Examples of Monte Carlo techniques applied for nuclear data uncertainty propagation

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5th Int. Workshop On Nuclear Data Evaluation for Reactor Applications (WONDER-2018), 8-12 October 2018
Abstract

The aim of this work is to review different Monte Carlo (MC) techniques used to propagate nuclear data uncertainties.

1. Uncertainty Quantification (UQ) studies
   - Required in safety calculations of large scale systems ~ PWR
   - ND uncertainty propagation on the main design parameters
   - Decomposition of uncertainty in: $^{235}\text{U} - ^{238}\text{U} - ^{239}\text{Pu}$ & XS-$\nu$-PFNS

2. Bayesian Monte Carlo approaches for data adjustment
   - Multivariate Normal Bayesian model relying on NUDUNA/SANDY codes
   - TMC + Bayesian MC Approach (BMC)
   - Selection of Benchmarks
     - BMC apply for Criticality + Shielding/Transmission
1. MC techniques to perform Uncertainty Quantification (UQ)

- Uncertainty Quantification (UQ) analysis in large scale systems
  
  e.g. the core design in a 3-loop PWR Westinghouse unit
  
  - In this work, we use our SEANAP system developed and applied for 3-D PWR core analysis which has demonstrated a very good agreement with the broad sets of parameters and cycles analysed at the Spanish PWR units.

- Methodologies for UQ:
  
  - Monte Carlo approaches which uses random samples of nuclear data libraries and perform a separate reactor calculation for each random sample
  
  - S/U method is based on first order perturbation theory approaches, which makes use of available covariance files.
1.1 S/U to perform to UQ in reactor calculations

- **S/U method** is based on first order perturbation theory approaches, which makes use of available covariance files.
  - Sensitivity coefficients -> sandwich rule
  - Easy decomposition of uncertainty in isotopic partial cross-section components
  - Low CPU time
  - Different theories [1]:
    - Standard Perturbation Theory (SPT) - > keff uncertainties
    - Generalized Perturbation Theory (GPT) -> power distribution uncertainty

- **S/U Weaknesses**
  1) low efficiency for a large number of response functions
  2) applicability for small uncertainties - > it is based on linear-approach
  3) severe limitations as consequence of the non-linearity of multi-physics calculations (neutronics, thermohydraulic, depletion, …) in reactor calculations
The Monte Carlo approaches which uses random samples of nuclear data libraries and perform a separate reactor calculation for each random sample.

- Large CPU time to perform enough sampling
  - The low execution times of SENAP code for a full scheme of PWR core cycle indicates that the parallelization of Monte Carlo sampling is reliable.
  - Nuclear data are sampled at the beginning of the simulation
  - Statistics of all SEANAP simulations yields the desired uncertainty quantification
  - SEANAP solves coupled multi-physics at different levels of approximation

- Different approaches by their nuclear data uncertainty input
  - “Total Monte Carlo” (TMC) relies on model parameter covariances
  - NUDUNA and SANDY take as input the information provided by ND evaluations
  - XSUSA takes the form of covariance matrices in multigroups
1.3 UQ in reactor calculations: SEANAP-SANDY

Scheme of the PWR Core Analysis SEANAP System

- Ref.: “Validation of PWR Core Analysis system SEANAP-86 with measurements in test and operation”, C. Ahnert et al., M&C87

SEANAP is integrated by 4 subsystems:

1. **MARIA** system for assembly calculations
2. **COBAYA** system for a detailed (pin-by-pin) core calculations at reference conditions
3. **SIMULA** system for 3D 1 group corrected-nodal core simulation
4. **CICLON** system for fuel management analysis of reload cycles

CPU Time/cycle ~ 5-10 min / i7 870@2.93GHz
1.3 UQ in reactor calculations: SEANAP-SANDY

**Validation of SEANAP in PWR Core Analysis**

- SEANAP system has been developed and implemented as an on-line simulator ~20 cycles of three PWRs (Vandellós-II, Ascó-I and Ascó-II).

