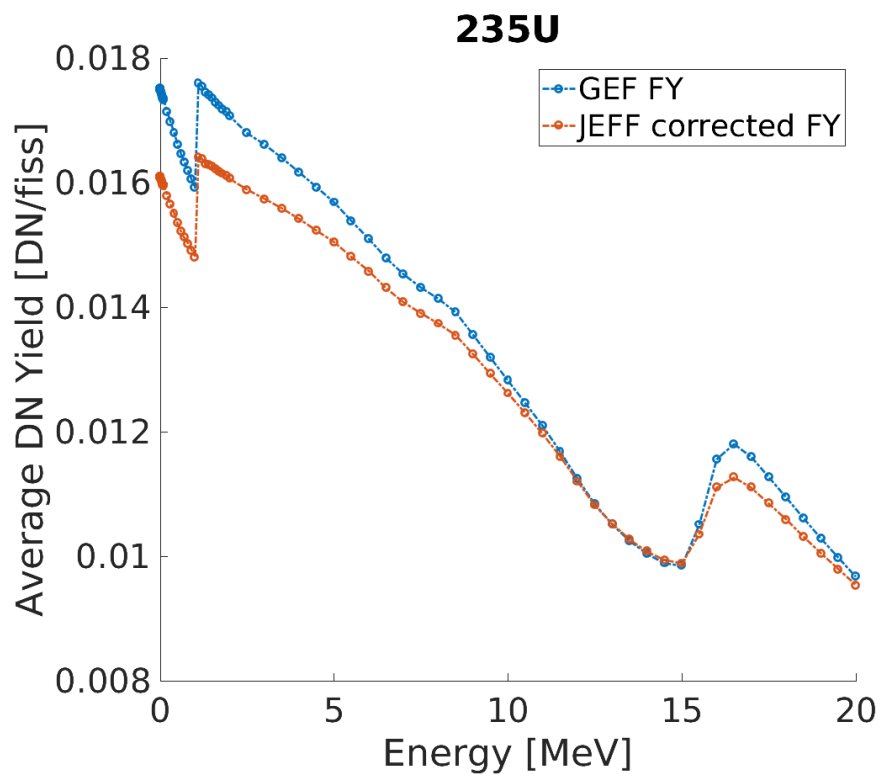
$$\nu_d(E) = Y_1(E) + \varphi(A_f, Z_f) \cdot \psi(E)$$

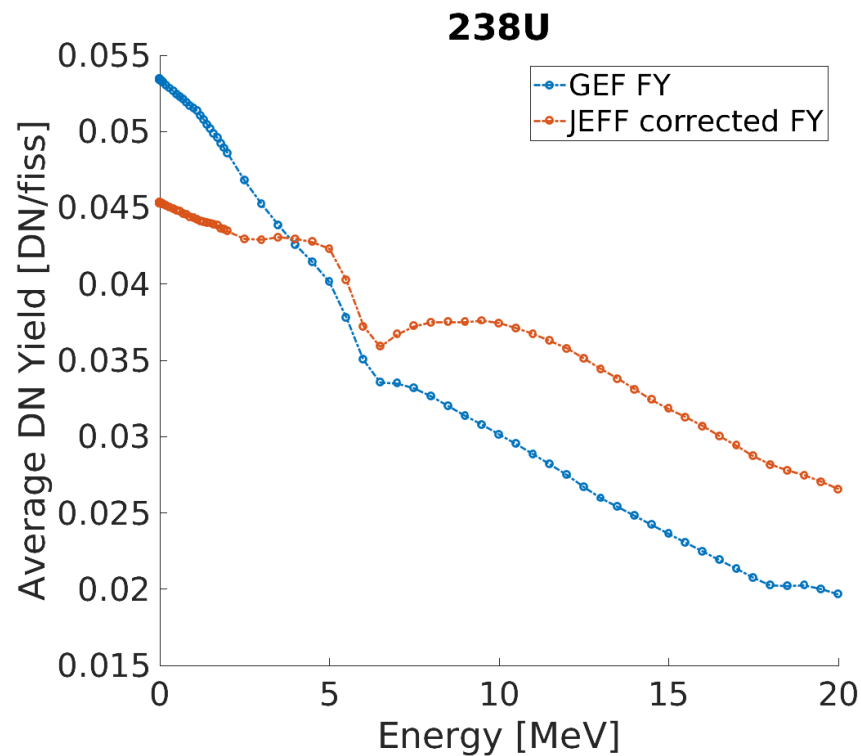
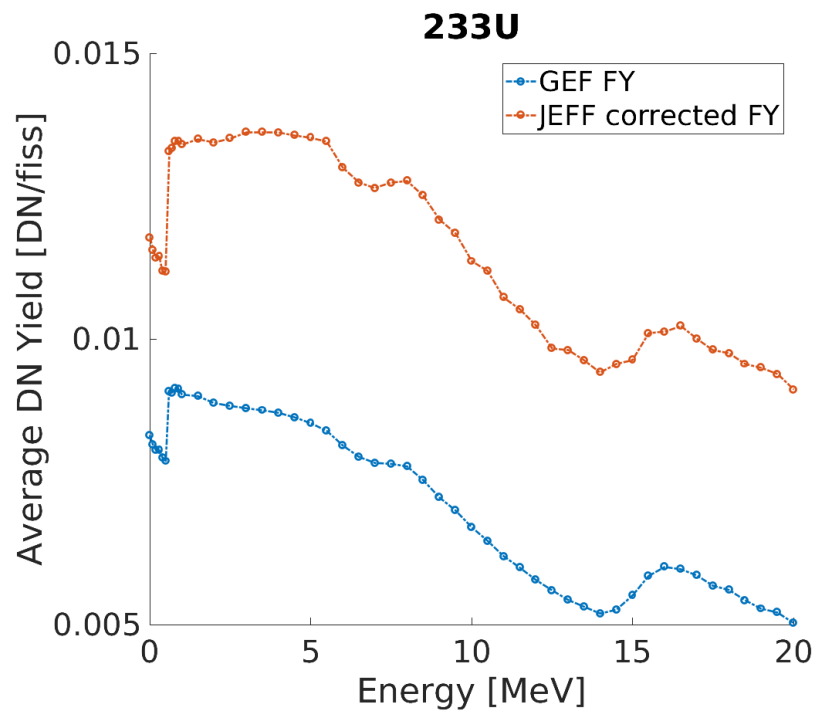
Direct macroscopic DN yield