**Figure:** Measured and Simulated Power vs Delta-I in return to Power after a Short Shutdown

Ref: “Upgraded SEANAP-PWR core simulator with JEFF-3.3: Impact of Nuclear Data Uncertainties for PWR cycle operation”, O. Cabellos, JEFFDOC-1917, April 2018

Fig. Scheme of SEANAP: WIMS-D5 (JEFF-3.3) + COBAYA + SIMULA
1.3 UQ in reactor calculations: SEANAP-SANDY

- SANDY: Numerical tool for nuclear data uncertainty quantification.
- Based on Monte Carlo sampling

Figure: First 20 JEFF-3.3 random files processed with NJOY/GROUPR in 69 energy groups at 293K with infinite dilution
### 1.4 Design and acceptance criteria for start-up and operation

<table>
<thead>
<tr>
<th>Core parameter</th>
<th>Design criteria</th>
<th>Acceptance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical boron concentration ARO</td>
<td>$</td>
<td>(C_B)^M_{ARO} - (C_B)^C_{ARO}</td>
</tr>
<tr>
<td>Isothermal temperature coefficient ARO at HZP</td>
<td>$</td>
<td>(a_{ISO_T})^M_{ARO} - (a_{ISO_T})^C_{ARO}</td>
</tr>
<tr>
<td>Moderator temperature coefficient ARO at HZP</td>
<td>$(a^{CTM})^{HZP}_{ARO} &lt; 9$ pcm/°C</td>
<td></td>
</tr>
<tr>
<td>Boron Worth Coefficient at HZP</td>
<td>$</td>
<td>(aC_B)^M - (aC_B)^C</td>
</tr>
<tr>
<td>Control banks worth for Reference Bank</td>
<td>$</td>
<td>(I_{REF})^M - (I_{REF})^C</td>
</tr>
<tr>
<td>Control Bank Worth value for other Banks using Rod Swap Technique</td>
<td>$</td>
<td>(I_{CBW})^M - (I_{CBW})^C</td>
</tr>
<tr>
<td>Total Control Bank Worth</td>
<td>$1.10 \times (I_{TOT})^C &gt; (I_{TOT})^M &gt; 0.9x(I_{TOT})^C$</td>
<td>$(I_{TOT})^M &gt; 0.9x(I_{TOT})^C$</td>
</tr>
<tr>
<td>Axial Offset</td>
<td>$</td>
<td>(AO)^M - (AO)^C</td>
</tr>
<tr>
<td>Max. Relative Assembly Power ($P_A$)</td>
<td>$%</td>
<td>(P_A)^M - (P_A)^C / (P_A)^C</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt; 15% if P&lt;90%$</td>
</tr>
</tbody>
</table>
## 1.5 UQ for Core Measurements: Boron Concentration (ppm)

<table>
<thead>
<tr>
<th>Core parameter</th>
<th>Design criteria</th>
<th>Acceptance criteria</th>
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</thead>
<tbody>
<tr>
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<td>$</td>
<td>(C_B)^M_{ARO} - (C_B)^C_{ARO}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power (%)</th>
<th>Burnup (GWd/THM)</th>
<th>Boron Meas. (ppm)</th>
<th>WIMS-D4 + ND-1981</th>
<th>WIMSD5 + JEFF-3.3</th>
<th>Uncertainties in ppm (Boron Concentration) due to JEFF-3.3 covariance data</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.015</td>
<td>1200</td>
<td>1150 -50</td>
<td>1165 -35</td>
<td>18 14 9 27 46 9 24</td>
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<td>0.031</td>
<td>1113</td>
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<td>0.134</td>
<td>985</td>
<td>1000 15</td>
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<td>897 27</td>
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<tr>
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<td>340 21</td>
<td>321 2</td>
<td>34 23 9 21 38 10 23</td>
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<td>101</td>
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<td>100</td>
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<td>4</td>
<td></td>
<td></td>
<td>39 26 9 20 34 10 23</td>
</tr>
</tbody>
</table>

C = Calculated (ppm Boron)  
M = Measured (ppm Boron)
### 1.5 UQ for Core Measurements: Axial Offset (%)