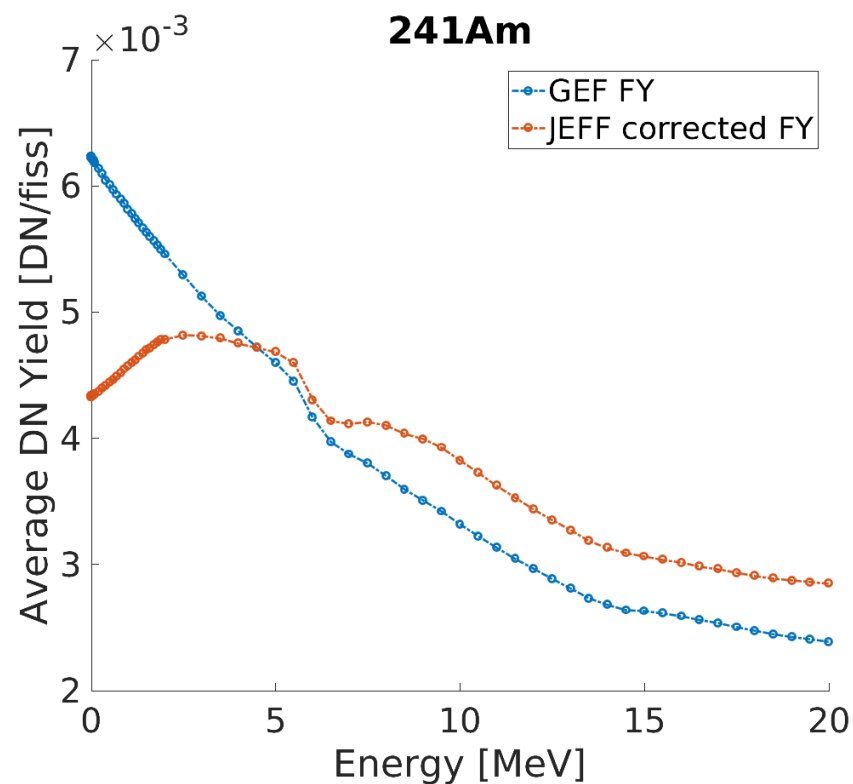
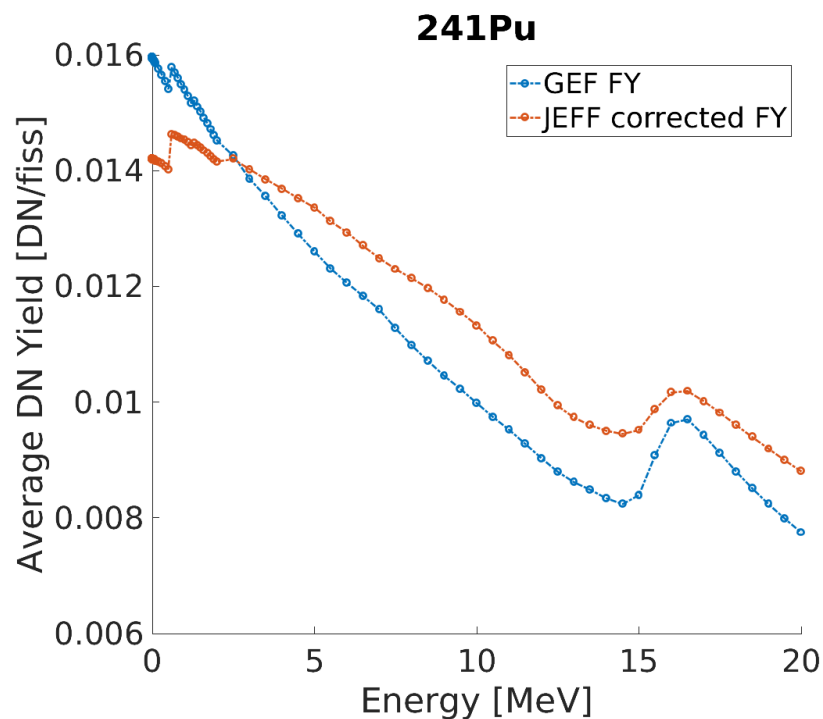
It describes the competition  
with prompt neutron  
emission

Correction

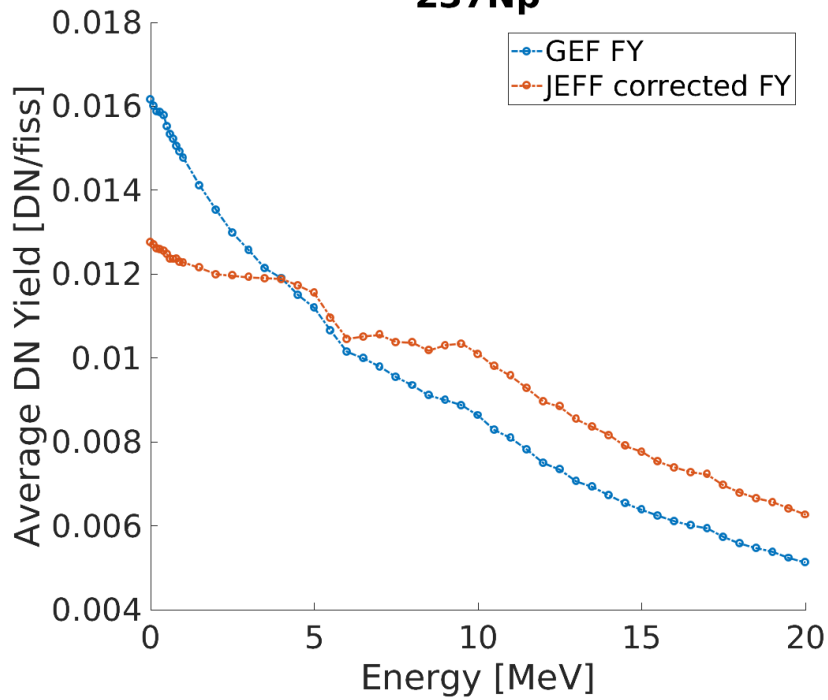
It takes into account the  
energy dependence of the  
odd-even effect in the first  
chance fissioning system



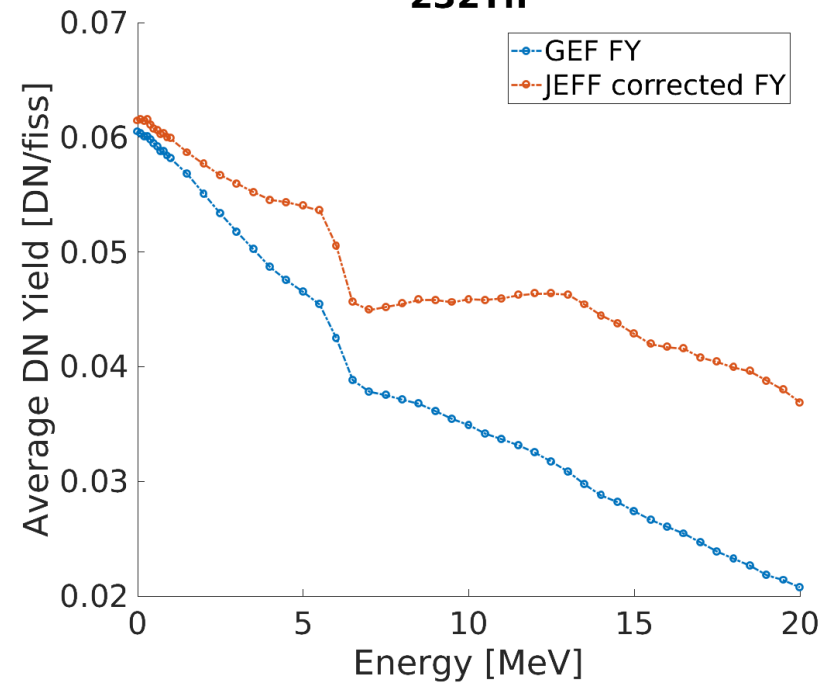




### **$^{237}\text{Np}$**



### **$^{232}\text{Th}$**



# New Bateman Solver

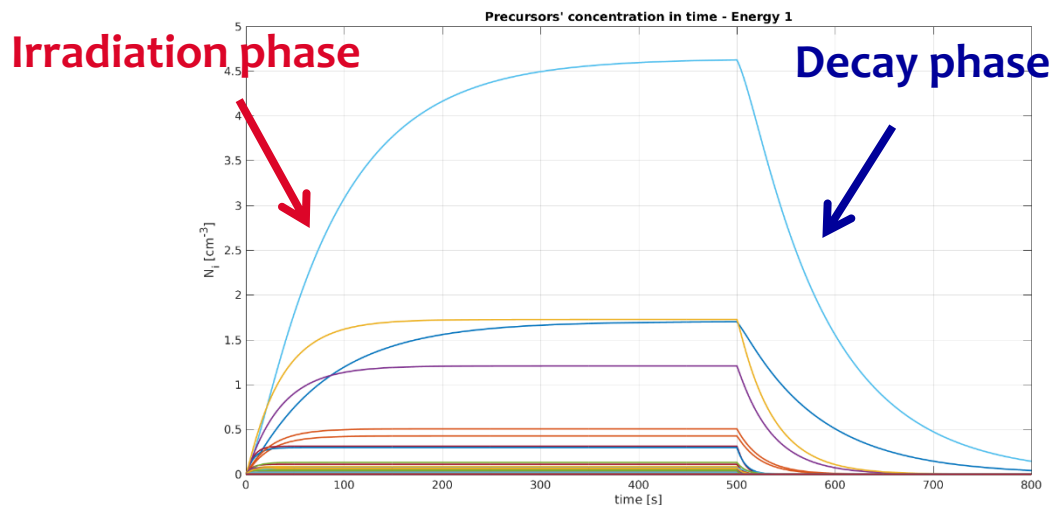
## Complete system of differential equations