<table>
<thead>
<tr>
<th>Core parameter</th>
<th>Design criteria</th>
<th>Acceptance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Offset</td>
<td>(AO)\textsuperscript{M}-(AO)\textsuperscript{C} &lt; 3%</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Core parameter</th>
<th>Design criteria</th>
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<th>Uncertainties in A.O. % due to JEFF-3.3 covariance data</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>P9-\textsuperscript{XS}</td>
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<tr>
<td>Power (%)</td>
<td>Burnup</td>
<td>Meas.</td>
<td>WIMS-D4 + ND-1981</td>
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<tr>
<td>----------------</td>
<td>----------------</td>
<td>---------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>50</td>
<td>0.015</td>
<td>7.7</td>
<td>5.6</td>
</tr>
<tr>
<td>75</td>
<td>0.031</td>
<td>3.8</td>
<td>3.7</td>
</tr>
<tr>
<td>100</td>
<td>0.134</td>
<td>-0.7</td>
<td>0.7</td>
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<tr>
<td>100</td>
<td>1.340</td>
<td>-1.6</td>
<td>-1.2</td>
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<tr>
<td>100</td>
<td>11.351</td>
<td>3.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

- C = AO Calculated (%)  
- M = AO Measured (%)
### 1.5 UQ for Core Measurements: Control Bank Worth (ppm)

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<tr>
<th>Control Bank Worth (ppm Boron)</th>
<th>WIMS-D4 + ND-1981</th>
<th>WIMSD5 + JEFF-3.3</th>
<th>Uncertainties in ppm Boron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pu9-XS</td>
<td>Pu9-ν</td>
<td>Pu9-χ</td>
</tr>
<tr>
<td>D-IN (REF)</td>
<td>113</td>
<td>114</td>
<td>0.4</td>
</tr>
<tr>
<td>C-IN</td>
<td>90</td>
<td>87</td>
<td>1.5</td>
</tr>
<tr>
<td>B-IN</td>
<td>132</td>
<td>135</td>
<td>0.4</td>
</tr>
<tr>
<td>A-IN</td>
<td>81</td>
<td>90</td>
<td>2.3</td>
</tr>
<tr>
<td>SB-IN</td>
<td>91</td>
<td>86</td>
<td>1.9</td>
</tr>
<tr>
<td>SA-IN</td>
<td>112</td>
<td>120</td>
<td>1.7</td>
</tr>
<tr>
<td>D+C-IN</td>
<td>224</td>
<td>221</td>
<td>1.6</td>
</tr>
<tr>
<td>D+C+B-IN</td>
<td>399</td>
<td>400</td>
<td>2.3</td>
</tr>
<tr>
<td>D+C+B+A-IN</td>
<td>526</td>
<td>542</td>
<td>1.7</td>
</tr>
<tr>
<td>D+C+B+A+SB-IN</td>
<td>657</td>
<td>664</td>
<td>1.2</td>
</tr>
<tr>
<td>ARI</td>
<td>868</td>
<td>884</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### Fig. Location of control rod banks

![Control Rod Bank Location Diagram](image-url)
2. Bayesian MC techniques to perform Data Adjustment

- **“Integral benchmarks are used for data validation, but should be avoided for the adjustment of general-purpose libraries“**
  
  - Why? This can lead to potential compensating effects due to both the impact of other isotopes included in the benchmark and defects in calculations attributed by complicated multi-physics.

- **“However, it is known that such integral data have been used to perform tune or fine adjustment of specific nuclear data to improve the overall performance of an entire general-purpose library”**
  
  - Nuclear data adjustments should rely on high-fidelity experiments that can be used as simple (e.g. one single isotope), well-understood and so-called clean benchmarks.
  - Consequently, these assumptions discharge other benchmarks (e.g. reactor calculations) for nuclear data adjustment into the evaluation procedure.

- **In this work, experimental data is referred to integral information**
  
  - Criticality integral benchmarks (e.g. $k_{eff}$ and spectral indices) in the ICSBEP
  - Shielding/transmission benchmarks (e.g. neutron leakage) in SINBAD/other databases
  - Delayed neutrons (e.g. beta), reactivity coefficients, etc…
2. Bayesian MC techniques to perform Data Adjustment

- Two distinct methods of nuclear data adjustment methodologies:
  - **Deterministic**
    - Generalized Linear Least Squares (GLLS)
      \[
      (E - C'(\sigma'))^T V_E^{-1} (E - C'(\sigma')) + (\sigma' - \sigma_0)^T V_\sigma^{-1} (\sigma' - \sigma_0) = \chi^2_{min}
      \]
      Assumptions:
      - Experimental and nuclear data are normally distributed
      - Linear approximations between all observables
      - Model and experimental data are uncorrelated
  