Delayed-neutron activity [DN/s]

$$n_d(t) = \sum_{i=1}^n x_i N_i(t) P_{n,i}$$

$$\left\{ \begin{array}{l} \frac{dN_1(t)}{dt} = -\lambda_1 N_1(t) + S_1 \\ \frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + S_2 + \lambda_1 BR_{2 \rightarrow 1} N_1(t) \\ \vdots \\ \frac{dN_i(t)}{dt} = -\lambda_i N_i(t) + S_i + \lambda_{i-1} BR_{(i-1) \rightarrow i} N_{i-1}(t) \\ \vdots \\ \frac{dN_n(t)}{dt} = -\lambda_n N_n(t) + S_n + \lambda_{n-1} BR_{(n-1) \rightarrow n} N_{n-1}(t) \end{array} \right.$$

### Individual precursor **build-up** and **decay**



$$N_n(t) = \sum_{i=1}^{i=n} \left[ \left( \prod_{j=i}^{j=n-1} \lambda_j \cdot P_{j \rightarrow (j+1)} \right) \cdot \sum_{j=i}^{j=n} \left( \frac{N_i^0 \cdot e^{-\lambda_j \cdot t}}{\prod_{\substack{p=i \\ p \neq j}}^{p=n} (\lambda_p - \lambda_j)} + \frac{P_i \cdot (1 - e^{-\lambda_j \cdot t})}{\lambda_j \cdot \prod_{\substack{p=i \\ p \neq j}}^{p=n} (\lambda_p - \lambda_j)} \right) \right]$$

## Validation with DARWIN<sup>®</sup>

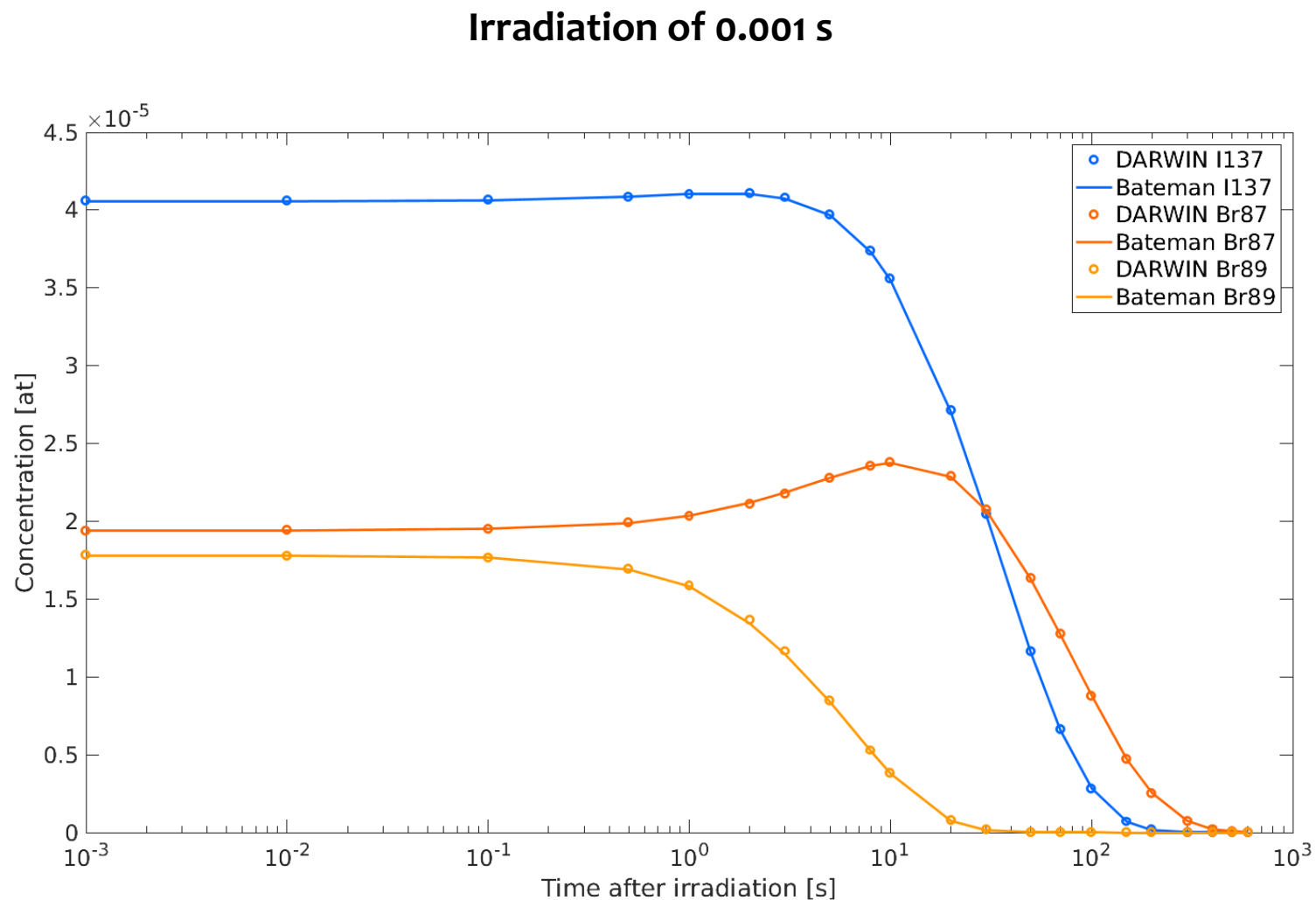
1. **Nuclear Data:** Fission Yields and Radioactive Decay Data from **JEFF-3.1**
2. **Fissioning System:** **1 g** ( $2.563\text{E}+21$  at) of **<sup>235</sup>U**
3. **Thermal Flux:** Constant **thermal** ( $E_n < 0.1$  eV) flux of  **$1 \text{ n}\cdot\text{cm}^{-2}\cdot\text{s}^{-1}$**
4. **Irradiation Durations:** 0.001 s, 10 s, 600 s
5. **Decay Duration:** 600 s
6. **Comparison:** 22 points of the decay-curve  
(0, 0.001, 0.01, 0.1, 0.5, 1, 2, 3, 5, 8, 10, 20, 30, 50, 70, 100, 150, 200, 300, 400, 500, 600 seconds after the end of the irradiation)

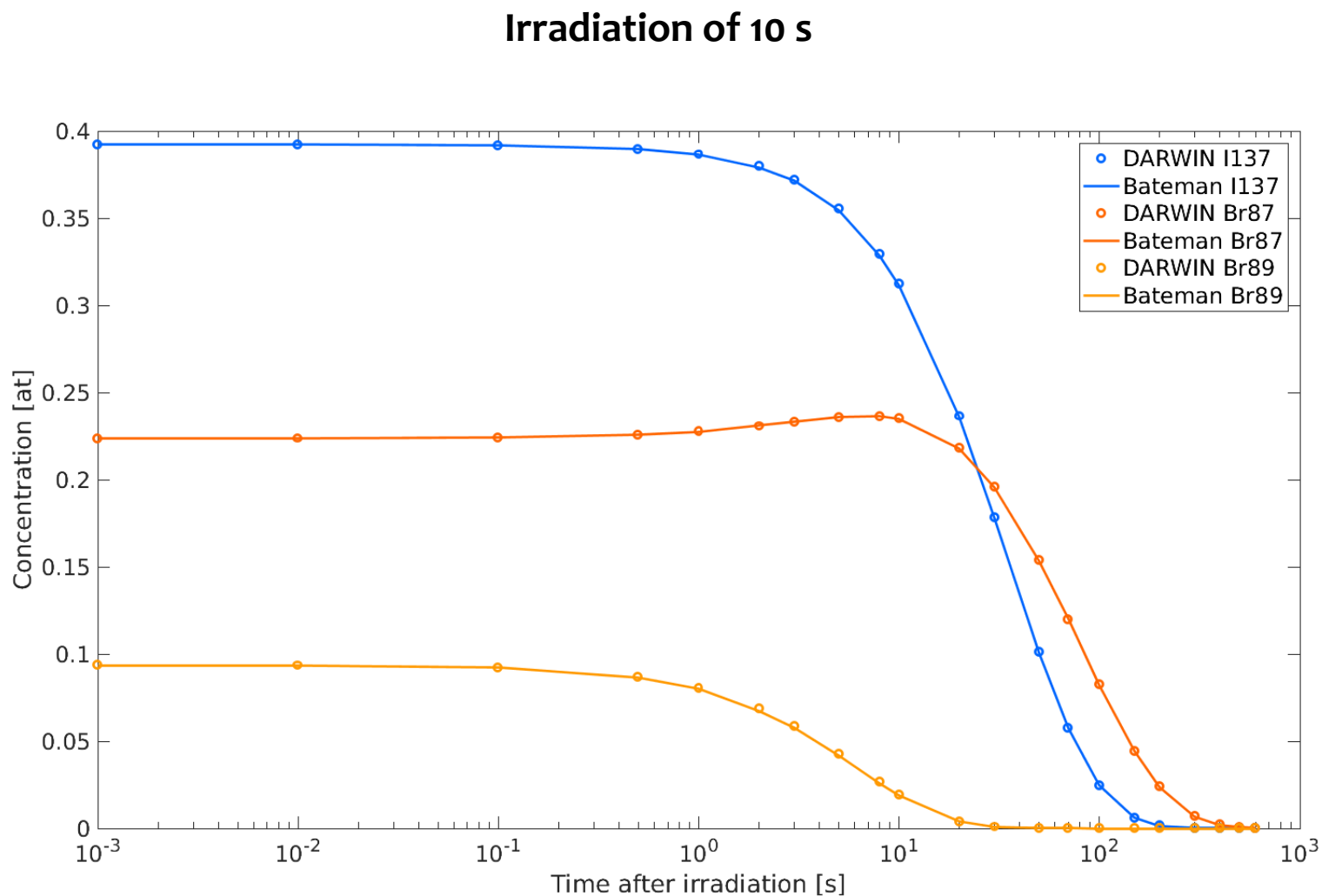
**Thanks to Jean-François Lebrat for the DARWIN simulations**