  - **Stochastic/Monte Carlo methods**
    - Bayesian MC techniques -> direct application of Bayes’ Theorem
      \[
      (\sigma|E) \propto p_0(\sigma|\sigma_C, V_C) \times L(y_E, V_E|\sigma)
      \]
    - To avoid the need to linearize non-linear models
    - To handle model which are not necessarily normally distributed
Generalized Linear Least Squares (GLLS)

- First-order Taylor series approximation
  \[ C(\sigma) \approx C(\sigma_0) + S(\sigma - \sigma_0) \]
  \[ V_C \approx SV_{\sigma_0}S^T \]

- “A posteriori” mean and variance-covariance matrix

\[ \sigma' = \sigma_0 + V_{\sigma_0}S^T [SV_{\sigma_0}S^T + V_E]^{-1} [E - C(\sigma_0)] \]

\[ V_{\sigma'} = V_{\sigma_0} - V_{\sigma_0}S^T [SV_{\sigma_0}S^T + V_E]^{-1} SV_{\sigma_0} \]

\[ C'(\sigma') \approx C(\sigma_0) + S(\sigma' - \sigma_0) = C(\sigma_0) + SV_{\sigma_0}S^T [SV_{\sigma_0}S^T + V_E]^{-1} [E - C(\sigma_0)] \]

\[ V_C' \approx SV_{\sigma'}S^T = SV_{\sigma_0}S^T - SV_{\sigma_0}S^T [SV_{\sigma_0}S^T + V_E]^{-1} SV_{\sigma_0}S^T \]

\[ V_E' = V_E - V_E [SV_{\sigma_0}S^T + V_E]^{-1} V_E \]

\[ V_{E-\sigma}' = V_E [SV_{\sigma_0}S^T + V_E]^{-1} SV_{\sigma_0} \]

GLLS: “This approach is a Bayesian approach in the sense that experimental data are used to adjust prior values. Although probability density functions are not considered explicitly.”
### Bayesian MC techniques

- “Prior probability” \( p_0(\sigma | \sigma_C, V_C) \) and “likelihood” \( L(y_E, V_E | \sigma) \) are independent \( pdfs \)
- The principle of maximum entropy -> normal distributions
  - Note: “In case the normality assumption is not acceptable, \( \sigma \) may be mapped onto an approximately normally distributed vector by an invertible transformation” \([10,16]\)
- Bayes’ Theorem: “posterior” normal distribution: \( p(\sigma | E) \sim N(\sigma', V_{\sigma'}) \)

#### Multivariate Normal Bayesian model

<table>
<thead>
<tr>
<th>( \sigma' = \sigma_C + M_{\sigma,C} [M_C + V_E]^{-1} [E - \bar{C}] )</th>
<th>( \sigma' = \sigma_0 + V_{\sigma_0} S^T [SV_{\sigma_0} S^T + V_E]^{-1} [E - C(\sigma_0)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\sigma'} = V_C - M_{\sigma,C} [M_C + V_E]^{-1} V_{\sigma,C}^T )</td>
<td>( V_{\sigma'} = V_{\sigma_0} - V_{\sigma_0} S^T [SV_{\sigma_0} S^T + V_E]^{-1} SV_{\sigma_0} )</td>
</tr>
<tr>
<td>( C'(\sigma') = \bar{C} + M_C [M_C + V_E]^{-1} [E - \bar{C}] )</td>
<td>( C'(\sigma') = C(\sigma_0) + SV_{\sigma_0} S^T [SV_{\sigma_0} S^T + V_E]^{-1} [E - C(\sigma_0)] )</td>
</tr>
<tr>
<td>( V_{\bar{C}}' = M_C - M_C [M_C + V_E]^{-1} V_C )</td>
<td>( V_{\bar{C}}' = SV_{\sigma_0} S^T - SV_{\sigma_0} S^T [SV_{\sigma_0} S^T + V_E]^{-1} SV_{\sigma_0} S^T )</td>
</tr>
</tbody>
</table>

... or MOCABA equations \([10]\)
### 2.2 Bayesian + MC Techniques

- **Bayesian MC (BMC) techniques** [3,4,5,6,7,8,9]
  - Provide many samples of nuclear data files, for each sampled nuclear data file simulation is performed