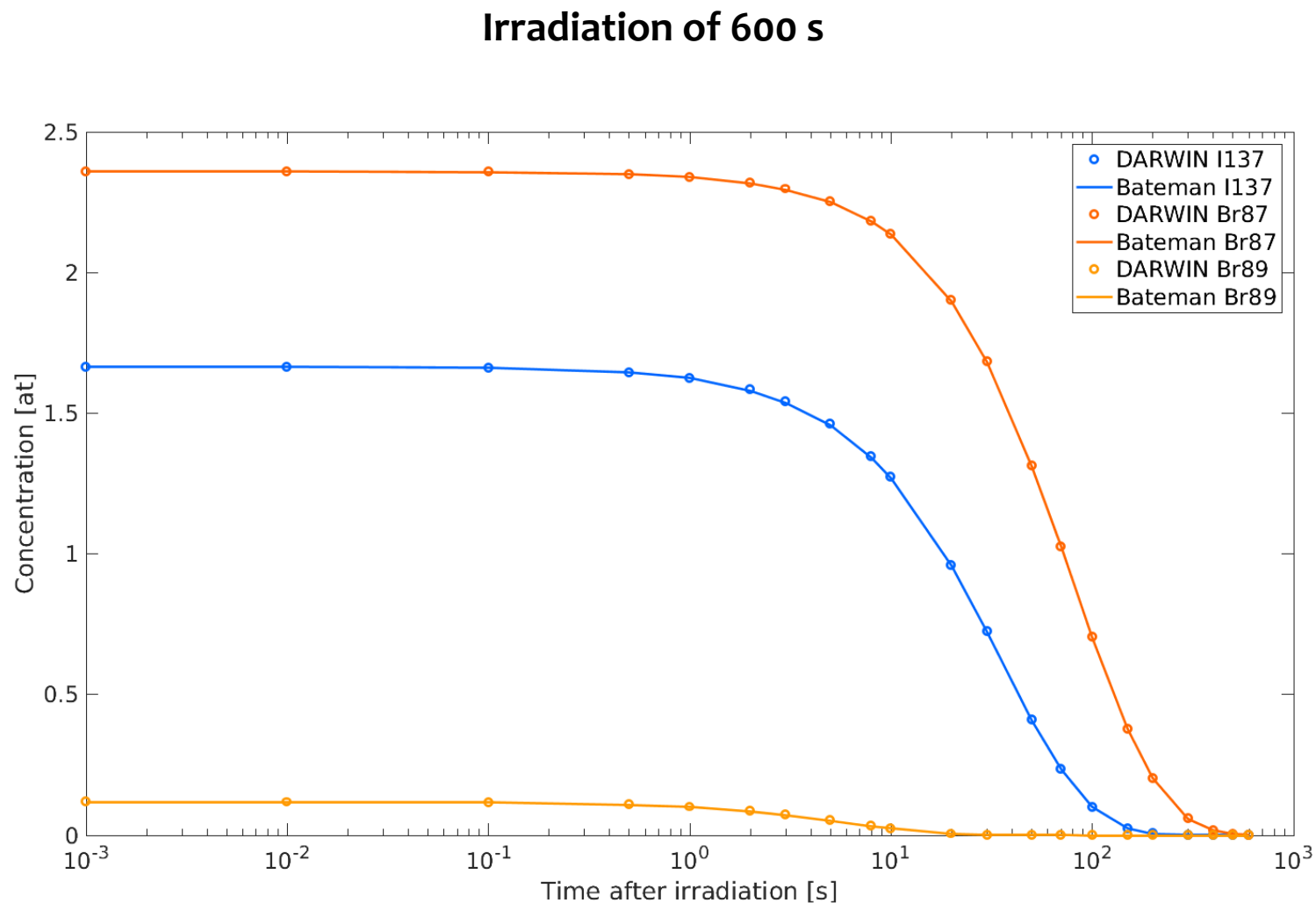
San-Felice L. et al. *Experimental validation of the DARWIN2.3 Package for fuel cycle applications* (2013)