- **Methodologies for randomly sampled “σ”**
  - **NUDUNA and SANDY + Bayesian Approach**
    - Samples of ND files for “a priori” $p_0(\sigma|\sigma_C, V_C)$ evaluated files
    - Based on normal (or log-normal) distributions because no further information on the distribution of the nuclear data are provided in evaluated files [11,12]
  - **“Total Monte Carlo” (TMC) + Bayesian MC Approach** [3,4,7,8]
    - Sampling performed at the level of nuclear parameters in nuclear reaction codes (e.g. TALYS or EMPIRE)
    - $p_0(\sigma|\sigma_C, V_C)$ will NOT be normal!?. Then, lost of information about the prior function if it is approximated by a normal pdf
2.3 Bayesian + Monte Carlo: MOCABA

- NUDUNA and SANDY + Bayesian Approach
  - NUDUNA/SANDY provides samples of nuclear data files
    \[
    \bar{\sigma}_i = \frac{\sum_{k=1}^{N} \sigma_{i,k}}{N}
    \]
    \[
    V_{\sigma_{ij}} = \frac{\sum_{k=1}^{N} (\sigma_{i,k} - \bar{\sigma}_i)^T (\sigma_{j,k} - \bar{\sigma}_j)}{N}
    \]
    \[
    \bar{C}_m = \frac{\sum_{k=1}^{N} C_m(\sigma_k)}{N}
    \]
    \[
    M_{C_{mn}} = \frac{\sum_{k=1}^{N} (C_m(\sigma_k) - \bar{C}_m)^T (C_n(\sigma_k) - \bar{C}_n)}{N}
    \]
    \[
    M_{\sigma_{i-c_m}} = \frac{\sum_{k=1}^{N} (\sigma_{i,k} - \bar{\sigma}_i)^T (C_m(\sigma_k) - \bar{C}_m)}{N}
    \]

- Multivariate Normal Bayesian model
  \[
  \sigma' = \sigma + M_{\sigma,C} [M_C + V_E]^{-1} [E - \bar{C}]
  \]
  \[
  V_{\sigma'} = \sigma' - M_{\sigma,C} [M_C + V_E]^{-1} V_{\sigma,C}^T
  \]
  \[
  C'(\sigma') = \bar{C} + M_C [M_C + V_E]^{-1} [E - \bar{C}]
  \]
  \[
  V_C' = M_C - M_C [M_C + V_E]^{-1} V_C
  \]

... or MOCABA equations
2.4 Bayesian Monte Carlo: BMC

- TMC + Bayesian MC Approach (BMC)
  - “The requirement of a well-defined prior pdf is a serious limitation of the previous approach if non-informative prior distributions are known [3,4]“
    - e.g. model parameters \( x \) which lead through a model transformation \( \sigma = \mathcal{M}(x) \)
  - BMC approach will use TMC method to generate “a priori” random files which are not explicitly well-defined normal pdfs
  - BMC incorporates integral “a priori” information through **likelihood factors**

\[
L(y_E, V_E | \sigma) \sim e^{-\chi^2_k/2} \quad \text{with:} \quad \chi^2_k = [E - C(\sigma_k)]^T V_E^{-1} [E - C(\sigma_k)]
\]

- One can calculate a “weight” for any k-sample set: \( \omega_k = e^{-\chi^2_k/2} \)

- “A posteriori” moments:

\[
\sigma_i' = \frac{\sum_{k=1}^{N} \omega_k \times \sigma_{i,k}}{\sum_{i=k}^{N} \omega_k} \quad V'_{\sigma_{ij}} = \frac{\sum_{k=1}^{N} \omega_k \times (\sigma_{i,k} - \sigma_i')^T (\sigma_{j,k} - \sigma_j')}{\sum_{k=1}^{N} \omega_k}
\]
### 2.4 Bayesian Monte Carlo: BMC

- **TMC + Bayesian MC Approach (BMC)**
  - “A posteriori” moments:
    
    \[
    \sigma'_i = \frac{\sum_{k=1}^{N} \omega_k \times \sigma_{i,k}}{\sum_{i=k}^{N} \omega_k} \\
    V'_{\sigma_{ij}} = \frac{\sum_{k=1}^{N} \omega_k \times (\sigma_{i,k} - \sigma'_i)^T (\sigma_{j,k} - \sigma'_j)}{\sum_{k=1}^{N} \omega_k}
    \]
    
    \[
    C'_m = \frac{\sum_{k=1}^{N} \omega_k \times C_m(\sigma_k)}{\sum_{k=1}^{N} \omega_k} \\
    V'_{C_{mn}} = \frac{\sum_{k=1}^{N} \omega_k \times (C_m(\sigma_k) - C'_m)^T (C_n(\sigma_k) - C'_n)}{\sum_{k=1}^{N} \omega_k}
    \]
  
  - Other “weights” definitions
    - Simpler definitions of \( \omega_k \) values can lead to very low small weights
    - So-called BFMC method, renormalization of \( \omega_k \)
      
      \[
      \omega_k = e^{-\chi_k^2 / \chi_{min}^2} \\
      \omega_k = e^{-\left(\frac{\chi_k^2}{\chi_{min}^2}\right)^2}
      \]
      - *Lost of normality assumption!? ... still Bayesian?* pre-requisite (not limitation) of any Bayesian to have a well-defined prior distribution and a well-defined likelihood function
2.5 Selection of Benchmarks

- **Scope of nuclear data adjustment (DA)**
  - Criticality Benchmarks
    - Nuclear data evaluations have used mainly criticality benchmarks to match C/E
    - However, $\textit{keff}$ is a global parameter that can be achieved through many possible combinations of nuclear data -> inherent compensating effects in DA
  - Other sources of integral data
    - Spectral index measurements
    - Delayed neutron fraction [14]
    - Shielding/transmission leakage neutron spectra [14]

- **Exercise:** BMC with criticality and transmission benchmarks jointly!!
  5000 random files 235U/TENDL2014
  Calculations with MCNP6.1.1

- LLNL-235U pulsed sphere
  - U-235, 0.7 mfp, fwhm=2.0 ns, NE213-B bias=1.6, FP=945.54 cm, 26-deg

Fig. Dimensions of the small 235U solid spherical target, Report: LLNL UCID-17332

Total $\chi^2=3.54$

- HMF1 – Godiva Benchmark

- Sensitivity Analysis
  - HMF1 (DICE database)
    - Large values for nubar and fission
    - Elastic and inelastic 10 times lower than fission/nubar
    - En < 5 MeV

- Sensitivity Analysis: “LLNL- 235U” pulsed sphere
  - Fission mainly around 14 MeV, other terms with lower values

Note: Sensitivities predicted with MCSEN code [15]

- Sensitivity Analysis: “LLNL- 235U” pulsed sphere
  - Nu-bar mainly 14 MeV

Note: Sensitivities predicted with MCSEN code [15]

- Sensitivity Analysis: “LLNL- 235U” pulsed sphere
  - **MT91**, between 1.8-14 MeV

Note: Sensitivities predicted with MCSEN code [15]
High sensitivity in criticality for HMF1 (and LLNL-235U)

- large MT18 “motive force”… cause of the cross-section alteration

Strong reduction of uncertainty: ~1.5%
High sensitivity in LLNL-235U: +2.4%
- large MT4 "motive force"… cause of the cross-section alteration
- Reduction of uncertainty: ~0.6%
3. Conclusion

- **Bayesian MC adjustment for “HMF1- Godiva” Benchmark**

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<th>HMF1</th>
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- **Bayesian MC adjustment for “LLNL-235U” Benchmark**

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<th>Total CHI^2 LLNL- 235U/0.7mfp Benchmark</th>
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3. Conclusion

The aim of this work is to review different Monte Carlo (MC) techniques used to propagate nuclear data uncertainties.

1. Uncertainty Quantification (UQ) studies
   - Required in safety calculations of large scale systems ~ PWR
   - ND uncertainty propagation on the main design parameters
   - Decomposition of uncertainty in: $^{235}$U-$^{238}$U-$^{239}$Pu & XS-$\nu$-PFNS
     - Assessing nuclear data/uncertainty trends
     - Determining contributors & uncertainty targets for a safety operation

2. Bayesian Monte Carlo approaches for data adjustment
   - Multivariate Normal Bayesian model relying on NUDUNA/SANDY codes
   - TMC + Bayesian MC Approach (BMC)
   - Selection of Benchmarks.
   - Exercise: BMC applied for Criticality + Shielding/Transmission
     - Importance of additional sensitivity analysis (…“motive force” [13] !?)
References


[16] A. Hoefer, private communication
Extra slides
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