Table 1: Discrepancies in the concentration at the end of the irradiation. The results are given in percentage. [1], [2] and [3] refer to an irradiation length of 0.001 s, 10 s and 600 s, respectively. The precursors are shown in order of importance with respect to the delayed-neutron emission

Z	A	I	Symbol	$(C - C_{ref})/C_{ref}$ [1]	$(C - C_{ref})/C_{ref}$ [2]	$(C - C_{ref})/C_{ref}$ [3]
53	137	0	I	0.002 %	0.008 %	-0.004 %
35	89	0	Br	-0.005 %	-0.009 %	-0.015 %
37	94	0	Rb	-0.003 %	-0.021 %	-0.023 %
35	88	0	Br	0.007 %	0.007 %	-0.003 %
35	90	0	Br	-0.031 %	-0.032 %	-0.036 %
53	138	0	I	-0.009 %	-0.005 %	-0.012 %
39	98	1	Y	-0.033 %	-0.032 %	-0.033 %
53	139	0	I	-0.028 %	-0.025 %	-0.029 %
37	95	0	Rb	-0.172 %	-0.178 %	-0.178 %
35	87	0	Br	0.007 %	0.013 %	-0.001 %
37	93	0	Rb	-0.001 %	-0.004 %	-0.009 %
39	99	0	Y	-0.031 %	-0.043 %	-0.043 %
33	85	0	As	-0.032 %	-0.031 %	-0.033 %
35	91	0	Br	-0.128 %	-0.125 %	-0.124 %
51	135	0	Sb	-0.040 %	-0.038 %	-0.039 %
55	143	0	CS	-0.029 %	-0.035 %	-0.036 %
33	86	0	AS	-0.071 %	-0.071 %	-0.072 %
37	96	0	RB	-0.169 %	-0.230 %	-0.230 %
55	145	0	CS	-0.118 %	-0.115 %	-0.114 %
53	140	0	I	-0.076 %	-0.077 %	-0.080 %
55	144	0	CS	-0.028 %	-0.044 %	-0.044 %







### Conclusions:

1. The code has been written to study **the delayed-neutron-precursors' behavior**
2. Discrepancies larger than 1% are present for irrelevant precursors (first occurrence is the 79<sup>th</sup> precursors in the sorted-by-importance ranking)
3. Only  **$IT, \beta^-, \beta^-_n, \beta^-_{2n}, \beta^-_{3n}, \beta^-_{4n}$**  are considered (no transmutation or absorption)
4. **The solver is validated for estimating delayed-neutron precursors' concentration**

